Assignment 14: Papoulis Chapter 11

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LaTeX

Outline

Question

Problem 11.13

Given a real process $x(t)$ with Fourier transform $X(\omega) = A(\omega) + jB(\omega)$, show that if the processes $A(\omega)$ and $B(\omega)$ satisfy $(11-79)$ and $E\{A(\omega)\} = E\{B(\omega)\} = 0$, then $x(t)$ is WSS.

Solution

$E\{A(u)\,A(v)\} = Q(u)\,\delta(u-v) = E\{B(u)\,B(v)\}$
$E\{A(u)\,B(v)\} = 0$
for $u \geq 0$, $u \geq 0$. We have to show that if the above is true and $E\{A(\omega)\} = E\{B(\omega)\} = 0$ then the process $x(t) = \frac{1}{\pi}\int_0^\infty (A(\omega)\cos\omega t - B(\omega)\sin\omega t)\,d\omega$ is WSS.

Solution

$E\left\{x\left(t\right)\right\} = 0$ and

$$= \frac{1}{\pi^2}\int_0^\infty \int_0^\infty E\left\{A\left(u\right)\cos u\left(t+\tau\right) - B\left(u\right)\sin u\left(t+\tau\right)\right\}\left(A\left(v\right)\cos vt - B \tag{1}$$

$$= \frac{1}{\pi}\int_0^\infty \int_0^\infty Q\left(u\right)\delta\left(u-v\right)\left(\cos u\left(t+\tau\right) + \sin u\left(t+\tau\right)\sin v\left(t\right)\right) dudv \tag{2}$$

$$= \frac{1}{\pi^2}\int_0^\infty Q\left(u\right)\left\{\cos u\left(t+\tau\right)\cos ut + \sin u\left(t+\tau\right)\sin ut\right\} du \tag{3}$$

$$= \frac{1}{\pi^2}\int_0^\infty Q\left(u\right)\cos u\tau du \tag{4}$$

Solution

Hence,
From this it follows that x(t) is WSS with $S_{xx} = \frac{Q(\omega)}{\pi}$