Assignment 7

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CBSE Probability Grade 12

Exercise 13.3.3: Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostler?

Solution: Let random variables $X, Y \in \{0, 1\}$ denote the following events in Table (I)

Event	Description
$X = 0$	Student is a hostler
$X = 1$	Student is a day scholar
$Y = 0$	Student gets an A grade
$Y = 1$	Student doesn't get an A grade

TABLE I

$$\Pr(X = 0) = 60\% \tag{1}$$
$$= 0.6 \tag{2}$$

$$\Pr(X = 1) = 40\% \tag{3}$$
$$= 0.4 \tag{4}$$

$$\Pr(Y = 0|X = 0) = 30\% \tag{5}$$
$$= 0.3 \tag{6}$$

$$\Pr(Y = 0|X = 1) = 20\% \tag{7}$$
$$= 0.2 \tag{8}$$

Probability	Value
$\Pr(X = 0)$	0.6
$\Pr(X = 1)$	0.4
$\Pr(Y = 0\|X = 0)$	0.3
$\Pr(Y = 0\|X = 1)$	0.2

TABLE II

Probability that the student selected is a hostler, if he has an A grade is

$$\Pr(X = 0|Y = 0) \tag{9}$$
$$= \frac{\Pr(X = 0, Y = 0)}{\Pr(Y = 0)} \tag{10}$$
$$= \frac{\Pr(Y = 0|X = 0)\Pr(X = 0)}{\sum_{i=0}^{1}\Pr(Y = 0, X = i)} \tag{11}$$
$$= \frac{\Pr(Y = 0|X = 0)\Pr(X = 0)}{\sum_{i=0}^{1}\Pr(Y = 0|X = i)\Pr(X = i)} \tag{12}$$

On substituting the values from Table (II) we get:

$$\Pr(X = 0|Y = 0) = \frac{0.3 \times 0.6}{0.3 \times 0.6 + 0.2 \times 0.4} \tag{13}$$
$$= \frac{0.18}{0.26} \tag{14}$$
$$= \frac{9}{13} \approx 0.69 \tag{15}$$

Probability that the student selected is a hostler, if he has an A grade is 0.69.