Assignment 7: CBSE Probability Grade 12

Dhatri Reddy

May 15, 2022

Outline

1. Question
2. Defining random variables
3. Values of probabilities
4. Table for probabilities
5. Solving
6. Substitution

Question

Exercise 13.3.3

Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostler?

Solution

Let random variables $X, Y \in \{0, 1\}$ denote the following events in Table (1)

Event	**Description**
$X = 0$	Student is a hostler
$X = 1$	Student is a day scholar
$Y = 0$	Student gets an A grade
$Y = 1$	Student doesn't get an A grade

Table 1: Random variables

$$\Pr(X = 0) = 60\% \tag{1}$$
$$= 0.6 \tag{2}$$

$$\Pr(X = 1) = 40\% \tag{3}$$
$$= 0.4 \tag{4}$$

$$\Pr(Y = 0|X = 0) = 30\% \tag{5}$$
$$= 0.3 \tag{6}$$

$$\Pr(Y = 0|X = 1) = 20\% \tag{7}$$
$$= 0.2 \tag{8}$$

Probability	**Value**
$\Pr(X = 0)$	0.6
$\Pr(X = 1)$	0.4
$\Pr(Y = 0 \mid X = 0)$	0.3
$\Pr(Y = 0 \mid X = 1)$	0.2
$\Pr(X = 0 \mid Y = 0)$	?

Table 2: Values of probabilities

Probability that the student selected is a hostler, if he has an A grade is

$$\Pr(X=0|Y=0) \tag{9}$$

$$= \frac{\Pr(X=0, Y=0)}{\Pr(Y=0)} \tag{10}$$

$$= \frac{\Pr(Y=0|X=0)\Pr(X=0)}{\sum_{i=0}^{1}\Pr(Y=0, X=i)} \tag{11}$$

$$= \frac{\Pr(Y=0|X=0)\Pr(X=0)}{\sum_{i=0}^{1}\Pr(Y=0|X=i)\Pr(X=i)} \tag{12}$$

On substituting the values from Table (2) we get:

$$\Pr(X = 0|Y = 0) = \frac{0.3 \times 0.6}{0.3 \times 0.6 + 0.2 \times 0.4} \tag{13}$$

$$= \frac{0.18}{0.26} \tag{14}$$

$$= \frac{9}{13} \approx 0.69 \tag{15}$$

Probability that the student selected is a hostler, if he has an A grade is 0.69.