Assignment 9: CBSE Probability Grade 12

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Outline

Question

Example 33

Coloured balls are distributed in four boxes as shown in the following table (1)

Box	Black	White	Red	Blue
I	3	4	5	6
II	2	2	2	2
III	1	2	3	1
IV	4	3	1	5

Table 1: Distribution of balls

A box is selected at random and then a ball is randomly drawn from the selected box. The colour of the ball is black, what is the probability that ball drawn is from the box III?

Solution

Let random variables $X = 0, Y = \{0, 1, 2, 3\}$ denote the following events as in Table (2)

Event	**Description**
$X = 0$	A black ball is selected
$Y = 0$	Box I is selected
$Y = 1$	Box II is selected
$Y = 2$	Box III is selected
$Y = 3$	Box IV is selected

Table 2: Random variables

Probability	**Value**
$\Pr(X=0\|Y=0)$	$\frac{3}{18}$
$\Pr(X=0\|Y=1)$	$\frac{2}{8}$
$\Pr(X=0\|Y=2)$	$\frac{1}{7}$
$\Pr(X=0\|Y=3)$	$\frac{4}{13}$
$\Pr(Y=2\|X=0)$	?

Table 3: Random variables

Bayes theorem

$$\Pr(Y=2|X=0) \tag{1}$$

$$= \frac{\Pr(X=0, Y=2)}{\Pr(X=0)} \tag{2}$$

$$= \frac{\Pr(Y=2) \times \Pr(X=0|Y=2)}{\sum_{i=0}^{3} \Pr(X=0, Y=i)} \tag{3}$$

$$= \frac{\Pr(X=0|Y=2)\Pr(X=0)}{\sum_{i=0}^{3} \Pr(Y=0|X=i)\Pr(X=i)} \tag{4}$$

On substituting the values from Table (3) we get:

$$\Pr(Y=2|X=0) = \frac{\frac{1}{7}\times\frac{1}{4}}{\frac{3}{18}\times\frac{1}{4}+\frac{2}{8}\times\frac{1}{4}+\frac{1}{7}\times\frac{1}{4}+\frac{4}{13}\times\frac{1}{4}} \tag{5}$$

$$= 0.165 \tag{6}$$

The probability that ball drawn is from the box III is 0.165.