MA5060: NUMERICAL ANALYSIS

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ASSIGNMENT-1

1) $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$, Let λ_1, λ_2 be the eigen values of A

Eigen values of $I+\alpha A+\alpha^2 A^2$ would be $1+\alpha\lambda_1+\alpha^2\lambda_1^2$, $1+\alpha\lambda_2+\alpha^2\lambda_2^2$

$$\begin{vmatrix} 3/2-\lambda & 1/2 \\ 1/2 & 3/2-\lambda \end{vmatrix} = 0 \Rightarrow (3/2-\lambda)^2 = 1/4$$

$\Rightarrow \lambda = 2, 1$ (eigen values of A)

Now eigen values of $I+\alpha A+\alpha^2 A^2$ are $1+\alpha+\alpha^2$, $1+2\alpha+4\alpha^2$

For sequence $\{y_n\}$ converge to 0 as $n \to \infty$

$$|1+\alpha+\alpha^2| < 1 \quad \text{and} \quad |1+2\alpha+4\alpha^2| < 1$$

$$\Rightarrow -2 < \alpha+\alpha^2 < 0 \quad \text{and} \quad -1 < \alpha+2\alpha^2 < 0$$

for real values of α

$$\alpha+\alpha^2 < 0 \Rightarrow \alpha(\alpha+1) < 0 \Rightarrow \alpha \in (-1, 0)$$

$$\alpha+2\alpha^2 < 0 \Rightarrow \alpha(2\alpha+1) < 0 \Rightarrow \alpha \in (-1/2, 0)$$

$$\alpha \in (-1,0) \cap (-1/2, 0)$$

$$\Rightarrow \alpha \in (-1/2, 0)$$

2) $\begin{bmatrix} -2 & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$

$-2x - y + 2z = -7$

$2x + y = 1$

$z = 1$

$y = 1 - 2x$

$x = \dfrac{-7 - y + 2z}{2}$

(a) Gauss-Jacobi

$x_0 = 0, \quad y_0 = 0, \quad z_0 = 0$

Step 1: $x_1 = \dfrac{-7-0+0}{2} = -3.5$

$y_1 = 1$

$z_1 = 1$

Step 2: $x_2 = \dfrac{-7-1+2(1)}{2} = -3$

$y_2 = 1 - 2(-7/2) = 8$

$z_2 = 1$

Step 3: $x_3 = \dfrac{-7-8+2}{2} = -13/2$

$y_3 = 1 - 2(-13/2) = 14$

$z_3 = 1$

(b) Gauss - Seidel

$x_0=0, \ y_0=0, \ z_0=0$

iteration 1: $x_1 = \dfrac{-7-0+0}{2} = -3.5$

$y_1 = 1-2(-7/2) = 8$

$z_1 = 1$

iteration 2: $x_2 = \dfrac{-7-8+2}{2} = -13/2$

$y_2 = 1-2(-13/2) = 14$

$z_2 = 1$

iteration 3: $x_3 = \dfrac{-7-14+2}{2} = -19/2$

$y_3 = 1-2(-19/2) = 20$

$z_3 = 1$