

9.2.4

EE24BTECH11015 - Dhawal

Question:

For the Differential Equation $y' = \frac{xy}{1+x^2}$, verify that $y = \sqrt{1+x^2}$ is a solution of the differential equation.

Solution: Solving the given D.E. , we get,

$$\frac{dy}{dx} = \frac{xy}{1+x^2} \quad (1)$$

$$\Rightarrow \frac{dy}{y} = \frac{x}{1+x^2} dx \quad (2)$$

Integrating both sides we get,

$$\Rightarrow \int \frac{dy}{y} = \int \frac{x}{1+x^2} dx \quad (3)$$

$$\Rightarrow \int \frac{dy}{y} = \int \frac{1}{2} \frac{2x}{1+x^2} dx \quad (4)$$

$$\Rightarrow \ln y = \frac{1}{2} \ln(1+x^2) \quad (5)$$

$$\Rightarrow \ln y = \ln \sqrt{1+x^2} \quad (6)$$

Taking antilog both sides we get,

$$\Rightarrow y = \sqrt{1+x^2} \quad (7)$$

Thus, $y = \sqrt{1+x^2}$ is a solution to the differential equation $y' = \frac{xy}{1+x^2}$.

Computational Solution:

Using classical definition of derivative we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \quad (8)$$

$$\Rightarrow f(x+h) = f(x) + hf'(x) \quad (9)$$

By increasing x_n each iteration by h , we are getting y by For,

$$x_0 = 0 \quad (10)$$

$$y_0 = 1 \quad (11)$$

$$h = 0.01 \quad (12)$$

$$n = 500 \quad (13)$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h \frac{dy}{dx} \Big|_{(x_n, y_n)} \quad (14)$$

$$y_{n+1} = y_n + h \frac{x}{\sqrt{1+x^2}} \quad (15)$$

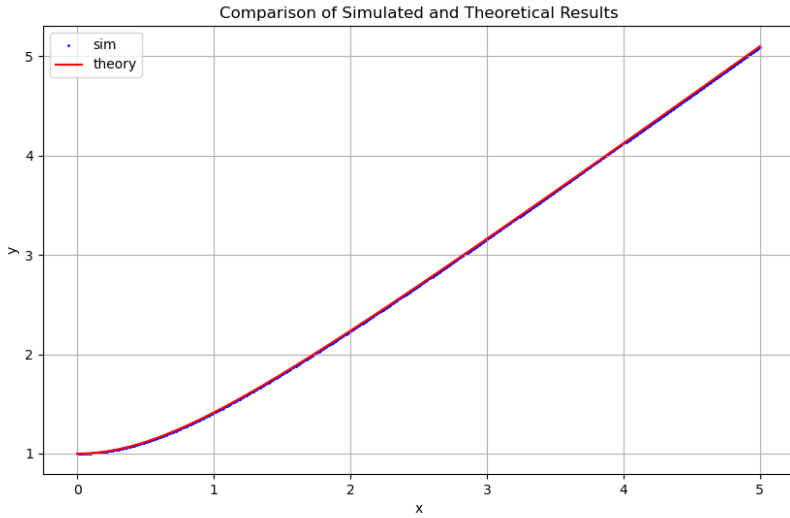


Fig. 0: Plot of the differential equation