10.4.3.9

EE24BTECH11015 - Dhawal

Question:

Two water taps together can fill a tank in $\frac{75}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution

Let time taken by each tap A, B to fill the tank be x, y respectively.
As tap A takes 10 hours less to fill the tank.

$$y = x + 10 \tag{1}$$

As total time taken by both to fill the tank is $\frac{75}{8}$

$$\left(\frac{1}{x} + \frac{1}{y}\right)\frac{75}{8} = 1 \tag{2}$$

$$\frac{1}{x} + \frac{1}{y} = \frac{8}{75} \tag{3}$$

Putting Eq. 1 in Eq. 3, we get

$$\frac{1}{x} + \frac{1}{x+10} = \frac{8}{75} \tag{4}$$

$$\frac{2x+10}{x(x+10)} = \frac{8}{75} \tag{5}$$

$$4x^2 - 35x - 375 = 0 \tag{6}$$

Theoretical Solution

Using quadratic formula, $a = 4, b = -35, c = -375$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{7}$$

We get $x = 15$ and $x = -6.25$
We can't take negative values so $x = 15$ and $y = 25$ is the solution.

So, time taken by each tap A, B to fill the tank is $15, 25$ hours.

Computational Solution We will use Newton's Method for solving equations.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{8}$$

Where we define $f(x)$ as,

$$f(x) = 4x^2 - 35x - 375 \tag{9}$$

$$f'(x) = 8x - 35 \tag{10}$$

Thus, the new update equation is,

$$x_{n+1} = x_n - \frac{4{x_n}^2 - 35x_n - 375}{8x_n - 35} \tag{11}$$

This is a quadratic equation, it can have 2 solutions. As at $x = 0, f(x) \leq 0$. So we will iterate it from $(-100, 0)$ and $(0, 100)$. Take initial guess as $x_0 = 0$, we can see that x_n converges at $x = 15$ and $x = -6.25$.