

9.4.9

EE24BTECH11015 - Dhawal

Question:

Find the solution of the differential equation $\frac{dy}{dx} = \sin^{-1} x$.

Solution: Solving the given D.E. , we get,

$$\frac{dy}{dx} = \sin^{-1} x \quad (1)$$

Integrate both sides with respect to x :

$$y = \int \sin^{-1} x \, dx \quad (2)$$

Using integration by parts:

$$\int u \, dv = uv - \int v \, du \quad (3)$$

Let:

$$u = \sin^{-1} x, \, dv = dx \quad (4)$$

Then:

$$du = \frac{1}{\sqrt{1-x^2}} \, dx, \, v = x \quad (5)$$

Substituting into the integration by parts formula:

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int x \frac{1}{\sqrt{1-x^2}} \, dx \quad (6)$$

For the remaining integral, let $u = 1 - x^2$, so $du = -2x \, dx$:

$$\int x \frac{1}{\sqrt{1-x^2}} \, dx = -\frac{1}{2} \int \frac{1}{\sqrt{u}} \, du \quad (7)$$

$$= -\sqrt{u} + C \quad (8)$$

$$= -\sqrt{1-x^2} + C \quad (9)$$

Thus, the solution to the differential equation is:

$$y = x \sin^{-1} x + \sqrt{1-x^2} + C \quad (10)$$

Computational Solution:

Using a classical definition of derivative, we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \quad (11)$$

$$\Rightarrow f(x+h) = f(x) + hf'(x) \quad (12)$$

By increasing x in each iteration by h and let $C = 0$, we are getting y by,

$$x_0 = 0 \quad (13)$$

$$y_0 = 1 \quad (14)$$

$$h = 0.01 \quad (15)$$

$$n = 100 \quad (16)$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h \left. \frac{dy}{dx} \right|_{(x_n, y_n)} \quad (17)$$

$$y_{n+1} = y_n + h \sin^{-1} x_n \quad (18)$$

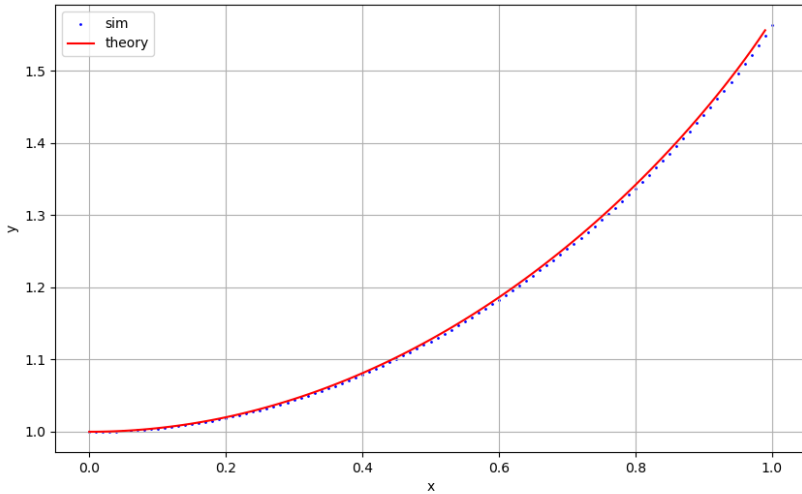


Fig. 0: Plot of the differential equation