9.2.4

EE24BTECH11015 - Dhawal

Question:
For the Differential Equation $y' = \frac{xy}{1+x^2}$, verify that $y = \sqrt{1+x^2}$ is a solution of the differential equation.

Solution: Solving the given D.E. , we get,

$$\frac{dy}{dx} = \frac{xy}{1+x^2} \tag{1}$$
$$\implies \frac{dy}{y} = \frac{x}{1+x^2}dx \tag{2}$$

Integrating both sides we get,

$$\implies \int \frac{dy}{y} = \int \frac{x}{1+x^2}dx \tag{3}$$
$$\implies \int \frac{dy}{y} = \int \frac{1}{2}\frac{2x}{1+x^2}dx \tag{4}$$
$$\implies \ln y = \frac{1}{2}\ln\left(1+x^2\right) \tag{5}$$
$$\implies \ln y = \ln\sqrt{1+x^2} \tag{6}$$

Taking antilog both sides we get,

$$\implies y = \sqrt{1+x^2} \tag{7}$$

Thus, $y = \sqrt{1+x^2}$ is a solution to the differential equation $y' = \frac{xy}{1+x^2}$.

Computational Solution:
Using classical defination of derivative we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{8}$$
$$\implies f(x+h) = f(x) + hf'(x) \tag{9}$$

By increasing x_n each iteration by h, we are getting y by For,

$$x_0 = 0 \tag{10}$$
$$y_0 = 1 \tag{11}$$
$$h = 0.01 \tag{12}$$
$$n = 500 \tag{13}$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h\frac{dy}{dx}\bigg|_{(x_n,y_n)} \tag{14}$$

$$y_{n+1} = y_n + h\frac{x_n}{\sqrt{1+{x_n}^2}} \tag{15}$$

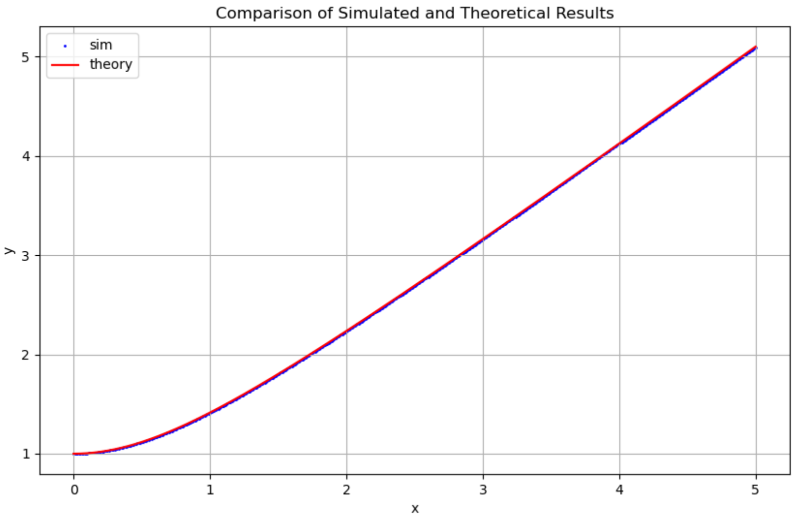

Fig. 0: Plot of the differential equation