12.9.4.9

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Problem Statement

Find the solution of the differential equation $\frac{dy}{dx} = \sin^{-1} x$.

Theoretical Solution

Using the given D.E. , we get,

$$\frac{dy}{dx} = \sin^{-1} x \tag{3.1}$$

Integrate both sides with respect to x:

$$y = \int \sin^{-1} x \, dx \tag{3.2}$$

Using integration by parts:

$$\int u \, dv = uv - \int v \, du \tag{3.3}$$

Let:

$$u = \sin^{-1} x, \; dv = dx \tag{3.4}$$

Then:

$$du = \frac{1}{\sqrt{1 - x^2}} \, dx, \; v = x \tag{3.5}$$

Theoretical Solution

Substituting into the integration by parts formula:

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int x \frac{1}{\sqrt{1-x^2}} \, dx \tag{3.6}$$

For the remaining integral, let $u = 1 - x^2$, so $du = -2x \, dx$:

$$\begin{aligned} \int x \frac{1}{\sqrt{1-x^2}} \, dx &= -\frac{1}{2} \int \frac{1}{\sqrt{u}} \, du && (3.7) \\ &= -\sqrt{u} + C && (3.8) \\ &= -\sqrt{1-x^2} + C && (3.9) \end{aligned}$$

Thus, the solution to the differential equation is:

$$y = x \sin^{-1} x + \sqrt{1-x^2} + C \tag{3.10}$$

Euler Formula

Using a classical definition of derivative, we get,

$$f'(x) = \frac{f(x+h) - f(x)}{h} \tag{3.11}$$

$$\implies f(x+h) = f(x) + hf'(x) \tag{3.12}$$

By increasing x n each iteration by h and let $C = 0$, we are getting y by,

$$x_0 = 0 \tag{3.13}$$

$$y_0 = 1 \tag{3.14}$$

$$h = 0.01 \tag{3.15}$$

$$n = 100 \tag{3.16}$$

Using Euler Method, we get difference equation,

$$y_{n+1} = y_n + h\frac{dy}{dx}\bigg|_{(x_n, y_n)} \tag{3.17}$$

$$y_{n+1} = y_n + h\sin^{-1} x_n \tag{3.18}$$

Bilinear transfrom

$$\frac{dy}{dx} = \sin^{-1} x \tag{3.19}$$

Taking Laplase of $\frac{dy}{dx}$ and putting $\sin^{-1} x$ as $F(s)$

$$sY(s) = F(s) \tag{3.20}$$

$$\frac{Y(s)}{F(s)} = \frac{1}{s} = H(s) \tag{3.21}$$

Now, convert it into Z-transform

$$Y(z) = \frac{h}{2}\frac{1+z^{-1}}{1-z^{-1}}F(z) \tag{3.22}$$

$$Y(z)\left(1-z^{-1}\right) = \frac{h}{2}\left(1+z^{-1}\right)F(z) \tag{3.23}$$

Applying inverse Z-transform, we get

$$y_n - y_{n-1} = \frac{h}{2}\left(\sin^{-1} x_n + \sin^{-1} x_{n-1}\right) \tag{3.24}$$

Plot

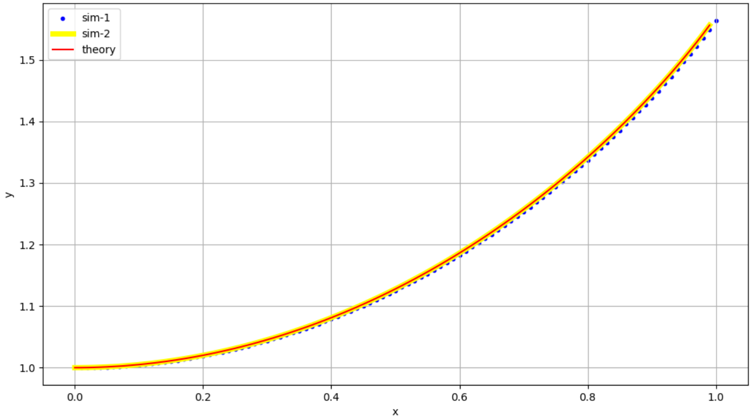

Figure: Plot of the differential equation

Codes

https://github.com/Dhawal24112006/EE1003/tree/main/NCERT/Q3/codes