JEE MAINS 13 April 2023 Shift-2

ee24btech11015 - Dhawal

1) The area of the region $\left\{(x, y) : x^2 \leq y \leq \left|x^2 - 4\right|, y \geq 1\right\}$ is

a) $\frac{3}{4}\left(4\sqrt{2} + 1\right)$ b) $\frac{4}{3}\left(4\sqrt{2} - 1\right)$ c) $\frac{3}{4}\left(4\sqrt{2} - 1\right)$ d) $\frac{4}{3}\left(4\sqrt{2} + 1\right)$

2) If $\lim_{x\to 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1-\cos 2x} = 17$, then $5a^2 + b^2$ is equal to

a) 76 b) 72 c) 64 d) 68

3) The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$,is

a) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ b) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$ c) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$ d) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

4) The plane, passing through the points $\begin{pmatrix}0\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\1\end{pmatrix}$ and parallel to the line passing through $\begin{pmatrix}5\\1\\-7\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-1\end{pmatrix}$, also passes through the point:

a) $\begin{pmatrix}0\\5\\-2\end{pmatrix}$ b) $\begin{pmatrix}-2\\5\\0\end{pmatrix}$ c) $\begin{pmatrix}2\\0\\1\end{pmatrix}$ d) $\begin{pmatrix}1\\-2\\1\end{pmatrix}$

5) Let for a triangle ΔABC,
$AB = -2\hat{i} + \hat{j} + 3\hat{k}$
$CB = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$
$CA = 4\hat{i} + 3\hat{j} + \delta\hat{k}$
If $\delta > 0$ and the area of the triangle ΔABC is $5\sqrt{6}$, then $CB \cdot CA$ is equal to

a) 108 b) 60 c) 54 d) 120

6) Let for A= $\begin{vmatrix}1 & 2 & 3\\ \alpha & 3 & 1\\ 1 & 1 & 1\end{vmatrix}$, $|A| = 2$. If $\left|2\ \text{adj}\left(2\ \text{adj}\left(2A\right)\right)\right| = 32^n$, then $3n + \alpha$ is equal to

a) 10 b) 9 c) 12 d) 11

7) The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$

a) $[0\ ,\ \pi)$ b) $[0, \pi]$ c) $[0\ ,\ 2\pi)$ d) $[0, 2\pi]$

8) Let $a_1, a_2, a_3, \ldots$ be a G. P. of increasing positive numbers. Let the sum of its 6^{th} and 8^{th} terms be 2 and the product of its 3^{rd} and 5^{th} terms be $\frac{1}{9}$. Then $6\,(a_2 + a_4)\,(a_4 + a_6)$ is equal to

a) 2 b) 3 c) $3\sqrt{3}$ d) $2\sqrt{2}$

9) If the system of equations
$2x + y - z = 5$
$2x - 5y + \lambda z = \mu$
$x + 2y - 5z = 7$
has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to

a) 904 b) 916 c) 912 d) 920

10) The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to .

a) $(\sim p) \vee (\sim q)$ b) $(\sim p) \wedge (\sim q)$ c) $p \vee (\sim q)$ d) $p \vee q$

11) Let $S = \left\{z \in C : \bar{z} = \iota\left(z^2 + \mathrm{Re}\,(\bar{z})\right)\right\}$. Then $\sum_{z \in S} |z|^2$ is equal to

a) 4 b) $\frac{7}{2}$ c) 3 d) $\frac{5}{2}$

12) Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, Then $\alpha^{14} + \beta^{14}$ is equal to

a) $-128\sqrt{2}$ b) $-64\sqrt{2}$ c) -128 d) -64

13) Let $|\mathbf{a}| = 2, |\mathbf{b}| = 3$ and the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ be $\frac{\pi}{4}$. Then $|(\mathbf{a} + 2\mathbf{b}) \times (2\mathbf{a} - 3\mathbf{b})|^2$ is equal to

a) 482 b) 841 c) 882 d) 441

14) The value of $\frac{e^{-\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x\, dx}{\int_0^{\frac{\pi}{4}} e^{-x}\left(\tan^{49} x + \tan^{51} x\right) dx}$ is

a) 25 b) 51 c) 50 d) 49

15) The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is

a) $\frac{80}{9}$ b) 8 c) 9 d) $\frac{26}{3}$