

# Chapter 6 Sequence and Series

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ee24btech11015 - Dhawal

## D. MCQs with One or More than One Correct

- 1) If the first and the  $(2n - 1)^{th}$  terms of an A.P., a G.P. and an H.P. are equal and their  $n^{th}$  terms are  $a, b$  and  $c$  respectively, then (1988 – 2Marks)

- a)  $a = b = c$       b)  $a \geq b \geq c$       c)  $a + b = c$       d)  $ac - b^2 = 0$

- 2) For  $0 < \phi < \pi/2$ , if

$$x = \sum_{n=0}^{\infty} \cos^{2n} \phi, \quad y = \sum_{n=0}^{\infty} \sin^{2n} \phi, \quad z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$$

then: (1993 – 2Marks)

- a)  $xyz = xz + y$       b)  $xyz = xy + z$       c)  $xyz = x + y + z$       d)  $xyz + yz + x$

- 3) Let  $n$  be a odd integer. If

$$\sin n\theta = \sum_{r=0}^n b_r \sin^r \theta,$$

for every value of  $\theta$ , then (1998 – 2Marks)

- a)  $b_0 = 1, b_1 = 3$       c)  $b_0 = -1, b_1 = 3$   
b)  $b_0 = 0, b_1 = n$       d)  $b_0 = 0, b_1 = n^2 - 3n + 3$

- 4) Let  $T_r$  be the  $r^{th}$  term of an A.P., for  $r = 1, 2, 3, \dots$ . If for some positive integers  $m, n$  we have  $T_m = \frac{1}{n}$  and  $T_n = \frac{1}{m}$ , then  $T_{mn}$  equals (1998 – 2Marks)

- a)  $\frac{1}{mn}$       b)  $\frac{1}{m} + \frac{1}{n}$       c) 1      d) 0

- 5) If  $x > 1, y > 1, z > 1$  are in G.P., then  $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$  are in (1998 – 2Marks)

- a) A.P.      b) H.P.      c) G.P.      d) None of these

- 6) For a positive integer  $n$ , let  $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n)-1}$ . Then (1999 – 2Marks)

- a)  $a(100) \leq 100$       b)  $a(100) > 100$       c)  $a(200) \leq 100$       d)  $a(200) > 100$

- 7) A straight line through the vertex **P** of a triangle **PQR** intersects the side **QR** at the point **S** and the circumcircle of the triangle **PQR** at the point **T**. If **S** is not the centre of the circumcircle, then (2008)

$$\begin{aligned} \text{a) } \frac{1}{PS} + \frac{1}{ST} &< \frac{2}{\sqrt{QS \cdot SR}} \\ \text{b) } \frac{1}{PS} + \frac{1}{ST} &> \frac{2}{\sqrt{QS \cdot SR}} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{1}{PS} + \frac{1}{ST} &< \frac{4}{QR} \\ \text{d) } \frac{1}{PS} + \frac{1}{ST} &> \frac{4}{QR} \end{aligned}$$

8) Let

$$S_n = \sum_{k=1}^n \frac{n}{n^2 + kn + k^2} \text{ and } T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$$

for  $n = 1, 2, 3, \dots$ . Then,

(2008)

$$\begin{aligned} \text{a) } S_n &< \frac{\pi}{3\sqrt{3}} & \text{b) } S_n &> \frac{\pi}{3\sqrt{3}} & \text{c) } T_n &< \frac{\pi}{3\sqrt{3}} & \text{d) } T_n &> \frac{\pi}{3\sqrt{3}} \end{aligned}$$

9) Let

$$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2.$$

Then  $S_n$  can take value(s)

(JEEAdv.2013)

$$\begin{aligned} \text{a) } 1056 & & \text{b) } 1088 & & \text{c) } 1120 & & \text{d) } 1332 \end{aligned}$$

10) Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - x - 1 = 0$ , with  $\alpha > \beta$ . For all positive integers  $n$ , define

$$a_n = \frac{\alpha_n - \beta_n}{\alpha - \beta}, n \geq 2, b_1 = 1 \text{ and } b_n = a_{n-1} + a_{n+1}, n \geq 1$$

Then which of the following options is/are correct?

(JEEAdv.2019)

$$\begin{aligned} \text{a) } \sum_{n=1}^{\infty} \frac{a_n}{10^n} &= \frac{10}{89} & \text{c) } a_1 + a_2 + a_3 + \dots + a_n &= a_{n+2} - 1 \forall n \geq 1 \\ \text{b) } B_n &= a^n + b^n \forall n \geq 1 & \text{d) } \sum_{n=1}^{\infty} \frac{b_n}{10^n} &= \frac{8}{89} \end{aligned}$$

### E. SUBJECTIVE PROBLEMS

1) The harmonic mean of two numbers is 4. Their arithmetic mean  $A$  and the geometric mean  $G$  satisfy the relation.

$$2A + G^2 = 27$$

Find the two numbers.

(1979)

2) The interior angles of a polygon are in arithmetic progression. The smallest angle is  $120^\circ$  and the common difference is  $5^\circ$ . Find the number of sides of the polygon.

(1980)

3) Does there exist a geometric progression containing 27, 8 and 12 as three of its terms? If it exists, how many such progressions are possible? (1982 – 3Marks)

4) Find three numbers  $a, b, c$  between 2 and 18 such that

a) their sum is 25

b) the numbers 2,  $a, b$  are consecutive terms of an A.P.  
and

c) the numbers  $b, c, 18$  are consecutive terms of a G.P.

(1983 – 2Marks)

5) If  $a > 0, b > 0, c > 0$ , prove that

$$(a + b + c) \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq 9$$

(1984 – 2Marks)