Question 9-9.3-3

EE24BTECH11015 - Dhawal

1) Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Variable	Description	Values
P	Parabola	$4y = 3x^2$
L	Line	$2y = 3x + 12$
A	Point of intersection	To find
B	Point of intersection	To find

TABLE 1: Variables given

Solution:
Parabola in terms of matrix:

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1.1}$$

Where:

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \qquad \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \qquad f = 0 \tag{1.2}$$

Point of intersection of line L

$$L: \quad \mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \quad \kappa \in \mathbb{R} \tag{1.3}$$

Where:

$$\mathbf{h} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} 1 \\ \frac{3}{2} \end{pmatrix} \tag{1.4}$$

is represented by:

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m} \tag{1.5}$$

Where:

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{1.6}$$

Finding $g(\mathbf{h})$:

$$g(\mathbf{h}) = -24 \tag{1.7}$$

Finding κ_i:

$$\kappa_i = 4 \text{ and } -2 \tag{1.8}$$

So Points of intersection are:

$$\mathbf{x}_i = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \text{ and } \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{1.9}$$

$$\mathbf{A} = \begin{pmatrix} 4 \\ 12 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{1.10}$$

Area between the curves:

$$\int_{-2}^{4} \left|1.5x + 6 - 0.75x^2\right| \, dx = 27 \tag{1.11}$$

So Area between the graphs is 27.

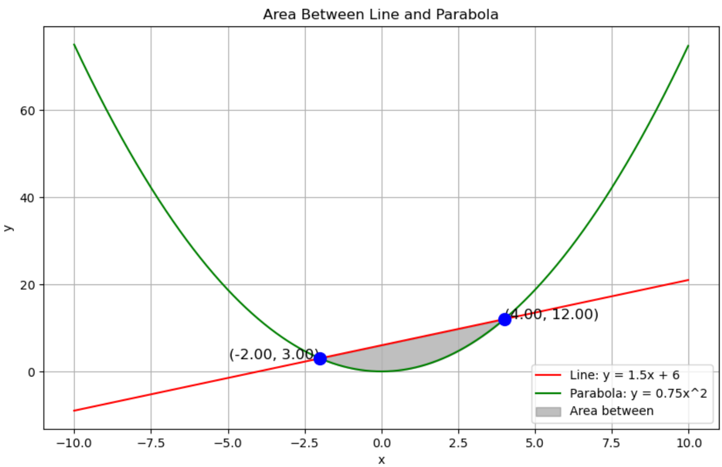

Fig. 1.1: Area Enclosed by parabola and line.