

JEE MAINS 13 April 2023 Shift-2¹

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1) The area of the region $\{(x, y) : x^2 \leq y \leq |x^2 - 4|, y \geq 1\}$ is (13/04/2023 - Shift - 2)

- a) $\frac{3}{4}(4\sqrt{2} + 1)$ b) $\frac{4}{3}(4\sqrt{2} - 1)$ c) $\frac{3}{4}(4\sqrt{2} - 1)$ d) $\frac{4}{3}(4\sqrt{2} + 1)$

2) If $\lim_{x \rightarrow 0} \frac{e^{ax} - \cos(bx) - \frac{cx e^{-cx}}{2}}{1 - \cos 2x} = 17$, then $5a^2 + b^2$ is equal to (13/04/2023 - Shift - 2)

- a) 76 b) 72 c) 64 d) 68

3) The line, that is coplanar to the line $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$, is (13/04/2023 - Shift - 2)

- a) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ b) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-5}{5}$ c) $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$ d) $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{4}$

4) The plane, passing through the points $\begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ and parallel to the line passing

through $\begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$, also passes through the point: (13/04/2023 - Shift - 2)

- a) $\begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}$ b) $\begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}$ c) $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

5) Let for a triangle ΔABC ,

$$AB = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$CB = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$CA = 4\hat{i} + 3\hat{j} + \delta\hat{k}$$

If $\delta > 0$ and the area of the triangle ΔABC is $5\sqrt{6}$, then $CB \cdot CA$ is equal to (13/04/2023 - Shift - 2)

- a) 108 b) 60 c) 54 d) 120

6) Let for $A = \begin{vmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 1 \end{vmatrix}$, $|A| = 2$. If $|2 \operatorname{adj}(2 \operatorname{adj}(2A))| = 32^n$, then $3n + \alpha$ is equal to (13/04/2023 - Shift - 2)

a) 10

b) 9

c) 12

d) 11

7) The range of $f(x) = 4\sin^{-1}\left(\frac{x^2}{x^2+1}\right)$ (13/04/2023 – Shift – 2)

a) $[0, \pi)$ b) $[0, \pi]$ c) $[0, 2\pi)$ d) $[0, 2\pi]$

8) Let a_1, a_2, a_3, \dots be a G. P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to (13/04/2023 – Shift – 2)

a) 2

b) 3

c) $3\sqrt{3}$ d) $2\sqrt{2}$

9) If the system of equations

$$\begin{aligned} 2x + y - z &= 5 \\ 2x - 5y + \lambda z &= \mu \\ x + 2y - 5z &= 7 \end{aligned}$$

has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to (13/04/2023 – Shift – 2)

a) 904

b) 916

c) 912

d) 920

10) The statement $(p \wedge (\sim q)) \vee ((\sim p) \wedge q) \vee ((\sim p) \wedge (\sim q))$ is equivalent to . (13/04/2023 – Shift – 2)

a) $(\sim p) \vee (\sim q)$ b) $(\sim p) \wedge (\sim q)$ c) $p \vee (\sim q)$ d) $p \vee q$

11) Let $S = \{z \in \mathbb{C} : \bar{z} = i(z^2 + \operatorname{Re}(\bar{z}))\}$. Then $\sum_{z \in S} |z|^2$ is equal to (13/04/2023 – Shift – 2)

a) 4

b) $\frac{7}{2}$

c) 3

d) $\frac{5}{2}$

12) Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, Then $\alpha^{14} + \beta^{14}$ is equal to (13/04/2023 – Shift – 2)

a) $-128\sqrt{2}$ b) $-64\sqrt{2}$

c) -128

d) -64

13) Let $|\mathbf{a}| = 2, |\mathbf{b}| = 3$ and the angle between the vectors \mathbf{a} and \mathbf{b} be $\frac{\pi}{4}$. Then $|(\mathbf{a} + 2\mathbf{b}) \times (2\mathbf{a} - 3\mathbf{b})|^2$ is equal to (13/04/2023 – Shift – 2)

a) 482

b) 841

c) 882

d) 441

14) The value of $\frac{e^{-\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^{-x} \tan^{50} x \, dx}{\int_0^{\frac{\pi}{4}} e^{-x} (\tan^{49} x + \tan^{51} x) \, dx}$ is (13/04/2023 – Shift – 2)

a) 25

b) 51

c) 50

d) 49

15) The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is (13/04/2023 – Shift – 2)

a) $\frac{80}{9}$

b) 8

c) 9

d) $\frac{26}{3}$