JEE MAINS 28 Jun 2022 Shift-2

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11) Let the plane $ax+by+cz = d$ pass through $\begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}$ and is perpendicular to the planes $2x+y-5z = 10$ and $3x+5y-7z = 12$. If a, b, c, d are integers $d > 0$ and $gcd(|a|, |b|, |c|, d) = 1$, then the value of $a + 7b + c + 20d$ is equal to :

a) 18 b) 20 c) 24 d) 22

12) The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

a) $\frac{1}{24}$ b) $\frac{1}{40}$ c) $\frac{1}{30}$ d) $\frac{1}{20}$

13) The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n} \tan^{-1}\left\{\frac{1}{r^2+3r+3}\right\}\right\}$ is equal to :

a) 1 b) 2 c) 3 d) 6

14) $\mathbf{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\mathbf{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is:

a) $\frac{1}{3}$ b) 1 c) $\frac{5}{3}$ d) $\frac{7}{3}$

15) If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are :

a) $\frac{-1}{7}$ and 4^{th} quadrant
b) 7 and 1^{st} quadrant
c) -7 and 4^{th} quadrant
d) $\frac{1}{7}$ and 1^{st} quadrant

B. Numericals

1) Let the image of the point $\mathbf{P}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in the line $L : \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be $\mathbf{Q}$. Let $\mathbf{R}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha+\beta+\gamma)$ is equal to :

2) Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of

students can fail is:

3) If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $\left(x - 2\sqrt{2}\right)^2 + \left(y - 2\sqrt{2}\right)^2 = r^2$, then the value of r^2 is equal to:

4) If $\lim_{x\to 1} \frac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b} = -2$, then the value of $(a-b)$ is equal to:

5) Let for $n = 1, 2, \ldots, 50, S_n$ be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$ is equal to:

6) If the system of linear equations $2x - 3y = \gamma + 5, \alpha x + 5y = \beta + 1,$ where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to:

7) Let $A = \begin{pmatrix} 1+\iota & 1 \\ -\iota & 0 \end{pmatrix}$ where $\iota = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A_n = A\}$ is:

8) Sum of squares of modulus of all the complex numbers z satisfying $z = \iota z^2 + z^2 - z$ is equal to:

9) Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \implies S : f \text{ is onto and } f(a, b) = f(b, a) \geq a \forall (a, b) =\in S \times S\}$ is:

10) The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to