JEE MAINS 28 Jun 2022 Shift-2

ee24btech11015 - Dhawal

11) Let the plane $ax+by+cz = d$ pass through $\begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}$ and is perpendicular to the planes $2x+y-5z = 10$ and $3x+5y-7z = 12$. If a, b, c, d are integers $d > 0$ and $gcd\,(|a|\,,|b|\,,|c|\,,d) = 1$, then the value of $a + 7b + c + 20d$ is equal to : $\quad$ $(28/06/2022 - Shift - 2)$

   a) 18 $\qquad\qquad$ b) 20 $\qquad\qquad$ c) 24 $\qquad\qquad$ d) 22

12) The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfies $f\,(a)+2f\,(b)-f\,(c) = f\,(d)$ is: $(28/06/2022 - Shift - 2)$

   a) $\frac{1}{24}$ $\qquad\qquad$ b) $\frac{1}{40}$ $\qquad\qquad$ c) $\frac{1}{30}$ $\qquad\qquad$ d) $\frac{1}{20}$

13) The value of $\lim_{n\to\infty} 6 \tan \left\{ \sum_{r=1}^{n} \tan^{-1} \left\{ \frac{1}{r^2+3r+3} \right\} \right\}$ is equal to :$(28/06/2022 - Shift - 2)$

   a) 1 $\qquad\qquad$ b) 2 $\qquad\qquad$ c) 3 $\qquad\qquad$ d) 6

14) $\mathbf{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\mathbf{a} \times \left(2\hat{i} + \hat{k}\right) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is: $(28/06/2022 - Shift - 2)$

   a) $\frac{1}{3}$ $\qquad\qquad$ b) 1 $\qquad\qquad$ c) $\frac{5}{3}$ $\qquad\qquad$ d) $\frac{7}{3}$

15) If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are : $(28/06/2022 - Shift - 2)$

   a) $\frac{-1}{7}$ and 4th quadrant $\qquad\qquad$ c) -7 and 4th quadrant
   b) 7 and 1st quadrant $\qquad\qquad\quad$ d) $\frac{1}{7}$ and 1st quadrant

B. Numericals

1) Let the image of the point $\mathbf{P}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ in the line $L : \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be $\mathbf{Q}$. Let $\mathbf{R}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22\,(\alpha + \beta + \gamma)$ is equal to : $\quad$ $(28/06/2022 - Shift - 2)$

2) Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the

examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is: $(28/06/2022 - Shift - 2)$

3) If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is a chord of the circle $\left(x - 2\sqrt{2}\right)^2 + \left(y - 2\sqrt{2}\right)^2 = r^2$, then the value of r^2 is equal to:
$(28/06/2022 - Shift - 2)$

4) If $\lim_{x \to 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the value of $(a - b)$ is equal to:
$(28/06/2022 - Shift - 2)$

5) Let for $n = 1, 2, \ldots, 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50} \left(S_n + \frac{2}{n+1} - n - 1\right)$ is equal to: $(28/06/2022 - Shift - 2)$

6) If the system of linear equations $2x - 3y = \gamma + 5$, $\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to:
$(28/06/2022 - Shift - 2)$

7) Let $A = \begin{pmatrix} 1 + \iota & 1 \\ -\iota & 0 \end{pmatrix}$ where $\iota = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A_n = A\}$ is: $(28/06/2022 - Shift - 2)$

8) Sum of squares of modulus of all the complex numbers z satisfying $z = \iota z^2 + z^2 - z$ is equal to: $(28/06/2022 - Shift - 2)$

9) Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \implies S : f \text{ is onto and } f(a, b) = f(b, a) \ge a \ \forall (a, b) \in S \times S\}$ is:
$(28/06/2022 - Shift - 2)$

10) The maximum number of compound propositions, out of $p \vee r \vee s$, $p \vee r\vee \sim s$, $p\vee \sim q \vee s$, $\sim p\vee \sim r \vee s$, $\sim p\vee \sim r\vee \sim s$, $\sim p \vee q\vee \sim s$, $q \vee r\vee \sim s$, $q\vee \sim r\vee \sim s$, $\sim p\vee \sim q\vee \sim s$ that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to: $(28/06/2022 - Shift - 2)$