

# Chapter 6 Sequence and Series

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## D. MCQs with One or More than One Correct

- 1) If the first and the  $(2n - 1)$ st terms of an A.P., a G.P. and an H.P. are equal and their  $n$ -th terms are  $a$ ,  $b$  and  $c$  respectively, then (1988-2 Marks)

- a)  $a = b = c$                       c)  $a + b = c$   
b)  $a \geq b \geq c$                       d)  $ac - b^2 = 0$

- 2) For  $0 < \phi < \pi/2$ , if

$$x = \sum_{n=0}^{\infty} \cos^{2n} \phi, y = \sum_{n=0}^{\infty} \sin^{2n} \phi, z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$$

then:

(1993-2 Marks)

- a)  $xyz = xz + y$                       c)  $xyz = x + y + z$   
b)  $xyz = xy + z$                       d)  $xyz + yz + x$

- 3) Let  $n$  be an odd integer. If

$$\sin n\theta = \sum_{r=0}^n b_r \sin^r \theta,$$

for every value of  $\theta$ , then (1998-2 Marks)

- a)  $b_0 = 1, b_1 = 3$   
b)  $b_0 = 0, b_1 = n$   
c)  $b_0 = -1, b_1 = 3$   
d)  $b_0 = 0, b_1 = n^2 - 3n + 3$

- 4) Let  $T_r$  be the  $r^{\text{th}}$  term of an A.P., for  $r=1,2,3,\dots$ . If for some positive integers  $m, n$  we have  $T_m = \frac{1}{n}$  and  $T_n = \frac{1}{m}$ , then  $T_{mn}$  equals (1998-2 Marks)

- a)  $\frac{1}{mn}$                                       c) 1  
b)  $\frac{1}{m} + \frac{1}{n}$                                 d) 0

- 5) If  $x > 1, y > 1, z > 1$  are in G.P., then  $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$  are in (1998-2 Marks)

- a) A.P.                                      c) G.P.  
b) H.P.                                      d) None of these

- 6) For a positive integer  $n$ , let  $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n)-1}$ . Then (1999-2 Marks)

- a)  $a(100) \leq 100$                       c)  $a(200) \leq 100$   
b)  $a(100) > 100$                       d)  $a(200) > 100$

- 7) A straight line through the vertex P of a triangle POR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then (2008)

- a)  $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \cdot XS \cdot R}}$                       c)  $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$   
b)  $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \cdot XS \cdot R}}$                       d)  $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

- 8) Let

$$S_n = \sum_{k=1}^n \frac{n}{n^2 + kn + k^2} \text{ and } T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$$

for  $n=1,2,3,\dots$ . Then,

(2008)

- a)  $S_n < \frac{\pi}{3\sqrt{3}}$                                       c)  $T_n < \frac{\pi}{3\sqrt{3}}$   
b)  $S_n > \frac{\pi}{3\sqrt{3}}$                                       d)  $T_n > \frac{\pi}{3\sqrt{3}}$

- 9) Let

$$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2.$$

Then  $S_n$  can take value(s) (JEE Adv.2013)

- a) 1056                                      c) 1120  
b) 1088                                      d) 1332

- 10) Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - x - 1 = 0$ , with  $\alpha > \beta$ . For all positive integers  $n$ , define

$$a_n = \frac{\alpha_n - \beta_n}{\alpha - \beta}, n \geq 2$$

$$b_1 = 1 \text{ and } b_n = a_{n-1} + a_{n+1}, n \geq 1$$

Then which of the following options is/are correct? (JEE Adv. 2019)

- a)  $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$   
b)  $B_n = a^n + b^n$  for all  $n \geq 1$   
c)  $a_1 + a_2 + a_3 + \dots + a_n = a_{n+2} - 1$  for all  $n \geq 1$   
d)  $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

### E. SUBJECTIVE PROBLEMS

- 1) The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation.  
 $2A + G^2 = 27$   
 Find the two numbers. (1979)
- 2) The interior angles of a polygon are in arithmetic progression. The smallest angle is  $120^\circ$  and the common difference is  $5^\circ$ . Find the number of sides of the polygon. (1980)
- 3) Does there exist a geometric progression containing 27, 8 and 12 as three of its terms? If it exists, how many such progressions are possible? (1982-3 Marks)
- 4) Find three numbers a, b, c, between 2 and 18 such that
  - a) their sum is 25
  - b) the numbers 2, a, b are consecutive terms of an A.P.  
and
  - c) the numbers b, c, 18 are consecutive terms of a G.P. (1983-2 Marks)
- 5) If  $a > 0, b > 0, c > 0$ , prove that  
 $(a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$  (1984-2 Marks)