

Question 1-1.8-5q

EE24BTECH11015 - Dhawal

- 1) If \mathbf{A} and \mathbf{B} be the points $(3, 4, 5)$ and $(-1, 3, -7)$ respectively, find the equation of the set of points \mathbf{P} such that $PA^2 + PB^2 = K^2$ where K is a constant.

Solution:

Solving the equation,

$$PA^2 + PB^2 = K^2 \quad (1.1)$$

$$\|\mathbf{P} - \mathbf{A}\|^2 + \|\mathbf{P} - \mathbf{B}\|^2 = K^2 \quad (1.2)$$

$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) + (\mathbf{P} - \mathbf{B})^T (\mathbf{P} - \mathbf{B}) = K^2 \quad (1.3)$$

$$\|\mathbf{P}\|^2 - \mathbf{P}^T \mathbf{A} - \mathbf{A}^T \mathbf{P} + \|\mathbf{A}\|^2 + \|\mathbf{P}\|^2 - \mathbf{P}^T \mathbf{B} - \mathbf{B}^T \mathbf{P} + \|\mathbf{B}\|^2 = K^2 \quad (1.4)$$

For vectors,

$$\mathbf{P}^T \mathbf{A} = \mathbf{A}^T \mathbf{P} \text{ and } \mathbf{P}^T \mathbf{B} = \mathbf{B}^T \mathbf{P} \quad (1.5)$$

Then,

$$2\|\mathbf{P}\|^2 - 2\mathbf{A}^T \mathbf{P} + \|\mathbf{A}\|^2 - 2\mathbf{B}^T \mathbf{P} + \|\mathbf{B}\|^2 = K^2 \quad (1.6)$$

Equation without putting the values of \mathbf{A} and \mathbf{B} ,

$$2\|\mathbf{P}\|^2 - 2\mathbf{A}^T \mathbf{P} + \|\mathbf{A}\|^2 - 2\mathbf{B}^T \mathbf{P} + \|\mathbf{B}\|^2 - K^2 = 0 \quad (1.7)$$

Finding $\|\mathbf{A}\|^2$,

$$\|\mathbf{A}\|^2 = \mathbf{A}^T \mathbf{A} = \begin{pmatrix} 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = 50 \quad (1.8)$$

Finding $\|\mathbf{B}\|^2$,

$$\|\mathbf{B}\|^2 = \mathbf{B}^T \mathbf{B} = \begin{pmatrix} -1 & 3 & -7 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix} = 59 \quad (1.9)$$

Putting the values in equation,

$$2\|\mathbf{P}\|^2 - 2\begin{pmatrix} 3 & 4 & 5 \end{pmatrix} \mathbf{P} + 50 - 2\begin{pmatrix} -1 & 3 & -7 \end{pmatrix} \mathbf{P} + 59 - K^2 = 0 \quad (1.10)$$

$$2\|\mathbf{P}\|^2 - \begin{pmatrix} 6 & 8 & 10 \end{pmatrix} \mathbf{P} - \begin{pmatrix} -2 & 6 & -14 \end{pmatrix} \mathbf{P} + 109 - K^2 = 0 \quad (1.11)$$

Final equation:

$$2\|\mathbf{P}\|^2 - \begin{pmatrix} 4 & 14 & -4 \end{pmatrix} \mathbf{P} + 109 - K^2 = 0$$

(1.12)

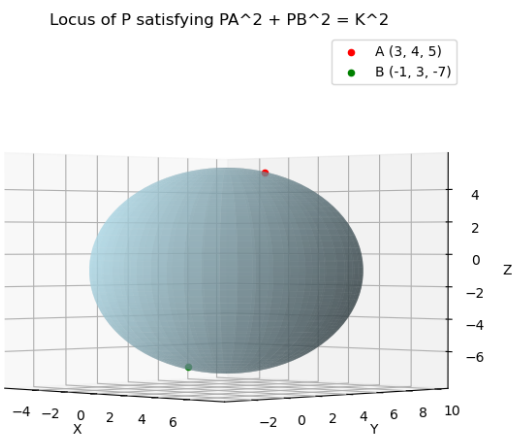


Fig. 1.1: Locus of P