Question 1-1.4-9q

EE24BTECH11015 - Dhawal

1) Let $\mathbf{A}(4,2)$,$\mathbf{B}(6,5)$ and $\mathbf{C}(1,4)$ be the vertices of $\Delta\ ABC$.Find the coordinates of points $\mathbf{Q}$ and $\mathbf{R}$ on medians BE and CF respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$. What do you observe?

Solution:
$\mathbf{F}$ is the mid point of AB

$$\mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}6\\5\end{pmatrix}}{2} = \begin{pmatrix}5\\\frac{7}{2}\end{pmatrix} \tag{1.1}$$

$\mathbf{E}$ is the mid point of AC

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{C}}{2} = \frac{\begin{pmatrix}4\\2\end{pmatrix}+\begin{pmatrix}1\\4\end{pmatrix}}{2} = \begin{pmatrix}\frac{5}{2}\\3\end{pmatrix} \tag{1.2}$$

By section formula,

$$\mathbf{R} = \frac{\mathbf{B}+K\mathbf{A}}{1+K} \tag{1.3}$$

It is given that $\frac{BQ}{QE} = \frac{2}{1}$
So,

$$\mathbf{Q} = \frac{\mathbf{B}+2\mathbf{E}}{1+2} = \frac{\begin{pmatrix}6\\5\end{pmatrix}+2\begin{pmatrix}\frac{5}{2}\\3\end{pmatrix}}{3} = \begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} \tag{1.4}$$

It is given that $\frac{CR}{RF} = \frac{2}{1}$
So,

$$\mathbf{R} = \frac{\mathbf{C}+2\mathbf{F}}{1+2} = \frac{\begin{pmatrix}1\\4\end{pmatrix}+2\begin{pmatrix}5\\\frac{7}{2}\end{pmatrix}}{3} = \begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} \tag{1.5}$$

Hence, Co-ordinates of $\mathbf{Q}$ and $\mathbf{R}$ are

$$\mathbf{Q}\begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} \text{and } \mathbf{R}\begin{pmatrix}\frac{11}{3}\\\frac{11}{3}\end{pmatrix} \tag{1.6}$$

It is observed that both points **Q** and **R** are same.

$$Hence, \mathbf{Q} = \mathbf{R} \tag{1.7}$$

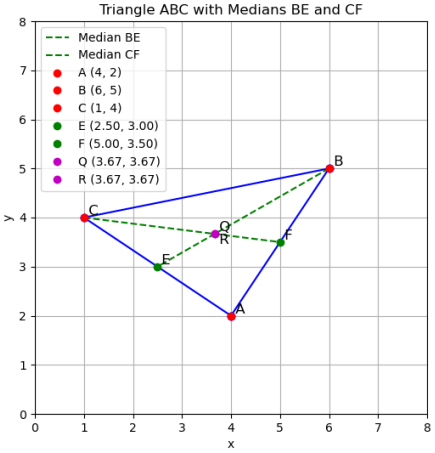

Fig. 1.1: Plot of Triangle *ABC*