

Question 1-1.4-9q

EE24BTECH11015 - Dhawal

- 1) Let $\mathbf{A}(4, 2)$, $\mathbf{B}(6, 5)$ and $\mathbf{C}(1, 4)$ be the vertices of $\triangle ABC$. Find the coordinates of points \mathbf{Q} and \mathbf{R} on medians BE and CF respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$. What do you observe?

Solution:

\mathbf{F} is the mid point of AB

$$\mathbf{F} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 5 \end{pmatrix}}{2} = \begin{pmatrix} 5 \\ \frac{7}{2} \end{pmatrix} \quad (1.1)$$

\mathbf{E} is the mid point of AC

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix}}{2} = \begin{pmatrix} \frac{5}{2} \\ 3 \end{pmatrix} \quad (1.2)$$

By section formula,

$$\mathbf{R} = \frac{\mathbf{B} + K\mathbf{A}}{1 + K} \quad (1.3)$$

It is given that $\frac{BQ}{QE} = \frac{2}{1}$
So,

$$\mathbf{Q} = \frac{\mathbf{B} + 2\mathbf{E}}{1 + 2} = \frac{\begin{pmatrix} 6 \\ 5 \end{pmatrix} + 2\begin{pmatrix} \frac{5}{2} \\ 3 \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \quad (1.4)$$

It is given that $\frac{CR}{RF} = \frac{2}{1}$
So,

$$\mathbf{R} = \frac{\mathbf{C} + 2\mathbf{F}}{1 + 2} = \frac{\begin{pmatrix} 1 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 5 \\ \frac{7}{2} \end{pmatrix}}{3} = \begin{pmatrix} \frac{11}{3} \\ \frac{11}{3} \end{pmatrix} \quad (1.5)$$

Hence, Co-ordinates of \mathbf{Q} and \mathbf{R} are

$$\mathbf{Q}\left(\frac{11}{3}, \frac{11}{3}\right) \text{ and } \mathbf{R}\left(\frac{11}{3}, \frac{11}{3}\right) \quad (1.6)$$

It is observed that both points **Q** and **R** are same.

$$\text{Hence, } \mathbf{Q} = \mathbf{R} \quad (1.7)$$

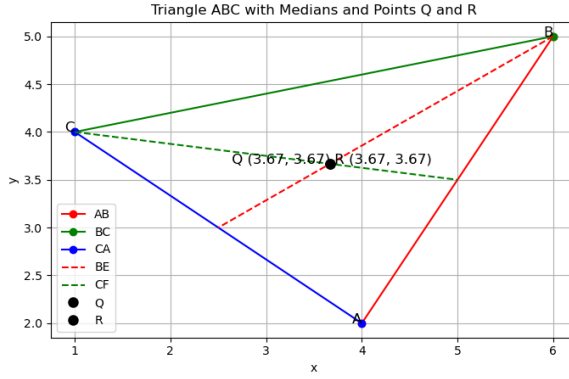


Fig. 1.1: Plot of Triangle *ABC*