

Chapter 6 Sequence and Series

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D. MCQs WITH ONE OR MORE THAN ONE CORRECT

- 1) If the first and the $(2n - 1)^{th}$ terms of an A.P., a G.P. and an H.P. are equal and their n -th terms are a , b and c respectively, then (1988-2 Marks)

- a) $a = b = c$ c) $a + b = c$
b) $a \geq b \geq c$ d) $ac - b^2 = 0$

- 2) For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n} \phi, y = \sum_{n=0}^{\infty} \sin^{2n} \phi, z = \sum_{n=0}^{\infty} \cos^{2n} \phi \sin^{2n} \phi$$

then:

(1993-2 Marks)

- a) $xyz = xz + y$ c) $xyz = x + y + z$
b) $xyz = xy + z$ d) $xyz + yz + x$

- 3) Let n be an odd integer. If

$$\sin n\theta = \sum_{r=0}^n b_r \sin^r \theta,$$

for every value of θ , then (1998-2 Marks)

- a) $b_0 = 1, b_1 = 3$
b) $b_0 = 0, b_1 = n$
c) $b_0 = -1, b_1 = 3$
d) $b_0 = 0, b_1 = n^2 - 3n + 3$

- 4) Let T_r be the r^{th} term of an A.P., for $r=1,2,3,\dots$. If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals (1998-2 Marks)

- a) $\frac{1}{mn}$ c) 1
b) $\frac{1}{m} + \frac{1}{n}$ d) 0

- 5) If $x > 1, y > 1, z > 1$ are in G.P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in (1998-2 Marks)

- a) A.P. c) G.P.
b) H.P. d) None of these

- 6) For a positive integer n , let $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n)-1}$. Then (1999-2 Marks)

- a) $a(100) \leq 100$ c) $a(200) \leq 100$
b) $a(100) > 100$ d) $a(200) > 100$

- 7) A straight line through the vertex P of a triangle POR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then (2008)

- a) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \cdot SR}}$ c) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$
b) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \cdot SR}}$ d) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

- 8) Let

$$S_n = \sum_{k=1}^n \frac{n}{n^2 + kn + k^2} \text{ and } T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$$

for $n=1,2,3,\dots$. Then, (2008)

- a) $S_n < \frac{\pi}{3\sqrt{3}}$ c) $T_n < \frac{\pi}{3\sqrt{3}}$
b) $S_n > \frac{\pi}{3\sqrt{3}}$ d) $T_n > \frac{\pi}{3\sqrt{3}}$

- 9) Let

$$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2.$$

Then S_n can take value(s) (JEE Adv.2013)

- a) 1056 c) 1120
b) 1088 d) 1332

- 10) Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n , define

$$a_n = \frac{\alpha_n - \beta_n}{\alpha - \beta}, n \geq 2$$

$$b_1 = 1 \text{ and } b_n = a_{n-1} + a_{n+1}, n \geq 1$$

Then which of the following options is/are correct? (JEE Adv. 2019)

- a) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$
b) $B_n = a^n + b^n$ for all $n \geq 1$
c) $a_1 + a_2 + a_3 + \dots + a_n = a_{n+2} - 1$ for all $n \geq 1$
d) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

E. SUBJECTIVE PROBLEMS

- 1) The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation.
 $2A + G^2 = 27$
 Find the two numbers. (1979)
- 2) The interior angles of a polygon are in arithmetic progression. The smallest angle is 120° and the common difference is 5° . Find the number of sides of the polygon. (1980)
- 3) Does there exist a geometric progression containing 27, 8 and 12 as three of its terms? If it exists, how many such progressions are possible? (1982-3 Marks)
- 4) Find three numbers a, b, c, between 2 and 18 such that
 - a) their sum is 25
 - b) the numbers 2, a, b are consecutive terms of an A.P.
and
 - c) the numbers b, c, 18 are consecutive terms of a G.P. (1983-2 Marks)
- 5) If $a > 0, b > 0, c > 0$, prove that
 $(a + b + c) \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \geq 9$ (1984-2 Marks)