Chapter 6 Sequence and Series

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D. MCQs with One or More than One Correct

1) If the first and the $(2n-1)^{th}$ terms of an A.P., a G.P. and an H.P. are equal and their n-th terms are a, b and c respectively, then $(1988-2Marks)$

a) $a=b=c$
b) $a \geq b \geq c$
c) $a+b=c$
d) $ac-b^2=0$

2) For $0<\phi<\pi/2$, if

$$x=\sum_{n=0}^{\infty}\cos^{2n}\phi, y=\sum_{n=0}^{\infty}\sin^{2n}\phi, z=\sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$$

then: $(1993-2Marks)$

a) $xyz=xz+y$
b) $xyz=xy+z$
c) $xyz=x+y+z$
d) $xyz+yz+x$

3) Let n be a odd integer. If

$$\sin n\theta=\sum_{r=0}^{n}b_r\sin^r\theta,$$

for every value of θ, then $(1998-2Marks)$

a) $b_0=1, b_1=3$
b) $b_0=0, b_1=n$
c) $b_0=-1, b_1=3$
d) $b_0=0, b_1=n^2-3n+3$

4) Let T_r be the r^{th} term of an A.P., for $r=1,2,3,\ldots$ If for some positive integers m,n we have $T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$,then T_{mn} equals $(1998-2Marks)$

a) $\frac{1}{mn}$
b) $\frac{1}{m}+\frac{1}{m}$
c) 1
d) 0

5) If $x>1, y>1, z>1$ are in G.P.,then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in $(1998-2Marks)$

a) A.P.
b) H.P.
c) G.P.
d) None of these

6) For a positive integer n, let $a_n=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots\frac{1}{(2^n)-1}$.Then $(1999-2Marks)$

a) $a(100)\leq 100$
b) $a(100)>100$
c) $a(200)\leq 100$
d) $a(200)>100$

7) A straight line through the vertex P of a triangle POR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle,then (2008)

a) $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\cdot SR}}$
b) $\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\cdot SR}}$
c) $\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$
d) $\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$

8) Let

$$S_n=\sum_{k=1}^{n}\frac{n}{n^2+kn+k^2} \quad and \quad T_n=\sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$$

for n=1,2,3,... Then, (2008)

a) $S_n<\frac{\pi}{3\sqrt{3}}$
b) $S_n>\frac{\pi}{3\sqrt{3}}$
c) $T_n<\frac{\pi}{3\sqrt{3}}$
d) $T_n>\frac{\pi}{3\sqrt{3}}$

9) Let

$$S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2.$$

Then S_n can take value(s) $(JEEAdv.2013)$

a) 1056
b) 1088
c) 1120
d) 1332

10) Let α and β be the roots of $x^2-x-1=0$, with $\alpha>\beta$. For all positive integers n, define

$$a_n=\frac{\alpha_n-\beta_n}{\alpha-\beta}, n\geq 2$$

$$b_1=1 \; and \; b_n=a_{n-1}+a_{n+1}, n\geq 1$$

Then which of the following options is/are correct? $(JEEAdv.2019)$

a) $\sum_{n=1}^{\infty}\frac{a_n}{10^n}=\frac{10}{89}$
b) $B_n=a^n+b^n\forall n\geq 1$
c) $a_1+a_2+a_3+.....a_n=a_{n+2}-1\forall n\geq 1$
d) $\sum_{n=1}^{\infty}\frac{b_n}{10^n}=\frac{8}{89}$

E. Subjective Problems

1) The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation.
   $2A + G^2 = 27$
   Find the two numbers. (1979)
2) The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°and the common difference is 5°Find the number of sides of the polygon. (1980)
3) Does there exist a geometric progression containing 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible? $(1982 - 3Marks)$
4) Find three numbers a, b, c between 2 and 18 such that
   a) their sum is 25
   b) the numbers $2, a, b$ are consecutive terms of an A.P.
      and
   c) the numbers $b, c, 18$ are consecutive terms of a G.P. $(1983 - 2Marks)$
5) If $a > 0, b > 0, c > 0$, prove that
   $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$ $(1984 - 2Marks)$