Chapter 6 Sequence and Series

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D. MCQs with One or More than One Correct

1) If the first and the $(2n-1)^{th}$ terms of an A.P., a G.P. and an H.P. are equal and their n^{th} terms are a, b and c respectively, then (1988 – 2$Marks$)

   a) $a = b = c$ b) $a \geq b \geq c$ c) $a + b = c$ d) $ac - b^2 = 0$

2) For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi,\ y = \sum_{n=0}^{\infty} \sin^{2n}\phi,\ z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$$

   then: (1993 – 2$Marks$)

   a) $xyz = xz + y$ b) $xyz = xy + z$ c) $xyz = x + y + z$ d) $xyz + yz + x$

3) Let n be a odd integer. If

$$\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta,$$

   for every value of θ, then (1998 – 2$Marks$)

   a) $b_0 = 1, b_1 = 3$
   b) $b_0 = 0, b_1 = n$
   c) $b_0 = -1, b_1 = 3$
   d) $b_0 = 0, b_1 = n^2 - 3n + 3$

4) Let T_r be the r^{th} term of an A.P., for $r = 1, 2, 3, \dots$ If for some positive integers m, n we have $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$,then T_{mn} equals (1998 – 2$Marks$)

   a) $\frac{1}{mn}$ b) $\frac{1}{m} + \frac{1}{m}$ c) 1 d) 0

5) If $x > 1, y > 1, z > 1$ are in G.P.,then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in (1998 – 2$Marks$)

   a) A.P. b) H.P. c) G.P. d) None of these

6) For a positive integer n, let $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \frac{1}{(2^n)-1}$. Then (1999 – 2$Marks$)

   a) $a(100 \leq 100$ b) $a(100) > 100$ c) $a(200) \leq 100$ d) $a(200) > 100$

7) A straight line through the vertex P of a triangle POR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle,then (2008)

a) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \cdot SR}}$
b) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \cdot SR}}$
c) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$
d) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

8) Let

$$S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2} \text{ and } T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$$

for $n = 1, 2, 3, \ldots$ Then, (2008)

a) $S_n < \frac{\pi}{3\sqrt{3}}$
b) $S_n > \frac{\pi}{3\sqrt{3}}$
c) $T_n < \frac{\pi}{3\sqrt{3}}$
d) $T_n > \frac{\pi}{3\sqrt{3}}$

9) Let

$$S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2.$$

Then S_n can take value(s) $(JEEAdv.2013)$

a) 1056
b) 1088
c) 1120
d) 1332

10) Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$. For all positive integers n, define

$$a_n = \frac{\alpha_n - \beta_n}{\alpha - \beta}, n \geq 2b_1 = 1 \; and \; b_n = a_{n-1} + a_{n+1}, n \geq 1$$

Then which of the following options is/are correct? $(JEEAdv.2019)$

a) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$
b) $B_n = a^n + b^n \; \forall /; n \geq 1$
c) $a_1 + a_2 + a_3 + ...a_n = a_{n+2} - 1 \forall n \geq 1$
d) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

E. Subjective Problems

1) The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation.
$2A + G^2 = 27$
Find the two numbers. (1979)
2) The interior angles of a polygon are in arithmetic progression. The smallest angle is 120° and the common difference is 5° Find the number of sides of the polygon. (1980)
3) Does there exist a geometric progression containing 27, 8 and 12 as three of its terms? If it exits, how many such progressions are possible? $(1982 - 3Marks)$
4) Find three numbers a, b, c between 2 and 18 such that
   a) their sum is 25
   b) the numbers $2, a, b$ are consecutive terms of an A.P.
   and
   c) the numbers $b, c, 18$ are consecutive terms of a G.P. $(1983 - 2Marks)$
5) If $a > 0, b > 0, c > 0$, prove that
$(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \geq 9$ $(1984 - 2Marks)$