Assignment – 1 : Implementation of Expectation Maximization Algorithm.

Artificial Intelligence Lab (CS:571) **Deadline : 13th Aug 2018**

Note:

1. **Read all the instructions carefully and adhere to them.**
2. You may choose any programming language for the implementation, however, ***Python*** is recommended.
3. Assignment has to be done in group of maximum three members.

Question: Assume there are two coins C_1 and C_2.

A. Choose a coin between C_1 and C_2 and toss it.
   a. p = probability of choosing C_1
   b. p_1 = probability of getting head from C_1
   c. p_2 = probability of getting head from C_2

B. Repeat Step A. for N number of times to generate a sequence of H and T using $\boldsymbol{p}$ (say 0.7), $\boldsymbol{p_1}$ (say 0.6), and $\boldsymbol{p_2}$ (say 0.3).

C. Understand the properties of random number generator such that probability of choosing $\mathbf{C_1}$ is 0.7. Generate another random number so that head is produced with probability 0.6 (for $\mathbf{C_1}$) and 0.3 (for $\mathbf{C_2}$).

D. Estimate p, p_1 and p_2 using Expectation Maximization (EM) Algorithm
   a. Initialize p, p_1, and p_2 randomly.
   b. **E step:** $E(z_i) = (p.p_1^{x_i}.(1-p_1)^{1-x_i}) / (p.p_1^{x_i}.(1-p_1)^{1-x_i} + (1-p).p_2^{x_i}.(1-p_2)^{1-x_i})$
   c. **M step:**

$$p = \frac{\sum_{i=1}^{N} E(z_i)}{N}$$

$$p_1 = \frac{\sum_{i=1}^{N} x_i E(z_i)}{\sum_{i=1}^{N} E(z_i)}$$

$$p_2 = \frac{M - \sum_{i=1}^{N} x_i E(z_i)}{N - \sum_{i=1}^{N} E(z_i)}$$

Where M is the number of heads in the generated sequence, and x_i and z_i are i^{th} ***indicator variable*** and i^{th} ***hidden variable***, respectively. $z_i = 1$, if C_1 is chosen, else 0, $x_i = 1$, if head, else 0.

   d. Repeat steps b-c until the p, p_1 and p_2 values converges.