HWO ASSIGNMENT

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1. function useless (n)
2. if n=1 then
3. return 1
4. else
5. return useless (random (1, n))
6. end if
7. end function

where random (1, n) returns uniformly distributed random integer in the range 1...n. Assume an initial call useless (m).

1. What value is returned? Prove your answer formally.

The value returned is always return 1.

Answer: 1

Proof: Let the initial call be useless (m)

Case1: If m=1, then the first if condition is checked and it returns 1.

Case2: If m≠1, then useless (random (1, n)) will be called. (ie) useless (random (1, m)) random (1, n) returns uniformly distributed random integer in the range 1...n.

So, the random (1, n) will generate 1 at some point. The ~~random~~ useless (random (1, n))

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will be called repeatedly until 1 is generated to satisfy the 'if' condition in the function. Only when '1' is generated in the random (1, n) the if condition n = 1 will be satisfied and it returns 1.

2. Calculate exactly the expected number of calls to random in line 5.

Let f(m) be the expected no. of calls to the random in line 5

<u>case 1:</u> Let m be 1.

If m = 1 then the first if condition will be satisfied and it returns 1. There will be no calls to random function. So in this case the number of calls to the random function is 0

$\therefore f(1) = 0$

<u>case 2:</u> Let m be 2

If m = 2, then the first if condition is failed and random function will be called atleast once. The random (1, n) will generate uniformly distributed random integer in the range 1....n. So the probability of getting 1 or 2 is equal. $\therefore f(2) = 1 + \frac{f(2)}{2}$ where $f(2) = \{1, 2\}$

So $f(2) = 2$

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Case 3 : Let m be greater than 2

There are two possibilities if m is greater than 2.

(i) If random $(1,n)$ is called m can be returned

(ii) If random $(1,n)$ is called other random number can be generated except m. So it is $(m-1)$

Eg: $f(3)$. Let $m=3$

$$f(m) = \frac{1+f(m)}{m} + \frac{1+f(m-1)}{m} + \dots + \frac{1+f(2)}{m} + \frac{1}{m}$$

(ie) Probability of all previous cases and probability of getting m

$f(3) = f(2) + \frac{1}{2}$ where $f(2) = \{1,2\}$

$f(3) = 2 + 1/2$

The probability of getting 3 from $\{\{1,2\},3\}$ is $5/2$ (ie $2 + 1/2$)

Similarly

$f(4) = f(3) + 1/3$

The probability of getting 4 from $\{\{1,2\},3,4\}$

$= 2 + 1/2 + 1/3$

So According to the generalisation

$$f(m) = f(m-1) + 1/m-1$$

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$$f(m) = f(m-1) + \frac{1}{m-1}$$

$\Rightarrow$ $f(m) = 1 + H_{m-1}$ where H_m is Harmonic number.

(ii) $H_{m-1} = \sum_{l=1}^{m-1} \frac{1}{l}$

Answer: $1 + H_{m-1}$

3. In the worst case, what is the number of calls to random in line 5.

Answer: m calls

Proof: random (1, n) ~~returns~~ will generate uniformly distributed random integer in the range ~~(x, ol)~~ 1 . . . n

If random(1, n) generates 1 in the first call then the number of call is 1.

Best case is 1.

If random(1, n) generates m in the first call then the probability to generate other numbers are equal since it is uniformly distributed.

So atleast m calls will be done.

In worst case, the random number generated will be in decreasing order eg: m, m-1, m-2 till 1. So the worst case is m calls.

The worst case is O(m). ~~Answer~~: O(m)

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