CS 522

Homework Assignment 3

NAME: DHAYALINI NAGARAJ

A.NO :A20359686

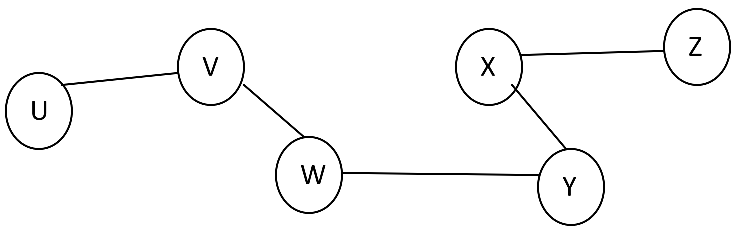
1. Exercise 10.1.1 : It is possible to think of the edges of one graph G as the nodes of another graph G′ We construct G′ from G by the dual construction:

1. If (X, Y) is an edge of G, then XY , representing the unordered set of X and Y is a node of G′ . Note that XY and Y X represent the same node of G′ , not two different nodes.

2. If (X, Y) and (X, Z) are edges of G, then in G′ there is an edge between XY and XZ.
That is, nodes of G′ have an edge between them if the edges of G that these nodes represent have a node (of G) in common.

(a) If we apply the dual construction to a network of friends, what is the interpretation of the edges of the resulting graph?

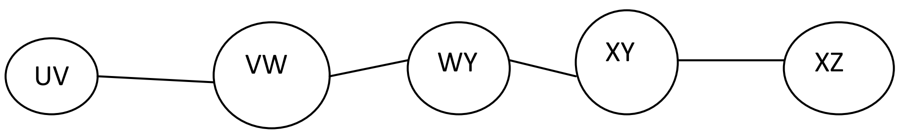
Let us consider a graph G



In this graph UV, VW, WY, XY,XZ are edges and represent network of friends.

Now let us apply dual construction to this network of friends.

Graph G^{l}



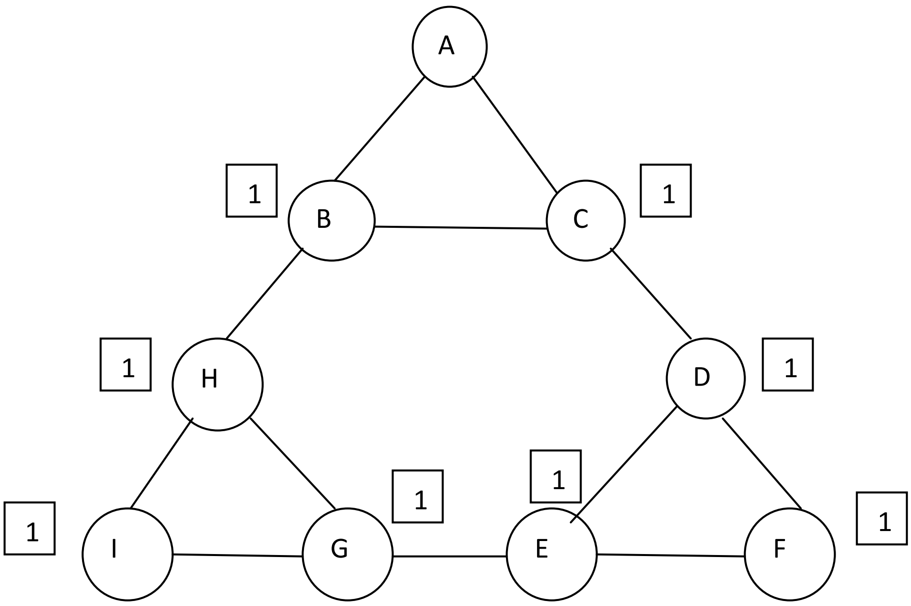
Here node XY and XZ have an edge since X is common. Similarly UV and VW has an edge, WY and XY has an edge and VW and WY has an edge.

This graph is a network of friends. So let us consider U,V, W, X,Y,Z as person and they are connected in social network. The dual construction of the graph G clearly shows that there is a edge connecting UV and VW. This means UV is two person connected through a link and VW is connected through a link called friends connection. In G^{l} graph these three person UVW are connected through a link. In this connection we can find the V is the common person who has friendship with U and W. So the two person U and W are connect through a mutual friend V.

So from this graph we get to know that there is a common or mutual friend between two friend relationship which are connected to the friendship link. So there is a relation between the other two person through a common friend. A network of interconnected friends can be got from this way of representation.

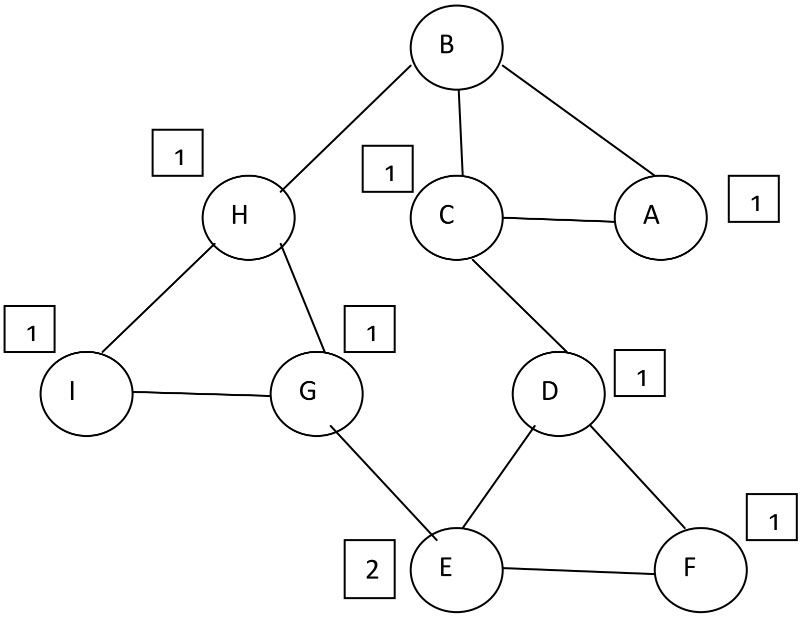
2. Exercise 10.2.1 : Figure 10.9 is an example of a social-network graph. Use the Girvan-Newman approach to find the number of shortest paths from each of the following nodes that pass through each of the edges.
(a) A (b) B.

a) **Breadth First Search from node A**



The shortest path from node A to all other nodes are calculated. All nodes in the graph has 1 shortest path from node A.

b) Breadth First Search from node B



The shortest path from node B to all other nodes are calculated. The node E has two shortest path from root node B and all the other nodes have 1 shortest path.

3. Exercise 10.2.2 : Using the symmetry of the graph, the calculations of Exercise 10.2.1 are all you need to compute the betweenness of each edge. Explain in a couple of sentences why you only need to compute the number of shortest paths for A and B and how do you use the symmetry property to get the # shortest paths for all other nodes. Also, explain very briefly, how do you updates the scores on the edges , Do the calculation for the betweenness, and explain in a couple of sentences the steps in your computation.

A graph G is said to be symmetric if any two pairs of adjacent vertices x1—y1 and x2—y2 of G, there is an automorphism.

f : V(G) → V(G)such that f(x1) = x2 and f(y1) = y2.

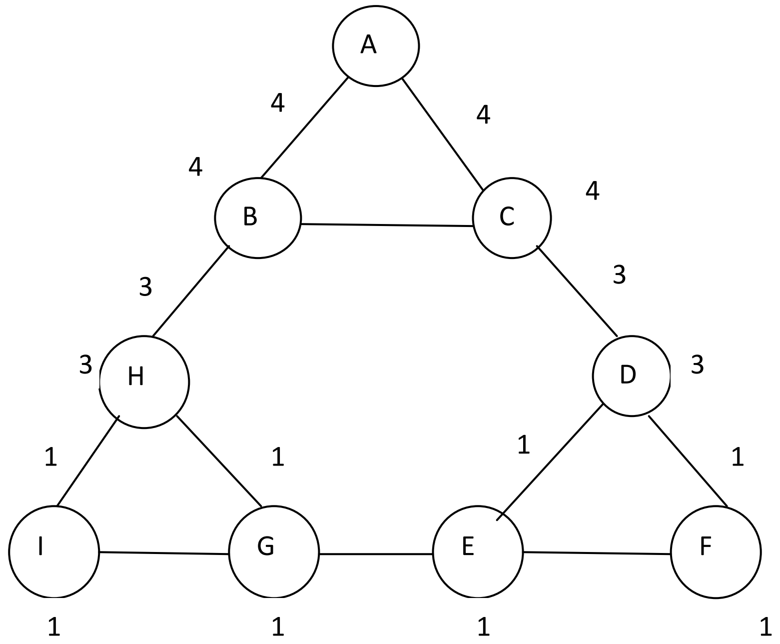
A graph is symmetric if its automorphism group acts transitively upon ordered pairs of adjacent vertices.

A symmetric graph must also be edge transitive and vertex transitive. A graph is edge transitive when any two edges are equivalent under some element of its automorphism group. That is for all pairs of edges there exists an element γ of the edge automorphism group. Similarly for vertex transitive graph every pair of vertices has some element of its vertex automorphism group.

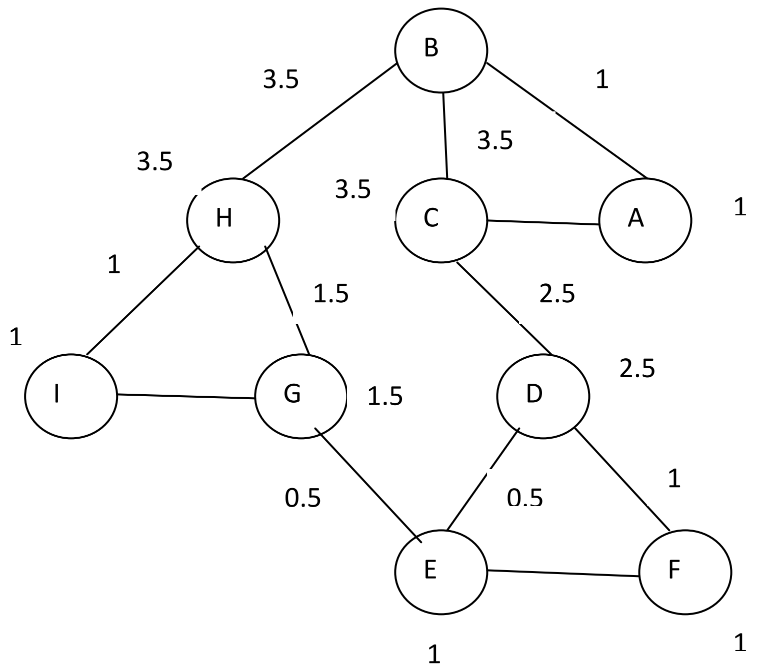
So based on the edge and vertex and its automorphism group the values of shortest path are gonna be repeated for a symmetric matrix.

Since the values of shortest path for each nodes are repeated based on symmetric property it is enough if we calculate the shortest path based on node A and node B from which we can calculate the betweenness and find the communities.

Betweenness of each edge for the graph when breadth first search is done from node A



Betweenness of each edge for the graph when breadth first search is done from node B



The betweenness is calculated based on the following steps:

- Each leaf node gets a credit 1.
- Other nodes gets a credit 1 plus the sum of the credits of the edges from that node to the level below.

- Let the parents of node X be Y1, Y2 ..... Yk. Let shortest path from root to node Y1 be sp .Then the credit for the edge (X, Y1) is calculated as X times sp.

From the above graph E has credit 1 and its parent is G and D. The shortest path to E is 2. So the edge credit to G and D are 1/2 where 1 is the E credit and 2 is the shortest path.

In the similar way all the credits for the edge and node are calculated.

In the first graph, all the nodes have shortest path as 1. So for each edge from below has credit 1 and the parent node has the sum of edge credit and the node credit. This process is repeated until we reach the root node A.

In the second graph, all the leaf nodes are assigned credit 1 and the edge from those nodes has the same credit. The parent node credit is the sum of the edge credit and the node credit. In second graph, since E has two shortest path, the credit of E is split or divided by the shortest path i.e 1/2 and the EG and ED edge has 0.5 credit which in turn is added to its parent node. The process is repeated until the root node B is reached.

4.Exercise 10.4.1 : For the graph of Fig. 10.9, construct:
(a) The adjacency matrix.
(b) The degree matrix.
(c) The Laplacian matrix.
Explain in 1-2 sentences the steps you take to construct each matrix.

a) **The Adjacency Matrix**

It is a nxn matrix. $A=[a_{ij}]$, $a_{ij} = 1$ if there is an edge between node i and j. The properties of adjacency matrix is, it is symmetric and Eigenvectors are real and orthogonal.

From Fig 10.9 we see that there is an edge from A to B, C.

Similarly there are edges from
From B to A, C, H
From C to A, B, D
From D to C, E, F
From E to G, D, F
From F to D, E
From G to H, I, E
From H to B, I, G
From I to H, G

So based on this the adjacency matrix for the given graph is

	A	B	C	D	E	F	G	H	I
A	0	1	1	0	0	0	0	0	0
B	1	0	1	0	0	0	0	1	0
C	1	1	0	1	0	0	0	0	0
D	0	0	1	0	1	1	0	0	0
E	0	0	0	1	0	1	1	0	0
F	0	0	0	1	1	0	0	0	0
G	0	0	0	0	1	0	0	1	1
H	0	1	0	0	0	0	1	0	1
I	0	0	0	0	0	0	1	1	0

b) The Degree Matrix

It is a nxn diagonal matrix. D =[d_{ij}] where d_{ij} = degree of node i.

The sum of incoming and outgoing edges of a node is the degree of the node. Based on the number of edges connected to the node, the degree matrix for the graph is found.

	A	B	C	D	E	F	G	H	I
A	2	0	0	0	0	0	0	0	0
B	0	3	0	0	0	0	0	0	0
C	0	0	3	0	0	0	0	0	0
D	0	0	0	3	0	0	0	0	0
E	0	0	0	0	3	0	0	0	0
F	0	0	0	0	0	2	0	0	0
G	0	0	0	0	0	0	3	0	0
H	0	0	0	0	0	0	0	3	0
I	0	0	0	0	0	0	0	0	2

c) The Laplacian matrix

It is a nxn symmetric matrix. The laplacian matrix is calculated from Adjacency(A) and Degree Matrix(D).

The Laplacian matrix L = D - A

The properties of laplacian matrix are Eigenvectors are real and orthogonal, eigenvalues are non-negative real numbers.

	A	B	C	D	E	F	G	H	I
A	2	-1	-1	0	0	0	0	0	0
B	-1	3	-1	0	0	0	0	-1	0
C	-1	-1	3	-1	0	0	0	0	0
D	0	0	-1	3	-1	-1	0	0	0
E	0	0	0	-1	3	-1	-1	0	0
F	0	0	0	-1	-1	2	0	0	0
G	0	0	0	0	-1	0	3	-1	-1
H	0	-1	0	0	0	0	-1	3	-1
I	0	0	0	0	0	0	-1	-1	2