An equation as x^2+3 x −10=0 is said to be a quadratic equation or a second degree equation because the exponent of x (which is the unknown) is 2 (an equation such as, for example, $4x^3$+2 x +10=0 would not be of the second degree, but of the third).

The general form of an equation of this type is:

$$ax^2+bx+c=0$$

Where x is the unknown and a, b, c are any numbers.

The formula that allows us to solve this type of equations is the following one:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this operation a sign ± appears, and the fact is that, in principle, a quadratic equation can have two different solutions, one of them is obtained when we use + and other one when we use −.

We are going to apply this formula to the equation $x^2 + 3x - 10 = 0$.

We write the values of a, b and c:

$$a = 1, b = 3 \text{ and } c = -10$$

and we replace them in the formula:

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot (-10)}}{2 \cdot 1} = \frac{-3 \pm \sqrt{9 + 40}}{2} = \frac{-3 \pm \sqrt{49}}{2} =$$

$$= \frac{-3 \pm 7}{2}$$

And we get two different solutions:

$$\frac{-3+7}{2} = \frac{4}{2} = 2$$
$$\frac{-3-7}{2} = \frac{-10}{2} = -5$$

Therefore, the proposed equation has the solutions 2 and -5.

In most of the textbooks the solutions are indicated by writing a subscript in the letter x, so that in our case we would have:

$$x_1 = 2$$
$$x_2 = -5$$

The solutions of the equation are called roots. It is the same to say that 2 and −5 are the solutions, than it is to say that the roots of the equation x2+3x−10=0 are 2 and −5.

Solve the equation $6x^2 - 5x - 4 = 0$. $a = 6, b = -5$ and $c = -4$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 6 \cdot (-4)}}{2 \cdot 6} = \frac{5 \pm \sqrt{25 + 96}}{12} = \frac{5 \pm 11}{12} =$$

$$= \begin{cases} x_1 = \frac{4}{3} \\ x_2 = -\frac{1}{2} \end{cases}$$

Find the solutions of the equation $x^2 + x - 2 = 0$. $a = 1, b = 1$ and $c = -2$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{-1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} = \begin{cases} x_1 = 1 \\ x_2 = -2 \end{cases}$$

Which are the roots of $2x^2 - 5x - 1 = 0$? $a = 2, b = -5$ and $c = -1$

$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{5 \pm \sqrt{25 + 8}}{4} = \frac{5 \pm \sqrt{33}}{4} =$$

$$= \begin{cases} x_1 = 2.69 \\ x_2 = -0.19 \end{cases}$$

Solve $x^2 - 16 = 0$. $a = 1, b = 0$ and $c = -16$

$$x = \frac{0 \pm \sqrt{0 - 4 \cdot (-16)}}{2} = \frac{\pm 8}{2} = \begin{cases} x_1 = 4 \\ x_2 = -4 \end{cases}$$

Find the roots of $2x^2 - 4x = 0$. $a = 2, b = -4$ and $c = 0$.

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 2 \cdot 0}}{2 \cdot 2} = \begin{cases} x_1 = 2 \\ x_2 = 0 \end{cases}$$

Sometimes the terms of the equation are grouped in a different way, as in 5−x=3x2 in which case we only need to move everything to the first member −3x2−x+5=0

In other cases it is possible that the unknown is not represented using the letter x, as in 3k2−8k+5=0, but this does not change things.

The solutions for this equation are:

$$k^1 = 1$$

$$k^2 = \frac{5}{3}$$

It is important to remember that the square root of a negative number does not exist within the set of the real numbers. If we find a case like this we will say that the equation has no solutions in R.

$x^2 + 2x + 5 = 0$. $a = 1, b = 2$ and $c = 5$.

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2 \cdot 2} = \frac{-2 \pm \sqrt{-16}}{4}$$

This equation has no solutions in $\mathbb{R}$.