As with the fractions, when we operate with algebraic fractions it is interesting that they share a common denominator. And so, the process of reduction of algebraic fractions to common denominator consists in finding two pairs of equivalent algebraic fractions with common denominator.

We will introduce a procedure to do this using the following example:

Consider the algebraic fractions $\frac{1}{x^2-1}$ and $\frac{x+2}{x-2}$, find two equivalent algebraic fractions with common denominator.

The procedure can be separated into three phases:

1) Factorize the polynomials of the denominator of both fractions:

In our case:

$x^2-1=(x-1)\cdot(x+1)$

$x-2$

2) Compute the least common multiple (l.c.m). of the polynomials in the denominators. Let's remember that to compute the l.c.m. we only need to combine all the distinct factors once we have factorized all the polynomials. In the case that there are equal terms, we have to take the one raised to the highest power.

In our case:

$lcm\{x^2-1,x-2\}=lcm\{(x-1)\cdot(x+1),x-2\}=(x-1)\cdot(x+1)\cdot(x-2)$

3) We divide the l.c.m. by every denominator and multiply it by the respective numerator. The result is the numerator of the algebraic fraction, the denominator is the l.c.m.

$\frac{(x-1)\cdot(x+1)\cdot(x-2)}{(x-1)\cdot(x+1)}=x-2\Rightarrow 1\cdot(x-2)=x-2\Rightarrow$

$\Rightarrow\frac{x-2}{(x-1)\cdot(x+1)\cdot(x-2)}$

$\frac{(x-1)\cdot(x+1)\cdot(x-2)}{x-2}=(x-1)(x+1)\Rightarrow(x+2)(x-1)(x+1)\Rightarrow$

$\Rightarrow\frac{(x+2)\cdot(x-1)\cdot(x+1)}{(x-1)\cdot(x+1)\cdot(x-2)}$

As we can see, now we have two fractions equivalent to the first ones who have a common denominator.

Consider the algebraic fractions $\frac{3x}{x^2-9}$ and $\frac{2x-1}{x+3}$, find two equivalent algebraic fractions with common denominator.

1) Factorize the polynomials of the denominator of both fractions:

$x^2-9=(x-3)\cdot(x+3)$

$x+3$

2) Compute the least common multiple (l.c.m). of the polynomials in the denominators.

$lcm\{x^2-9,x+3\}=lcm\{(x-3)\cdot(x+3),x+3\}=(x-3)\cdot(x+3)$

3) We divide the l.c.m. by every denominator and multiply it by the respective numerator. The result is the numerator of the algebraic fraction; the denominator is the l.c.m.

$\frac{(x-3)\cdot(x+3)}{(x-3)\cdot(x+3)}=1\Rightarrow 1\cdot 3x=3x\Rightarrow$

$\Rightarrow\frac{3x}{(x-3)\cdot(x+3)}$

$\frac{(x-3)\cdot(x+3)}{(x+3)}=(x-3)\Rightarrow(2x-1)\cdot(x+3)=2x\cdot(x+3)-1\cdot(x+3)=$

$=2x^2+5x-3\Rightarrow\frac{2x^2+5x-3}{(x-3)\cdot(x+3)}$

As we can see, now we have two fractions equivalent to the first ones who have a common denominator.

Consider the algebraic fractions $\frac{x^2+3}{x-1}$ and $\frac{x-1}{x+1}$, find two equivalent algebraic fractions with common denominator.

1) In this case, the denominators are already factorized.

$x-1$

$x+1$

2) Compute the least common multiple (l.c.m). of the polynomials in the denominators. In this case that there are no common factors, it is enough to calculate the product of both:

$lcm\{x-1,x+1\}=(x-1)\cdot(x+1)$

3) We divide the l.c.m. by every denominator and multiply it by the respective numerator. The result is the numerator of the algebraic fraction, the denominator is the l.c.m.

$\frac{(x-1)\cdot(x+1)}{(x-1)}=(x+1)\Rightarrow(x^2+3)(x+1)=x^2\cdot(x+1)+3\cdot(x+1)=$

$=x^3+x^2+3x+3\Rightarrow\frac{x^3+x^2+3x+3}{(x-1)\cdot(x+1)}$

$\frac{(x-1)\cdot(x+1)}{(x+1)}=(x-1)\Rightarrow(x-1)(x-1)=x^2-1\Rightarrow$

$\Rightarrow\frac{x^2-1}{(x-1)\cdot(x+1)}$

As we can see, now we have two fractions equivalent to the first ones who have a common denominator.

Consider the algebraic fractions $\frac{2}{x^2+1}$ and $\frac{x}{x+2}$, find two equivalent algebraic fractions with common denominator.

1) In this case, the denominators are already factorized. Remember that we can have a polynomial of second degree whenever it does not have real roots.

x^2+1

$x+2$

2) Compute the least common multiple (l.c.m). of the polynomials in the denominators. In this case there are no common factors, so it is enough to calculate the product of both:

$lcm\{x^2+1,x+2\}=(x^2+1)\cdot(x+2)$

3) We divide the l.c.m. by every denominator and multiply it by the respective numerator. The result is the numerator of the algebraic fraction, while the denominator is the l.c.m.

$\frac{(x^2+1)\cdot(x+2)}{(x^2+1)}=x+2\Rightarrow 2\cdot(x+2)=2x+4\Rightarrow$

$\Rightarrow\frac{2x+4}{(x^2+1)\cdot(x+2)}$

$\frac{(x^2+1)\cdot(x+2)}{(x+2)}=x^2+1\Rightarrow x\cdot(x^2+1)=x^3+x\Rightarrow$

$\Rightarrow\frac{x^3+x}{(x^2+1)\cdot(x+2)}$

As we can see, now we have two fractions equivalent to the first ones which have a common denominator.