

An experiment can be modeled with a binomial distribution whenever :

1. There are only two possible events resulting from the experiment :  $A, \bar{A}$  (success and defeat).
2. The probability of every event  $A, \bar{A}$  are the same in any happening of the experiment ( $p$  and  $q = 1 - p$ , respectively). Namely if a coin is flipped several times, the probability of having 'heads' does not change.
3. Any realization of the experiment is independent from the rest.

A binomial random variable will give the number of successes when having happened a certain number of experiments.

It turns out to be useful to analyze the number of times that 'heads' is obtained when flipping a coin  $n$  times.

The binomial distribution is usually represented by  $B(n,p)$ , with :

1.  $n$  : number of happenings of the random experiment.
2.  $P$  : probability of success in doing an experiment.

So if we want to study the binomial distribution that models 10 flips of a coin (in which the 'heads' and 'tails' are equally probable) we have :

$$B(10, \frac{1}{2})$$

The probability function of the binomial distribution is :

$$p(X = k) = \binom{n}{k} p^k \cdot q^{n-k}$$

1.  $n$  : number of experiments
2.  $k$  : number of success
3.  $p$  : success probability
4.  $q$  : defeat probability

The combinatorial number is defined :

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Calculate the probability of obtaining 8 'heads' when flipping a coin ten times.

Distribution  $B(10, \frac{1}{2})$

Number of experiments :  $n = 10$

Number of successful result:  $k = 8$

Probability of each success and each defeat :  $p = q = \frac{1}{2}$

$$p(X = 8) = \binom{10}{8} \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 = 0.044$$

The average of a binomial distribution is :

$$\mu = n \cdot p$$

The variance is :

$$\sigma^2 = n \cdot p \cdot q = n \cdot p (1 - p)$$

The standard deviation is :

$$\sigma = \sqrt{n \cdot p \cdot q}$$