Definition and classification of polynomials

When we multiply a number (coefficient) for an unknown (variable) is a monomial. But what if we add instead of multiply?

$$x^6 + 10$$

$$x + 1$$

What happens when we add monomials that are similar? and if we subtract them?

When we join not similar monomials by adding or subtracting them we get a polynomial.

$$2x^2 + x - 1$$

that is the result of adding the monomials $2x^2$ and x, and subtracting the monomial 1.

Or

$$3x^5 - x^2 + x - 5$$

that is the result of adding the monomials $3x^5$ and x, and subtracting the monomials x^2 and 5.

In mathematics, to call polynomials we use one letter followed by a parenthesis with the variable (or variables, separated by commas). So the above examples would be:

p(x)=2x2+x−1 and q(x)=3x5−x2+x−5

If there is more than one variable, as we said:

$p(x) = 2x^2 + x - 1$ and $q(x) = 3x^5 - x^2 + x - 5$

If there is more than one variable, as we said:

$p(x, y) = x^6y + xy - x$

$q(x, y, z) = xyz^2 + xyz - xy^3z - zyz + zy - z$

$r(x, y, z, t) = xyzt$

Be careful in the way we represent polynomials because it is easy to make notation mistakes.

$q(x, y) = 3x^2y + 4x, q(x) = 3x^2y + 4x$

In the first polynomial, "y" acts as a variable. However, in the second, the "y" is a coefficient (which value is y, a number that we don't know a priori).

So they are two different polynomials (For example, the first one has degree 3 and the second one has degree 2).

Now, using as an example the polynomial p(x)=2x2+x−1, we define the following characteristics of a polynomial:

1. Variable/s of the polynomial: unknown or unknowns that we find in the polynomial. In the polynomial p(x),x.

2. Degree of the polynomial: the greatest exponent of all monomials which has the polynomial. In our example max{2,1,0}=2

3. Leading coefficient: the coefficient of the monomial which has the higher degree. In our case, 2.

4. Independent term: the coefficient of the monomial with exponent zero. If there is no such monomial then is equal to 0. In our case, it is −1.