From now on we are going to take the radians as units, instead of the sexagesimal degrees. To move from degrees to radians we only need to use the following relationship 180∘=π rad.

The trigonometric ratios of α, if $\frac{\pi}{2} < \alpha < \pi$

Let's suppose we want to calculate the trigonometrical ratios of an angle α with π2<α<π.

Then we have:

$$\begin{aligned} \sin\alpha &= \sin(\pi-\alpha) \\ \cos\alpha &= -\cos(\pi-\alpha) \\ \tan\alpha &= -\tan(\pi-\alpha) \end{aligned}$$

Therefore, from these equalities we already have defined the trigonometric ratios for angles 0<α<π, since π−α is an acute angle and, therefore, we can calculate its sine, cosine and tangent.

Now we can calculate the trigonometric ratios of the angle of 120 degrees that, in radians is $\frac{2}{3}\pi$, and therefore:

$$\begin{aligned} \sin\left(\frac{2}{3}\pi\right) &= \sin\left(\pi-\frac{2}{3}\pi\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \\ \cos\left(\frac{2}{3}\pi\right) &= -\cos\left(\pi-\frac{2}{3}\pi\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2} \\ tan\left(\frac{2}{3}\pi\right) &= -\tan\left(pi-\frac{2}{3}\pi\right) = -\tan\left(\frac{\pi}{3}\right) = \sqrt{3} \end{aligned}$$

The trigonometric ratios of α, if $\pi < \alpha < \frac{3\pi}{2}$

If $\pi < \alpha < \frac{3\pi}{2}$, we have:

$$\begin{aligned} \sin\alpha &= -\sin(\alpha-\pi) \\ \cos\alpha &= -cos(\alpha-\pi) \\ \tan\alpha &= tan(\alpha-\pi) \end{aligned}$$

Therefore, from these equalities and those of the previous point, we have defined the trigonometric ratios for the angles 0<α<3π2.

Now we can compute the trigonometric ratios of the angle of 225 degrees that, in radians is $\frac{5\pi}{4}$:

$$\begin{aligned} \sin\left(\frac{5}{4}\pi\right) &= -\sin\left(\frac{5}{4}\pi-\pi\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \\ \cos\left(\frac{5}{4}\pi\right) &= -\cos\left(\frac{5}{4}\pi-\pi\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \\ \tan\left(\frac{5}{4}\pi\right) &= \tan\left(\frac{5}{4}\pi-\pi\right) = \tan\left(\frac{\pi}{4}\right) = 1 \end{aligned}$$

The trigonometric ratios of α, if $\frac{3\pi}{2} < \alpha < 2\pi$

If $\frac{3\pi}{2} < \alpha < 2\pi$, we have:

$$\begin{aligned} \sin\alpha &= -\sin(2\pi-\alpha) \\ \cos\alpha &= cos(2\pi-\alpha) \\ \tan\alpha &= -tan(2\pi-\alpha) \end{aligned}$$

Therefore, from these equalities and all the previous ones, we can compute the trigonometric ratios for the angles 0<α<2π.

Now we can calculate the trigonometric ratio of the angle of 330 degrees that, in radians is $\frac{11\pi}{6}$ and, therefore:

$$\begin{aligned} \sin\frac{11\pi}{6} &= -\sin\left(2\pi-\frac{11\pi}{6}\right) = -\sin\left(\frac{pi}{6}\right) = -\frac{\sqrt{3}}{2} \\ \cos\frac{11\pi}{6} &= cos\left(2\pi-\frac{11\pi}{6}\right) = \cos\left(\frac{pi}{6}\right) = \frac{1}{2} \\ \tan\frac{11\pi}{6} &= -tan\left(2\pi-\frac{11\pi}{6}\right) = -\tan\left(\frac{pi}{6}\right) = \sqrt{3} \end{aligned}$$

Special angles

Now we define the trigonometrical ratios for the angles

Now we define the trigonometrical ratios for the angles of $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ and 2π rad.

$$\begin{aligned} \sin 0 &= \sin(2\pi) = 0 \\ \cos 0 &= \cos(2\pi) = 1 \\ \tan 0 &= \tan(2\pi) = 0 \end{aligned}$$

$$\begin{aligned} \sin\left(\tfrac{\pi}{2}\right) &= 1 \\ \cos\left(\tfrac{\pi}{2}\right) &= 0 \end{aligned}$$

$$\begin{aligned} \sin\pi &= 0 \\ \cos\pi &= -1 \\ \tan\pi &= 0 \end{aligned}$$

$$\begin{aligned} \sin\left(\tfrac{3\pi}{2}\right) &= -1 \\ \cos\left(\tfrac{3\pi}{2}\right) &= 0 \end{aligned}$$

It is necessary to notice that the tangent is not defined for the angles $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

Periodicity

We have defined the trigonometric ratios for any angle α with 0≤α≤2π. Let's extend this definition for every real α through this:

$$\begin{aligned} \sin\alpha &= \sin(\alpha+2\pi) \\ \cos\alpha &= \cos(\alpha+2\pi) \\ \tan\alpha &= \tan(\alpha+2\pi) \end{aligned}$$

bearing in mind that the tangent will not be defined at all the points that come from adding a multiple of 2π to π2 or 3π2.

For this reason we say that the trigonometric functions are periodic functions of period 2π.

For example, we can now find the value of the trigonometric ratios of $\alpha = \frac{13}{6}\pi$ since $\alpha = \frac{13}{6}\pi = 2\pi + \frac{\pi}{6}$, therefore:

$$\begin{aligned} \sin\frac{13}{6}\pi &= \sin\left(\frac{13}{6}\pi+2\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \\ \cos\frac{13}{6}\pi &= \cos\left(\frac{13}{6}\pi+2\pi\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \\ \tan\frac{13}{6}\pi &= \tan\left(\frac{13}{6}\pi+2\pi\right) = \tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3} \end{aligned}$$