

We start by posing a problem: The Sangakoo family has a rectangular plot of land. Only half of the plot is used, and the three fourths of the harvest is corn. We want to know what part of the entire surface of the plot is cultivated by corn.

Let's consider a rectangular plot:



Only half of the plot is used:



And on this half, 35 are corn (it is a blue corn!)



Altogether, the area dedicated to cultivating corn represents 310 of the whole plot.

The fraction 310 is the result of multiplying 12 by 35.

The product of two fractions is another fraction which numerator is the product of the numerators of the given fractions, and its denominator is the product of its denominators.

Let's see an example:

The product $\frac{1}{2} \cdot \frac{3}{5}$ is calculated by multiplying the numerators and the denominators by themselves:

$$\frac{1}{2} \cdot \frac{3}{5} = \frac{1 \cdot 3}{2 \cdot 5} = \frac{3}{10}$$

This can be written according to the following formula: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

The multiplication of an integer by a fraction, as in this example:

$$7 \cdot \frac{3}{4}$$

is done in the same way.

$$7 \cdot \frac{3}{4} = \frac{7}{1} \cdot \frac{3}{4} = \frac{7 \cdot 3}{1 \cdot 4} = \frac{21}{4}$$

When the same number is multiplying the numerator and the denominator of the operation, it is possible to eliminate it from both positions since the actions of multiplying and dividing by the same number are annulled mutually.

$$\frac{1}{2} \cdot \frac{2}{5} = \frac{1 \cdot 2}{2 \cdot 5} = \frac{1}{5}$$

Properties of the product of fractions

The multiplication of fractions has the following properties:

1. Commutative property: if $\frac{a}{b}$ and $\frac{c}{d}$ are any two fractions, it is satisfied that:
- $$\frac{a}{b} \cdot \frac{c}{d} = \frac{c}{d} \cdot \frac{a}{b}$$
- Namely, changing the order of the factors, the result is not modified.
2. Associative property: if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{n}{m}$ are any three fractions, it is satisfied that:
- $$\left(\frac{a}{b} \cdot \frac{c}{d}\right) \cdot \frac{n}{m} = \frac{a}{b} \cdot \left(\frac{c}{d} \cdot \frac{n}{m}\right)$$
- Namely, calculating the product of three or more fractions, we can group them as we want: the result will always be the same.
3. Neutral element: the integer 1 is the neutral element of the multiplication of fractions since it is a fraction: $1 = \frac{1}{1} = \frac{a}{a}$, for any integer $a \neq 0$, and it is satisfied that:
- $$\frac{a}{b} \cdot 1 = 1 \cdot \frac{a}{b} = \frac{a}{b}$$
4. Inverse element: for any fraction different from zero, there exists another fraction which product is the unit.