

The discriminant of a quadratic equation  $ax^2+bx+c=0$  is a number, indicated with the letter D (in some texts the Greek letter  $\Delta$  is used) whose value is calculated as follows:  $D=b^2-4ac$

$$x^2 + 3x - 10 = 0 \rightarrow D = 3^2 - 4 \cdot 1 \cdot (-10) = 9 + 40 = 49$$
$$x^2 + 2x + 5 = 0 \rightarrow D = 2^2 - 4 \cdot 1 \cdot 5 = 4 - 20 = -16$$
$$x^2 - 16 = 0 \rightarrow D = -4 \cdot 1 \cdot (-16) = 64$$

So the discriminant is the expression underneath the square root in the general solution of the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

When the discriminant is zero, the equation will have just one solution (it is also said that the equation has a double solution).

If it is less than zero, since there are not square roots of negative numbers, the equation will have no solutions.

- $D>0$  two solutions
- $D=0$  one solution
- $D<0$  no solutions in  $\mathbb{R}$

In the previous examples we can say, with no need to solve the equations, that:

1.  $x^2 + 3x - 10 = 0$  has two solutions, since  $D = 49 > 0$
2.  $x^2 + 2x + 5 = 0$  has no solutions, since  $D = -16 < 0$
3.  $x^2 - 4x + 4 = 0$  has one solution, since  $D = 0$