Definition, analytical expression and properties of scalar product

The scalar product between two vectors u⟶ and v⟶, that is represented by u⟶·v⟶, is a real number that is obtained by multiplying the magnitude of u⟶ by the magnitude of v⟶ and by the cosine of the angle that is formed

From the definition of the scalar product we have:

1. If $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$, then $\vec{u} \cdot \vec{v} = 0$.
2. If $\vec{u}$ and $\vec{v}$ are perpendicular vectors and since $\cos(\widehat{\vec{u}\vec{v}}) = \cos(90^\circ) = 0$, we have $\vec{u} \cdot \vec{v} = 0$.

If $\vec{u} = (0,2)$, $\vec{v} = (3,3)$ and $\widehat{\vec{u}\vec{v}} = 45^\circ$:

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos(45^\circ) = 2 \cdot \sqrt{18}\frac{\sqrt{2}}{2} = \sqrt{36} = 6$

If $|\vec{u}| = 3$, $|\vec{v}| = 2$ and $\vec{u} \cdot \vec{v} = 0$. What angle is formed by $\vec{u}$ and $\vec{v}$?

Since the formula of the scalar product is $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos(\widehat{\vec{u}\vec{v}})$, by replacing the information that we have, we will obtain:

$$\cos(\widehat{\vec{u}\vec{v}}) = 0 \Rightarrow \widehat{\vec{u}\vec{v}} = 90^\circ$$

These two vectors are perpendicular.

Analytical expression of the scalar product:

Given $\vec{u} = (u_1, u_2)$ and $\vec{v} = (v_1, v_2)$, its scalar product can be written as:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$$

If $\vec{u} = (3,1)$ and $\vec{v} = (2,-1)$, then:

$$\vec{u} \cdot \vec{v} = 3 \cdot 2 + 1 \cdot (-1) = 6 - 1 = 5$$