

Product of matrices

In a strict sense, the rule to multiply matrices is:

"The matrix product of two matrixes A and B is a matrix C whose elements a_{ij} are formed by the sums of the products of the elements of the row i of the matrix A by those of the column j of the matrix B ."

Actually, this is not a very encouraging statement, but in the end it is simple and only requires a little practice, since it is a question of multiplying the rows of the first matrix by the columns of the second one.

Let's start with a simple example.

$$(5 \quad 2 \quad 1) \cdot \begin{pmatrix} 4 \\ -3 \\ 6 \end{pmatrix} = 5 \cdot 4 + 2 \cdot (-3) + 1 \cdot 6 = 20 - 6 + 6 = 20$$

That is, we have to multiply the first element of the row in the first matrix by the first element from the column in the second matrix, then add up this product and the product of the second element in the row by the second element in the column, and finally add up the product of the third element in the row by the third element in the column.

This is easier done than said!

$$(2 \quad 3 \quad 5) \cdot \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = 2 \cdot 3 + 3 \cdot 2 + 5 \cdot 4 = 6 + 6 + 20 = 32$$

Let's see a product of two square matrices 2×2

$$\begin{pmatrix} 1 & 5 \\ 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 1 & 6 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 + 5 \cdot 1 & 1 \cdot 4 + 5 \cdot 6 \\ 2 \cdot 3 + 2 \cdot 1 & 2 \cdot 4 + 2 \cdot 6 \end{pmatrix} = \begin{pmatrix} 8 & 34 \\ 8 & 20 \end{pmatrix}$$