Root and factorization of a polynomial

Concept of root

The root or zero of a polynomial p(x) is that value a that

$$p(a)=0$$

Mathematicians, throughout history, have always been fascinated by finding the roots of any polynomial. In general, this is a very complicated problem.

So, using the remainder theorem and the factor theorem, we can deduce some properties of the roots of a polynomial:

1) The roots of a polynomial are divisors of the independent term. If it does not have an independent term, it means that it is divisible by x−a, where a=0, this is, it is divisible by x.

$p(x)=x^5+2x^4-3x^3+x^2-1$ has as a root 1,

$$p(1)=1^5+2\cdot 1^4-3\cdot 1^3+1^2-1=0$$

and 1 divides the independent term -1.

The polynomial $p(x)=2x^5+5x^4+4x^3-x^2+x$ has the independent term equal to 0.

Then, using the factor theorem, 0 is a root of $p(x)$ and therefore $x-0=x$ divides the polynomial $p(x)$ exactly.

2) Being ai the i roots of a polynomial, we can express this polynomial as product of polynomials like x−ai.

The polynomial $p(x)=x^2-3x+2$ has roots $x=2$ and $x=1$. Therefore, it can be expressed as

$$p(x)=(x-2)\cdot(x-1)$$

The polynomial $p(x)=x^2+5x+6$ has root $x=-2$ and $x=-3$. Therefore, it can be expressed as

$$p(x)=(x+2)\cdot(x+3)$$

3) A polynomial is called irreducible or prime if it does not have any rational number that is a root.

The polynomials $p(x)=x^2+x+1$ and $q(x)=x^2+1$ do not have any root in the rational numbers.