

Imagine you have the two fractions $\frac{-3}{4}$ and $\frac{6}{-8}$ and we apply it to an integer, 32 for instance, and we get the same result:

$$(32 : 4) \cdot (-3) = 8 \cdot (-3) = -24$$

$$(32 : (-8)) \cdot 6 = -4 \cdot 6 = -24$$

In this case, we can say that $\frac{-3}{4}$ and $\frac{6}{-8}$ are equivalent fractions.

In this case, we can say that $-3/4$ and $6/-8$ are equivalent fractions.

In this example, we can check that if we multiply the numerator of the first fraction by the denominator of the second, the result is the same as if we multiply the denominator of the first fraction by the second numerator:

$$-3 \cdot (-8) = 4 \cdot 6$$

In general, we say that two fractions a/b and c/d , with $b \neq 0$ and $d \neq 0$, are equivalent if: $a \cdot d = b \cdot c$

To express it we write down: $a/b \sim c/d$ or $a/b = c/d$.

The relation "be equivalent to" has the following properties:

Reflexive property

Every fraction is equivalent to itself, $a/b = a/b$ because $a \cdot b = a \cdot b$.

Symmetric property

If a/b is equivalent to c/d , then the fraction c/d is equivalent to a/b .

If $a/b = c/d$ means that $a \cdot d = b \cdot c$, then $c \cdot b = d \cdot a$ that means $c/d = a/b$.

Transitive property

If one fraction is equivalent to another, and this last one is equivalent to a third fraction, then the first one is equivalent to the third one:

$$a/b = c/d \quad c/d = n/m \quad \} \text{ then } a/b = n/m$$

The equivalence between fractions is an equivalence relation that classifies the fractions into classes of equivalent fractions.

A class of equivalent fractions is a set of fractions where all of them are equivalent, and any other fraction which isn't into the set is not equivalent to any of them.

Every class of equivalence is a rational number.

Obtain equivalent fractions

Consider the fraction a/b and an integer m nonzero.

If we multiply the numerator and denominator of the fraction a/b by m the result is: $a \cdot m / b \cdot m$

This new fraction is equivalent to a/b , ie: $a \cdot m / b \cdot m = a/b$ due to: $(a \cdot m) \cdot b = a \cdot (b \cdot m)$

A very important case of equivalent fractions appears when we have negative denominators, because if we want to interpret the fraction as an integer division, having a negative divisor makes it very difficult, so if we multiply numerator and denominator by -1 then we obtain an equivalent fraction with positive denominator, and therefore much easier to operate.

To simplify a fraction means to divide numerator and denominator by the same integer. Those fractions that can not be simplified are called irreducible.

Formally, we say that a fraction a/b is irreducible if the numerator and denominator are coprime, ie, $m.c.d(a,b)=1$.