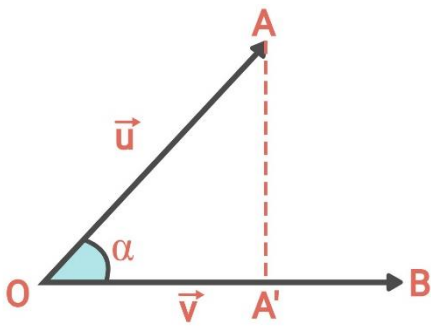
Geometric interpretation of the scalar product

The product of two non zero vectors is equal to the magnitude of one of them times the projection of the other onto it.



In the picture, OA' is the projection of the vector $\vec{u}$ on $\vec{v}$. If we observe the OAA' triangle and apply the cosinus definition, we have:

$$\cos(\alpha) = \frac{OA'}{|\vec{u}|} \Rightarrow |OA'| = |\vec{u}| \cdot \cos(\alpha)$$

Finally, applying to the scalar product formula what we just have found:

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos(\alpha) = |\vec{u}| \text{proj}_{\vec{v}}(\vec{u})$$

Find the projection of the vector $\vec{u} = (2, 3)$ on $\vec{v} = (-1, 4)$.

$$\text{proj}_{\vec{v}}(\vec{u}) = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \frac{2 \cdot (-1) + 3 \cdot 4}{\sqrt{(-1)^2 + 4^2}} = \frac{10}{\sqrt{17}}$$