Ruffini's rule

To calculate the quotient of two polynomials the procedure used needs many intermediate calculations. A rule that can help us to simplify them is Ruffini's rule. This rule will only be valid when the divisor is a polynomial, such as $x-a$, with a being a real number.

We will use an example to explain the methodology:

Do the division $\dfrac{p(x)}{q(x)}$, where $p(x)=x^4-3x^2+x+5$ and $q(x)=x+2$.

1) Complete and arrange the dividend polynomial.

Write the dividing polynomial as $x-a$, if necessary.

In our case:

$p(x)=x^4+0x^3-3x^2+x+5$

$q(x)=x-(-2)$

Notice that in this example the value of $a=-2$.

2) We write down the elements in a table like the following one.

	1	0	-3	1	5
-2					

In the top row, we write the coefficients of the polynomial (arranged and completed!) $p(x)$.

In the left cell, we write the value of a.

3) We put the first coefficient in, and multiply it by the value of a. The result of which we write just under the second coefficient:

	1	0	-3	1	5
-2		$1\cdot(-2)=-2$			
	1				

4) We add up the second column and put the obtained result in, repeating the process until the last column:

	1	0	-3	1	5
-2		$1\cdot(-2)=-2$	$(-2)\cdot(-2)=4$	$1\cdot(-2)=-2$	$(-1)\cdot(-2)=2$
	1	$0+(-2)=-2$	$(-3)+4=1$	$1+(-2)=-1$	$5+2=7$

5) The digit on the bottom-right corner is the remainder. The other digits of the last row are the coefficients, arranged, for the polynomial quotient.

And so, in our case:

quotient: x^3-2x^2+x-1

remainder: 7

As we can see, the relation od degrees is satisfied:

$3=$degree$(x^3-2x^2+x-1)=$degree$(x^4-3x^2+x+5)-$degree$(x+2)=4-1=3$

degree$(7)=0<1=$degree$(x+2)$

Do the division $\dfrac{p(x)}{q(x)}$, where $p(x)=x^5+2x^4-3x^3+x^2-1$ and $q(x)=x-1$.

1) $p(x)=x^5+2x^4-3x^3+x^2+0x-1$

$q(x)=x-1$

$a=1$.

2)

	1	2	-3	1	0	-1
1						

3)

	1	2	-3	1	0	-1
1		1				
	1	3				

4)

	1	2	-3	1	0	-1
1		1	3	0	1	1
	1	3	0	1	1	0

5)

quotient: x^4+3x^3+x+1

remainder: 0

And it is satisfied that:

$4=$degree$(x^4+3x^3+x+1)=$degree$(x^5+2x^4-3x^3+x^2-1)-$

$-$degree$(x-1)=5-1=4$

degree$(0)=0<1=$degree$(x-1)$