The following expression is a linear equation (it has no exponents) with two unknowns x and y: x+y=1.

To find the solution we need another equation that is not equivalent, so that there we have a system of two linear equations with two unknowns, as for example:

$$\left.\begin{array}{r} x+y=1 \\ 2x-y=5 \end{array}\right\}$$

When we link two equations that are equivalent, like the following:

$$\left.\begin{array}{r} x+y=1 \\ 2x+2y=2 \end{array}\right\}$$

we have an indeterminate system, something we will not tackle here.

There are at least three methods to solve systems of linear equations. We will call them substitution, equalization and reduction.

The substitution method consists in clearing x in one of the equations - basically in the one that turns out to be easier - and replacing it, or substituting it, in the other one.

In the following case, it is easy to clear x in the first equation, since it does not have any coefficient:

$$\left.\begin{array}{r} x+y=2 \\ -2x-3y=5 \end{array}\right\}$$

So that:

$$\left.\begin{array}{r} x=2-y \\ -2x-3y=5 \end{array}\right\}$$

Now it is possible to replace x into the second equation with the expression $x=2-y$. So we have:

$$\left.\begin{array}{r} x=2-y \\ -2(2-y)-3y=5 \end{array}\right\}$$

By doing this we obtain an equation with only one unknown.

$$-2(2-y)-3y=5 \Rightarrow -4+2y-3y=5 \Rightarrow -y=5+4 \Rightarrow -y=9 \Rightarrow$$
$$y=-9$$

Once we have the value y, we can put it back into the first equation. That is, the equation where we have x in terms of y. We then compute the value of x:

$$x=2-y \Rightarrow x=2-(-9) \Rightarrow x=2+9=11$$

Note that we obtain two values $x=11, y=-9$, which are the solution to the system.

In order to see if the result is correct we can plug these values into both equations and see if the identities are satisfied or not. Let's verify the first equation:

$$x+y=2 \Rightarrow 11-9=2 \Rightarrow 2=2$$

So, the first equation is satisfied. Let's verify the second one:

$$-2x-3y=5 \Rightarrow -2 \cdot 11-3 \cdot (-9)=5 \Rightarrow -22+27=5 \Rightarrow 5=5$$

We observe that this second equation is also satisfied, so the solution is correct.

$$\left.\begin{array}{r} 2x-4y=8 \\ -3x+y=3 \end{array}\right\}$$

We can follow the same steps as in the previous example, but first it is necessary to see if the equations can be simplified.

In the case of the first equation $2x-4y=8$, all the terms are divisible by 2, so that the whole equation can be divided by 2 to simplify it. We then obtain:

$$x-2y=4$$

This equation is completely equivalent to the previous one, that is, it has the same solution (when combined with the second equation in our system), and the clearing of x is almost immediate since it has no coefficient.

This new equivalent equation is replaced - instead of the first one - in our system and we can then look for a solution to the following system:

$$\left.\begin{array}{r} x-2y=4 \\ -3x+y=3 \end{array}\right\}$$

First, we obtain x as a function of y using the first equation:

$$\left.\begin{array}{r} x=4+2y \\ -3x+y=3 \end{array}\right\}$$

Then we replace x in the second equation and we solve for y:

$$-3(4+2y)+y=3 \Rightarrow -12-6y+y=3 \Rightarrow -5y=3+12 \Rightarrow$$

$$\Rightarrow -5y=15 \Rightarrow y=\frac{15}{-5}=-3$$

We finally use the value for y that we obtained to back up the value of x:

$$x=4+2(-3) \Rightarrow x=4-6=-2$$

The solution to the system is $x=-2, y=-3$.

We can now verify that the solution we found is in fact correct by putting the values for x and y into the original system of equations.

$$x-2y=4 \Rightarrow -2-2(-3)=4 \Rightarrow -2+6=4 \Rightarrow 4=4$$

$$-3x+y=3 \Rightarrow -3(-2)+(-3)=3 \Rightarrow 6-3=3 \Rightarrow 3=3$$

In both case the identities are satisfied, so the result is correct.

$$\left.\begin{array}{r} x+1-y=-2 \\ y+1=x-4 \end{array}\right\}$$

The first thing that we have to do is to put all the variables on one side (the left hand side) and all the numbers on the other side:

$$\left.\begin{array}{r} x-y=-2-1 \\ -x+y=-4-1 \end{array}\right\} \Rightarrow \left.\begin{array}{r} x-y=-3 \\ -x+y=-5 \end{array}\right\}$$

We can then obtain an expression of x in terms of y: $x=y-3$. This equation is then used in the second equation to obtain a single equation with a single unknown. We can, thus, solve for y:

$$-(y-3)+y=-5 \Rightarrow -y+3+y=-5 \Rightarrow -y+y=-5-3 \Rightarrow 0=-8$$

Note that in this case there seems to be no solution and in fact there is not.

The problem we have encountered is that this is an incompatible system: the unknowns are cleared and the system fails to have a solution. On the other hand, when a system has a solution it is called a compatible system.

Finally, let's summarize the steps we need to solve a system with the substitution method:

1. Express one of the variables in terms of the other one using one of the equations and replace it into the other equation so as to obtain one linear equation with one unknown.
2. Solve the system of that equation and plug the solution into the first equation in order to obtain the value of the other variable.

When it comes to create systems of equations, specially if we want the solutions to be integers, we better start from unknowns of known values and then lay out equetions where equalities are true.

For example, if $x = 1, y = -1$, any of the following equations is satisfied when replacing x and y by 1 and -1:

$$x + y = 0$$

$$2x - 2y = 4$$

$$x + 3(2y - 1) = -8$$

So a possible system could be done by taking the first two equations:

$$\left.\begin{array}{r} x + y = 0 \\ 2x - 2y = 4 \end{array}\right\}$$