Simplification of algebraic fractions

Consider an algebraic fraction, if the numerator and the denominator have some factor in common it can be simplified. The result can be an equivalent algebraic fraction, or a polynomial.

Simplify the following algebraic fraction and conclude if it is a polynomial or an algebraic fraction.

$$\frac{x^2-1}{x^2-2x+1}$$

We factorize the numerator and the denominator:

$x^2-1=(x+1)\cdot(x-1)$

$x^2-2x+1=(x-1)^2$

We see that the numerator and the denominator have a common factor, therefore:

$$\frac{x^2-1}{x^2-2x+1}=\frac{(x+1)(x-1)}{(x-1)^2}=\frac{x+1}{x-1}$$

And the result is an algebraic fraction.

Simplify the following algebraic fraction and conclude if it is a polynomial or an algebraic fraction.

$$\frac{x^2-4x+4}{x-2}$$

We factorize the numerator and the denominator:

$x^2-4x+4=(x-2)^2$

$x-2$

We see that the numerator and the denominator have a common factor, therefore:

$$\frac{x^2-4x+4}{x-2}=\frac{(x-2)^2}{x-2}=x-2$$

And the result is a polynomial.

Expansion of algebraic fractions

As in a fraction, we can always multiply the numerator and the denominator by any polynomial. This strategy is called an enlargement or expansion and it can be useful on some occasions.

Expand the following algebraic fraction $\frac{x-1}{x+2}$ in such a way that it has a polynomial with a root at $x=-1$ in the denominator.

It is enough to multiply the algebraic fraction by the expression $x+1$ both in the numerator and in the denominator:

$$\frac{x-1}{x+2}=\frac{x-1}{x+2}\cdot\frac{x+1}{x+1}$$

Now, we can expand the polynomials:

$$\frac{x-1}{x+2}\cdot\frac{x+1}{x+1}=\frac{(x-1)(x+1)}{(x+2)(x+1)}=\frac{x^2-1}{x\cdot(x+1)+2\cdot(x+1)}=$$

$$=\frac{x^2-1}{x^2+3x+2}$$

The result is an algebraic fraction equivalent to the initial.

Expand the following algebraic fraction $\frac{x+2}{x-2}$ in such a way that it has a polynomial with a root at $x=3$ in the denominator.

It is enough to multiply the algebraic fraction by the expression $x-3$ both in the numerator and in the denominator:

$$\frac{x+2}{x-2}=\frac{x+2}{x-2}\cdot\frac{x-3}{x-3}$$

Now, we can expand the polynomials:

$$\frac{x+2}{x-2}\cdot\frac{x-3}{x-3}=\frac{(x+2)(x-3)}{(x-2)(x-3)}=\frac{x\cdot(x-3)+2\cdot(x-3)}{x\cdot(x-3)-2\cdot(x-3)}=$$

$$=\frac{x^2-x-6}{x^2-5x+6}$$

The result is an algebraic fraction equivalent to the initial.