Now let’s see how to compute the determinant of a 3×3 matrix. Sarrus' rule is useful for third-order determinants only.

We have our determinant of any 3×3 matrix, for instance:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

We rewrite the first two rows while occupying hypothetical fourth and fifth rows, respectively:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$\begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix}$$

Once this is done the calculation of the determinant is computed as follows:

1. Multiply the diagonal elements.
2. The descending diagonal from left to right has a sign +, while the descending diagonal from right to left has the sign −.

$$\begin{matrix} \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \\ \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} \end{matrix} = 1\cdot5\cdot9+4\cdot8\cdot3+7\cdot2\cdot6-3\cdot5\cdot7-6\cdot8\cdot1-9\cdot2\cdot4=0$$

Now take a look at the following example

$$\begin{vmatrix} 9 & 1 & 5 \\ 3 & 4 & 7 \\ 8 & 2 & 0 \end{vmatrix} \rightarrow \begin{matrix} \begin{vmatrix} 9 & 1 & 5 \\ 3 & 4 & 7 \\ 8 & 2 & 0 \end{vmatrix} \\ \begin{matrix} 9 & 1 & 5 \\ 3 & 4 & 7 \end{matrix} \end{matrix} = 9\cdot4\cdot0+3\cdot2\cdot5+8\cdot1\cdot7-5\cdot4\cdot8-7\cdot2\cdot9-0\cdot1\cdot3=$$

$$=86-286=-200$$

As you can see the method is very simple, although the number of operations to perform is high, as is the possibility of error in the calculations.

There are certain properties that speed up the calculations, although it is also habitual to use powerful calculators to compute determinants.