## Linear Combination of vectors

Given two vectors $\vec{u}$ and $\vec{v}$ we name linear combination of $\vec{u}$ and $\vec{v}$ to any expression of the form: $\lambda\vec{u}+\mu\vec{v}$ where λ and μ are real numbers.

A vector $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$ if real (scalar) numbers (escalars) λ and μ exist such that we can express $\vec{w}$ as follows: $\vec{w}=\lambda\vec{u}+\mu\vec{v}$.

The vectors we have been working with until now are vectors on the plane, so they have two components. In this case we can express any vector $\vec{w}$ as a linear combination of two non parallel vectors $\vec{u}$ and $\vec{v}$. This combination is unique.

Is the vector $\vec{w}=(-1,3)$ a linear combination of the vectors of $\vec{u}=(1,2)$ and $\vec{v}=(0,3)$?

We want to find $\lambda$ and $\mu$ so as $\vec{w}=\lambda\vec{u}+\mu\vec{v}$. We have:

$$(-1,3)=\lambda(1,2)+\mu(0,3)=(\lambda,2\lambda)+(0,3\mu)=(\lambda,2\lambda+3\mu)$$

Therefore:

$$\left.\begin{array}{rcl}-1 & = & \lambda \\ 3 & = & 2\lambda+3\mu\end{array}\right\} \Rightarrow \lambda=-1,\ \mu=\frac{5}{3}$$

We have just found values for $\lambda$ and $\mu$ for which $\vec{w}=\lambda\vec{u}+\mu\vec{v}$ is true. So the answer is "yes", we can express $\vec{w}=(-1,3)$ as a linear combination of $\vec{u}=(1,2)$ and $\vec{v}=(0,3)$.