

Sum and subtraction

In order to compute the sum or subtraction of algebraic fractions, first we have to convert the fractions to a common denominator, and then we can compute the sum or subtraction as if it was a fraction.

Once the denominator is computed, the numerator is just the sum or subtraction of the different terms.

Compute the sum of the following algebraic fractions $\frac{x-1}{x+4}$ and $\frac{x^2+2}{x+4}$

In this case, both fractions have the same denominator, and we can then compute straight away:

$$\frac{x-1}{x+4} + \frac{x^2+2}{x+4} = \frac{x-1+(x^2+2)}{x+4} = \frac{x^2+x+1}{x+4}$$

Compute the subtraction of the following algebraic fractions $\frac{x^2+1}{x-2}$ and $\frac{x+1}{x-1}$

First, we have to convert the algebraic fractions into fractions with common denominator:

$$lcm\{x-2, x-1\} = (x-2) \cdot (x-1)$$

$$\frac{(x-2) \cdot (x-1)}{(x-2)} = x-1 \Rightarrow (x-1) \cdot (x^2+1) = x \cdot (x^2+1) - 1 \cdot (x^2+1) =$$

$$= x^3 - x^2 + x - 1 \Rightarrow \frac{x^3 - x^2 + x - 1}{(x-2) \cdot (x-1)}$$

$$\frac{(x-2) \cdot (x-1)}{(x-1)} = x-2 \Rightarrow (x-1) \cdot (x+1) = x^2 - 1 \Rightarrow \frac{x^2 - 1}{(x-2) \cdot (x-1)}$$

Now we compute:

$$\frac{x^3 - x^2 + x - 1}{(x-2)(x-1)} + \frac{x^2 - 1}{(x-2)(x-1)} = \frac{x^3 - x^2 + x - 1 + (x^2 - 1)}{(x-2)(x-1)} =$$

$$= \frac{x^3 + x - 2}{(x-2)(x-1)}$$

Compute the subtraction of the following algebraic fractions $\frac{x-2}{x+3}$ and $\frac{x-1}{(x+1)^2}$

First, we have to convert the algebraic fractions into fractions with common denominator:

$$lcm\{x+3, (x+1)^2\} = (x+3) \cdot (x+1)^2$$

$$\frac{(x+3) \cdot (x+1)^2}{x+3} = (x+1)^2 \Rightarrow (x-2) \cdot (x+1)^2 = x \cdot (x+1)^2 + 1 \cdot (x+1)^2 =$$

$$= x \cdot (x^2 + 2x + 1) + 1 \cdot (x^2 + 2x + 1) = x^3 + 3x^2 + 3x + 1 \Rightarrow$$

$$\Rightarrow \frac{x^3 + 3x^2 + 3x + 1}{(x+3) \cdot (x+1)^2}$$

$$\frac{(x+3) \cdot (x+1)^2}{(x+1)^2} = x+3 \Rightarrow (x-1) \cdot (x+3) = x^2 + 2x - 3 \Rightarrow$$

$$\Rightarrow \frac{x^2 + 2x - 3}{(x+3) \cdot (x+1)^2}$$

Now we compute:

$$\frac{x^3 + 3x^2 + 3x + 1}{(x+3)(x+1)^2} - \frac{x^2 + 2x - 3}{(x+3)(x+1)^2} = \frac{x^3 + 3x^2 + 3x + 1 - (x^2 + 2x - 3)}{(x+3)(x+1)^2} =$$

$$= \frac{x^3 + 2x^2 - x + 4}{(x+3)(x+1)^2}$$