Basis and coordinates

A vector space is a mathematical structure formed by a set of vectors, which can be added up and multiplied by a scalar. Will work on vector spaces, and we will operate with vectors and will define the concept of basis.

On the plane, two vectors u→ and v→ form a basis if they are linearly independent, since any vector w→ can be expressed as a linear combination of these two vectors.

The basis formed by u→ and v→ is represented like B={u→,v→}.

Given any basis B={u→,v→},w→=λu→+μv→

This expression is unique, or, in other words, λ and μ are uniquely determined.

The coordinates of w→ in the basis B are λ and μ. We can say that w→=(λ,μ) in the base B.

From the infinite number of basis that we can find among the vectors of the plane there is one that is especially simple: it is the one that is formed by two vectors i→ and j→ perpendicular to each other and with module 1. This basis is named the canonical basis of the plane.

Remember that two vectors are perpendicular when they form an angle of 90∘.

1. The vector $\vec{v} = (2,3)$ expressed in the canonical basis $B = \{\vec{\imath}, \vec{\jmath}\}$ is $\vec{v} = 2\vec{\imath} + 3\vec{\jmath}$.
2. Do the following vectors form a basis in the plane?
    1. $\vec{u} = (1,1), \vec{v} = (-3,-3)$. Com que $\frac{1}{-3} = \frac{1}{-3}$ Therefore, they are l.d. (linearly dependent) vectors, so they cannot form a base.
    2. $\vec{u} = (-1,2), \vec{v} = (2,3)$. Com que $\frac{-1}{2} \neq \frac{2}{3}$ They are l.i. (linearly independent) vectors, therefore they form a basis in the plane.
    3. $\vec{u} = (4,2), \vec{v} = (2,1)$. Com que $2 = \frac{4}{2} = \frac{2}{1} = 2$ Therefore, they are l.d. (linearly dependent) vectors, so they cannot form a base.
3. To express the vector $\vec{w} = (4,5)$ in the basis $B = \{\vec{u}, \vec{v}\}$ where $\vec{u} = (1,1)$ and $\vec{v} = (2,3)$.

   We want to find λ and μ such that:

   $$(4,5) = \lambda(1,1) + \mu(2,3) = (\lambda,\lambda) + (2\mu,3\mu) = (\lambda+2\mu, \lambda+3\mu)$$

   therefore,

   $$\left.\begin{array}{ll} 4 = \lambda + 2\mu & (a) \\ 5 = \lambda + 3\mu & (b) \end{array}\right\} \Rightarrow \text{ subtracting } (a)-(b) \Rightarrow 1 = \mu \Rightarrow \lambda = 4 - 2\mu = 4 - 2 = 2$$

The vector $\vec{w} = (4,5)$ will be $(2,1)$ in the basis $B = \{\vec{u}, \vec{v}\}$.