There are some fractions whose denominator is less than the numerator, we call them improper, in comparison with the proper fractions, whose numerator is less than the denominator. Any proper fraction is less than the unit, while the improper fractions are bigger.

The problem that appears with the improper fractions is that sometimes, on having operated with them, there appear numbers that are very big and difficult to deal with.

If we want to add up the fractions $\frac{113}{17}$ and $\frac{94}{9}$:

$$\frac{113}{17}+\frac{94}{9}=\frac{113\cdot 9}{17\cdot 9}+\frac{94\cdot 17}{9\cdot 17}=\frac{1017}{153}+\frac{1598}{153}=\frac{2615}{153}$$

Then we have to simplify the fraction or verify whether it is possible or not. With the numerator and the denominator that we have found, it is difficult to see that:

$$153=17\cdot 3^2$$
$$2615=5\cdot 523$$

And therefore mcd (153,2615)=1, and consequently the fraction cannot be simplified.

To facilitate these operations, we define for every improper fraction its mixed number that is formed by an integer part and a proper fraction. Given an improper fraction, its mixed number is obtained doing making the integer division, or using the remainder, associated with the fraction. If we have the improper fraction Dd, we do the division of the dividend D by the divisor d, and we obtain a quotient q and a remainder r, then the mixed number associated to Dd is:qrd

In the previous fractions, we have:

$$\begin{array}{r|l} 113 & \underline{17} \\ 11 & 6 \end{array}$$

So the associated mixed number is

$$6\frac{11}{17}$$

$$\begin{array}{r|l} 94 & \underline{9} \\ 4 & 10 \end{array}$$

therefore, we have as a compound associated to

$$10\frac{4}{9}$$

The implicit operation that there is between the integer and the fraction of a mixed number is the sum. In this way, to pass from a mixed number to the corresponding improper fraction, it is just necessary to do the above mentioned sum. If we have the mixed number: qrd We will have:qrd=q+rd=q·dd+rd=q·d+rd=Dd

$$q\frac{r}{d}=q+\frac{r}{d}=q\cdot\frac{d}{d}+\frac{r}{d}=\frac{q\cdot d+r}{d}=\frac{D}{d}$$

If we start with the mixed number $6\frac{11}{17}$, we will have:

$$6\frac{11}{17}=6+\frac{11}{17}=6\cdot\frac{17}{17}+\frac{11}{17}=\frac{102}{17}+\frac{11}{17}=\frac{113}{17}$$

That is, to pass from an improper fraction to a mixed number, it is necessary to do the integer division, while to pass from a mixed number to a fraction, it is necessary to do the sum.

The main application of the mixed numbers is to simplify the fractions when there are sums and subtractions. The sum or subtraction of two mixed numbers is a mixed number whose integer part is the sum of integer parts, and whose fraction is equal to the sum of fractions. In the event that the resultant fraction is not proper, it is passed to mixed and its integer part is added to the sum of integer parts. Namely, to add or subtract the mixed numbers abc and dmn, we will do:abc+dmn=a+bc+d+mn=(a+d)+(bc+mn)abc−dmn=a+bc−(d+mn)=(a−d)+(bc−mn)

$$a\frac{b}{c}+d\frac{m}{n}=a+\frac{b}{c}+d+\frac{m}{n}=(a+d)+\left(\frac{b}{c}+\frac{m}{n}\right)$$
$$a\frac{b}{c}-d\frac{m}{n}=a+\frac{b}{c}-\left(d+\frac{m}{n}\right)=(a-d)+\left(\frac{b}{c}-\frac{m}{n}\right)$$

In the previous example, we wanted to add the fractions $\frac{113}{17}$ and $\frac{94}{9}$. We have seen that its associated mixed numbers were $6\frac{11}{17}$ and $10\frac{4}{9}$ respectively. To do the sum:

$$6\frac{11}{17}+10\frac{4}{9}=6+10+\left(\frac{11}{17}+\frac{4}{9}\right)=16+\left(\frac{99+68}{153}\right)=$$
$$=16+\frac{167}{153}=16\frac{167}{153}=16+1+\frac{14}{153}=17\frac{14}{153}$$