The rank of a matrix can also be calculated using determinants. We can define rank using what interests us now.

The rank of a matrix is the order of the largest non-zero square submatrix.

See the following example.

$$A = \begin{pmatrix} 2 & 1 & 3 & 2 & 0 \\ 3 & 2 & 5 & 1 & 0 \\ -1 & 1 & 0 & -7 & 0 \\ 3 & -2 & 1 & 17 & 0 \\ 0 & 1 & 1 & -4 & 0 \end{pmatrix}$$

1) Given A, we eliminate rows or columns acording to the criterion to calculate the rank using the Gaussian elimination method. Thus,

Column 5 can be discarded because all its elements are zero.

Column 3 can be discarded because it is a linear combination of column 1 and column 2. Specifically, $c3 = c1 + c2$.

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & -7 \\ 3 & -2 & 17 \\ 0 & 1 & -4 \end{pmatrix}$$

2) Is there any non-zero square submatrix of order 1?

Any non-zero element is a non-zero square submatrix, therefore we will look at those of higher order.

Is there any non-zero square submatrix of order 2?

$$\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 1 \neq 0$$

Yes, there is, therefore we will look for higher orders.

4) Is there any non-zero square submatrix of order 3?

$$\begin{vmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ -1 & 1 & -7 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & 2 & 1 \\ -1 & 1 & -7 \\ 3 & -2 & 17 \end{vmatrix} = 0$$

$$\begin{vmatrix} -1 & 1 & -7 \\ 3 & -2 & 17 \\ 0 & 1 & -4 \end{vmatrix} = 0$$

No, there is not. Therefore, rank$(A) = 2$, which is the order of the largest non-zero square submatrix.