The determinants have certain properties that should be known. These properties are very useful to convert the determinants calculation into something a little less slow and tedious.

Let's see some of these properties:

1. Any matrix and its transpose (*the transpose matrix is the result of rotating the rows of a matrix to turn them into columns*) have the same determinant. $|A| = |A^t|$
2. The determinant of a matrix is zero, |A|=0,

   if:

   1. The matrix has two equal rows. It is easy to prove this in an exercise for a 3×3 case, for example:
   $$\begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} = a\cdot e\cdot c + d\cdot b\cdot c + a\cdot b\cdot f - c\cdot e\cdot a - f\cdot b\cdot a - c\cdot b\cdot d = 0$$
   2. All the elements of a row are zeros.
   3. The elements of a row are a linear combination of other rows. That is:
   $$\begin{vmatrix} 2 & 3 & 2 \\ 1 & 2 & 4 \\ 3 & 5 & 6 \end{vmatrix}$$

   The 3rd row is a linear combination of the other two (f3=f1+f2). Without calculating anything, we know that the determinant will be zero.

3. If we swap two parallel rows the determinant changes its sign:
$$\begin{vmatrix} 0 & 5 & 1 \\ 1 & 2 & 7 \\ 3 & 1 & 2 \end{vmatrix} = -\begin{vmatrix} 1 & 2 & 7 \\ 0 & 5 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$
4. If we add the elements of a row to the elements of a parallel row that have previously been multiplied by a real number, the value of the determinant does not change.
$$\begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} \rightarrow C_3 = 2\cdot C_1 + C_3 \rightarrow \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$
5. Multiplying a determinant by a real number is the same as multiplying one of its rows by that real number.
6. The determinant of a product is equal to the product of determinants.
$$|A\cdot B| = |A|\cdot|B|$$

Knowing these properties the determinants calculation can be faster. Bearing in mind the 4th property, we can go on modifying our determinant by means of linear combinations in such a way that we can get the largest number of possible 0 or 1, which would reduce the calculations a lot.

$$\begin{vmatrix} 1 & 3 & 3 & 6 \\ 1 & 3 & 6 & 7 \\ 2 & 4 & 0 & 3 \\ 1 & 5 & 2 & 3 \end{vmatrix} \rightarrow \begin{matrix} f_1 \rightarrow f_1 \\ f_2 \rightarrow f_2 - f_1 \\ f_3 \rightarrow f_3 - 2f_1 \\ f_4 \rightarrow f_4 - f_1 \end{matrix} \rightarrow \begin{vmatrix} 1 & 3 & 3 & 6 \\ 0 & 0 & 3 & 1 \\ 0 & -2 & -6 & -9 \\ 0 & 2 & -1 & -3 \end{vmatrix}$$

And as the first column is zero, except for the first element, we will have to calculate the determinant $\begin{vmatrix} 0 & 3 & 1 \\ -2 & -6 & -9 \\ 2 & -1 & -3 \end{vmatrix}$ because the other contributions would be zero.