Adding and subtracting polynomials

To add or to subtract polynomials is equivalent to adding or subtracting the similar monomials (of the polynomial) two by two .

It will be easier to understand with an example .

If we want to add

$$p(x) = x^2 - x + 1$$

and

$$q(x) = 3x^2 + x - 2$$

we put the similar monomials into two by two groups and we do the operation.

Let's see it in the following table:

	$p(x)$	$q(x)$	$p(x)+q(x)$
degree 0	$+1$	-2	-1
degree 1	$-x$	$+x$	0
degree 2	x^2	$3x^2$	$4x^2$

Now we join the monomials that result from the sum, so the result will be

$$p(x) + q(x) = 4x^2 - 1$$

In this example, the polynomials were ordered, but it would be possible that they were not. Also, they might not have been finished polynomials, which would mean that the box that corresponds to the coefficient one should have been filled in with a zero.

Then, in general, the steps that we would follow to add or to subtract polynomials are:

1. Put the polynomials in order, if they are not.
2. Add or subtract the similar monomials two by two.
3. Put into groups the resultant monomials to generate the adding or subtracting polynomial.

The degree of the adding or subtracting polynomial is the maximum of the degree of the polynomials involved in the operation. That is:degree(p(x)±q(x))=max{degree(p(x)),degree(q(x))}

Let's see some other examples:

Calculate $p(x) - q(x)$, where

$$p(x) = -x - 4x^3 + x^2$$

and

$$q(x) = -2 + x^2 + 5x$$

1. We put them in order:

$$p(x) = -4x^3 + x^2 - x$$

$$q(x) = x^2 + 5x - 2$$

2. We use the table to subtract the monomials two by two:

	$p(x)$	$q(x)$	$p(x)-q(x)$
degree 0	0	-2	2
degree 1	$-x$	$+5x$	$-6x$
degree 2	x^2	x^2	0
degree 3	$-4x^3$	0	$-4x^3$

3. And, then, the final result:

$p(x) - q(x) = -4x^3 - 6x + 2$

And the rule is satisfied:

$$\text{degree}(-4x^3 - 6x + 2) = \max\{\text{degree}(-x - 4x^3 + x^2), \text{degree}(-2 + x^2 + 5x)\} = 3$$

Calculate $p(x) + q(x)$, where

$$p(x) = x^4 - x^3 + x^2 - 2$$

and

$$q(x) = 4x^2 + 6x - 10$$

1. Both polynomials are already ordered.
2. We use the table to add the monomials two to two:

	$p(x)$	$q(x)$	$p(x)+q(x)$
degree 0	-2	-10	-12
degree 1	0	$6x$	$6x$
degree 2	x^2	$4x^2$	$5x^2$
degree 3	x^3	0	x^3
degree 4	x^4	0	x^4

3. And the final result is:

$p(x) + q(x) = x^4 + x^3 + 5x^2 + 6x - 12$

And the rule is, also, satisfied:

$$\text{degree}(x^4 + x^3 + 5x^2 + 6x - 12) =$$

$$= \max\{\text{degree}(x^4 - x^3 + x^2 - 2), \text{degree}(4x^2 + 6x - 10)\} = 4$$