1. Magnitude of a vector.

The scalar product can be used to determine the length of a vector $\vec{u}$ since:

$$\vec{u}\cdot\vec{u} = |\vec{u}||\vec{u}|\cos(\widehat{\vec{u}\vec{u}}) = |\vec{u}|^2$$

from which: $\vec{u} = \sqrt{\vec{u}\cdot\vec{u}}$

So, we obtain, using the coordinates of the vector $\vec{u} = (u_1, u_2)$,

$$\vec{u} = \sqrt{u_1^2 + u_2^2}$$

For $\vec{u} = (3,4)$, we have that

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

1. Angle between two vectors.

From the definition of the scalar product, $\vec{u}\cdot\vec{u} = |\vec{u}||\vec{u}|\cos(\widehat{\vec{u}\vec{u}})$ we can convert the cosine to another value:

$$\cos(\widehat{\vec{u}\vec{v}}) = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

Applying the function arcosine to both sides of the equality we obtain (ang=angle):

$$\text{ang}(\vec{u},\vec{v}) = \arccos\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$$

So, if we have two vectors $\vec{u} = (u_1, u_2)$ and $\vec{v} = (v_1, v_2)$ we have:

$$\text{ang}(\vec{u},\vec{v}) = \text{ang}(\vec{v},\vec{u}) = \arccos\left(\frac{u_1v_1 + u_2v_2}{\sqrt{u_1^2 + u_2^2}\cdot\sqrt{v_1^2 + v_2^2}}\right)$$

Find the angle formed by $\vec{u} = (2,3)$ and $\vec{v} = (-1,4)$. In this case, applying the previous formula, we obtain:

$$\text{ang}(\vec{u},\vec{v}) = \arccos\left(\frac{2\cdot(-1) + 3\cdot 4}{\sqrt{2^2 + 3^2}\cdot\sqrt{(-1)^2 + 4^2}}\right) = \arccos(0.67267) = 47^\circ 43' 35''$$