

We are going to learn how to create and solve an exercise of the following type (and others more complicated than this without a doubt):

A carpenter has to construct rectangular tables whose sides do not exceed 2 meters and that the sum of its biggest side and the double of the minor does not exceed 4 meters. What is the maximum value of the perimeter of the above mentioned tables?

In this type of problem, the new part regarding what we have already seen in the previous levels, is that there appears a certain quantity that has to be maximized (it might also have to be minimized, but here this is not the case). In this example, it is the perimeter of the tables, which is subject to certain restrictions. The function to maximize (minimize) is called the objective function. The maximum value (or minimum) of the objective function is in the margins of the feasible area delimited by the restrictions of the problem. This value is called the ideal value.

Let's see how to express mathematically the problem of the example:

The first thing to be identified are the variables or unknowns. In this case those will be the long side of the table (that we will call  $x$ ) and the short side of the table (that we will call  $y$ ).

We identify the objective function. What we are asked to maximize / minimize? And: How does this quantity express itself according to the variables of the problem?

In our case we have been asked to maximize the perimeter (which we will call  $P$ ) of the tables. The perimeter can be expressed as a function of the variables of the problem (long side and short side), since it is the sum of the double of every side. Mathematically:  $P(x,y)=2x+2y$

Following the steps of the first level, we write the restrictions as inequations. These restrictions are:

- The sides cannot be bigger than two meters (not even less than zero, since it would not make sense):  $x \geq 0$   $x \leq 2$   $y \geq 0$   $y \leq 2$
- The sum of the biggest side and the double of the minor must not exceed 4 meters:  $x+2y \leq 4$

We identify the region of validity defined by the restrictions and calculate the apexes of the above mentioned region. The straight lines associated with the restrictions are:

- $x=0$  and  $x=2$ , which are straight lines parallel to the axis  $y$ , which cross  $x=0$  and  $x=2$  respectively. The inequality from which they come defines a stripe of feasible solutions between  $x=0$  and  $x=2$ .
- $y=0$  and  $y=2$ , which are straight lines parallel to the axis  $x$ , which cross  $y=0$  and  $y=2$  respectively. The inequality from which they come defines a stripe of feasible solutions between  $y=0$  and  $y=2$ .
- $y=-\frac{1}{2}x+2$ , whose validity area is below the straight line (it is possible to see verifying that the point  $(0,0)$ , which is below the straight line, fulfills the inequation  $x+2y \leq 4$ ).

