

The equalization method consists in isolating the same unknown in both equations and then equaling the obtained expressions.

$$\left. \begin{array}{l} x + y = 3 \\ x - y = -1 \end{array} \right\}$$

If we isolate  $x$  in both expression we have:

$$\left. \begin{array}{l} x = 3 - y \\ x = -1 + y \end{array} \right\}$$

This system is equivalent to the first one, since only some terms have changed position. Now, since the value of  $x$  has to be the same in both equations, they necessarily have the same value, so that:

$$3 - y = -1 + y$$

This is a linear system of one equation and one unknown

$$-y - y = -1 - 3 \Rightarrow -2y = -4 \Rightarrow y = \frac{-4}{-2} = 2$$

We then put the value we found for  $y$  into any of the original equations to obtain the value of  $x$ . If we use the first equation:

$$x = 3 - y \Rightarrow x = 3 - 2 = 1$$

So that the solution to this system is  $x = 1, y = 2$ .

Sometimes it will turn out to be easier to simplify the system a little bit before clearing the unknowns.

$$\left. \begin{array}{l} 2(x - 1) + 3(y + 2) = -3 \\ x + 5 - 2y = 1 - 2x \end{array} \right\}$$

We can get rid of the brackets in the first equation, while we can put all the variables to one side and all the constants to the other side on the second equation:

$$2x - 2 + 3y + 6 = -3 \Rightarrow 2x + 3y = -3 + 2 - 6 \Rightarrow 2x + 3y = -7$$

$$x - 2y + 2x = 1 - 5 \Rightarrow 3x - 2y = -4$$

We then obtain a completely equivalent system, but much simpler:

$$\left. \begin{array}{l} 2x + 3y = -7 \\ 3x - 2y = -4 \end{array} \right\}$$

Now we can clear out one of the two variables in both equations, for example  $x$ :

$$\left. \begin{array}{l} 2x = -7 - 3y \\ 3x = -4 + 2y \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{-7 - 3y}{2} \\ x = \frac{-4 + 2y}{3} \end{array} \right\}$$

We then combine the two values of  $x$  (in terms of  $y$ ) to obtain one single equation and one unknown ( $y$ ).

This can be easily solved:

$$\begin{aligned} \frac{-7 - 3y}{2} &= \frac{-4 + 2y}{3} \Rightarrow 3(-7 - 3y) = 2(-4 + 2y) \Rightarrow \\ \Rightarrow -21 - 9y &= -8 + 4y \Rightarrow -9y - 4y = -8 + 21 \Rightarrow -13y = 13 \Rightarrow \\ y &= -\frac{13}{13} = -1 \end{aligned}$$

Finally we only need to put  $y$  back into any of the two original equations, for example into the second one:

$$x = \frac{-4 + 2(-1)}{3} = \frac{-4 - 2}{3} = \frac{-6}{3} = -2$$

So that the solution to this system is  $x = -2, y = -1$ .

In short, the steps we have to follow in order to solve the system are:

1. Clear the same unknown in both equations.
2. Equate the obtained expressions in order to have one single equation with one unknown that can be easily solved.
3. Put the solution back into any of the two original equations in order to obtain the value of the second variable.