We are going to learn how to solve equations

$$ax^4 + bx^2 + c = 0$$

of this type:

that is, 4-degree equations in which we do not have terms of an odd degree. These equations are called biquadratic.

To solve them we will convert them into quadratic equations.

Let's see an example that will help us to better understand the process:

We want to solve the following equation:

$$x^4 - 8x^2 + 12 = 0$$

If we change the variable $x^2 = t$, we get the equation:

$$t^2 - 8t + 12 = 0$$

This equation can be solved:

$$t = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{8 \pm \sqrt{64 - 48}}{2} = \frac{8 \pm \sqrt{16}}{2} = \\ = \begin{cases} t_1 = \frac{8+4}{2} = \frac{12}{2} = 6 \\ t_2 = \frac{8-4}{2} = \frac{4}{2} = 2 \end{cases}$$

Therefore we have two solutions:

$$\begin{matrix} t_1 = 6 \\ t_2 = 2 \end{matrix}$$

But we want to find the value of x; if we undo the first change we will have:

$$\begin{array}{rcl} x^2 & = & t \\ x & = & \pm\sqrt{t_1} \\ x & = & \pm\sqrt{t_2} \end{array} \quad \begin{array}{c} \longrightarrow \\ \longrightarrow \\ \longrightarrow \end{array} \quad \begin{array}{rcl} x & = & \pm\sqrt{t} \\ x & = & \pm\sqrt{6} \\ x & = & \pm\sqrt{2} \end{array}$$

Therefore we obtain 4 solutions:

$$\begin{matrix} x_1 = \sqrt{6} & x_3 = \sqrt{3} \\ x_2 = -\sqrt{6} & x_4 = -\sqrt{2} \end{matrix}$$

Now that we have seen an example of how to solve this type of equations, we could wonder if we will always obtain 4 solutions.

The answer is no, and let's see why.

The number of solutions of the equation will depend on the number of solutions of the quadratic equation since for every positive solution of the quadratic equation we will have 2 solutions in the biquadratic one.

This way we can make sure that we will not have more than 4 solutions in the biquadratic equation.