

In order to solve a trigonometric equation we will follow these steps:

1) We develop the expressions until we obtain only one trigonometric expression equaling to a number.

2) We will obtain one of the following equalities:

$$\begin{aligned}\sin u &= a \\ \cos u &= b \\ \tan u &= c\end{aligned}$$

3) We solve each of them by taking the arc of the corresponding functions in the two sides of the equations:

$$\sin u = a \Rightarrow \arcsin(\sin u) = \arcsin a \Rightarrow$$

$$u = \begin{cases} \arcsin a + 2k \cdot \pi \\ (\pi - \arcsin a) + 2k \cdot \pi \end{cases}, k \in \mathbb{Z}$$

$$\cos u = b \Rightarrow \arccos(\cos u) = \arccos b \Rightarrow$$

$$u = \begin{cases} \arccos b + 2k \cdot \pi \\ (2\pi - \arccos b) + 2k \cdot \pi \end{cases}, k \in \mathbb{Z}$$

$$\tan u = c \Rightarrow \arctan(\tan u) = \arctan c \Rightarrow u = \arctan c + \pi \cdot k$$

4) Once we have u, we isolate x.

Let's solve the following trigonometric equation:

$$\sin^2 x - \cos^2 x = \frac{1}{2}$$

First, we isolate  $\sin^2 x$ :

$$\sin^2 x = \frac{1}{2} + \cos^2 x$$

From the relation:

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$$

whereby we substitute in our equation:

$$\sin^2 x = \frac{1}{2} + \cos^2 x = \frac{1}{2} + 1 - \sin^2 x = \frac{3}{2} - \sin^2 x \Rightarrow 2\sin^2 x = \frac{3}{2} \Rightarrow$$
$$\Rightarrow \sin^2 x = \frac{3}{4} \Rightarrow \sin x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$$

Now we have already managed to obtain a trigonometric ratio which equals to a number.

We apply now the relation 3.i in the two possible situations:

Case (a):

$$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = \begin{cases} \frac{\pi}{3} + 2\pi \cdot k \\ \pi - \frac{\pi}{3} + 2\pi \cdot k = \frac{2\pi}{3} + 2\pi \cdot k \end{cases}, k \in \mathbb{Z}$$

Case (b):

$$\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = \begin{cases} -\frac{\pi}{3} + 2\pi \cdot k \\ \pi + \frac{\pi}{3} + 2\pi \cdot k = \frac{4\pi}{3} + 2\pi \cdot k \end{cases}, k \in \mathbb{Z}$$

So we obtain the following solution:

$$x = \begin{cases} \frac{\pi}{3} + 2\pi \cdot k \\ \frac{2\pi}{3} + 2\pi \cdot k \\ -\frac{\pi}{3} + 2\pi \cdot k \\ \frac{4\pi}{3} + 2\pi \cdot k \end{cases}, k \in \mathbb{Z}$$