It is said that a system is equivalent to another one if they have the same solution.

The Gaussian method uses the idea of equivalent systems to solve the given system. To do it, it uses certain rules of systems transformation:

1. If we add (or subtract) the same expression to both members of an equation of a system, the resultant system is equivalent.

2. If we multiply (or divide) both members of a system by a number different from zero the resulting system is equivalent. For example, $3x+2y-z=2$ is equivalent to $15x+10y-5z=10$.

3. If we add (or substract) an equation of the system to (or from) another equation of the system the resulting one is equivalent.

$$\begin{pmatrix} 1 & 2 & 1 & -1 \\ -1 & 3 & 0 & 2 \\ 1 & 1 & -1 & 3 \end{pmatrix} \to \text{row1-row2} \to \begin{pmatrix} 2 & -1 & 1 & -3 \\ -1 & 3 & 0 & 2 \\ 1 & 1 & -1 & 3 \end{pmatrix}$$

These two systems are equivalent.

4. If we replace an equation with another one

$$\begin{pmatrix} 1 & 2 & 1 & -1 \\ -1 & 3 & 0 & 2 \\ 1 & 1 & -1 & 3 \end{pmatrix} \to \text{row1-3row2} \to \begin{pmatrix} 4 & -7 & 1 & -7 \\ -1 & 3 & 0 & 2 \\ 1 & 1 & -1 & 3 \end{pmatrix}$$

   that is the result of doing a linear combination of other equations of the system, the resultant system is equivalent.

In this case we substract 3 times the second row from the first row. The resulting system is equivalent.

5. If we change the order of the equations or the order of the unknowns in a system, the resulting system is equivalent.

$$\begin{pmatrix} 1 & 2 & 1 & -1 \\ -1 & 3 & 0 & 2 \\ 1 & 1 & -1 & 3 \end{pmatrix} \to \begin{pmatrix} -1 & 3 & 0 & 2 \\ 1 & 2 & 1 & -1 \\ 1 & 1 & -1 & 3 \end{pmatrix}$$

We changed row1 and row2, but the resulting system is equivalent to the first one.

In other words, if we are given a system of equations, we can use the 5 previous rules to modify it and to write another system with the same solution.