We must distinguish two cases:

1. product of a monomial by a polynomial
2. product of a polynomial by a polynomial

## Product of a polynomial by a monomial

The monomial multiplies all the monomials that form the polynomial.

The degree of the product is the adding of the degrees of the factors.

Let's consider, $p(x) = 4x^2$ and $q(x) = x^2 + 3x - 2$.

Then,

$$p(x) \cdot q(x) = (4x^2)(x^2 + 3x - 2) = (4x^2) \cdot x^2 + (4x^2) \cdot 3x - (4x^2) \cdot 2 =$$

$$= 4x^4 + 12x^3 - 8x^2$$

And it is satisfied that

$$\text{degree}(4x^4 + 12x^3 - 8x^2) = \text{degree}(4x^2) + \text{degree}(x^2 + 3x - 2) =$$

$$= 2 + 2 = 4$$

Let's consider, $p(x) = -2x$ and $q(x) = 5x^3 + 3x^2 - 1$.

Then,

$$p(x) \cdot q(x) = (-2x)(5x^3 + 3x^2 - 1) = -2x \cdot 5x^3 - 2x \cdot 3x^2 + 2x \cdot 1 =$$
$$= -10x^4 - 6x^3 + 2x$$

And it is also satisfied that:

$$\text{degree}(-10x^4 - 6x^3 + 2x) = \text{degree}(-2x) + \text{degree}(5x^3 + 3x^2 - 1) =$$

$$= 1 + 3 = 4$$

## Product of a polynomial for a polynomial

Every monomial of the first polynomial multiplies all the monomials that form the second polynomial. Then, if necessary, we add or subtract all the similar monomial (only, that is, if they exist).

The degree of the product is the sum of the degrees of the factors.

Do the multiplication of $p(x)$ and $q(x)$ where

$$p(x) = 4x^2 - 1$$

$$q(x) = x^2 + 3x - 2$$

We multiply the first monomial of $p(x)$ by $q(x)$:

$$4x^2 \cdot q(x) = 4x^2(x^2 + 3x - 2) = 4x^4 + 12x^3 - 8x^2$$

Now we multiply the second monomial of $p(x)$ by $q(x)$:

$$(-1) \cdot q(x) = (-1)(x^2 + 3x - 2) = -x^2 - 3x + 2$$

Finally, we put together both expressions and we add those that are similar:

$$p(x) \cdot q(x) = (4x^4 + 12x^3 - 8x^2) + (-x^2 - 3x + 2) =$$

$$= 4x^4 + 12x^3 - 9x^2 - 3x + 2$$

It is satisfied:

$$\text{degree}(4x^4 + 12x^3 - 9x^2 - 3x + 2) =$$

$$= \text{degree}(4x^2 - 1) + \text{degree}(x^2 + 3x - 2) = 2 + 2 = 4$$

Do the multiplication of $p(x)$ by $q(x)$ where

$$p(x) = x + 2$$

$$q(x) = 3x^3 - 2x - 1$$

We multiply the first monomial of $p(x)$ by $q(x)$:

$$x \cdot q(x) = x(3x^3 - 2x - 1) = 3x^4 - 2x^2 - x$$

Now we multiply the second monomial of $p(x)$ by $q(x)$:

$$2 \cdot q(x) = 2(3x^3 - 2x - 1) = 6x^3 - 4x - 2$$

Finally, we put together both expressions and we add those that are similar:

$$p(x) \cdot q(x) = (3x^4 - 2x^2 - x) + (6x^3 - 4x - 2) =$$

$$= 3x^4 + 6x^3 - 2x^2 - 5x - 2$$

It is fulfilled:

$$\text{degree}(3x^4 + 6x^3 - 2x^2 - 5x - 2) =$$
$$= \text{degree}(x + 2) + \text{degree}(3x^3 - 2x - 1) = 1 + 3 = 4$$