Equivalent fractions

An algebraic fraction is a division of the polynomial quotient.

Let's see some examples:

$$\frac{x^2-3}{x+1}$$

$$\frac{x^4+x^3+x-1}{x^3+2x+3}$$

$$\frac{x^6}{x-2}$$

In a similar way as with fractions, we can define two algebraic fractions as equivalent if its cross product is equal. That is, if we have:p(x)q(x) and r(x)s(x)two pairs of algebraic fractions, they will be equivalent if, and only if:p(x)·s(x)=r(x)·q(x)

Let's see if this pair of algebraic fractions is equivalent:

$$\frac{x^2-1}{x} \text{ and } \frac{(x-1)^2}{x}$$

To verify, we will compute the cross products:

$$(x^2-1)\cdot x = x^3-x$$

$$x\cdot(x-1)^2 = x\cdot(x^2-2x+1) = x^3-2x^2+x$$

They obviously are not equal. Therefore, the previous fractions are not equivalent.

Let's see if this pair of algebraic fractions is equivalent:

$$\frac{x-1}{x+1} \text{ and } \frac{(x-1)^2}{x^2-1}$$

To verify, we will operate the cross products:

$$(x-1)\cdot(x^2-1) = x\cdot(x^2-1)-1\cdot(x^2-1) =$$

$$= x^3-x-x^2+1 = x^3-x^2-x+1$$

$$(x+1)\cdot(x-1)^2 = (x+1)\cdot(x^2-2x+1) =$$

$$x\cdot(x^2-2x+1)+1\cdot(x^2-2x+1) = x^3-2x^2+x+x^2-2x+1 =$$

$$= x^3-x^2-x+1$$

We can see that they are equal, and therefore, the previous fractions are equivalent.