We are going to see now the way we can construct a quadratic equation when the solutions are known.

The solutions of the equation $x^2+2x-3=0$ are:

$$x=\frac{-2\pm\sqrt{4+12}}{2}=\frac{-2\pm4}{2}=\begin{cases}x_1=1\\x_2=-3\end{cases}$$

Now let's look at what happens when we do the product $(x-x_1)\cdot(x-x_2)$

$$(x-1)\cdot(x+3)=x^2-x+3x-3=x^2+2x-3$$

We have returned to the original equation.

So "the product of x minus a root multiplied by x minus the other root is equal to the quadratic equation that has these roots as a solution".

If the solutions of the equation are $x_1=4, x_2=2$ the corresponding quadratic equation is:

$$(x-4)(x-2)=x^2-6x+8=0$$

If the roots of the equation are $x_1=-2, x_2=-5$ the corresponding quadratic equation is:

$$(x+1)(x+5)=x^2+6x+5=0$$

If the solutions of the equation are $x_1=3, x_2=-\frac{2}{3}$ the corresponding quadratic equation is:

$$(x-3)(x+\frac{2}{3})=x^2-\frac{7}{3}x-2=0$$

If the roots of the equation are $x_1=0, x_2=16$ the corresponding quadratic equation is:

$$(x-0)(x-16)=x^2+16x=0$$

Reconstruction of the quadratic equation from the sum and product of roots

We know that (x−x1)·(x−x2) leads to the equation that has x1,x2 as its solutions. If we do the product:

an expression in which appear the sum and the product of the roots, let's call them s and p.

Write a quadratic equation knowing that the sum of its roots is 5 and its product 6.

We know that $s=5,\ p=6$, then the equation will be:

$$x^2-5x+6=0$$

This method is faster than doing the product of roots.

Let's see some other examples:

The quadratic equation that has solutions 4 and 9 is:

$$x^2-13x+36=0$$

The quadratic equation that has solutions -3 and -5 is:

$$x^2+8x+15=0$$

Let's say it is not easy to lay out an exercise that ends with a quadratic equation. The easiest way would be writing literally what the equation says.

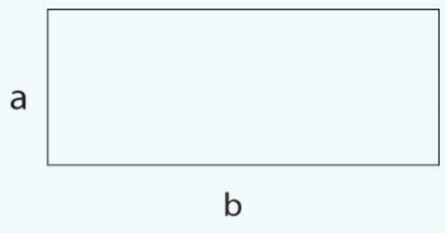
If we want to get the equation $x^2-5x+6=0$ as a solution to a problem, we can formulate a statement like: If we raise an amount to the square and we subtract 5 times this amount the result is -6. What is the value of that amount?

The following statement is clearly much more interesting: "Find two numbers knowing that their sum is 5 and their product is 6" , a statement that ends with the same equation and whose solutions can be found solving the proposed equation:

$$x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm1}{2}=\begin{cases}x_1=3\\x_2=2\end{cases}$$

With these same values we can approach it geometrically.

We know that the perimeter of a rectangle is 10 and its area 6. Calculate the sides of this rectangle.



The perimeter of a rectangle is the sum of all its sides, then it is $a+a+b+b=2a+2b=2(a+b)=10$, that is, $a+b=5$

On the other side, the area of the rectangle is $a\cdot b=6$.

Then, to solve this problem we have to solve a quadratic equation in which the sum of its roots is 5 and its product 6. This equation is $x^2-5x+6=0$.

And the solution is:

$$x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm1}{2}$$

Then, the sides of the rectangle will be, $a=2$ and $b=3$