The expression:

$$x-1=7$$

is an equation. That is, an equality that is satisfied for some value of x.

The left side of the equality is named the first member of the equation and the right one, the second member.

In this equality there are known numbers (-1 and 7) and one not known number (x).

These are the terms of the equation: x is the unknown since it is the number that it is necessary to find, and -1 and 7 are the constants or independent terms because they are not associated with any unknown.

All the equations that we will study in this unit are named linear or first degree equations because the unknown is raised to the first power, 1, or in other words, the unknown or unknowns have no exponents.

Returning to the example, the equation is asking: what number is equal to 7 if we subtract 1 from that number?

The almost immediate answer is 8. It is possible to verify whether this number satisfies the equality by substituting, in the equation, x with 8:

$$8-1=7 \Rightarrow 7=7$$

And, indeed, 8 is the solution since the equality is satisfied.

Might -6 be another solution? Again, let's verify it by substituting x with -6:

$$-6-1=7 \Rightarrow -7=7$$

The equality is not satisfied, so -6 is not a solution to the equation.

We can apply the same reasoning to the following equation:

$$2x=12$$

Namely: what number multiplied by 2 is 12? We don't need to think very long to conclude that it is 6. We replace x by this number to verify that our deduction is true:

$$2\cdot 6=12 \Rightarrow 12=12$$

The equality is satisfied, so 6 is a solution for the equation.

Usually, the equations are not so simple in the sense that it is not always so easy to deduce its solution as in the previous cases.

To solve equations there is a quite effective method that is summed up in the following points:

1. Group the terms with the unknown on one side of the equality, usually the first member, and the constants on the other side.
2. Operate whenever it is possible to simplify the expression. This means removing brackets and denominators if there are any.
3. Isolate the unknown.

Applying this method to the previous examples:

$$x-1=7 \Rightarrow x=7+1 \Rightarrow x=8$$

$$2x=12 \Rightarrow x=\frac{12}{2} \Rightarrow x=6$$

To move elements from side to side of the equality it is necessary to bear in mind that:

1. If they were adding up or subtracting they will have the opposite sign on the other side.

Example

$$-2-3x=7 \Rightarrow -3x=7+2$$

1. If they were multiplying they will be dividing and vice versa, but the sign is not modified on having changed side.

Continuing with the same equation of the example:

$$-3x=7+2 \Rightarrow x=\frac{7+2}{-3}=\frac{9}{-3}=-3$$

Let's apply those steps to solve equations to the following example:

$$-2x+3=5$$

The first step is to group the terms with x in the first member.

To do this, we will move the 3 to the second member, bearing in mind that we have to change its sign:

$$-2x=5-3$$

We operate the second member:

$$-2x=2$$

Now it is necessary to get rid of the -2 that it is multiplying x.

The product, in this case, goes to the other side of the equality becoming a quotient, but without changing its sign, so that:

$$x=\frac{2}{-2}=-1$$

To verify if the result is correct we replace the value we have found by x:

$$-2x+3=5 \Rightarrow -2\cdot(-1)+3=5 \Rightarrow 2+3=5 \Rightarrow 5=5$$

The result is correct, since the equality is satisifed.

$$1+\frac{x}{2}=-3$$

We start by isolating the terms with x in the first member. To do this the 1 goes to the second member, with the opposite sign:

$$\frac{x}{2}=-3-1$$

If we calculate the second member:

$$\frac{x}{2}=-4$$

And now we move the element that it is dividing x by 2. For this it is necessary to bear in mind that a quotient goes to the other side by multiplying (and without changing sign), so that:

$$x=-4\cdot 2=-8$$

We can prove that the result is correct by replacing the value found for x:

$$1+\frac{x}{2}=-3 \Rightarrow 1+\frac{-8}{2}=-3 \Rightarrow 1-4=-3 \Rightarrow -3=-3$$

The obtained value is, again, valid.

Sometimes there are linear equations with one unknown that have no solution. For instance:

$$\frac{x}{2}-1=\frac{3x}{2}-x$$

If the method is applied and all the terms with x are isolated in the first member and the constant in the second one, we obtain:

$$\frac{x}{2}-1=\frac{3x}{2}-x \Rightarrow \frac{x}{2}-\frac{3x}{2}+x=1$$

Now it is necessary to apply the least common multiple:

$$\frac{x}{2}-\frac{3x}{2}+\frac{2x}{2}=1 \Rightarrow \frac{3x-3x}{2}=1 \Rightarrow \frac{0}{2}=1 \Rightarrow 0=2$$

The unknown disappears after performing the operations. When this happens it is said that the equation has no solution.

A useful tool when we want to set out problems in a written form, is being able to write an equation from its solution. We are going to see how to write an equation for which we want to have a specific solution.

We want to write an equation that should have the value 7 as a solution. It is written as follows: $x=7$.

If we subtract 3 from both sides of the equality, we will get the same solution: $x-3=7-3$.

Now we multiply by 2 on both sides of the equality:

$$\begin{aligned}2\cdot(x-3)&=2\cdot(7-3)\\2\cdot(x-3)&=2\cdot 4\\2\cdot(x-3)&=8\end{aligned}$$

If we evaluate 7 in the equation we will see that it is a solution:

$$\begin{aligned}2\cdot(7-3)&=8\\2\cdot 4&=8\\8&=8\end{aligned}$$

This equation $2\cdot(x-3)=8$ might appear after reading the following exercise: "3 years ago, the double of my age was 8. How old am I?".