Remainder theorem

The remainder of dividing a polynomial p(x) by another one of the form x−a, coincides with the value of p(a).

Notice that this kind of division satisfies the hypotheses of the Ruffini's rule.

Calculate the remainder of the division $\frac{p(x)}{q(x)}$, where $p(x) = x^4 + 3x^2 - x + 4$ and $q(x) = x + 2$.

We apply the remainder theorem. Notice that, in this case $a = -2$.

$$p(-2) = (-2)^4 + 3 \cdot (-2)^2 - (-2) + 4 = 16 + 3 \cdot 4 + 2 + 4 = 34$$

To verify it we use Ruffini:

	1	0	3	-1	4
-2		-2	4	-14	30
	1	-2	7	-15	34

And, it is the same as the previous solution.

Calculate the remainder of the division $\frac{p(x)}{q(x)}$, where $p(x) = x^5 - 2x^2 + x + 3$ and $q(x) = x + 1$.

We apply the remainder theorem. Notice that, in this case $a = -1$.

$$p(-1) = (-1)^5 - 2 \cdot (-1)^2 + (-1) + 3 = -1 - 2 - 1 + 3 = -1$$

To verify it we use Ruffini:

	1	0	0	-2	1	3
-1		-1	1	-1	3	-4
	1	-1	1	-3	4	-1

And it is the same than the previous solution.

Factor theorem

Its statement is the following one:

A polynomial $p(x)$ is divisible by another of the form $x - a$ if, and only if, $p(a) = 0$. In this case, we will say that a is a root or zero of the polynomial $p(x)$.

Calculate the remainder of the division $\frac{p(x)}{q(x)}$, where $p(x) = x^5 + 2x^4 - 3x^3 + x^2 - 1$ and $q(x) = x - 1$.

We apply the remainder theorem

$$p(1) = 1^5 + 2 \cdot 1^4 - 3 \cdot 1^3 + 1^2 - 1 = 0$$

We verify the result using Ruffini:

	1	2	-3	1	0	-1
1		1	3	0	1	2
	1	3	0	1	1	0

Indeed, the remainder is 0. And so, according to the factor theorem, the division of $p(x)$ by $q(x)$ is exact.