

Quotient of two rational numbers

The quotient of the integer -6 by the integer 2 is the integer -3 , since: $-6=2\cdot(-3)$

This exercise of multiplying integers can be written as a division:

(-6) $:$ 2 $=$ -3

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\uparrow

\nwarrow

dividend

divisor

quotient

In the same way, the rational number $\frac{3}{20}$ can be expressed as the product of the rational number $\frac{3}{4}$ by another one. Which rational is this another one?

We can prove that this rational is

$\frac{3}{20}$ $=$ $\frac{3}{4}$ \cdot $\frac{1}{5}$ $=$ $\frac{3\cdot 1}{4\cdot 5}$

And then we say that the quotient of the division of $\frac{3}{20}$ by $\frac{3}{4}$ is equal to $\frac{1}{5}$. In the same way as done with integers, the exercise of multiplying $\frac{3}{20}=\frac{3}{4}\cdot$ can be written as a division:

Calculating the quotient of two rational numbers

1. The exercise:

$\frac{3}{20} : \frac{3}{4} = ?$

can be written as:

$? \cdot \frac{3}{4} = \frac{3}{20}$

2. Multiplying both terms of the equality by the inverse of the divisor: $\left(? \cdot \frac{3}{4}\right) \cdot \frac{4}{3} = \left(\frac{3}{20} \cdot \frac{4}{3}\right)$

3. Bearing in mind the properties of the product of fractions, we obtain:

$? \cdot \frac{3}{4} \cdot \frac{4}{3} = \frac{3}{20} \cdot \frac{4}{3}$

And as $\frac{3}{4} \cdot \frac{4}{3} = 1$, we have

$? \cdot 1 = \frac{3}{20} \cdot \frac{4}{3} = \frac{3\cdot 4}{20\cdot 3} = \frac{4}{20} = \frac{1}{5}$

Therefore:

$\frac{3}{20} : \frac{3}{4} = \frac{3}{20} \cdot \frac{4}{3} = \frac{1}{5}$

Namely, to find the quotient of two rational numbers $\frac{a}{b}$ (dividend) and $\frac{c}{d}$ (divisor), the divisor being other than zero, it is necessary to multiply the dividend by the inverse of the divisor:

$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

Calculate the quotient of $-\frac{4}{5}$ by $-\frac{3}{2}$:

$-\frac{4}{5} : \left(-\frac{3}{2}\right)$

We multiply the dividend $-\frac{4}{5}$ by the inverse of the divisor $-\frac{2}{3}$, that is $-\frac{2}{3}$:

$-\frac{4}{5} : \left(-\frac{3}{2}\right) = -\frac{4}{5} \cdot \left(-\frac{2}{3}\right) = \frac{-4}{5} \cdot \frac{-2}{3} = \frac{8}{15}$

Quotient of a rational number and an integer

To divide an integer a by a rational number $\frac{m}{n}$ we have to express the integer a as $\frac{a}{1}$ and proceed as in the previous case:

$a : \frac{m}{n} = \frac{a}{1} : \frac{m}{n} = \frac{a}{1} \cdot \frac{n}{m}$

And, in the same way, to divide a rational number $\frac{m}{n}$ by an integer

$\frac{m}{n} : a = \frac{m}{n} : \frac{a}{1} = \frac{m}{n} \cdot \frac{1}{a}$