Now we will explain a method to divide polynomials of one variable. We will use an example to illustrate the procedure:

Let's consider,

$p(x) = x^5 - 3x^3 + 2x - 1$

$q(x) = x^2 - 1 - 2x$

Calculate this quotient $\frac{p(x)}{q(x)}$.

1) Complete and put in order both polynomials.

In our case,

$p(x) = x^5 + 0x^4 - 3x^3 + 0x^2 + 2x - 1$

$q(x) = x^2 - 2x - 1$

2) Write both polynomials as if we wanted to solve a traditional division of two numbers (the dividend into the left, the divisor into the right). Let's consider that every monomial is a number.

Here we will use the following table:

x^5	0	$-3x^3$	0	$2x$	-1	$x^2 - 2x - 1$

3) Divide the first monomial of the dividend by the first monomial of the divisor.

In our case: $\frac{x^5}{x^2} = x^3$

4) Multiply the result by every monomial of the dividing polynomial and subtract the result from the polynomial dividend.

The result of the product is $x^3 \cdot q(x) = x^3(x^2 - 2x - 1) = x^5 - 2x^4 - x^3$

And we subract it by the dividend. Then, we schematize it:

x^5	0	$-3x^3$	0	$2x$	-1	$x^2 - 2x - 1$
$-x^5$	$+2x^4$	$+x^3$	0	0	0	x^3
0	$+2x^4$	$-2x^3$	0	$2x$	-1	

The result of the subtraction appears in the third line. We take note of the result of the division of monomials placed just under the divisor: this will be our quotient.

Let's focus on the box of the degree of the polynomial that we have divided. In this case, we find a 0. This must happen in each one of the steps that we make.

5) Repeat steps 3 and 4 until the degree of the polynomial by which we need to divide is lower than the degree of the dividing polynomial.

Let's see how we continue: $\frac{2x^4}{x^2} = 2x^2$

$2x^2(x^2 - 2x - 1) = 2x^4 - 4x^3 - 2x^2$

x^5	0	$-3x^3$	0	$2x$	-1	$x^2 - 2x - 1$
$-x^5$	$+2x^4$	$+x^3$	0	0	0	$x^3 + 2x^2$
0	$+2x^4$	$-2x^3$	0	$2x$	-1	
	$-2x^4$	$4x^3$	$2x^2$	0	0	
	0	$2x^3$	$2x^2$	$2x$	-1	

Now, we have a 0 in the degree 4 monomial. Let's continue:

$\frac{2x^3}{x^2} = 2x$

$2x(x^2 - 2x - 1) = 2x^3 - 4x^2 - 2x$

x^5	0	$-3x^3$	0	$2x$	-1	$x^2 - 2x - 1$
$-x^5$	$+2x^4$	$+x^3$	0	0	0	$x^3 + 2x^2 + 2x$
0	$+2x^4$	$-2x^3$	0	$2x$	-1	
	$-2x^4$	$4x^3$	$2x^2$	0	0	
	0	$2x^3$	$2x^2$	$2x$	-1	
		$-2x^3$	$+4x^2$	$+2x$	0	
		0	$6x^2$	$4x$	-1	

We can see, again, that we find a 0 in the degree 3 monomial. We repeat the operation:

$\frac{6x^2}{x^2} = 6$

$6(x^2 - 2x - 1) = 6x^2 - 12x - 6$

x^5	0	$-3x^3$	0	$2x$	-1	$x^2 - 2x - 1$
$-x^5$	$+2x^4$	$+x^3$	0	0	0	$x^3 + 2x^2 + 2x + 6$
0	$+2x^4$	$-2x^3$	0	$2x$	-1	
	$-2x^4$	$+4x^3$	$+2x^2$	0	0	
	0	$2x^3$	$2x^2$	$2x$	-1	
		$-2x^3$	$+4x^2$	$+2x$	0	
		0	$6x^2$	$4x$	-1	
			$-6x^2$	$+12x$	$+6$	
			0	$16x$	$+5$	

Again, a 0 appears in the monomial of second degree. Now, the polynomial that we want to divide has degree 1, which is less than the degree of the divisor (degree 2). At this point, the division is finished. Then:

1. The quotient will be the polynomial placed just under the divisor: $x^3 + 2x^2 + 2x + 6$
2. The remainder will be the polynomial located at the end, which degree will be always lower than the one of the divisor: $16x + 5$

VERIFICATION

To verify that we have done the division correctly, we will calculate:

$$\text{quotient} \times \text{divisor} + \text{remainder}$$

The result, if we have done the operation correctly, should be the dividend.

So, in our example:

$$(x^3 + 2x^2 + 2x + 6) \cdot (x^2 - 2x - 1) + (16x + 5)$$

We calcule the multiplication:

$x^3 \cdot (x^2 - 2x - 1) = x^5 - 2x^4 - x^3$

$2x^2 \cdot (x^2 - 2x - 1) = 2x^4 - 4x^3 - 2x^2$

$2x \cdot (x^2 - 2x - 1) = 2x^3 - 4x^2 - 2x$

$6 \cdot (x^2 - 2x - 1) = 6x^2 - 12x - 6$

$(x^5 - 2x^4 - x^3) + (2x^4 - 4x^3 - 2x^2) + (2x^3 - 4x^2 - 2x) +$

$+(6x^2 - 12x - 6) = x^5 - 3x^3 - 14x - 6$

Then, we add the remainder:

$(x^5 - 3x^3 - 14x - 6) + (16x + 5) = x^5 - 3x^3 + 2x - 1$

As we can see, the result coincides with our dividend.

We can also verify that:

degree(quotient)=degree(dividend)-degree(divisor)

degree(remainder)

Calculate the quotient 3 where $x^3 + 2x^2 + 2x + 6$ and $x^5 - 3x^3 + 2x - 1$.

1. We complete and put in order

$x^2 - 2x - 1$

$5 - 2 = 3$

2. We define the initial table

$16x + 5$	$1 < 2$	$x^2 - 2x - 1$	$\frac{p(x)}{q(x)}$	$p(x) = 1 - x^3$

Continuing with the operation: $q(x) = x + 2$ $p(x) = -x^3 + 0x^2 + 0x + 1$

$q(x) = x + 2$	$-x^3$	0	0	1
$x + 2$	$\frac{-x^3}{x} = -x^2$	$-x^2(x+2) = -x^3 - 2x^2$	$-x^3$	0
0	1	$x + 2$	$+x^3$	

And then, the next step: $+2x^2$ 0

0	$-x^2$	0	$+2x^2$	0
1	$\frac{2x^2}{x} = 2x$	$2x(x+2) = 2x^2 + 4x$	$-x^3$	0
0	1	$x + 2$	$+x^3$	
	$+2x^2$	0	0	
	$-x^2 + 2x$	0	$+2x^2$	

Third step: 0 1

$-2x^2$	$-4x$	0	0	$-4x$
1	$\frac{-4x}{x} = -4$	$-4(x+2) = -4x - 8$	$-x^3$	0
0	1	$x + 2$	$+x^3$	
	$+2x^2$	0	0	
	$-x^2 + 2x - 4$	0	$+2x^2$	
		0	1	
		$-2x^2$	$-4x$	