<u>**SYSTEM OPTIMIZATION METHOD**</u>

COURSE WORK PROJECT REPORT

Title: **OPTIMIZATION OF PID CONTROLLER FOR INTERVAL PLANT**

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SUMMARY:
The advent of Control System has resulted in the phenomenal transformation of our life style. Optimization plays an important role in it. Its adaptive nature has resulted in increased accuracy, precision, reliability and it also reduces the errors and time depending on the situation. The most widely used controllers in industries are of the Proportional Integral (PI), proportional Integral Derivative (PID) and the lead/lag controllers. This Project “Optimization of PID controller for interval plant” deals with the PID controller by minimizing the objective function for determining the optimal gain Kp, Ki, Kd, in order to maximize the stability of a system.

INTRODUCTION:
This Project involves in Optimization of PID controller for interval plants. The optimization based design of robust PID controller guarantees both the stability and the performance of an interval plant. The objective is to find the optimum Kp and Ki value for stable First Order Plus Time Delay (FOPTD) process. Optimization is done by using Non-Linear Programming (NLP). Two methods are used for solving the optimization problem in this project. One is done by Lagrange Multipliers method and other is done by converting the nonlinear equations to linear equation and solving it by simplex method, along with their MATLAB simulation.

PID CONTROLLER DESIGN:

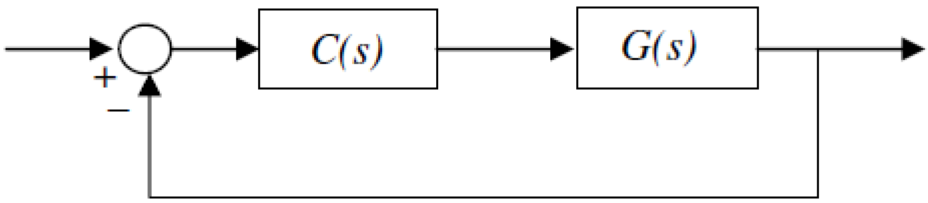


Fig. 1. A SISO control system.

Consider an interval plant in which the system parameters are bound to vary within closed limits then the system has structured uncertainty also called as parametric uncertainty. Kharitonov theorem is generally used to analysis the stability of these systems. The Kharitonov theorem is an extension of Routh Stability criterion. The Kharitonov polynomials are given by

$$P(s) = P_0 + P_1 s + P_2 s^2 + \ldots\ldots + P_n s^n$$

Where the coefficients are given by

$$P_0 \in [a_0,b_0],\ P_1 \in [a_1,b_1], \ldots\ldots P_n \in [a_n,b_n]$$

$$P_1(s) = a_0 + a_1 s + b_2 s^2 + \ldots\ldots + a_n s^n$$
$$P_2(s) = a_0 + b_1 s + b_2 s^2 + \ldots\ldots + a_n s^n$$
$$P_3(s) = b_0 + a_1 s + a_2 s^2 + \ldots\ldots + b_n s^n$$
$$P_4(s) = b_0 + b_1 s + a_2 s^2 + \ldots\ldots + b_n s^n$$

Let us consider a control system with PID controller and the transfer function is given by

$$G(s) = \frac{\mathrm{N(s)}}{\mathrm{D(s)}}$$

$$\mathrm{C(s)} = Kp + \frac{Ki}{s} + Kds$$

Controller should satisfy the gain margin and phase margin requirements which are found by plotting the Kp-Ki curve. The design specification for the FOTP process should have Minimum Phase Margin of 2.12 and Minimum Gain Margin of 1.1.
The nominal controller parameter be K^0. Therefore the nominal value will be K_p^0 and K_i^0.

STABLE FOPTD PROCESS:
Let us consider a stable FOTPD process with given uncertainties,

$$G(s) = \frac{Ke^{-Ls}}{(Ts + 1)}$$

Where the nominal value be $K_p^o = 1.4375$, $K_i^o = 1.25$,
T = [0.9, 1.1], K = [0.8, 1.2], L = [0.29, 0.31]

The time delay is given as

$$e^{-Ls} = \frac{1}{1 + Ls}$$

Objective Function:
To obtain the objective function the interval polynomial P(s) should satisfy the following necessary and sufficient conditions. Let $P_i \in [x_i,y_i]$, for all i.
Necessay conditions;
$y_i > x_i > 0$, i=1,2….n
$x_i x_{i+1} > y_{i-1} y_{i+2} > 0$, i=1,2….n-2
Sufficient conditions:
$y_i > x_i > 0$, i=1,2….n
$0.4655 x_i x_{i+1} > y_{i-1} y_{i+2} > 0$, i=1,2….n-2

To obtain the objective function for an interval block the controller parameters (K_p^0, K_i^0, K_d^0) are obtained by pole placement method.
Therefore
$J = \min \|K-K^0\|^2$
$J = (K_p - K_p^o)^2 + (K_i - K_i^o)^2 + (K_d - K_d^o)^2$
Since K_d is fixed we can neglect it
Finally,

$$\mathbf{J = (K_p - K_p^o)^2 + (K_i - K_i^o)^2}$$

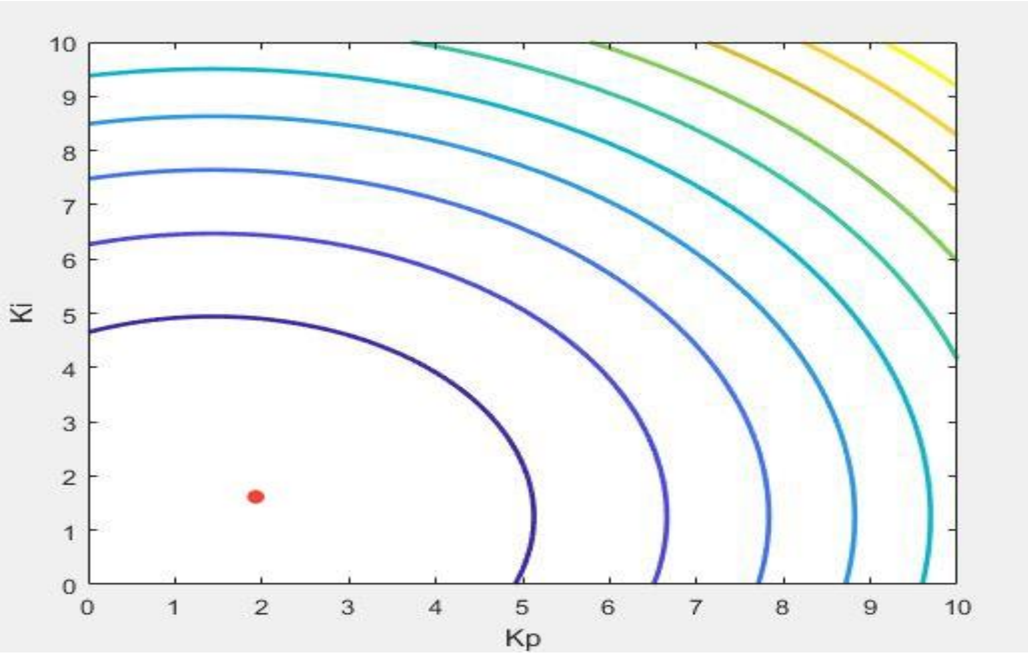
Constraints:
Hurwitz Criteria are used to frame the Constraints. These are designed to **minimize** the objective function. Therefore the constraints can be formulated as

$$\mathbf{-0.952\ K_p + 0.4092\ K_i - 1.19 + \in \leq 0}$$
$$\mathbf{-0.952\ K_p + 0.8790\ K_i - 1.19 + \in \leq 0}$$
$$\mathbf{-0.8\ K_p - 1 + \in \leq 0}$$
$$\mathbf{-1.2\ K_p - 1 + \in \leq 0}$$
$$\mathbf{-0.8\ K_i + \in \leq 0}$$
$$\mathbf{-1.2\ K_i + \in \leq 0}$$

Where $\in$ is the small variation in the uncertainties. Let $\in = 1.5$.

NON-LINEAR PROGRAMMING:

The Objective function is a nonlinear equation we can solve this optimization problem using Nonlinear Programming with Linear Inequality constraints. The contour of the objective function and the optimum point is shown below,



Objective function J

METHOD OF LAGRANGE MULTIPLIERS:

The method of Lagrange multipliers is used for finding the local minima and maxima of a objective function given the equality or inequality linear or non-linear constraints.

Let let us consider the objective function and constraints be

maximize $f(x)$

subject to $Ax \geq 0$

where i = 1….m, m = number of constraints

Changed to

min $f(x)$

subject to $Ax = b$, all the active constraints

For the inactive constraints, we associate with zero Lagrange multipliers:

$\lambda_i (a^T x_* - b_i) = 0$, I = 0,1, 2….,

For the points to be optimum it should satisfy the necessary and sufficient conditions (KKT)

Necessary conditions:

- $\nabla f(x) = A^T \lambda_*$
- $\lambda_* \geq 0$
- $\lambda_*^T (Ax_* - b) = 0$
- $Z^T \nabla^2 f(x_*) Z$ is a positive semi-definite

Where Z is the null space matrix of active constraints.

Sufficient conditions:

- It should satisfy necessary conditions.

- $A_i^T x_* - b_i = 0$ or $\lambda_* = 0$
- $Z^T \nabla^2 f(x_*) Z$ is a positive definite

If the lambda is degenerative (i.e. degenerative constraints) then it should satisfy the following necessary conditions

- It should satisfy the necessary conditions
- $\lambda_*^T (Ax_* - b) = 0$
- $Z^T \nabla^2 f(x_*) Z$ is a positive definite, where Z is the null space with respect to non-degenerate active constraints

After simplification of the Objective junction and Constraints,

Min $\quad$ $\mathbf{J = (K_p - 1.435)^2 + (K_i - 1.25)^2}$

Subject to $\quad$ $\mathbf{0.952\, K_p - 0.4092\, K_i \geq 0.31}$

$\mathbf{0.952\, K_p - 0.8790\, K_i \geq 0.31}$

$\mathbf{0.8\, K_p \geq 0.5}$

$\mathbf{1.2\, K_p \geq 0.5}$

$\mathbf{0.8\, K_i \geq 1.5}$

$\mathbf{1.2\, K_i \geq 1.5}$

Since there are 6 lagrangain variable, $2^n = 2^6 = 64$ combinations are possible

By using the method of lagrange the optimum value can be found at point Kp = 2.0569, KI = 1.8750

The MATLAB code for solving the objective function is given below

MATLAB Code:

```
fun = @(k)100*(k(1)-1.4375)^2 + (k(2)-1.25)^2;
k0 = [0,0];
A = [-0.952, 0.4092; -0.952, 0.8790; -0.8, 0; -1.2, 0; 0 -0.8; 0, -1.2];
b = [-0.31; -0.31; -0.5; -0.5; -1.5; -1.5];
k = fmincon(fun,k0,A,b)
```

OUTPUT:

```
 k = 2.0569    1.8750
```

QUADRATIC PROGRAMMING: (BY CONVERTING TO LINEAR EQUATIONS):

In this method the nonlinear objective method is converted to linear equations using KKT (Karush-Kuhn-Tucker) formulas. Then the linear problem is solved using Simplex method (Two-Phase Simplex method).

Objective Function,

$\text{Max } Z = CX + X^T DX$

$AX \geq B$

$X \geq 0$

$\lambda \geq 0$

Conversion to linear programming can be done using the below formulas

$$\begin{bmatrix} -2D & A' & -I & 0 \\ A & 0 & 0 & I \end{bmatrix} \begin{bmatrix} X \\ \lambda \\ \mu \\ s \end{bmatrix} = \begin{bmatrix} C' \\ B \end{bmatrix}$$

λ_i , i = 1,2,….m, m = number of constraints (6 in this problem)
μ_j, j = 1,2,……n, n = number of variables (2 in this problem)
S = Slack variables

After converting to linear problem based on the above conversion, we can use any linear programming method to solve this. In this simplex method is used to find the optimum value.
2 phase method:

$$\text{Min } P = C^T x$$
$$\text{subject to } Ax=b,$$
$$x \geq 0$$

Phase 1

$$\text{Min } P = \sum_i ai$$
$$\text{Subject to } Ax + a = b,$$
$$x, a \geq 0$$

After solving phase 1 we can proceed to phase 2 (similar to simplex method)

The MATLAB code for quadratic programming is given below

MATLAB Code:

```
[Kp,Ki] = meshgrid(0:.1:10, 0:.1:10);
J=(Kp-1.4375).^2+(Ki-1.25).^2
figure(1)
contour(Kp,Ki,J,10,'LineWidth', 2)
xlabel('Kp')
ylabel('Ki')
zlabel('J')
hold on
H = [2 0; 0 2];
f = [-2.875; -2.5];
A = [-0.952 0.4092; -0.952 0.8790; -0.8 0; -1.2 0; 0 -0.8; 0 -1.2];
b = [-0.31; -0.31; -0.5; -0.5; -1.5; -1.5];
LB =[0,0];
UB =[10,10];
[K,P] = quadprog(H,f, A, b,[],[],LB,UB);
```

OUTPUT:
K = 2.0569 1.8750
P = - 2.8547

CONCLUSION:

Thus by using Nonlinear Programming on the objective function for a control system such as in FOPTD process we can improve the stability and also the performance of an interval block. Thus Optimization techniques play an important role in control systems. Also it can be inferred that the nonlinear programming methods are much simpler in computation than the linear programming methods for a larger problems.

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