Assignment:- 2

AI1110: Probability and Random Variables
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Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that:

(i) the youngest is a girl,

(ii) at least one is a girl?

Solution: The probability of having a boy or a girl is the same, which means that the probability of having two girls is 1/4, having two boys is 1/4, and having one boy and one girl is 1/2. We can use the conditional probability formula to calculate the probabilities requested:

(i) Let A be the event that both children are girls, and B be the event that the youngest child is a girl. We want to calculate $\Pr(A|B)$.

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{1}$$

The event A and B means that both children are girls and the youngest is a girl. The probability of this happening is 1/4 because the only possibility is GG. The probability of the youngest child being a girl is 1/2 because there are two possible outcomes: GG and GB. Therefore:

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{2}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}} \tag{3}$$

$$= \frac{1}{2} \tag{4}$$

(ii) Let C be the event that at least one child is a girl. We want to calculate $\Pr(A|C)$.

$$\Pr(A|C) = \frac{\Pr(AC)}{\Pr(C)} \tag{5}$$

The event A and C means that both children are girls and at least one is a girl. The probability of this happening is 1/4 + 1/4 = 1/2, because the possible outcomes are GG, GB, and BG (which are the same as GB). The probability of at least one child being a girl is 3/4, because the only outcome that doesn't have a girl is BB. Therefore:

$$\Pr(A|C) = \frac{\Pr(AC)}{\Pr(C)} \tag{6}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} \tag{7}$$

$$= \frac{1}{3} \tag{8}$$

Therefore, the conditional probability that both children are girls given that the youngest is a girl is 1/2, and given that at least one is a girl is 1/3.