Assignment:- 2

AI1110: Probability and Random Variables
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Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

(1) the youngest is a girl
(2) at least one is a girl

Solution: Let X_1 and X_2 be independent random variables that represent the gender of the first and second child, respectively. We assume that $X_i = 1$ if the i-th child is a girl and $X_i = 0$ if the i-th child is a boy. Then, we have $\Pr(X_i = 1) = \Pr(X_i = 0) = 1/2$ for $i = 1, 2$. We want to find the conditional probability that both children are girls given two different conditions:

(1) The younger child is a girl.
(2) At least one of the children is a girl.

For (1), we want to find $\Pr(X_1 = 1, X_2 = 1 | X_1 = 1)$. By the definition of conditional probability, we have:

$$\Pr(X_1 = 1, X_2 = 1 | X_1 = 1) = \frac{\Pr(X_1 = 1, X_2 = 1, X_1 = 1)}{\Pr(X_1 = 1)} = \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1)} = \frac{\Pr(X_1 = 1)\Pr(X_2 = 1)}{\Pr(X_1 = 1)} \tag{1}$$

$$= \Pr(X_2 = 1) \tag{2}$$

$$= \frac{1}{2}. \tag{3}$$

For (2), we want to find $\Pr(X_1 = 1, X_2 = 1 | X_1 = 1 \text{ or } X_2 = 1)$. By the law of total probability, we have:

$$\Pr(X_1 = 1, X_2 = 1 | X_1 = 1 \text{or} X_2 = 1) = \frac{\Pr(X_1 = 1, X_2 = 1)}{\Pr(X_1 = 1) + \Pr(X_2 = 1) - \Pr(X_1 = 1, X_2 = 1)} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{4}} = \frac{1}{3}. \tag{4}$$

Therefore, the conditional probability that both children are girls given that the younger one is a girl is $\frac{1}{2}$, and the conditional probability that both children are girls given that at least one is a girl is $\frac{1}{3}$.