

Assignment:- 2

AI1110: Probability and Random Variables

Indian Institute of Technology, Hyderabad

Dudekula Dheeraj
CS22BTECH11019

NCERT(12.13.1.12)

Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

- (1) the youngest is a girl
- (2) at least one is a girl

Solution: Let B be the event that a child is a boy, and G be the event that a child is a girl. We are given that the probability of each of these events is $\Pr(B) = \Pr(G) = \frac{1}{2}$.

We want to find the conditional probability that both children are girls given two different conditions:

- (1) The youngest child is a girl.
- (2) At least one of the children is a girl.

Let's start with (1) first.

Let Y be the event that the younger child is a girl. Then we want to find $\Pr(GG|Y)$, the probability that both children are girls given that the younger one is a girl.

Using Bayes' theorem, we have:

$$\Pr(GG|Y) = \frac{\Pr(Y|GG) \Pr(GG)}{\Pr(Y)} = \frac{1 \cdot \frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \quad (1)$$

Therefore, the conditional probability that both children are girls given that the youngest is a girl is $\frac{1}{2}$.

Now let's consider (2).

Let O be the event that at least one of the children is a girl. Then we want to find $\Pr(GG|O)$, the probability that both children are girls given that at least one is a girl.

Using Bayes' theorem again, we have:

$$\Pr(GG|O) = \frac{\Pr(O|GG) \Pr(GG)}{\Pr(O)} = \frac{1 \cdot \frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \quad (2)$$

Therefore, the conditional probability that both children are girls given that at least one is a girl is $\frac{1}{3}$.