Assignment:- 2

AI1110: Probability and Random Variables
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Question: Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that

(1) at least one is a girl
(2) the youngest is a girl

Solution: Lets assume two independent random variables, X_1 and X_2, which represent the gender of the first and second child in a family. We define the variables as follows:

X_i	Gender of i-th child
1	Girl
0	Boy

Both X_1 and X_2 have an equal probability of $\frac{1}{2}$ of being a boy or a girl.

Now, let's address the two parts separately:

(1) We want to find the probability that both children are girls, given that at least one of them is a girl ($X_1 = 1$ or $X_2 = 1$). We can use the law of total probability to calculate this. The probability can be expressed as:

$$\Pr\left(X_1 = 1 \wedge X_2 = 1 | X_1 = 1 \vee X_2 = 1\right) = \frac{\Pr\left(X_1 = 1 \wedge X_2 = 1\right)}{\Pr\left(X_1 = 1 \vee X_2 = 1\right)} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{2} - \frac{1}{4}} = \frac{1}{3}. \tag{1}$$

Therefore, the conditional probability that both children are girls, given that at least one of them is a girl, is $\frac{1}{3}$.

(2) We want to find the probability that both children are girls, given that the older child is a girl ($X_1 = 1$). Using the definition of conditional probability, we have:

$$\Pr\left(X_1 = 1 \wedge X_2 = 1 | X_1 = 1\right) = \frac{\Pr\left(X_1 = 1 \wedge X_2 = 1\right)}{\Pr\left(X_1 = 1\right)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}. \tag{2}$$

Therefore, the conditional probability that both children are girls, given that the older child is a girl, is $\frac{1}{2}$.