PART 3 : MATHEMATICS

SECTION-I

1. Ans (C)

$$\int \frac{x+\cos 2x+1}{x\cos^2 x}dx = f(x) + K\log|x| + C \text{ and}$$

$f(\pi/4) = 1$

$$\int \frac{x+2\cos^2 x}{x\cos^2 x}dx = \int \frac{x}{x\cos^2 x}dx + \int \frac{2}{x}dx =$$

$\tan x + 2\log|x| + C$

$f(x) = \tan x,\ K = 2$ (by comparing)

Now, $f(0) + 10K = \tan 0 + 10 \cdot 2 = 0 + 20 = 20$

2. Ans (C)

$$I = \int e^{\tan^{-1}x}(1+x+x^2)\left(-\left(\frac{1}{1+x^2}\right)dx\right)$$

$$= -\int e^{\tan^{-1}x}\left(1+\frac{x}{1+x^2}\right)dx$$

$$= -\int e^{\tan^{-1}x}dx - \int x\frac{e^{\tan^{-1}x}}{1+x^2}dx$$

$$= -\int e^{\tan^{-1}x}dx - \left[x.e^{\tan^{-1}x} - \int e^{\tan^{-1}x}dx\right]$$

$$= -xe^{\tan^{-1}x} + c$$

3. Ans (A)

Let

$$A = \lim_{n\to\infty}\left(\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \ldots\ldots + \frac{n}{n^2}\sec^2 1\right)$$

$$\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{n}\sec^2\frac{1}{n^2} + \frac{2}{n}\sec^2\frac{4}{n^2} + \ldots\ldots + \frac{n}{n}\sec^2 1\right)$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2$$

$$\therefore\ A = \int_0^1 x\sec^2(x^2)\,dx$$

Put $x^2 = t$

$$\Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$$

$$\therefore\ A = \frac{1}{2}\int_0^1 \sec^2 t\,dt$$

$$= \frac{1}{2}\tan 1$$

4. Ans (C)

Put $y = vx$

$$v + x\frac{dv}{dx} = -\left(\frac{1+3v^2}{3+v^2}\right)$$

$$x\frac{dv}{dx} = -\frac{(v+1)^3}{3+v^2}$$

$$\frac{(3+v^2)\,dv}{(v+1)^3} + \frac{dx}{x} = 0$$

$$\int\frac{4dv}{(v+1)^3} + \int\frac{dv}{v+1} - \int\frac{2dv}{(v+1)^2} + \int\frac{dx}{x} = 0$$

$$\frac{-2}{(v+1)^2} + \ln(v+1) + \frac{2}{v+1} + \ln x = c$$

$$\frac{-2x^2}{(x+y)^2} + \ln\left(\frac{x+y}{x}\right) + \frac{2x}{x+y} + \ln x = c$$

$$\frac{2xy}{(x+y)^2} + \ln(x+y) = c$$

$\therefore c = 0$, as $x = 1, y = 0$

$$\therefore\ \frac{2xy}{(x+y)^2} + \ln(x+y) = 0$$

5. Ans (D)

$$I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{x+\frac{\pi}{4}}{2-\cos 2x}dx \qquad (1)$$

$x \longrightarrow -x$

$$I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{-x+\frac{\pi}{4}}{2-\cos 2x}dx \qquad (2)$$

(1) + (2)

$$2I = \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{\frac{\pi}{2}}{2-\cos 2x}dx$$

$$I = \frac{\pi}{4}.2\int_0^{\frac{\pi}{4}}\frac{dx}{2-\cos 2x}dx$$

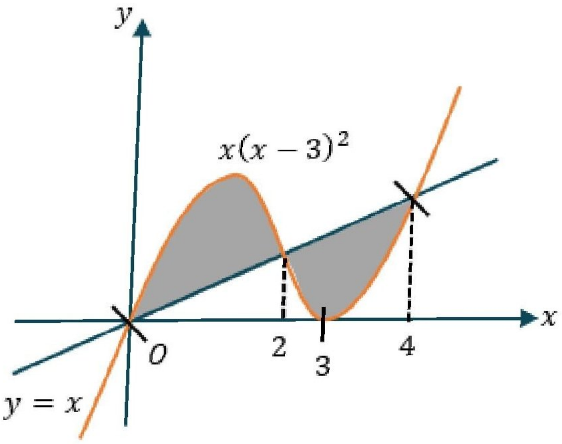
$$I = \frac{\pi}{4}.2\int_0^{\frac{\pi}{4}}\frac{(1+\tan^2 x)\,dx}{2(1+\tan^2 x) - (1-\tan^2 x)}$$

$$I = \frac{\pi}{4}\int_0^1\frac{dt}{3t^2+1}$$

$$\Rightarrow I = \frac{\pi}{2\sqrt{3}}\tan^{-1}\sqrt{3}$$

$$I = \frac{\pi^2}{6\sqrt{3}}$$

6. **Ans (D)**



$x(x-3)^2 = x$

$\Rightarrow$ x 0, 2, 4

Required area

$$= \int_0^2 \left(x(x-3)^2 - x\right)dx + \int_2^4 \left(x - x(x-3)^2\right)dx$$

$$= \int_0^2 \left(x^3 - 6x^2 + 8x\right)dx - \int_2^4 \left(x^3 - 6x^2 + 8x\right)dx$$

$$= \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_0^2 - \left[\frac{x^4}{4} - 2x^3 + 4x^2\right]_2^4$$

$= 4 + 4 = 8$ sq units

7. **Ans (A)**

$$x = \frac{1-t^2}{1+t^2}, \; y = \frac{2t}{1+t^2}$$

$t = \tan\theta$

$$x = \frac{1-\tan^2\theta}{1+\tan^2\theta}, \quad y = \frac{2\tan\theta}{1+\tan^2\theta}$$

$x = \cos 2\theta$,$y = \sin 2\theta$

$x^2 + y^2 = 1$

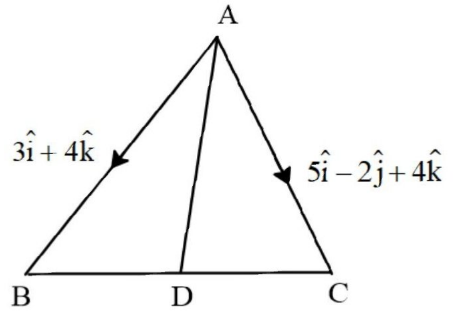
Area $= \pi r^2 = \pi$ .... $\because r = 1$

8. **Ans (D)**

$$\sin^{-1}\frac{|2\times 3 + (-1)\times 6 + 2\times(-2)|}{\sqrt{2^2 + (-1)^2 + 2^2} \cdot \sqrt{3^2 + 6^2 + (-2)^2}}$$

9. **Ans (C)**

Let D be the mid-point of BC. Then,



$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\Rightarrow \overrightarrow{AD} = \frac{\left(3\hat{i} + 4\hat{k}\right) + \left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)}{2} = 4\hat{i} + \hat{j} + 4\hat{k}$$

$$\Rightarrow \left|\overrightarrow{AD}\right| = \sqrt{16 + 1 + 16} = \sqrt{33}$$

10. **Ans (D)**

It is given that $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and

$\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$.

$\therefore \vec{a} + 2\vec{b} = x\vec{c}$, and $\vec{b} + 3\vec{c} = y\vec{a}$ for some

$x, y \in R$

$\therefore \vec{a} + 2\vec{b} + 6\vec{c} = (x+6)\vec{c}$

Also, $\vec{a} + 2\vec{b} + 6\vec{c} = (1+2y)\vec{a}$

$\therefore (x+6)\vec{c} = (1+2y)\vec{a}$

$\Rightarrow x + 6 = 0$ and $1 + 2y = 0$ [$\because \vec{a}, \vec{c}$ are non-

collinear]

$\Rightarrow x = -6$ and $y = -1/2$

$\Rightarrow \vec{a} + 2\vec{b} + 6\vec{c} = \vec{0}$

11. Ans (C)

$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$

$\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$

Unit vector perpendicular to plane of $\vec{a}$ and $\vec{b}$ is given by :-

$$\pm\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$$

$$\bar{a}\times\bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix}$$

$= 15\hat{i} - 10\hat{j} + 30\hat{k}$

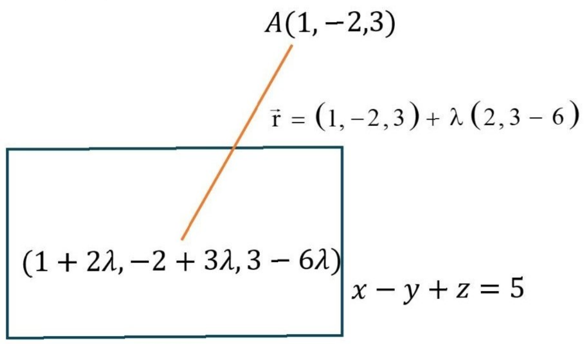
$= 5\left(3\hat{i} - 2\hat{j} + 6\hat{k}\right)$

$|\bar{a}\times\bar{b}| = 5.\sqrt{3^2 + (-2)^2 + 36}$

$= 5 \times 7$

$\Rightarrow$ Unit vector $= \dfrac{3\hat{i} - 2\hat{j} + 6\hat{k}}{7}$

12. Ans (D)



$(1 + 2\lambda) + 2 - 3\lambda + 3 - 6\lambda = 5$

$\Rightarrow 6 - 7\lambda = 5 \;\Rightarrow \lambda = \dfrac{1}{7}$

so, $P = \left(\dfrac{9}{7}, -\dfrac{11}{7}, \dfrac{15}{7}\right)$

$AP = \sqrt{\left(1 - \dfrac{9}{7}\right)^2 + \left(-2 + \dfrac{11}{7}\right)^2 + \left(3 - \dfrac{15}{7}\right)^2}$

$AP = \sqrt{\left(\dfrac{4}{49}\right) + \dfrac{9}{49} + \dfrac{36}{49}} = 1$

13. Ans (C)

$\because A : B : C = 3 : 5 : 4$

$\Rightarrow A + B + C = 12x = 180^\circ \Rightarrow x = 15^\circ$

$\therefore A = 45^\circ, B = 75^\circ, C = 60^\circ$

$\dfrac{a}{\sin 45^\circ} = \dfrac{b}{\sin 75^\circ} = \dfrac{c}{\sin 60^\circ} = K$ (say)

$\therefore a = \dfrac{1}{\sqrt{2}}K, b = \dfrac{\sqrt{3}+1}{2\sqrt{2}}K, c = \dfrac{\sqrt{3}}{2}K$

$\therefore a + b + c\sqrt{2} = 3b.$

14. Ans (A)

We have

$\dfrac{2\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{2\cos C}{c} = \dfrac{a}{bc} + \dfrac{b}{ac}$

Multiplying both sides of abc, we get

$\Rightarrow \quad 2bc\cos A + ac\cos B + 2ab\cos C = a^2 + b^2$

$\Rightarrow \quad (b^2 + c^2 - a^2) + \dfrac{(a^2 + c^2 - b^2)}{2} + (a^2 + b^2 - c^2) = a^2 + b^2$

$\Rightarrow \quad c^2 + a^2 - b^2 = 2a^2 - 2b^2$

$\Rightarrow \quad b^2 + c^2 = a^2$

$\therefore \quad \Delta ABC$ is right angled at A.

$\Rightarrow \quad \angle A = 90^\circ$

15. Ans (A)

Let the equation be $x^2 + y^2 + 2gx + 2fy + c = 0$

it passes through $(-1, -3)$ and $(3, 0)$ therefore

$10 - 2g - 6f + c = 0$ ....(i)

$9 + 6g + c = 0$ ....(ii)

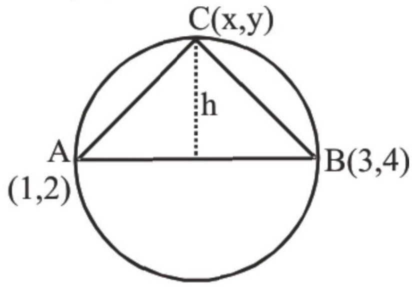
Slope of tangent $= -4/3$

$\left(\dfrac{0+f}{3+g}\right)\left(\dfrac{-4}{3}\right) = -1$

$\Rightarrow 3g - 4f + 9 = 0$ ....(iii)

solving $g = -1, f = 3/2, c = -3$

16. Ans (B)



$\because$ A & B are end's of diameter, diameter $= 2\sqrt{2}$

$\therefore$ radius $= \sqrt{2}$

Let height of ΔABC is h.

Now, $\dfrac{1}{2}$.Base $\times$ h = 1 {Base=daimeter

$\Rightarrow \quad \dfrac{1}{2} \times 2\sqrt{2} \times h = 1$

$\Rightarrow \quad h = \dfrac{1}{\sqrt{2}}$

$\because \quad h < r$

therefore no. of position of C is 4.

17. Ans (C)

Let PQ be a diameter of the circle

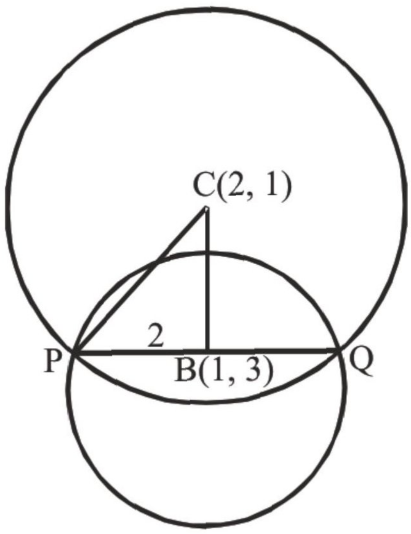
$x^2 + y^2 - 2x - 6y + 6 = 0$ such that PQ is a

chord of the circle having its centre at C(2, 1).

Clearly,

Radius = CP = $\sqrt{BC^2 + BP^2}$

$\Rightarrow$ Radius = $\sqrt{(2-1)^2 + (1-3)^2 + 2^2} = 3$



18. Ans (B)

$$y = \frac{1}{x} \quad \Rightarrow \frac{dy}{dx} = -\frac{1}{x^2}$$

$$\Rightarrow x^2 dy + dx = 0$$

$$\Rightarrow \frac{x^2}{\sqrt{1+x^4}} dy + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{\frac{1}{x^4}+1}} + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} = 0$$

$$\Rightarrow \frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} + 4 = 4$$

19. Ans (A)

$$f''(x) = -\frac{1}{x^2} - \pi^2 \sin(\pi x)$$

$$\Rightarrow f'(x) = \int -\frac{1}{x^2} - \pi^2 \sin(\pi x) \cdot dx$$

$$\Rightarrow f'(x) = \frac{1}{x} + \pi^2 \cdot \frac{\cos(\pi x)}{\pi} + c$$

$$\left\{ \begin{array}{l} \because f'(2) = \pi + \dfrac{1}{2} \\ \dfrac{1}{2} + \pi\cos(2x) + c = \pi + \dfrac{1}{2} \\ \Rightarrow c = 0 \end{array} \right\}$$

$$\Rightarrow f'(x) = \frac{1}{x} + \pi\cos(\pi x)$$

$$\Rightarrow f(x) = \int \frac{1}{x} + \pi\cos(\pi x) \cdot dx$$

$$\Rightarrow f(x) = \ln(x) + \frac{\pi\sin(\pi x)}{\pi} + C_1$$

$$\left\{ \begin{array}{l} \because f(1) = 0 \\ \Rightarrow \sin\pi + c_1 = 0 \\ \Rightarrow c_1 = 0 \end{array} \right\}$$

$$\Rightarrow f(x) = \ln(x) + \sin(\pi x)$$

$$\therefore f\left(\frac{1}{2}\right) = \ln\left(\frac{1}{2}\right) + \sin\left(\frac{\pi}{2}\right) = 1 - \ell n(2)$$

20. Ans (C)

$$\int_6^{f(x)} 4t^3 dt = (x-2)\, g(x)$$

Differentiate both sides

$$\lim_{x\to 2} g(x) = \frac{\lim\limits_{x\to 2} \int_6^{f(x)} 4t^3 dt}{x-2}$$

Use L'hopital's rule

$$= \lim_{x\to 2} \frac{f'(x) 4f^3(x)}{1} = 18$$

PART 3 : MATHEMATICS

SECTION-II

1. Ans (1)

$$\int_{-1}^{1} \frac{dx}{\left(1+x^3+\sqrt{1+x^6}\right)}$$

$$\frac{1}{(1+x^3)+\left(\sqrt{(1+x^6)}\right)} \times \frac{1+x^3-\sqrt{(1+x^6)}}{1+x^3-\sqrt{(1+x^6)}}$$

$$= \frac{(1+x^3)-\sqrt{(1+x^6)}}{(1+x^3)^2-(1+x^6)}$$

$$\frac{1+x^3-\sqrt{1+x^6}}{1+x^6+2x^3-1-x^6} = \frac{1}{2x^3}+\frac{x^3}{2x^3}-\frac{\sqrt{1+x^6}}{2x^3}$$

$$\int_{-1}^{1} \frac{1}{2x^3}dx + \int_{-1}^{1} \frac{1}{2}dx - \int \frac{\sqrt{1+x^6}}{2x^3}dx$$

$$\Rightarrow \int_{-1}^{1} \frac{1}{2x^3}dx + \frac{1}{2}\int_{-1}^{1} dx + \int_{-1}^{1} \underbrace{\frac{\sqrt{1+x^6}}{x^3}}_{\text{odd function}} dx = 1$$

2. Ans (8)

$x dy - (y^2 - 4y)dx = 0, x > 0$

$$\int \frac{dy}{y^2-4y} = \int \frac{dx}{x}$$

$$\int \left(\frac{1}{y-4}-\frac{1}{y}\right) dy = 4\int \frac{dx}{x}$$

$$\log_e|y-4| - \log_e|y| = 4\log_e x + \log_e c$$

$$\frac{|y-4|}{|y|} = cx^4 \xrightarrow{(1,2)} c = 1$$

$$|y-4| = |y|x^4$$

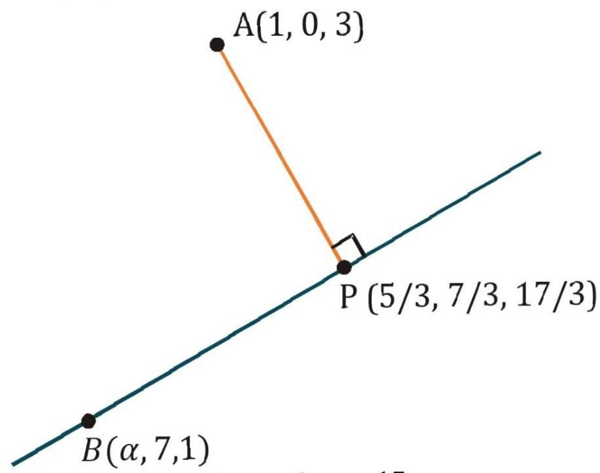
C-1 and C-2

$y - 4 = yx^4$ $\qquad$ $y - 4 = -yx^4$

$y = \frac{4}{1-x^4}$ $\qquad$ $y = \frac{4}{1+x^4}$

$y(1)$ ND (rejected) $\qquad$ $y(1) = 2$

3. Ans (4)



D.R. of BP $= < \frac{5}{3} - \alpha, \frac{7}{3} - 7, \frac{17}{3} - 1 >$

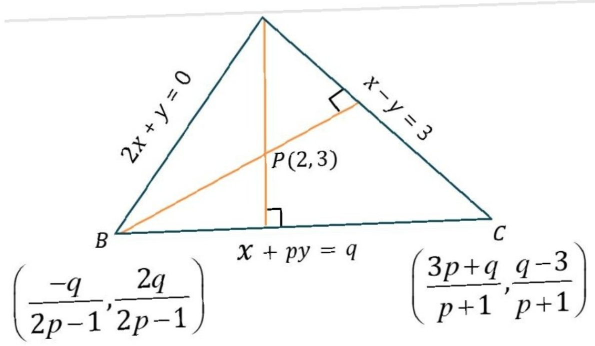
D.R. of AP $= < \frac{5}{3} - 1, \frac{7}{3} - 0, \frac{17}{3} - 3 >$

$BP \perp^r AP$

$\Rightarrow \alpha = 4$

4. Ans (5)

P is orthocentre



$\Rightarrow AP \perp BC$

$$\Rightarrow \left(-\frac{1}{p}\right)\left(\frac{3+2}{2-1}\right) = -1$$

$$\Rightarrow \frac{5}{p} \Rightarrow p = 5$$

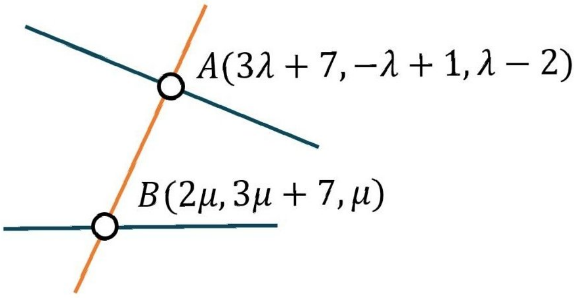
$\because BP \perp AC$

$$\Rightarrow \frac{27-2q}{18+q} = -1 \qquad \Rightarrow q = 27 + 18$$

$\Rightarrow q = 45$

$\therefore p + q = 5 + 45 = 50.$

5. **Ans (84)**



DR's of AB

$(3\lambda - 2\mu + 7, -\lambda - 3\mu - 6, \lambda - \mu - 2)$

$$\frac{3\lambda - 2\mu + 7}{1} = \frac{-\lambda - 3\mu - 6}{-4} = \frac{\lambda - \mu - 2}{2}$$

Taking first (2) $-12\lambda + 8\mu - 28 = -\lambda - 3\mu - 6$

$\lambda - \mu + 2 = 0$

Taking second & third

$-2\lambda - 6\mu - 12 = -4\lambda + 4\mu + 8$

$\lambda - 5\mu - 10 = 0$

After solving above two equation

$\lambda = -5, \mu = -3$

$A = (-8, 6, -7)$

$B = (-6, -2, -3)$

$(AB)^2 = 4 + 64 + 16 = 84$