# Analyzing the Impact of Voting Rules and External Factors on Election Outcomes

#### Abstract

This journal report delves into the mathematical analysis of different voting rules, specifically focusing on Plurality, Approval, and Borda voting systems. Additionally, the impact of external factors, such as pre-opinion polls and voter turnout rates, on the final poll results is thoroughly examined. Through rigorous mathematical modeling and analysis, this report aims to provide valuable insights into the dynamics of democratic elections.

#### 1 Introduction

Democratic elections serve as the cornerstone of modern societies. Understanding the intricacies of voting systems and the influence of external factors is crucial for ensuring fair and transparent electoral processes. This report explores various voting rules and investigates their behavior under different conditions.

### 2 Plurality Voting

In Plurality voting, each voter selects one candidate, and the candidate with the most votes wins. Mathematically, if n candidates are competing, the winner (W) is the candidate with the maximum votes  $(V_i)$ :

$$W = \operatorname{argmax}_{i}(V_{i})$$

### 3 Approval Voting

Approval voting allows voters to approve multiple candidates. The candidate with the most approvals wins. Mathematically, let  $A_i$  represent the approval set for candidate i. The winner is the candidate maximizing the approval count:

$$W = \operatorname{argmax}_{i}(|A_{i}|)$$

### 4 Borda Voting System

In the Borda voting system, each candidate receives points based on their position in the voter's preference list. Mathematically, for n candidates, the Borda score  $(B_i)$  for candidate i is calculated as:

$$B_i = \sum_{i=1}^{n} (n - \text{position of } i \text{ in voter's preference list})$$

The candidate with the highest Borda score wins.

### 5 Impact of Pre-opinion Polls

Pre-opinion polls can influence voters' decisions. Let  $P_i$  represent the pre-opinion poll percentage for candidate i. A weighted voting system can be formulated:

$$V_i = (1 - \alpha) \times P_i + \alpha \times B_i$$

Here,  $\alpha$  represents the weight assigned to the Borda score, allowing the model to adjust the influence of the pre-opinion polls.

## 6 Voter Turnout Rates

Voter turnout rates (T) significantly affect the legitimacy of election results. A turnout-adjusted voting score  $(V'_i)$  can be calculated as:

$$V_i' = T \times V_i$$

This adjusted score reflects the impact of voter participation on the final outcome.

#### 7 Conclusion

Through rigorous mathematical analysis, this report highlighted the behavior of various voting rules – Plurality, Approval, and Borda systems. The incorporation of pre-opinion polls and voter turnout rates into the models provided a comprehensive understanding of their influence on election outcomes. Such analyses are crucial for enhancing the democratic process, ensuring transparency, and facilitating informed decision-making in electoral contexts.

#### References

[List academic references and sources here]