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Section - CST

Roll No. - 23

Subject :- Design and Analysis of Algorithm (DAA)

Tutorial - 1 (Answers)

~~Ans~~ Ques 1 ⇒ What do you understand by Asymptotic notations. Define different Asymptotic notation with examples.

Answer →

(i) Big O(n)

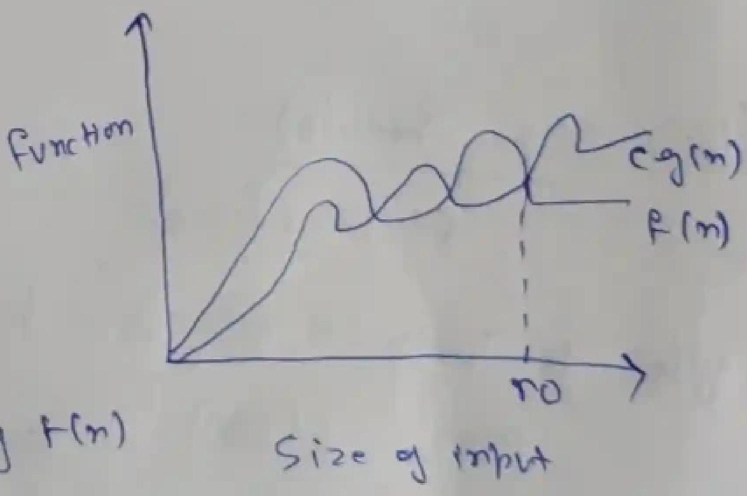
$f(n) = O(g(n))$

iff

$f(n) \le c\, g(n)$

$\forall\ n \ge n_0$

for some constant, $c > 0$



g(n) is "tight" upper bound of f(n)

Ex :→ $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \le c . n^3$

$n^2 + n = O(n^3)$

(ii) Big Omega (Ω)

Dhevraj

$$f(n) = \Omega(g(n))$$

$g(n)$ is "tight" lower bound of function $f(n)$

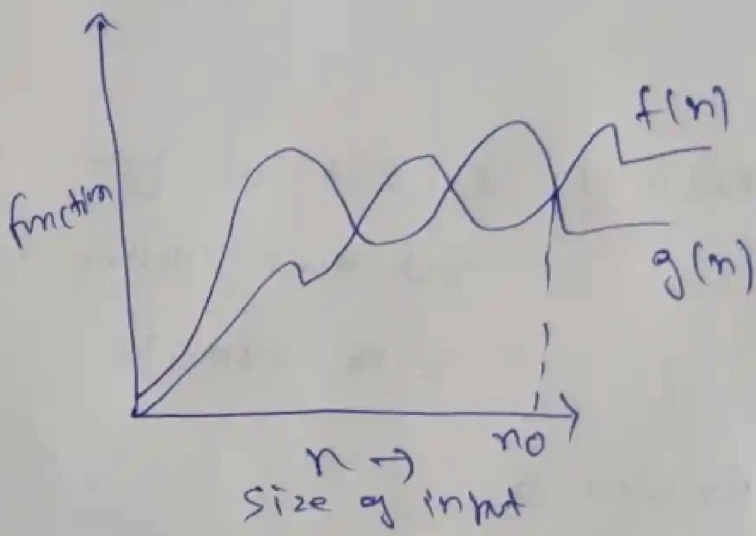
$$f(n) = \Omega(g(n))$$

iff

$$f(n) \geq c\,g(n)$$

$$\forall \ n \geq n_0$$

for some constant, $c > 0$



Example :- $f(n) = n^3 + 4n^2$

$g(n) = n^2$

$n^3 + 4n^2 = \Omega(n^2)$

(iii) Big Theta (θ)

$$f(n) = \theta(g(n))$$

$g(n)$ is both "tight" upper bound & lower bound of function $f(n)$

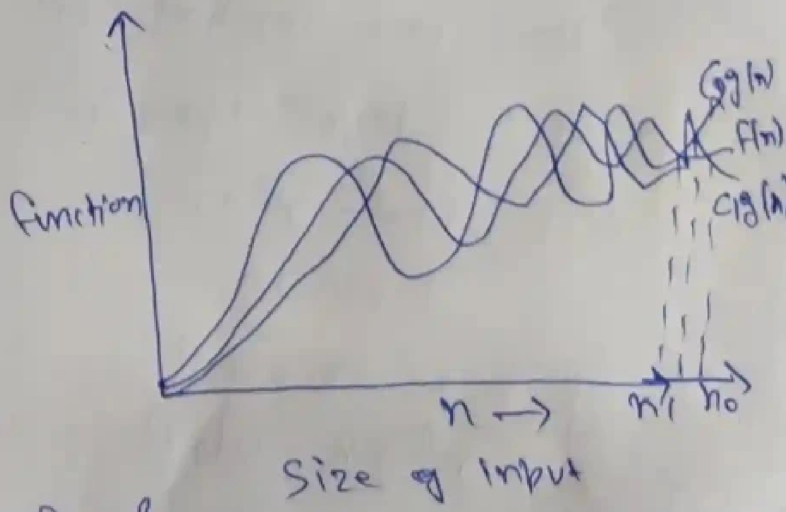
$$f(n) = \theta(g(n))$$

iff

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$\forall \ n \geq \max(n_1, n_2)$$

for some constant $c_1 > 0$ & $c_2 > 0$



Ex :- $3n+2 = \theta(n)$ as $3n+2 \geq 3n$ &

$3n+2 \leq 4n$ for n, $k1 = 3$, $k2 = 4$ & $n_0 = 2$

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(iv) Small o(o) Dheeraj

$f(n) = o(g(n))$

g(n) is upper bound of function f(n)

$f(n) = o(g(n))$

when $f(n) < c\,g(n)$

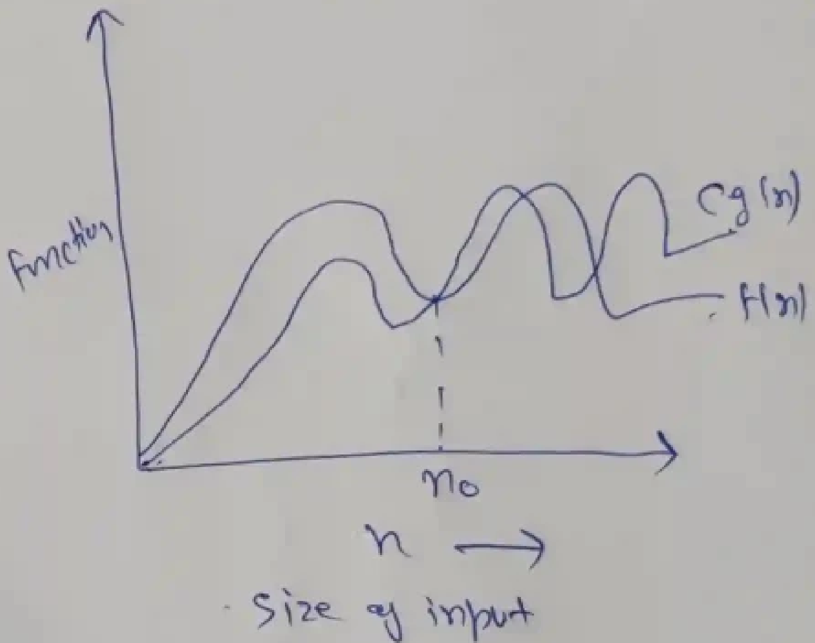
$\forall\ n > n_0$

& $\forall$ Constants, $c > 0$

Ex → $f(n) = n^2$

$g(n) = n^3$

$n^2 = o(n^3)$



(v) Small Omega(ω)

$f(n) = \omega(g(n))$

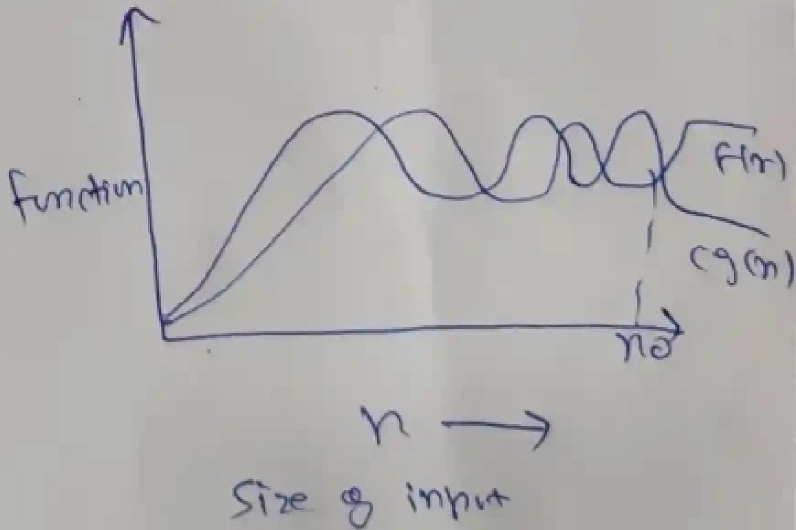
g(n) is lower bound of function f(n)

$f(n) = \omega(g(n))$

when $f(n) > c\,g(n)$

$\forall\ n > n_0$

& $\forall$ Constants $c > 0$



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Ques 2 ⇒ What should be time Complexity of -
for(i=1 to n) { i = i*2; }

Ans ⇒

```
for(i=1 to n)
{
    i = i*2;        → O(1)
}
```

$$i = \overbrace{1, 2, 4 \ldots\ldots n}^{k}$$

$$a = 1, \quad r = \frac{b2}{b1} = 2$$

G.P. k^{th} value, $t_k = ar^{k-1}$

$$t_k = 2^{k-1}$$

$$t_k = \frac{2^k}{2} \qquad \{t_k = n\}$$

$$2n = 2^k$$

$$\log_2(2n) = k\log_2 2 \qquad \{\log_a a = 1\}$$

$$k = \log_2 2n$$

$$k = \log_2 2 + \log_2 n \qquad \{\log ab = \log a + \log b\}$$

$$k = 1 + \log_2 n$$

$$O(1 + \log_2 n)$$

$$\Rightarrow \underline{\underline{O(\log n)}}$$

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Ques 3 ⇒ What should be time complexity of. Dheeraj

$$T(n) = \{3T(n-1) \text{ if } n>0, \text{ otherwise } 1\}$$

Ans ⇒ $T(n) = 3T(n-1)$ —— (1)

put $n = n-1$ in eqⁿ (1)

$T(n-1) = 3T(n-2)$ —— (2)

put value of $T(n-1)$ from (2) to (1)

$T(n) = 3[3T(n-2)] \Rightarrow 9T(n-2)$ —— (3)

put $n = n-2$ in eqⁿ (1)

$T(n-2) = 3T(n-3)$ —— (4)

put value of $T(n-2)$ from eqⁿ (4) to (3)

$\cancel{T(n) = 3[9T}$

$T(n) = 9[3T(n-3)]$

$T(n) = 27T(n-3)$

By generalizing, $T(n) = 3^k T(n-k)$ —— (5)

Let $n-k = 1$

$k = n-1$

put value of k in eqⁿ (5)

$T(n) = 3^{n-1} T(n-n+1)$

$T(n) \Rightarrow 3^{n-1} T(1)$

$T(n) = \frac{3^n}{3^1}$

$O(3^n)$

Ques4 ⇒ Time Complexity of -

Dheeraj

$T(n) = \{ 2T(n-1) - 1$ if $n > 0$, otherwise $1 \}$

Ans ⇒ $T(n) = 2T(n-1) - 1$ — (1)

put $n = n-1$ in eqn (1)

$T(n-1) = 2T(n-2) - 1$ — (2)

put value of $T(n-1)$ from (2) to (1)

$T(n) = 2[2T(n-2) - 1] - 1$

$T(n) = 4T(n-2) - 2 - 1$ — (3)

~~$T(n) = 4T(n-2) - 2$~~

put $n = n-2$ in eqn (1)

$T(n-2) = 2T(n-3) - 1$ — (4)

put value of $T(n-2)$ from eqn (4) to (3)

$T(n) = 4[2T(n-3) - 1] - 2 - 1$

$T(n) = 8T(n-3) - 4 - 2 - 1$ — (5)

~~$T(n) = 2^k T(n-k) - 2^{k-1} - 2^{k-2} - 1$~~

$T(n) = 8T(n-3) - 7$

By Generalizing, we get

$T(n) = 2^k T(n-k) - (2^k - 1)$. — (6)

let $n - k = 1$

$k = n - 1$

Put k in eqn (6)

$T(n) \Rightarrow 2^{n-1} T(n-n+1) - (2^{n-1} - 1)$

$\Rightarrow 2^{n-1} T(1) - 2^{n-1} - 1$

$\Rightarrow 2^{n-1} - 2^{n-1} + 1$

$\Rightarrow 1$

$O(1)$

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Ques 5 ⇒ What should be time Complexity of -

Dheeraj

```
Int i=1, s=1;
While (s<=n)
{
    i++;
    s=s+i;
    printf("#");

}
```

Answer ⇒

$i = 1 \quad 2 \quad 3 \quad 4 \quad 5$

$S = 1 \quad 3 \quad 6 \quad 10 \quad 15 \; \ldots \; n$

Sum of $S = 1+3+6+10+\ldots+n$ — (1)

Also $S = 1+3+6+10+\ldots$ — (2)

from (1) - (2)

$0 = 1+2+3+4+\ldots\ldots n - \sqrt{n}$

$T_k = 1+2+3+4+\ldots k$

$T_k = \frac{1}{2} k(k+1)$

for k titrations

$1+2+3+\ldots+k <= n$

$\frac{k(k+1)}{2} <= n$

$\frac{k^2+k}{2} <= n$

$O(k^2) <= n$

$k = O(\sqrt{n})$

$T(n) = O(\sqrt{n})$

Ques 6 ⇒ Time Complexity of -

Dheeraj

```
Void function(int n)
{
    int i, count=0;
    for(i=1; i*i<=n; i++)
        count++;
}
```

Answer ⇒ Here,

$i*i <= n$

$i^2 <= n$

$i <= \sqrt{n}$

$i = 1, 2, 3, 4 \ldots\ldots \sqrt{n}$

$\sum_{i=1}^{n} 1+2+3+4+\ldots+\sqrt{n}$

$T(n) = \frac{\sqrt{n} \times (\sqrt{n}+1)}{2} \Rightarrow \frac{n+\sqrt{n}}{2}$

$T(n) = O(n)$

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Ques 7 ⇒ Time Complexity of -

Dheeraj

```
Void function(int n)
{
    int i, j, k, Count = 0;
    for(i=n/2; i<=n; i++)
    {
        for(j=1; j<=n; j=j*2)
        {
            for(k=1; k<=n; k=k*2)
                Count++
        }
    }
}
```

Answer ⇒ For $k = k*2$

$k \Rightarrow \overbrace{1, 2, 4, 8 \; --- \; n}^{k}$

$a = 1, \; r = 2$

G.P. , $k \Rightarrow \frac{a(r^n - 1)}{r - 1} \Rightarrow \frac{1(2^k - 1)}{1}$

$n = 2^k$

$k = \log n$

i	j	k
$n/2$	$\log n$	$\log n * \log n$
$(n+2)/2$	$\log n$	$\log n * \log n$
⋮	⋮	⋮
n	$\log n$	$\log n * \log n$

$n * \log n * \log n$ $\quad$ $O(n \log^2 n)$ or $O((\log n)^2)$

Ques 8 ⇒ Time Complexity of -

Dheeraj

```
function(int n)
 {
     if(n==1)
       return;
     for(i=1 to n)
      {
         for(j=1 to n)
           printf("*");
      }
    function(n-3);
 }
```

Answer ⇒ for:- for(i=1 to n)

we get j = n times every turn

$\therefore i * j = n^2$

Now,

$$\left.\begin{aligned} T(n) &= n^2 + T(n-3); \\ T(n-3) &= (n-3)^2 + T(n-6); \\ T(n-6) &= (n-6)^2 + T(n-9); \end{aligned}\right\} k \text{ times}$$

$\vdots$

$T(1) = 1$

Now subs. each value in $T(n)$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

Let $n - 3k = 1$

$k = (n-1)/3$

Total terms ⇒ $k+1$

$T(n) = n^2 + (n-3)^2 + (n-6)^2 + \dots + 1$

$T(n) \approx k n^2$

$T(n) \approx \frac{(n-1)}{3} \times n^2$

$T(n) = O(n^3)$

Ques 9 => Time Complexity of -

Dheeraj

```
void function(int n)
{
    for(i = 1 to n)
    {
        for(j = 1; j < n; j = j + i)
            printf("*")
    }
}
```

Answer => for $i = 1$ $\quad j = 1 + 2 + \ldots \quad (n \geq j + i)$

$i = 2 \quad j = 1 + 3 + 5 \ldots$

$i = 3 \quad j = 1 + 4 + 7 + \ldots$

m^{th} term of A.P. is

$T(m) = a + d \times m$

$T(m) = 1 + d \times m$

$(n-1)/d = m$

for $i = 1 \quad (n-1)/1$ times

$i = 2 \quad (n-1)/2$ times

$\vdots \quad \vdots$

$i = (n-1) \quad 1$

We get, $T(n) = i_1 j_1 + i_2 j_2 \ldots \ldots i_{n-1} j_{n-1}$

$\Rightarrow \frac{(n-1)}{1} + \frac{(n-2)}{2} + \frac{(n-3)}{3} + \ldots \ldots + 1$

$\Rightarrow n + \frac{n}{2} + \frac{n}{3} + \ldots \frac{n}{n-1} - n \times 1 +$

$\Rightarrow n\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{n-1}\right] - n + 1$

$\Rightarrow n + \log n - n + 1$ $\qquad T(n) = O(n \log n)$

Since $\int \frac{1}{x} = \log x$

<u>Ques 10</u> For the functions n^k & c^n, what is the asymptotic relationship b/w these functions. Assume that k>=1 & c>1 are constants. Find out the ~~value~~ value of c & no. for which relation holds.

<u>Ans</u> As Given, n^k & c^n

relation between n^k & c^n is

$$n^k = O(c^n)$$

$$n^k \le a(c^n)$$

$$\forall\ n \ge n_0 \ \&\ \text{Constant},\ a > 0$$

for $n_0 = 1$

$$c = 2$$

$$1^k \le a^{2.1}$$

$n_0 = 1$ & $c = 2$

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