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Best Case time Complexity of Insertion sort

Given array 2, 3, 4, 5

i	0	1	2	3	4
Element					
i					
Key			3	4	5
no. of comparison			0	0	0
movements			1	1	1

Time complexity $\Rightarrow 1 + 1 + 1 = 3$
 $= O(n-1)$
 $\approx O(n)$

Modification in Insertion sort (Shell sort)

We observed that if the list is already sorted then time complexity in case of Insertion sort is $O(n)$. So we first sort the elements with some gap taken in decreasing order. Then after apply Insertion sort

50 40 30 20 10

Total elements = $n = 5$

$$\text{gap} = \left\lceil \frac{5}{2} \right\rceil = 2$$

Heery

50 40 30 20 10 ($50 > 30$ exchange)
↑ ↑

30 40 50 20 10 ($40 > 20$ exchange)
 ↑ ↑

30 20 50 40 10 ($50 > 10$ exchange)
 ↑ ↑

30 20 10 40 50 ($30 > 10$ exchange)
↑ ↑

10 20 30 40 50

~~(Now apply insertion sort)~~

$$\text{gap} = \frac{2}{2} = 1$$

Now apply insertion sort

No: of element scanned in each
pass (or gap) = $n-1$

Heapsort no: of passes performed = 2 = $\log_2(n-1)$

Time complexity = $(n-1) \log(n-1)$

$\approx O(n \log n)$

Quick Sort (Best or Avg Case)

50 60 40 10 20

main
50
i

find the greater element than 50

(1)

60 40 10 20

j

find the smaller element than 50

(2)

Swap elements

50 60 40 10 20

i

j

repeat step 1 repeat step 2

cheers

50 20 40 10 60

if $i > j$

Swap j and main element

10 20 40 50 60

$$\begin{aligned} \text{time complexity} &= \underbrace{1+1+1}_{\text{compare}} + \underbrace{3+1+1}_{\text{swap}} \\ &= 8 \approx n \log_2 n \end{aligned}$$

$$\text{Time} \approx n \log n$$

Quick Sort

Worst Case



10 20 30 40 50 ∞

① find element greater than 10

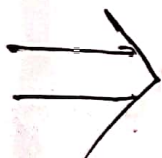
② find element smaller than 10

main 10 20 30 40 50 ∞
j i

③ swap the main element with jth element

$\frac{1}{\text{swap}} + n$ comparisons

Cheers



20 30 40 50 ∞
do step 1 with 20 do step 2 with 20

20 30 40 50 ∞
j i

do step 3

$\frac{1}{\text{swap}} + \frac{n-1}{\text{comparisons}}$



30 40 50
do step 1 with 30 do step 2 with 30

30 40 50
j i

do step 3 $\frac{1}{\text{swap}} + \frac{n-2}{\text{comparisons}}$

⇒

do step 1
with 40

do step 2
with 40

40
1

50
1

1 + n-3
swap comparisons

So

If a list contain n element then
Time complexity

$$\begin{aligned}
 &= 1 + n + 1 + n-1 + 1 + n-2 + \dots + 1 + 1 \\
 &= (1 + 1 + 1 \dots n \text{ times}) + (n + n-1 + \dots + 1) \\
 &= n + \frac{n(n+1)}{2} \\
 &= \frac{2n + n^2 + n}{2} \\
 &= \frac{3n + n^2}{2} \\
 &\approx O(n^2) \quad (\text{Worst case})
 \end{aligned}$$

Dheeraj

As it doesnot need an extra space
so it is inplace algorithm.

Algo name	Cases	Time
	Best Avg Worst	$n \log n$ n^2
Quick Sort		
Merge Sort	Best case	$n \log n$
	Avg	
	Worst	
insertion Sort	Best	n
	Avg	n^2
	Worst	n^2
Bubble Sort	Best	$O(n)$
	Avg	$O(n^2)$
	Worst	$O(n^2)$

Q2 Bubble Sort

```
void Bubble Sort(int arr[], int n)
{
    int i, j;
    for (i = 0; i < n-1; i++)
    {
        for (j = 0; j < n-i-1; j++)
        {
            if (arr[i] > arr[j+1])
                swap(arr[i], arr[j+1])
        }
    }
}
```

Check

Worst Case

when	$i = 0$	inner loop run
	$i = 1$	n times
	$i = 2$	$n-1$ times
	$i = 3$	$n-2$ times
	\vdots	\vdots
	$i = n-2$	2 times

$$\begin{aligned} \text{Total} &= n + n-2 + n-3 + \dots + 2 + 1 - 1 \\ &= \frac{n(n+1)}{2} - 1 \end{aligned}$$

Best case $\approx O(n^2)$
 $\approx O(n)$ (when element are already sorted)

Avg case $\approx O(n^2)$

Q4

BST for String

- Step 1: Insert the first string in tree
- Step 2: Before insert second string
- Step 3: compare $key[i]$ & $node \rightarrow key[i]$
- Step 4: If $key[i]$ is equal to $node \rightarrow key[i]$
then again do Step 3 with $i++$
- Step 5: If it is $key[i] < node \rightarrow key[i]$
Insert that node to the left
- Step 6: If it is $key[i] > node \rightarrow key[i]$
Insert that node to the right