14 February 2022 08:44 AM

Q1) Let there is sphere is Radius 'R' with total charge 'q'

$$Q = \int \rho \, dv = \rho \int_0^R 4\pi r^2 dr = \rho\left(\tfrac{4}{3}\pi R^3\right)$$

Applying Gauss law

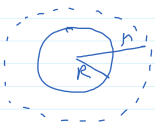
Let consider a Gauss area inside a surface at a distance 'r'

$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{in}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{Q_{in}}{\varepsilon_0} \Rightarrow E = \frac{Q_{in}}{4\pi\varepsilon_0 r^2}$$

$$\left[Q_{in} = \rho\left(\tfrac{4}{3}\pi r^3\right)\right] \text{ so } \quad E = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{E = \frac{\rho r}{3\varepsilon_0}}$$

When $r > R$

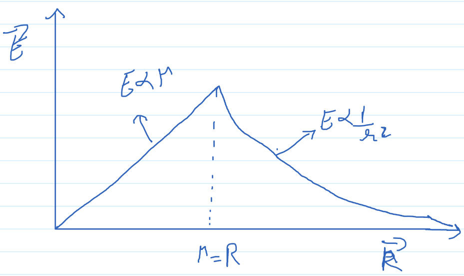
$$Q = \rho\left(\tfrac{4}{3}\pi R^3\right)$$



Applying Gauss law

here total charge enclosed $= Q$,

$$\oint \vec{E} \cdot \vec{dA} = \frac{q_{en}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$



Q2) i) T.P $\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A}) = 0$

for any field $\vec{A}$ divergence of a curl of $\vec{A}$ is like this

$$(\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A})) = \left(\hat{i}_x \frac{\delta}{\delta x} + \frac{\delta}{\delta y}\hat{i}_y + \frac{\delta}{\delta z}\hat{i}_z\right) \times \left[\left(\frac{\delta A_z}{\delta y} - \frac{\delta A_y}{\delta z}\right)\hat{i}_x + \left(\frac{\delta A_x}{\delta z} - \frac{\delta A_z}{\delta x}\right)\hat{i}_y + \left(\frac{\delta A_x}{\delta y} + \frac{\delta A_y}{\delta x}\right)\hat{i}_z\right]$$

$$[\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A})] = \frac{\delta}{\delta x}\left(\frac{\delta A_z}{\delta y} - \frac{\delta A_y}{\delta z}\right) + \frac{\delta}{\delta y}\left(\frac{\delta A_x}{\delta z} - \frac{\delta A_z}{\delta x}\right) + \frac{\delta}{\delta z}\left(\frac{\delta A_x}{\delta y} - \frac{\delta A_y}{\delta x}\right) = 0$$

Hence proof

Q3) Given Electric potential $= v(r) = \dfrac{A e^{-\lambda r}}{r}$

(i) we can write E.Field $(E) = -\nabla v = -A \dfrac{\delta}{\delta r}\left(\dfrac{e^{-\lambda r}}{r}\right)\hat{r} = -A\left[\dfrac{r(-\lambda) e^{-\lambda r} - e^{-\lambda r}}{r^2}\right]\hat{r}$

$$= A e^{-\lambda r}(1 + \lambda r)\frac{\hat{r}}{r^2}$$

(ii) To find charge density 'ρ'

$$\frac{\rho}{\varepsilon_0} = \vec{\nabla} \cdot \vec{E} = A e^{-\lambda r}(1 + r\lambda)\nabla \cdot \frac{\hat{r}}{r^2} + A\frac{\hat{r}}{r^2} \cdot \vec{\nabla} e^{-\lambda r}(1 + r\lambda)$$

$$= A e^{-\lambda r}(1 + r\lambda)\nabla \cdot \frac{\hat{r}}{r^2} = A e^{-\lambda r}(1 + r\lambda) 4\pi\delta(\vec{r}) = A 4\pi\delta(\vec{r})$$

$$\frac{\rho}{\varepsilon_0} = A e^{-\lambda r}(1 + r\lambda) 4\pi\delta(\vec{r}) - A\frac{1}{r}e^{-\lambda r}\lambda^2$$

$$\boxed{\rho = \varepsilon_0 A\left[4\pi\delta(\vec{r}) - \frac{e^{-r\lambda}}{r}\lambda^2\right]}$$

ε_0 r

$$\boxed{\rho = \varepsilon_0 A \left[4\pi \delta(\vec{r}) - \frac{e^{-\lambda r}}{r} \lambda^2 \right]}$$

Q4) b) Given plates are infinite along y z plane, then potential will varies in x coordinate only

as it is 1-D so $E = -\frac{dV}{dx}$ & we know $E = \frac{\sigma}{A\varepsilon_0}\hat{x}$ where σ is charge density of plate

$$\int_0^x -E \cdot dx = \int_0^V dV \Rightarrow$$

The problem is in 1-D because only x coordinate will varies potential