11 February 2022 04:13 PM

Dheeraj, 202014

Q1) (i) $e^{3\theta}(dr + 3r\,d\theta) = 0$

$$\underbrace{e^{3\theta}}_{M}dr + \underbrace{3e^{3\theta}r}_{N}\,d\theta = 0$$

$$\Rightarrow \frac{dM}{d\theta} = e^{3\theta} \Rightarrow 3e^{3\theta} \quad (1)$$

$$\frac{dN}{dr} = 3e^{3\theta} \Rightarrow 3e^{3\theta} \quad (2)$$

(1) = (2) hence it is exact

$$\frac{dU}{dr} = M \Rightarrow \int du = \int M\,dr \Rightarrow U = \int e^{3\theta}dr \Rightarrow U = e^{3\theta}\times r + g(\theta) \quad (3)$$

$$\frac{dU}{d\theta} = N \Rightarrow \cancel{3e^{3\theta}\times r} + g'(\theta) = \cancel{3e^{3\theta}} \Rightarrow g'(\theta) = 0 \quad \text{so } \boxed{g(\theta) = C}$$

$$\boxed{U = e^{3\theta}\times r + C = 0}$$

(ii) $\underbrace{2x\tan y}_{M}\,dx + \underbrace{\sec^2 y}_{N}\,dy = 0$

$$\frac{dM}{dy} = 2x\sec^2 y \quad (1), \quad \frac{dN}{dx} = 0 \quad (2) \qquad \text{here } (1) \neq (2) \text{ so it is not exact}$$

Now finding integrating factor:

$$I.f(y) = e^{\int \frac{1}{N}\left(\frac{dM}{dy} - \frac{dN}{dx}\right)dx}$$

$$= e^{\int \frac{1}{\sec^2 y}(2x\sec^2 y - 0)\,dx} \Rightarrow e^{\int 2x\,dx} \Rightarrow e^{x^2}$$

$\rightarrow$ Now $e^{x^2}(\underbrace{2x\tan y\,dx}_{M = \frac{dM}{dy}} + \underbrace{\sec^2 y\,dy}_{N = \frac{dN}{dx}}) = 0$ <u>is exact</u> and solving this

$$\frac{dU}{dy} = N \Rightarrow \int du = \int N\,dy \Rightarrow U = e^{x^2}\int \sec^2 y\,dy + g(x) \Rightarrow e^{x^2}\tan y + g(x)$$

$$\frac{dU}{dx} = M \Rightarrow \frac{d(e^{x^2}\tan y + g(x))}{dx} = 2x\tan y \Rightarrow \cancel{2x}\,\cancel{e^{x^2}}\,\cancel{\tan y} + g'(x) = \cancel{2x}(\cancel{\tan y})\times \cancel{e^{x^2}}$$

$$g'(x) = 0 \quad \text{so } \boxed{g(x) = C}$$

$$\boxed{U = e^{x^2}\tan y + C = 0}$$

(iii) $e^{2x}(2\cos(y)\,dx - \sin y\,dy) = 0, \quad y(0) = 0$

$$\underbrace{e^{2x}2\cos(y)}_{M}\,dx - \underbrace{e^{2x}\sin(y)}_{N}\,dy = 0$$

$$\frac{dM}{dy} = -2e^{2x}\sin(y) \quad (1), \quad \frac{dN}{dx} = -2e^{2x}\sin(y) \quad (2); \quad (1) = (2) \text{ so it is exact}$$

$$\frac{du}{dx} = M \Rightarrow \int du = \int M\,dx \Rightarrow U = \frac{e^{2x}}{\cancel{2}}\cancel{2}\cos(y) + g(y)$$

$$\frac{dU}{dy} = N \Rightarrow d(e^{2x}\cos(y) + g(y)) = -e^{2x}\sin(y) \Rightarrow -e^{2x}\cancel{\sin(y)} + g'(y) = -e^{2x}\cancel{\sin(y)}$$

$$\frac{dU}{dy} = N \Rightarrow \frac{d\left(e^{2x}\cos(y) + g(y)\right)}{dy} = -e^{2x}\sin(y) \Rightarrow -e^{2x}\cancel{\sin(y)} + g'(y) = -e^{2x}\cancel{\sin(y)}$$

$$g'(y) = 0 \quad so \quad \boxed{g(y) = C}$$

$$\boxed{U = e^{2x}\cos(y) + C = 0}$$

at $y(0) = 0 \Rightarrow \quad 0 = e^{2x}\cos(0) + C \Rightarrow 1 + C = 0 \Rightarrow \boxed{C = -1}$

$\boxed{e^{2x}\cos(y) - 1 = 0} \rightarrow$ particular sol$^n$

(iv) $2\cosh x \cos y \, dx = \sinh x \sin y \, dy$

$M =$ $\qquad$ $N =$

$\frac{dM}{dy} = 2\cosh x(-\sin y)$ ①, $\frac{dN}{dx} = -\cosh x \sin y$ ② ; here ① ≠ ② so not a exact

Now finding I.F

$$I.F = e^{\int\left(\frac{\frac{dM}{dy} - \frac{dN}{dx}}{N}\right)dx} \Rightarrow e^{\int\left(\frac{-2\cosh x \sin y + \cosh x \sin y)dx}{-\sinh x \sin y}\right)} \Rightarrow e^{\int \frac{-\cosh x \cancel{\sin y}\, dx}{\sinh x \cancel{\sin y}}}$$

$$I.F = e^{\ln(\sinh x)} \Rightarrow \boxed{I.F = \sinh x}$$

Now Exact ODE is

$\rightarrow \sinh x \,(2\cosh x \cos y - \sinh x \sin y \, dy) = 0$

$$\frac{dU}{dx} = M \Rightarrow U = \int (2\cosh x \sinh x \cos y)dx \Rightarrow \cos y\left(\frac{-\cos 2hx}{2}\right) + g(y)$$

$$\frac{du}{dy} = N \Rightarrow + \cancel{\sin y}\left(+\frac{\cos 2hx}{2}\right) + g(y) = -\sinh^2 x \cancel{\sin y} \Rightarrow \left(\frac{1 - 2\sinh^2 x}{2}\right) + g(y) = -\sinh^2 x$$

$$g'(y) = -\frac{\sin y}{2} \Rightarrow g(y) = -\frac{1}{2}\int \sin y \, dx \Rightarrow \boxed{g(y) = \frac{\cos y}{2} + C}$$

$$so \quad \boxed{U = -\cos y \frac{\cos 2hx}{2} + \frac{\cos y}{2} + c = 0}$$

Q2) b)(i) $y' + ky = e^{-kx}$

It is in the form of $y' + p(x)y = r(x)$