H-w 2

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Q1) ~~Ans~~) i) $y' = (y+4x)^2$

$\left[\text{let } y+4x = v \Rightarrow \text{diff. w.r.t } x \Rightarrow y'+4 = \frac{dv}{dx} \Rightarrow y' = \frac{dv}{dx} - 4 \right]$

$\frac{dv}{dx} - 4 = v^2 \Rightarrow \int \frac{dv}{v^2+4} = \int dx \Rightarrow \left[\text{using } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c \right]$

$\frac{1}{2} \tan^{-1} \frac{v}{2} + c = x \Rightarrow \boxed{x = \frac{1}{2} \tan^{-1}(y+4x) + c}$

~~Ans~~ ii)

$xy' = x+y \Rightarrow y' = 1 + \frac{y}{x}$ $\left[\text{let } \frac{y}{x} = u \Rightarrow y = ux \Rightarrow y' = u + u'x \right]$

$\cancel{u} + u'x = 1 + \cancel{u}$

$u' = \frac{1}{x} \Rightarrow \int du = \int \frac{1}{x} dx \Rightarrow u = \ln x + c \Rightarrow u - c = \ln x \Rightarrow x = e^{u-c} \Rightarrow x = \frac{e^u}{e^c}$

$\left[\frac{1}{e^c} = c_1 \leftarrow \text{let} \right]$ $x = e^u c \Rightarrow \boxed{x = e^{y/x} c}$

Q2) ~~Ans~~) $xy' = y + 3x^4 \cos^2(y/x)$, $y(1) = 0$

$y' = \frac{y}{x} + 3x^3 \cos^2(y/x)$ $\left[\text{let } \frac{y}{x} = u \Rightarrow y' = u + u'x \right]$

$\cancel{u} + u'x = \cancel{u} + 3x^3 \cos^2(u) \Rightarrow \frac{du}{dx} = 3x^2 \cos^2(u) \Rightarrow \int \frac{du}{\cos^2(u)} = \int 3x^2 dx \Rightarrow \int \sec^2(u) = \left| \frac{3x^3}{3} + c \right|$

$\tan(u) = x^3 + c \Rightarrow u = \tan^{-1}(x^3+c) \Rightarrow \frac{y}{x} = \tan^{-1}(x^3+c) \Rightarrow y = x(\tan^{-1}(x^3+c))$ ①

A.T.Q $y(1) = 0$ putting this in ①

$0 = 1(\tan^{-1}(1+c)) \Rightarrow \tan(0) = 1+c \Rightarrow \boxed{c = -1}$

IVP $\Rightarrow \boxed{y = x \times \tan^{-1}(x^3-1)}$

Q3) ~~Ans~~) (i) $2xy\,dx + x^2 dy = 0$

here $M = 2xy$, $N = x^2$

$\frac{dM}{d\cancel{x}y} = 2x$, $\frac{dN}{dx} = 2x$

since $\frac{dM}{dy} = \frac{dN}{dx}$ it is exact ODE

solving $\frac{du}{dx} = \cancel{2x}\, M = 2xy \Rightarrow \int du = \int 2xy\, dx \Rightarrow U = x^2y + g(y)$ ① $\left[\text{here } \frac{g(y)}{dy} \text{ is some fn} \right]$

Now we know if it is exact ODE so $\frac{du}{dy} = N$

$\frac{du}{dy} = \cancel{x^2} + g'(y) = \cancel{x^2}$ so $g'(y) = 0$ $\boxed{\text{hence } g(y) = C}$

so $\boxed{x^2y = C}$

(ii) $(\sin x \cos(y))dx + (\cos x \sin y)dy = 0$

$$\underset{M}{\underbrace{\sin x \cos(y)}} \qquad \underset{N}{\underbrace{\cos x \sin y}}$$

$\frac{dM}{dy} = -\sin x \sin(y)$, $\frac{dN}{dx} = -\sin x \sin y$ since $\frac{dM}{dy} = \frac{dN}{dx}$ it is exact ODE dy dx dy dx

$$\frac{du}{dx} = \sin x \cos y = M \Rightarrow \int du = \int dx(\sin x \cos y) \Rightarrow \boxed{u = -\cos x \cos y + g(y)} \text{①} \; [g(y) \text{ is fn of } y]$$

$$\frac{du}{dy} = N \Rightarrow -(\cos x)-(\sin y)+g'(y) = N \Rightarrow +\cancel{\cos x \sin y} + g'(y) - \cancel{\cos x \sin y} \Rightarrow g'(y) = 0$$

so $\boxed{g(y) = C}$

$$\boxed{-\cos x \cos y + C = 0}$$

(iii) $2\cosh x \cos y \, dx = \sinh(x) \sin y \, dy$

$$\underset{M}{\underset{\downarrow}{(2\cosh x \cos y)}} dx - \underset{N}{\underset{\downarrow}{(\sinh(x) \sin y)}} dy = 0$$

$$\frac{dM}{dy} = -2\cosh x \, \cancel{\cos} \sin y \, , \quad \frac{dN}{dx} = \cancel{2y} - \cosh x \sin y$$

$$\frac{dM}{dy} \neq \frac{dN}{dx}$$ so it is not exact ODE

here we will find I.F $[I.F = e^{\int f(x) dx}]$

$$\frac{M_y - N_x}{N} = f(x)$$

$$\frac{\cosh x}{\sinh x} = f(x).$$

$$I.F = e^{\int \frac{\cosh(x) dx}{\sinh(x)}} \Rightarrow e^{\ln \sinh x}$$

$$\boxed{I.F = \sinh x}$$

Now Multiply $\sinh x$ both side

$$\boxed{2 \underset{M}{\underset{\downarrow}{\sinh x \cosh x \cos y}} \, dx - \underset{N}{\underset{\downarrow}{\sinh^2 x \sin y}} \, dy = 0}$$ Now it is ODE

solving ODE [using thom]

$$P = \int M dx \Rightarrow \int 2 \sinh x \cosh x \cos y \, dx \Rightarrow 2 \cos y \int (\sinh x \cosh x) dx \Rightarrow \boxed{\cos y \sinh^2 x}$$

$$N - \frac{\delta P}{\delta y} \Rightarrow \sinh^2 x \sin y - \frac{\delta}{\delta y}(\cos y \sinh^2 x) \Rightarrow \sinh^2 x \sin y - \sinh^2 x \sin y = 0$$

Now

$$\cos y \sinh^2 x + \int 0 (dy) = C$$

$$\boxed{\cos y \sinh^2 x = C}$$

(iv) $(2xy \, dx + dy) e^{x^2} = 0 \, , \; y(0) = 2$

As)

$$\underset{M}{\underset{\downarrow}{2xy e^{x^2}}} dx + \underset{N}{\underset{\downarrow}{e^{x^2}}} dy = 0$$

$$\frac{dM}{dy} = 2x e^{x^2} \, , \quad \frac{dN}{dx} = 2e^{x^2} x$$, since $\frac{dM}{dy} = \frac{dN}{dx}$ hence exact ODE

$$U = \int du = \int M dx \Rightarrow U = 2y \int x e^{x^2} dx$$ [let $x^2 = t \Rightarrow dx \, 2x = dt$]

$$U = y \int e^t dt \Rightarrow \boxed{y e^{x^2} + g(y)}$$ [where $g(y)$ is ~~fn~~ of y]

$$\frac{du}{dy} = N \Rightarrow \frac{d(y e^{x^2} + g(y))}{dy} = e^{x^2} \Rightarrow \cancel{e^{x^2}} + g'(y) = \cancel{e^{x^2}} \Rightarrow g'(y) = 0$$

so $g(y) = c$

$$\boxed{y e^{x^2} + c = 0}$$

Now A.T.Q $y(0) = 2$

$$2e^{0} + c = 0 \Rightarrow \boxed{c = -2}$$

$$\boxed{y e^{x^2} - 2 = 0}$$

Q4) As) $(ax + by)\,dx + (kx + ly)\,dx = 0$

$$\underbrace{(ax+by)}_{M} \qquad \underbrace{(kx+ly)}_{N}$$

$\frac{dM}{dy} = b$, $\frac{dN}{dx} = k$ $\boxed{\text{so to be exact ODE } b = k \text{ is must}}$ ①

also $\frac{du}{dx} = M \Rightarrow \int du = \int dx\,M \Rightarrow u = \int (ax + by)\,dx \Rightarrow u = \frac{ax^2}{2} + bxy + g(y)$

$\frac{du}{dy} = N \Rightarrow bx + g'(y) = kx + ly \Rightarrow g'(y) = x(k-b) + ly$ from ①

$g'(y) = ly \Rightarrow g(y) = \frac{ly^2}{2}$

so solution is

$$\boxed{\frac{ax^2}{2} + bxy + \frac{ly^2}{2} + c = 0}$$