Assigment

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Problem - II

(i) $y(t) = \sin(t x(t))$;

1st checking time variant

putting $x(t) \longrightarrow x(t-t_0)$

so $\sin(t \cdot x(t-t_0)) \neq y(t-t_0)$ $\boxed{\text{therefore its a time variant}}$

Checking linearity

$y_1(t) = \sin(t(x_1(t))$; $y_2 = \sin(t \cdot x_2(t))$

$y_3(t) = y_1(t) + y_2(t) \Rightarrow y_3(t) = \sin(t(x_1(t))) + \sin(t(x_1(t)))$ ①

let $y_x(t) = x_1(t) + x_2(t)$ is system

$y_x(t) = \sin(t x_1(t) + t x_2(t))$ ②

since ① $\neq$ ② that is $y_x(t) \neq y_3(t)$ $\boxed{\text{hence not a linear system}}$

(ii) $y(t) = (3-2i) x(1-t)$

. 1st checking linearity

$y_1(t) = (3-2i)(x_1(1-t))$, $y_2(t) = (3-2i)(x_2(1-t))$

$y_3 = T[x_1(t) + x_2(t)] \Rightarrow (3-2i)(x_1(1-t) + x_2(1-t)) \Rightarrow$

$(3-2i)(x_1(1-t)) + 3(-2i)(x_2(1-t))$

$\boxed{\text{because } y_3 = y_1 + y_2}$. hence it is linear system

Checking for time variant

delay output : $y(t-t_0) = (3-2i) \cdot (x(1+t_0-t))$ ①

delay input : $x(t-t_0) = (3-2i)(x(1+t_0-t))$ ②

as ① & ② are equal hence time invariant system

(iii) $y(t) = \int_{-\infty}^{t} x(\tau) e^{(-(t-\tau))}$

. 1st checking linearity

Given time delay to to input $x_1(t) \longrightarrow \text{system} \rightarrow y_1(t) = \int_{-\infty}^{t} x_1(\tau) e^{(-(t-\tau))} dt$

Given time delay to to output $x_2(t) \longrightarrow \text{system} \rightarrow y_2(t) = \int_{-\infty}^{t} x_2(\tau) e^{(-(t-\tau))} dt$

$y_1(t) + y_2(t) = \int_{-\infty}^{t} x_1(t) e^{-(t-\tau)} dt + \int_{-\infty}^{t} x_2(t) e^{-(t-\tau)} dt$ ①

$a_1 x(t) \longrightarrow \text{system} \longrightarrow a y(t) = \int_{-\infty}^{t} a(x(t)) e^{-(t-\tau)} dt = a \int_{-\infty}^{t} x(t) e^{-(t-\tau)} dt$ ②

from ① & ② we can say it is linear system [because it follow additivity , homogenity]

Checking time variant

delay output : $y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau - t_0) e^{(-(t-t_0-\tau)} dt$ ①

delay input : $x(t-t_0) = \int_{-\infty}^{t} x(\tau - t_0) e^{-(t-\tau)} d\tau$ (2)

since (1) ≠ (2) it is a time variable system

Q2) b) $s(t) = \cos(2\pi f_0 t + \theta)$ Given: $h(t) \to$ impulse response.

$y(t) = (s * h)(t) = |H(f_0)| \cos(2\pi f_0 t + \theta + \phi)$

$y(t) = \int_{-\infty}^{\infty} h(t) * s(t-\tau) dt \Rightarrow \int_{-\infty}^{\infty} h(\tau) * s(t-\tau) dt$

$\Rightarrow \int_{-\infty}^{\infty} h(\tau) \cos(2\pi f_0 (t-\tau) + \phi) dt \Rightarrow \int_{-\infty}^{\infty} h(\tau) \cos(2\pi f_0 t + \phi - 2\pi f_0 \tau) dt$

[using : $\cos(x-y) = \cos x \cos y + \sin x \sin y$]

$\Rightarrow \int_{-\infty}^{\infty} h(\tau)[\cos(2\pi f_0 t + \phi)\cos(2\pi f_0 \tau) + \sin(2\pi f_0 t + \theta)\sin(2\pi f_0 \tau) d\tau$

$\Rightarrow \cos(2\pi f_0 t + \theta) \int_{-\infty}^{\infty} h(\tau)\cos(2\pi f_0 \tau) + \sin(2\pi f_0 t + \theta) \int_{-\infty}^{\infty} h(\tau) \sin(2\pi f_0 \tau) d\tau$

let $H_c(f_0) = \int_{-\infty}^{\infty} h(\tau) \cos(2\pi f_0 \tau) d\tau$

$H_s(f_0) = \int_{-\infty}^{\infty} -h(\tau) \sin(2\pi f_0 \tau) d\tau$

$y(t) = H_c(f_0) \cos(2\pi f_0 t + \theta) - H_s(f_0) \sin(2\pi f_0 t + \theta)$ (1)

$|H(f_0)| = \sqrt{H_c(f_0)^2 + H_s(f_0)^2}$ & $\tan\phi = \dfrac{-H_s(f_0)}{H_c(f_0)}$

[X, ÷ by $|H(f_0)|$ in (1) we will get .]

$|H(f_0)|(\cos\phi \cos(2\pi f_0 t + \theta) - \sin\phi \sin(2\pi f_0 t + \theta))$

$y(t) = |H(f_0)| \cos(2\pi f_0 t + \theta + \phi)$

hence proved

Q3) $V_p(t) = 2\cos(2\pi f_m t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)$ when $f_m << f_c$

Given: passband signal we can say $V_c(t) = 2\cos(2\pi f_m t)$, $V_s(t) = \sin(2\pi f_m t)$

→ Time domain: $V_p(t) = 2\cos(2\pi f_m t)\cos(2\pi f_c t) - \sin(2\pi f_m t)\sin(2\pi f_c t)$

→ polar form: $V_p(t) = e(t)\cos(2\pi f_c t + \theta(t))$ (3)

$e(t) = \sqrt{V_c^2(t) + V_s^2(t)} \Rightarrow \sqrt{4\cos^2(2\pi f_m t) + \sin^2(2\pi f_m t)} \Rightarrow \sqrt{1 + 3\cos^2(2\pi f_m t)}$ (1)

$\theta(t) = \tan^{-1}\left(\dfrac{V_s(t)}{V_c(t)}\right) = \tan^{-1}\left(\dfrac{\tan(2\pi f_m t)}{2}\right)$ (2)

putting (2) & (1) in (3)

$V_p(t) = \sqrt{1 + 3\cos^2(2\pi f_m t)} \times \cos\left(2\pi f_c t + \tan^{-1}\left(\dfrac{\tan(2\pi f_m t)}{2}\right)\right)$

→ Complex baseband now $= V_p(t) = \mathrm{Re}\{V(t) e^{j2\pi f_c t}$

$V(t) = V_c(t) + j V_s(t) = \boxed{2\cos(2\pi f_m t) + j\sin(2\pi f_m t)}$

A.T.Q Part 2

we have to compute $2V_p(t)\sin(2\pi f_c t)$

$2[V_c(t)\cos(2\pi f_c t)\sin(2\pi f_c t) - V_s(\sin(2\pi f_c t)^2)$

$2\left[\dfrac{V_c(t)\sin(4\pi f_c t)}{2} - V_c\dfrac{(1-\cos(4\pi f_c t))}{2}\right] \Rightarrow V_c(t)\sin(4\pi f_c t) - V_s(t) + V_s(t)\cos(4\pi f_c t)$

via low pass filter the output signal is passed having frequency f_c

$-V_s$ component $= -\sin(2\pi f_m t)$

A.T.Q Part 2 : $V_p \longleftrightarrow v(t)$, V_f = Complex baseband frequency

$$V_p(t) = Re\{v(t)e^{j2\pi f_c t}\} = Re\{c(t)\}$$

$$c(t) = v(t)e^{j2\pi f_c t} \longleftrightarrow C(f) = V(f - f_c)$$

$$v_p(t) = \frac{c(t) + c^*(t)}{2} \leftrightarrow V_p(f) = \frac{C(f) + C^*(-f)}{2} \Rightarrow$$

$$\boxed{V_p(f) = \frac{V(f - f_c)}{2} + \frac{V^*(-f - f_c)}{2}}$$

A.T.Q Part 4

Given

$$H(f) = \begin{cases} 2 - \frac{f}{f_m} & -f_m \le f \le -f_m \\ 1 & f_m \le f \le 2f_m \\ 3 & -2f_m \le f \le -f_m \\ 0 & \text{otherwise.} \end{cases}$$

$$y(t) = v(t) * h(t)$$

frequency domain

$$Y(f) = V(f) * H(f)$$

$$v(t) = 2\cos(2\pi f_m t) + j\sin(2\pi f_m t) \Leftrightarrow V(f)$$

$$\boxed{V(f) = \frac{3}{2}\delta(f - f_m) + \frac{1}{2}\delta(f + f_m)}$$

Acc. to question

at f_m $H(f) = 1$

at $-f_m$ $H(s) = 3$

$$Y(f) = \frac{3}{2}\delta(f - f_m) + \frac{3}{2}\delta(f + f_m)$$

$$y(t) = 3\cos(2\pi f_m t) \Rightarrow \boxed{Re\{y(t)\} = 3\cos(2\pi f_m t) \ \& \ Img\{y(t)\} = 0}$$

Now)

Rectangular representation $= y(t) = 3\cos(2\pi f_m t)$

polar representation $= y(t) = 3\cos(2\pi f_m t)e^{0j}$

Problem 4 Let the passband signal be

$$\tilde{v}_p(t) = 2cos(2\pi f_m t)cos(2\pi f_c t + \theta(t)) - sin(2\pi f_m t)sin(2\pi f_c t + \theta(t))$$

where $f_m << f_c$ and $\theta(t) = 2\pi f_o t + \gamma$.

- Write down the I and Q components of the above passband signal in terms of I and Q components of $v_p(t)$ as given in the equation above.
- Write the frequency domain representation (separate frequency and phase plots) of $\tilde{v}_p(t)$.
- Say $\tilde{v}_p(t) = Re\{\tilde{v}(t)e^{j2\pi f_c t}\}$ and $c(t) = \tilde{v}(t)e^{-j\theta(t)}$. Provide the rectangular and polar representation of $c(t)$.

a) $$\left.\begin{aligned} \tilde{v}_p(t) &= 2\cos(2\pi f_m t)\cos(2\pi f_c t + \theta(t)) - \sin(2\pi f_m t)\sin(2\pi f_c t + \theta(t)) \\ f_m &<< f_c \ \& \ \theta(t) = 2\pi f_o t + \gamma \end{aligned}\right\} \text{given}$$

$$\tilde{v}_p(t) = 2\cos(2\pi f_m t)[\cos(2\pi f_c t)\cos(\theta t) - \sin(2\pi f_c t)\sin(\theta(t))] - \sin(2\pi f_m t)$$
$$\times [\sin(2\pi f_c t)\cos(\theta(t)) + \cos(2\pi f_c t)\sin(\theta(t))]$$

here combining terms $\cos(2\pi f_c t)$ & $\sin(2\pi f_c t)$ so we will get

$$\tilde{v}_p(t) = \tilde{v}_c(t)\cos(2\pi f_c t) - \tilde{v}_s(t)\sin(2\pi f_c t)$$

$$\tilde{v}_c(t) = 2\cos(2\pi f_m t)\cos(\theta(t)) - \sin(2\pi f_m t)\sin(\theta(t))$$

$$\tilde{v}_s(t) = 2\cos(2\pi f_m t)\sin(\theta(t)) + \sin(2\pi f_m t)\cos(\theta(t))$$

in term of $v_p(t)$ $(v_c'(t), v_s'(t))$

we will get

$\dot{V}_c(t) = 2\cos(2\pi f_m t)$, $v_s(t) = \sin(2\pi f_m t)$

$\bar{v}_c(t) = v_c(t)\cos(\theta(t)) - v_s(t)\sin(\theta(t))$

$\bar{v}_s(t) = v_c(t)\sin(\theta(t)) + v_s(t)\cos(\theta(t))$

(b) Part)

frequency domain representation

$V_p(t) = Re\{v(t)\, e^{(2\pi j f_c t + \theta(t))}\} \Rightarrow \bar{v}_p(t) = Re\{\bar{v}(t) e^{j2\pi f_c t}\}$ & $\bar{v}(t) = v(t) e^{j\theta(t)}$

$\bar{V}_p(f) = \frac{1}{2}[\bar{V}(f - f_c) + \bar{V}^*(-f - f_c)]$

(c) part) $\bar{v}_p(t) = 2\cos(2\pi f_m t)\cos(2\pi f_c t + \theta) - \sin(2\pi f_m t)\sin(2\pi f_c t + \theta(t))$

[take $z(t) = 2\cos(2\pi f_m t) + j\sin(2\pi f_m t) = z(t)$

$\bar{v}_p(t) = Re\{z(t)\, e^{j2\pi f_c t + \theta(t)}\}$]

so now $\bar{z}(t) = z(t)\, e^{j\theta}$

on comparing $\bar{v}(t) = \bar{z}(t) = z(t)\, e^{j\theta(t)}$

Rectangular form $= c(t) = 2\cos(2\pi f_m t) + j\sin(2\pi f_m t)$

Polar $= c(t) = e^{r(t)}\, e^{j\theta(t)} = \sqrt{1 + 3\cos^2(2\pi f_m t)} \cdot e^{j\tan^{-1}\left[\frac{\tan(2\pi f_m t)}{2}\right]}$