Work sheet 1

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Q1) Given: $y = Cx - C^2$

$y' = C$ ①

$y'^2 - xy' + y = 0$ putting ①

$C^2 - xC + y = 0 \Rightarrow \cancel{C^2} - \cancel{xC} + \cancel{Cx} - \cancel{C^2} = 0 \Rightarrow 0 = 0$

from this we can say $y = Cx - C^2$ is a sol^n of ODE $y'^2 - xy^2 + y = 0$

$y = \frac{x^2}{4} \Rightarrow y' = \frac{x}{2} \Rightarrow \frac{x^2}{4} - \frac{x^2}{2} + \frac{x^2}{4} = 0$

Q2) $y' = -ky$

$y(t) = Ce^{-kt}$, $y(0) = 1$ gm (given) so $C = 1$

$y(t) = 1e^{-kt} \Rightarrow \frac{1}{2} = 1e^{-k \times 3.6} \Rightarrow \ln(\frac{1}{2}) = -k \times 3.6 \Rightarrow \boxed{\frac{\ln 2}{3.6} = k}$ ①

a) $y(1) = 1e^{-\ln 2/3.6} \times 1 = 0.8249$

b) $y(365) = 1 \cdot e^{-\frac{\ln 2}{3.6}(365)} \Rightarrow 2.54 \times 10^{-31}$

Q3) $mv' = mg - kv^2$ $\left[k = 1, m = 1 kg \right]$

$m\frac{dv}{dt} = mg - kv^2$

$v' = g - \frac{kv^2}{m} \Rightarrow v' = 9.8 - v^2 \Rightarrow \int_{10}^{v} \frac{dv}{9.8 - v^2} = \int_0^1 dt$

$t = 1.5 \times 6 \ln \left| \frac{3.13 + v}{3.13 - v} \right| + C \Rightarrow \ln \left| \frac{3.13 + v}{3.13 - v} \right| = 0.64t + C \Rightarrow$

$\frac{3.13 + v}{3.13 - v} = C_1 e^{0.64t} \Rightarrow C_1 = e^C$

$3.13 + v = (3.13 - v) Ce^{0.64t} \Rightarrow v(1 + Ce^{0.64t}) = 3.13(Ce^{0.64t} - 1)$

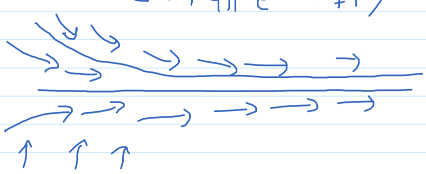
$v = 3.13 \frac{Ce^{0.64t} - 1}{Ce^{0.64} + 1}$

Putting $t \to \infty$

$v \simeq 3.13 \simeq \sqrt{g}$ also $v(0) = 10$

$10 = 3.13 \left(\frac{C - 1}{C + 1} \right)$ $\left[C = -1.911 \right]$

$v = 3.13 \left(\frac{-1.91 e^{-0.64t} - 1}{-1.911 e^{0.64t} + 1} \right)$



Q4) a) Birth rate → k_1 proportionality constant, similarly k_2

$y' = k_1 y - k_2 \Rightarrow \frac{dy}{dt} = (k_1 - k_2)y \Rightarrow dt = \frac{dy}{(k_1 - k_2)y}$

$(k_1 - k_2)t + C = \ln y \Rightarrow y = e^{(k_1 - k_2)t + C} \Rightarrow y = Ce^{(k_1 - k_2)t}$

(b) Now According to question

population rate depends on $k_1 - k_2$, if $k_1 = k_2$, then number of bacteria for $t = 0$

if $k_1 > k_2$, population grows exponentially

if $k_1 < k_2$, the no. of bacteria decreases exponentially

Q5) $\frac{dv}{dP} = -\frac{V}{P} \Rightarrow \int \frac{dv}{v} = -\int \frac{dP}{P} \Rightarrow \ln v = -\ln P + C \Rightarrow \ln v = \ln(C/p)$

$V = \frac{C}{p}$ [where C is constant]

Q6) b) According to question

$y' = -Ay\ln(y) \Rightarrow \frac{dy}{dt} = -Ay\ln y \Rightarrow \frac{dy}{y\ln(y)} = -Adt$

let $\ln y = K$

$\frac{1}{y} = \frac{dK}{dy} \Rightarrow dy = y\,dK \Rightarrow \int \frac{dK}{K} = \int -Adt \Rightarrow \ln K = -At + C$

$K = Ce^{-At}$

Now $\ln y = Ce^{-At} \Rightarrow y = me^{(e^{-At})}$

So when $y > 1$, $y\ln y > 0$ [decreasing growth rate of tumor]

when $0 < y < 1$, $\ln y < 0$ increasing constant increase in growth of tumor

when $y < 0$ $\ln y$ is not defined