- $E(t)$ is the external voltage provided to circuit. Here, $E(t) = E_0 \sin(\omega t)$.
- Relationship in the capacitor between charge and current is

$$I = \frac{dQ}{dt}$$

Q.1 Write a 2nd order ODE model for the current $I(t)$ in this RLC circuit.

Q.2. Solve the characteristic eq. of homogeneous part (find roots in terms of $R, L \& C$).

Q1 From Kirchhoff's law

$$\Rightarrow E_0 \sin(\omega t) - IR - \int \frac{I\,dt}{C} - \frac{L\,dI}{dt}$$

$$\Rightarrow E_0 \cdot \omega \cdot \cos(\omega t) - I'R - \frac{I}{C} - L \cdot I'' = 0$$

Q2 Now

$$L \cdot I'' + I'R + \frac{I}{C} = 0$$

$$L \cdot \lambda^2 + R \cdot \lambda + \frac{1}{C} = 0 \Rightarrow \lambda = \frac{-R \pm \sqrt{R^2 - 4\,L/C}}{2L}$$

Q3) $y_p = a\cos\omega t + b\sin\omega t$

$y_p' = -a\omega\sin\omega t + b\omega\cos\omega t$

$y_p'' = -a\omega^2\cos\omega t - b\omega^2\sin\omega t$

ODE $\rightarrow L I'' + R I' + \frac{I}{C} = \omega E_0 \cos\omega t$

Comp

$\omega E_0 = R\omega b + a - a\omega^2 L$

$0 = b - aR\omega - \omega^2 L b$

$E_0 = Rb + a\left(-\omega L + \frac{1}{C\omega}\right)$ — (1)

$0 = b\left(\frac{1}{C\omega} - L\omega\right) - Ra$ — (2)

let $S = -L\omega + \frac{1}{C\omega}$

equations: $Sa + Rb = E_0$ $\quad Sb = Ra$

$Sb - Ra = 0$ $\quad b = \frac{R}{S}a \longrightarrow$ put in eq

$Sa + \frac{R^2}{S}a = E_0 \Rightarrow a\left(S + \frac{R^2}{S}\right) = E_0$

$a\left(\frac{S^2 + R^2}{S}\right) = E_0$ $\quad \boxed{a = \frac{SE_0}{R^2+S^2}}$ $\quad \boxed{b = E_0 R/(R^2+S^2)}$

Q4) $y_p = a\cos(\omega t) + b\sin(\omega t)$

$A \cdot T \cdot 0$ $\quad y_p = C\sin(\omega t \cdots$