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Given: $y_1 = B_0 + B_1 x_i + u_i$ where $i = 1, 2, \ldots N$

1) $E(u_i) = 0$
2) $E(u_i | x_i) = 0$
3) $E(u_i^2 | x_i) = \sigma^2$

T.P = $\mathrm{var}(B_0 | x_i) = \dfrac{\sigma^2 n^{-1} \sum_{i=1}^{N} x_i^2}{\sum_{n=1}^{N} (x_i - \bar{x})^2}$

Ans) We know $\hat{B}_0 = \bar{y} - \hat{B}_1 \bar{x}$ and $\mathrm{var}(\hat{B}_1 | x_i) = \dfrac{\sigma^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$

$\Rightarrow \mathrm{var}(\hat{B}_0 | x_i) = \mathrm{var}(\bar{y} - \hat{B}_1 \bar{x} / x_i) = \mathrm{var}(\bar{y} / x_i) + \bar{x}^2 \mathrm{var}\left(\dfrac{\hat{B}_1}{x_i}\right) - 2\bar{x}\, \mathrm{Cov}(\bar{y}, \hat{B}_1)$

$\Rightarrow \mathrm{var}\left(\dfrac{1/n \sum_{i=1}^{N} y_i}{x_i}\right) + \bar{x}^2 \cdot \dfrac{\sigma^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2} - 2\bar{x}\, \mathrm{Cov}(\bar{y} \cdot \hat{B}_1) - (1)$

Now doing calculation of cov

$$\mathrm{Cov}(\bar{y}, \hat{B}_1) = \mathrm{Cov}\left(1/n \sum_{i=1}^{N} y_i, \sum_{j=1}^{N} \frac{(x_j - \bar{x}) \cdot y_j}{\sum_{j=1}^{N} (x_j - \bar{x})^2}\right)$$

$$= n^{-1} \left(\frac{1}{\sum_{j=1}^{N} (x_i - \bar{x})^2}\right) \mathrm{Cov}\left(\sum_{i=1}^{N} y_i, \sum_{j=1}^{N} (x_j - \bar{x}) \cdot y_j\right)$$

[so $\mathrm{Cov}(y_i, y_j) = 0 \ \forall i, j$
$i: 1 \to n$; $j: 1 \to n$]

$\mathrm{Cov}(\bar{y}, \hat{B}_1) = 0$

$\rightarrow$ putting this in (1)

$\mathrm{var}(\hat{B}_0 | x_i) = \dfrac{1}{n^2} \sum_{i=1}^{n} \mathrm{var}\left(\dfrac{y_i}{x_i}\right) + x^{-2} \cdot \dfrac{\sigma^2}{\sum_{i=1}^{N} (x_i - \bar{x})^2}$ [from assumption 3 $\mathrm{var}(y_i | x_i) = \sigma^2$]

$\mathrm{var}(\hat{B}_0 | x_i) = \left(\dfrac{1}{n} + \dfrac{\bar{x}^2}{\sum (x_i - \bar{x})^2}\right) \sigma^2 \Rightarrow \sigma^2 \left(\dfrac{n\bar{x}^2 - 2n\bar{x}^2 + \sum_{i=1}^{N} x_i^2 + n\bar{x}^2}{n \cdot \sum_{i=1}^{N} (x_i - \bar{x})^2}\right) =$

$\Rightarrow \boxed{\dfrac{\sigma^2 \cdot \sum_{i=1}^{N} x_i^2}{n \cdot \sum_{i=0}^{N} (x_i - \bar{x})^2}}$

Hence prove