Q1) a) (i) $4y'' - 25y = 0$

so charact. equations : $4\lambda^2 - 25 = 0$

$\lambda^2 = \frac{25}{4} \Rightarrow \boxed{\lambda = \pm 5/2}$ real & distinct root

$\therefore y_1 = e^{5x/2}$, $y_2 = e^{-5x/2}$

so General sol^n $y = c_1 e^{5x/2} + c_2 e^{-5x/2}$

(ii) $y'' + 2\pi y' + \pi^2 y = 0$

charact. equations : $\lambda^2 + 2\pi\lambda + \pi^2 = 0$

$(\lambda + \pi)^2 = 0$

$\boxed{\lambda = -\pi}$ Real root

$\therefore y_1 = e^{-\pi x}$

$\therefore y_2 = C \int \frac{e^{-\int 2\pi \, dx}}{(e^{-\pi x})^2} \cdot dx \Big] y_1 = x y_1$

General sol^n $y = C_1 e^{-\pi x} + C_2 x e^{-\pi x}$

(iii) $y'' + 1.8y' - 2.08y = 0$

Char. eq : $\lambda^2 + 1.8\lambda - 2.08 = 0$

$\lambda = \frac{-1.8 \pm \sqrt{11.56}}{2(1)} = 1.7$

so $\lambda_1 = 0.8$, $\lambda_2 = -2.6$ real & distinct

General sol^n : $y = C_1 e^{0.8x} + C_2 e^{-2.6x}$

(iv) $y'' + 2e^2 y' + k^2 y = 0$

charact. eq : $\lambda^2 + 2k^2\lambda + k^4 = 0$

$(\lambda + k^2)^2 = 0$ so $\boxed{\lambda = -k^2}$ real & same

$\therefore \lambda_1 = \lambda_2 = -k^2$

$\therefore y_1 = e^{-k^2 x}$

$y_2 = \left[\int \frac{e^{-\int 2k^2 \, dx}}{e^{-2k^2 x}} \, dx \right] y_1 = x e^{-2k^2 x}$

General sol^n $y = C_1 e^{-k^2 x} + C_2 x e^{-2k^2 x}$

Q2)

Q(2) Solve the IVP. Check that your answer satisfies the ODE as well as initial conditions.

i) $4y'' - 4y' - 3y = 0$, $y(2) = e$, $y'(2) = -e/2$ (4 marks)

ii) $9y'' - 30y' + 25y = 0$, $y(0) = 3.3$, $y'(0) = 10$

(i) $4y'' - 4y' - 3y = 0$

A-T-O

Charc eq : $4\lambda^2 - 4\lambda - 3 = 0$

$$\lambda = \frac{4 \pm \sqrt{16 - 8(4)(-3)}}{8} = \frac{1 \mp 2}{2}$$

$\therefore \lambda_1 = \frac{3}{2}, \quad \lambda_2 = -\frac{1}{2}$

$\therefore y_1 = e^{3x/2}, \quad y_2 = e^{-x/2}, \quad y = c_1 e^{3x/2} + c_2 e^{-x/2}$

using IVP

$$e = c_1 e^{-4} + c_2$$
$$1 = -3c_1 e^{-4} + c_2 \quad -(1)$$

so $c_1 = 0$, $c_2 = 1$

Now Particular solⁿ : $y = e^{-x/2}$

$y' = -\frac{1}{2}e^{-x/2}, \quad y'' = \frac{1}{4}e^{-x/2}$

Now, $4\left(\frac{1}{4}e^{-x/2}\right) - 4\left(-\frac{1}{2}e^{-x/2}\right) - 3\left(e^{-x/2}\right) = 0$

$0 = 0$

$\boxed{\therefore LHS = RHS}$

(ii) given: $y(0) = 3.3$, $y'(0) = 10$

Character. eq : $9\lambda^2 - 30\lambda + 25 = 0$

$$\lambda = \frac{30 \pm \sqrt{900 - 4(25)(9)}}{2(9)} = \frac{5}{3}$$

$- \lambda_1 = \lambda_2 = \frac{5}{3}$

$- y_1 = e^{5x/3}, \quad y_2 = \left[\int \frac{e^{-\int -(30/9)\,dx}}{e^{30x/9}}\right] y_1 = x e^{5x/3}$

General solⁿ : $y = c_1 e^{5x/3} + c_2 x e^{5x/3}$

by putting value $c_1 = 3.3$
$c_2 = 4.5$

Now Particular solⁿ : $y = 3.3e^{5x/3} + 4.5x e^{5x/3} = e^{5x/3}[3.3 + 4.5x]$

so $y' = e^{5x/3}(10 + 7.5)x \quad -(1)$

$y'' = \frac{50}{3}e^{5x/3} + 12.5x e^{5x/3} + 7.5e^{5x/3}$

putting ① & ② in ODE

$$150 e^{5x/3} + 112.5 e^{5x/3} x + 67.5 e^{5x/3} \cdot x - 300 e^{5x/3} + 82.5 e^{5x/3} + 112.5 e = 0$$

$$\boxed{LHS = RHS}$$

Q3) Ans) $(D-2I)(D+3I)$;

$$D^2 + D - 6I$$

$$y''(x) + y'(x) - 6y(x) \quad —①$$

(i) $y(x) = e^{2x}$

from ①

$$4e^{2x} + 2e^{2x} - 6e^{2x} = 6e^{2x} - 6e^{2x} = 0$$

(ii) $y(x) = xe^{2x}$

from ①

$$= 2xe^{2x} + e^{2x} + 2e^{2x} + 2e^{2x} + 4xe^{2x} - 6xe^{2x} = 5e^{2x}$$

(iii) $y(x) = e^{-3x}$

from ① $= -3e^{-3x} - 6e^{-3x} + 9e^{-3x}$

$= 0$

Q4) Ans) $5x^2y'' + 23xy' + 16.2y = 0$ ①

(Euler Cauchy from)

$y = x^m$ ②

differentiate w.r.t x

putting ③ ④ ② in ① $\qquad y' = mx^{m-1}$ ③ $\Rightarrow y'' = m(m-1)x^{m-2}$ ④

$$5x^2 \cdot m(m-1)x^{m-2} + 23x(mx^{m-1}) + 16.2x^m = 0$$

$$\Rightarrow 5m^2 - 5m + 23m + 16.2 = 0$$

$$\Rightarrow 5m^2 + 18m + 16.2 = 0$$

solving: $m = \dfrac{18 \pm \sqrt{324 - 4(5)(16.2)}}{2(5)} = 1.8$

$m_1 = -1.8$

$m_2 = -1.8$

$$y_1 = x^m = x^{1.8}$$

$$y_2 = xy_1 = \ln x\,(x^{-1.8})$$

General soln $y = c_1 x^{-1.8} + c_2 x^{-1.8} \ln x$

Q5) Ans) $(x^2D^2 - xD - 15I)y = 0$, $y(1) = 0.1$, $y'(1) = -4.5$

divide by x^2

$$\left(D^2 - \frac{D}{x} - \frac{15}{x^2}\right)y = 0 \Rightarrow D^2 \cdot y - \frac{D}{x}y - \frac{15}{x^2}y = 0$$

$$= (y'') - \frac{1}{x}(y') - \frac{15}{x^2}(y) = 0$$

Multiply x^2

$$x^2 y'' - xy' - 15y = 0$$

(Euler Cauchy form),

Let $y = x^m$

$$y' = m x^{m-1} \Rightarrow y'' = m(m-1)x^{m-2}$$

$$\Rightarrow x^2 m(m-1)x^{m-2} - x(m)x^{m-1} - 15x^m = 0$$

$$m(m-1)x^m - m x^m - 15x^m = 0$$

$$m(m-1) - m - 15 = 0$$

$$m^2 - 2m - 15 = 0$$

$$m^2 - 5m + 3m - 15 = 0$$

$$\boxed{m = 5, -3}$$

Now $y = C_1 x^5 + C_2 x^{-3}$

using IVP $\quad C_1 + C_2 = 0.1$ — (1)

$\qquad\qquad\quad 5C_1 - 3C_2 = -4.5$ — (2)

$\therefore$ from (1) & (2)

$$C_1 = -0.52$$

$$C_2 = 0.62$$

$$\boxed{\text{Particular sol}^n \quad y = -0.52x^5 + 0.62x^{-3}}$$

Graph