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Q1) (i) $e^{3\theta}(dr + 3r\,d\theta) = 0$

$\underbrace{e^{3\theta}}_{M}dr + \underbrace{3e^{3\theta}r}_{N}d\theta = 0$

$\Rightarrow \frac{dM}{d\theta} = e^{3\theta} \Rightarrow 3e^{3\theta}$ (1)

$\frac{dN}{dr} = 3e^{3\theta} \Rightarrow 3e^{3\theta}$ (2)

(1) = (2) hence it is exact

$\frac{dU}{dr} = M \Rightarrow \int du = \int M\,dr \Rightarrow U = \int e^{3\theta}dr \Rightarrow U = e^{3\theta} \times r + g(\theta)$ (3)

$\frac{dU}{d\theta} = N \Rightarrow \cancel{3e^{3\theta}} \times r + g'(\theta) = \cancel{3e^{3\theta}} \Rightarrow g'(\theta) = 0$ so $\boxed{g(\theta) = C}$

$\boxed{U = e^{3\theta} \times r + C = 0}$

(ii) $\underbrace{2x\tan y}_{M}\,dx + \underbrace{\sec^2 y}_{N}\,dy = 0$

$\frac{dM}{dy} = 2x\sec^2 y$ (1), $\frac{dN}{dx} = 0$ (2) here (1) $\neq$ (2) so it is not exact

Now finding integrating factor:

$If(y) = e^{\int \frac{1}{N}\left(\frac{dM}{dy} - \frac{dN}{dx}\right)dx}$

$= e^{\int \frac{1}{\sec^2 y}(2x\sec^2 y - 0)\,dx} \Rightarrow e^{\int 2x\,dx} \Rightarrow e^{x^2}$

$\rightarrow$ Now $e^{x^2}(\underbrace{2x\tan y\,dx}_{M = \frac{dM}{dy}} + \underbrace{\sec^2 y\,dy}_{N = \frac{dN}{dx}}) = 0$ is exact and solving this

$\frac{dU}{dy} = N \Rightarrow \int du = \int N\,dy \Rightarrow U = e^{x^2}\int \sec^2 y\,dy + g(x) \Rightarrow e^{x^2}\tan y + g(x)$

$\frac{dU}{dx} = M \Rightarrow \frac{d(e^{x^2}\tan y + g(x))}{dx} = 2x\tan y \Rightarrow \cancel{2x}\cancel{e^{x^2}}\cancel{\tan y} + g'(x) = \cancel{2x\tan y} \times \cancel{e^{x^2}}$

$g'(x) = 0$ so $\boxed{g(x) = C}$

$\boxed{U = e^{x^2}\tan y + C = 0}$

(iii) $e^{2x}(2\cos(y)\,dx - \sin y\,dy) = 0$, $y(0) = 0$

$\underbrace{e^{2x}2\cos(y)}_{M}dx - \underbrace{e^{2x}\sin(y)}_{N}dy = 0$

$\frac{dM}{dy} = -2e^{2x}\sin(y)$ (1), $\frac{dN}{dx} = -2e^{2x}\sin(y)$ (2); (1) = (2) so it is exact

$\frac{du}{dx} = M \Rightarrow \int du = \int M\,dx \Rightarrow U = \frac{e^{2x}}{\cancel{2}}\cancel{2}\cos(y) + g(y)$

$\frac{dU}{dy} = N \Rightarrow d(e^{2x}\cos(y) + g(y)) = -e^{2x}\sin(y) \Rightarrow -e^{2x}\cancel{\sin(y)} + g'(y) = -e^{2x}\cancel{\sin(y)}$

$$\frac{dU}{dy} = N \Rightarrow \frac{d\left(e^{2x}\cos(y) + g(y)\right)}{dy} = -e^{2x}\sin(y) \Rightarrow -e^{2x}\cancel{\sin(y)} + g'(y) = -e^{2x}\cancel{\sin(y)}$$

$$g'(y) = 0 \quad \text{so} \quad \boxed{g(y) = c}$$

$$\boxed{U = e^{2x}\cos(y) + c = 0}$$

$$\text{at } y(0) = 0 \Rightarrow \quad 0 = e^{2x}\cos(0) + c \Rightarrow 1 + c = 0 \Rightarrow \boxed{c = -1}$$

$$\boxed{e^{2x}\cos(y) - 1 = 0} \rightarrow \text{particular sol}^n$$

(iv) $2\cosh x \cos y \, dx = \sinh x \sin y \, dy$

$M =$ $\qquad$ $N =$

$$\frac{dM}{dy} = 2\cosh x(-\sin y) \ (1), \quad \frac{dN}{dx} = -\cosh x \sin y \ (2); \ \text{here } (1) \neq (2) \ \text{so not a exact}$$

Now finding I.F

$$I.F = e^{\int\left(\frac{\frac{dM}{dy} - \frac{dN}{dx}}{N}\right)dx} \Rightarrow e^{\int\left(\frac{(-2\cosh x \sin y + \cosh x \sin y)dx}{-\sinh x \sin y}\right)} \Rightarrow e^{\int\frac{-\cosh x \sin y \, dx}{\sinh x \sin y}}$$

$$I.F = e^{\ln(\sinh x)} \Rightarrow \boxed{I.F = \sinh x}$$

Now Exact ODE is

$$\rightarrow \sinh x\,(2\cosh x \cos y - \sinh x \sin y \, dy) = 0$$

$$\frac{dU}{dx} = M \Rightarrow U = \int(2\cosh x \sinh x \cos y)dx \Rightarrow \cos y\left(\frac{-\cos 2hx}{2}\right) + g(y)$$

$$\frac{dU}{dy} = N \Rightarrow +\cancel{\sin y}\left(+\frac{\cos 2hx}{2}\right) + g(y) = -\sinh^2 x \cancel{\sin y} \Rightarrow \left(\frac{1 - 2\sinh^2 x}{2}\right) + g(y) = -\sinh^2 x$$

$$g'(y) = -\frac{\sin y}{2} \Rightarrow g(y) = -\frac{1}{2}\int \sin y \, dx \Rightarrow \boxed{g(y) = \frac{\cos y}{2} + c}$$

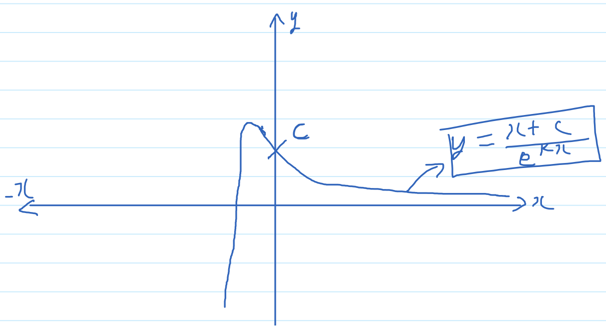
$$\text{so} \quad \boxed{U = -\cos y \frac{\cos 2hx}{2} + \frac{\cos y}{2} + c = 0}$$

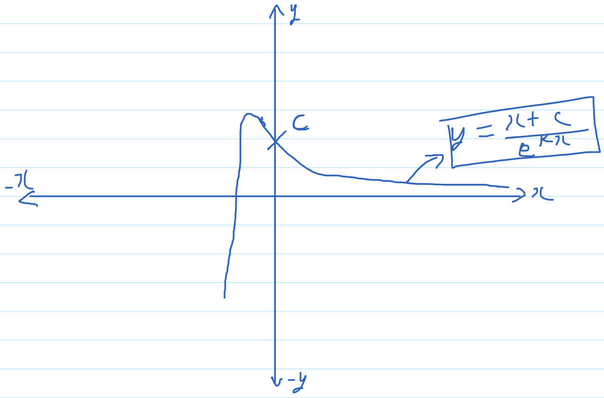
Q2) b) (i) $y' + ky = e^{-kx}$

It is in the form of $y' + p(x)y = r(x)$

$$I.F = e^{\int p\,dx} \Rightarrow e^{\int k\,dx} \Rightarrow e^{kx}$$

$$\text{sol}^n \Rightarrow \quad y \cdot IF = \int(I.f \cdot r(x))dx \Rightarrow y \cdot e^{kx} = \int(e^{kx} \cdot e^{-kx})dx \Rightarrow y \cdot e^{kx} = \int 1 \cdot dx \Rightarrow \boxed{y = \frac{x + c}{e^{kx}}}$$



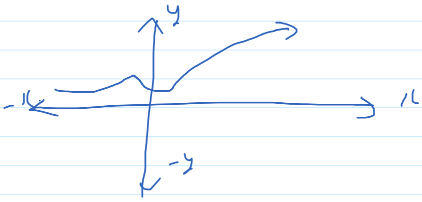


(ii) $xy' = 2y + x^3 e^x$

$$\frac{dy}{dx} = \frac{2y}{x} + x^2 e^x \Rightarrow \frac{dy}{dx} - \frac{2y}{x} = x^2 \cdot e^x \qquad \left[\text{here } P = \frac{-2}{x},\ Q = x^2 e^x\right]$$

$$I.F = e^{\int -2/x} \Rightarrow e^{-\ln x^2} \Rightarrow \boxed{I.F = \frac{1}{x^2}}$$

sol^n of ODE is $y \cdot I.F = \int I.F \times Q \Rightarrow \frac{y}{x^2} = \int \cancel{x^2} \cdot e^x \cdot \frac{1}{\cancel{x^2}} \Rightarrow \boxed{y = (e^x + c)\, x^2}$



(iii) $y' + y \tan x = e^{-0.01x} \cos x \quad , y(0) = 0$

$[P = \tan x \ , \ Q = e^{-0.01x} \cos x]$

$I.F = e^{\int \tan x} \Rightarrow e^{\log(\sec x)} \Rightarrow I.F = \sec(x)$

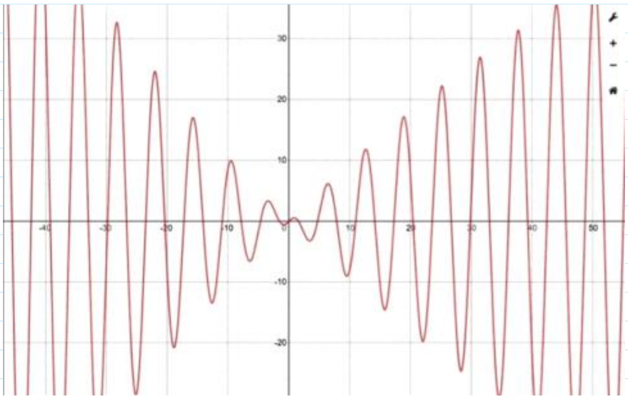
sol^n $\Rightarrow y \cdot I.F = \int (I.F \cdot Q)\, dx \Rightarrow y \sec(x) = \int (\cancel{\sec x} \times e^{0.01x} \cancel{\cos x})\, dx$

$$\boxed{y \sec x = -\frac{e^{-0.01x}}{0.01} + c}$$

When $y(0) = 0$.

$0(\sec x) = -\frac{e^{-0.0(0)}}{0.01} + c \Rightarrow \boxed{100 = c}$

$$\boxed{y \sec(x) = \frac{-e^{-0.01(x)}}{0.01} + 100}$$ particular sol^n



Q3) b) (i) $y' + xy = xy^{-1}$, $y(0) = 3$

$$\frac{dy}{dx} + xy = xy^{-1} \qquad [n=-1,\ Q = x,\ P = x]$$

$$u(x) = e^{\int (1-n)P(x)dx} \Rightarrow e^{\int (1+1)x\,dx} \Rightarrow e^{x^2}$$

Sol^{n})

$$y^{(1-n)} e^{\int (1-n)P(x)dx} = \int \left((1-n)\,Q(x)\, e^{\int (1-n)P(x)dx}\right)dx + c$$

$$y^{1-(-1)} \cdot e^{x^2} = \int (2x e^{x^2})dx + c \qquad [\text{let } x^2 = t \Rightarrow 2x\,dx = dt]$$

$$y^2 \cdot e^{x^2} = \int e^t dt \Rightarrow y^2 e^{x^2} = e^t + c \Rightarrow \boxed{y^2 = 1 + \frac{c}{e^{x^2}}}$$

at $y(0) = 3$

$$9 = 1 + \frac{c}{e} \Rightarrow \boxed{c = 8}$$

$$\boxed{y^2 = 1 + \frac{8}{e^{x^2}}}$$

(ii) $y' = 3.2y - 10y^2$

$$y' - 3.2y = -10y^2 \Rightarrow \frac{1}{y^2}y' - \frac{3.2}{y} = -10 \qquad \left[\frac{1}{y} = t \Rightarrow -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}\right]$$

$$-\frac{dt}{dx} - 3.2t = -10 \Rightarrow e^{\int 3.2dx} = e^{3.2x}$$

$$t \cdot e^{3.2x} = \int 10 \cdot e^{3.2x} + c \Rightarrow \frac{1}{y} = \frac{10}{3.2} + ce^{-3.2x}$$

$$\boxed{y = \frac{3.2}{10 + 3.2xc\,(e^{-3.2x})}}$$

(iii) $2xyy' + (x-1)y^2 = x^2e^x$ $\quad (y^2 = z)$

$$[\text{let } y^2 = z \Rightarrow z' = 2yy']$$

$$x\frac{dz}{dx} + (x-1)z = x^2e^x \Rightarrow \frac{dz}{dx} + \frac{(x-1)}{x}z = xe^x \Rightarrow \frac{dz}{dx} + \left(1 - \frac{1}{x}\right)z = xe^x$$

$$\left[P = 1 - \frac{1}{x},\quad Q = xe^x\right]$$

$$I.f = e^{\int (1-\frac{1}{x})dx} \Rightarrow e^{(x - \ln x)} \Rightarrow e^{x} \cdot e^{-\ln x} \Rightarrow \boxed{\frac{e^{x}}{x} = I f}$$

sdn

$$z \cdot \frac{e^{x}}{x} = \int \frac{e^{x}}{\cancel{x}} \cancel{x} e^{x} \Rightarrow z \cdot \frac{e^{x}}{x} = \frac{e^{2x}}{2} + c \Rightarrow z = \frac{e^{2x}}{2} \times \frac{x}{e^{x}} + \frac{c}{e^{x}} \times x$$

$$\boxed{y^{2} = \frac{x e^{x}}{2} + c x e^{-x}}$$