H4

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Q1) (i) $2xy'' = 3y'$

[let $y' = v$, $y'' = v'$]

$2xv' = 3v \Rightarrow 2x\frac{dv}{dx} = 3v \Rightarrow \int \frac{dv}{v} = \frac{3}{2}\int \frac{dx}{x} \Rightarrow \ln(v) = 1.5(\ln x + \ln c)$

$\ln(v) = 1.5(\ln(c+x)) \Rightarrow \ln(y') = 1.5[\ln(c+x)] \Rightarrow y' = (xC_0)^{3/2}$

$\int dy = \int (x^{3/2} C_0^{3/2})\, dx \Rightarrow y = \left(\frac{5}{2} C^{3/2}\right) x^{5/2} + C_2$

$\boxed{y = C_1 x^{5/2} + C_2}$

(ii) $xy'' + 2y' + xy = 0$ when $y_1 = \frac{\cos x}{x}$

$y'' + \left(\frac{2}{x}\right)y' + y = 0$ [here $\frac{2}{x} = P(x)$, $g(x) = 1$]

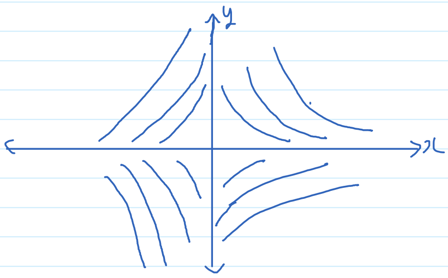
$y_2 = \left[\int \frac{1}{y_1^2} e^{-\int P(x)\,dx} dx\right] y_1 \Rightarrow y_2 = \left[\frac{x^2}{\cos^2 x} e^{-\int 2/x\, dx} dx\right] \times \frac{\cos x}{x}$

$y_2 = \int \frac{x^2}{\cos^2 x}(x^{-2})\, dx \times \frac{\cos x}{x} \Rightarrow \frac{\cos x}{x}\int \sec^2 x\, dx \Rightarrow y_2 = \frac{\cos x}{x}\tan x = \frac{\sin x}{x}$

$\boxed{y_2 = \frac{\sin x}{x}}$

Now we know sol^n is $\Rightarrow C_1 y_1 + C_2 y_2 = C_1 \frac{\cos x}{x} + C_2 \frac{\sin x}{x}$

Q1) b) (i) $y = \frac{C}{x^2}$

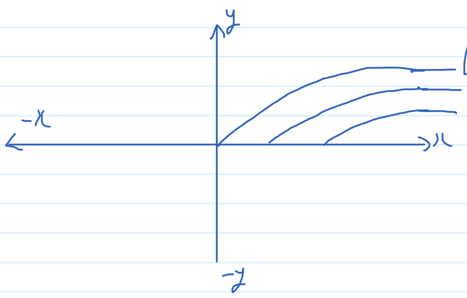


$y' = \frac{-2C}{x^3}$ so $y'_{(OT)} = \frac{x^3}{2C}$ [we know $y = \frac{C}{x^2} \Rightarrow C = yx^2$] putting value of C

(O.T may be hyperbola) $y'_{(OT)} = \frac{x}{2y} \Rightarrow \int 2y\, dy = \int x\, dx \Rightarrow \boxed{2y^2 = x^2 + C}$

(ii) $y = \sqrt{x + C}$

so $y^2 - x = C$ (1)



[O.T may be e^x something]

$y' = \frac{1}{2\sqrt{x+c}} = \frac{1}{2y}$;

Now $y'_{(o.T)} = -2y \Rightarrow \int \frac{dy}{y} = -2\int dx \Rightarrow \ln y = -2x + C$

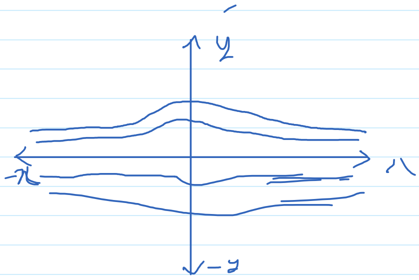
$$\boxed{y = Ce^{-2x}}$$

(iii) $y = ce^{-x^2}$

so $C = y \cdot e^{x^2}$

$y' = -2x \cdot C \cdot e^{-x^2}$ [putting C value]

$y' = -2x \cdot y$



Now $y'_{(o.T)} = \frac{1}{2xy}$

$\Rightarrow 2y \cdot dy = \frac{dx}{x} \Rightarrow \boxed{y^2 = \ln x + C}$

Q2) b) $(x^2 - x)y' = (2x-1)y$

A.T.Q [y' has no solⁿ means y' is discontinous] so also $y \neq 0$

$y' = \left(\frac{2x-1}{x^2-x}\right)y$

[let $x^2 - x = u \Rightarrow 2x - 1 = \frac{du}{dx}$]

$\frac{dy}{y} = \frac{du}{u} \Rightarrow \ln y = \ln(u) + \ln c$

$y = C \cdot u$

$$\boxed{y = C \cdot (x^2 - x)}$$

for $x = 0, 1$ we can see $y_0 = 0$ always $\longrightarrow$ infinite many solⁿ

for $x_0 \neq 0, 1$ we can see that it has unique solⁿ [because $x^2 - x \neq 0$]

for $y \neq 0$ and $x_0 = 0, 1$ it has no solⁿ

Q4) b) i) $y'' + 0.6y' + 0.09y = 0$ (1) , $y(0) = 2.2$, $y'(0) = 0.14$, $e^{-0.3x}$, $xe^{-0.3x}$

$\frac{y_1}{y_2} = \frac{x\cancel{e^{-0.3x}}}{\cancel{e^{-0.3x}}} = x$ [not constant] so $f(x)$ is linearly independent

$y_1' = -0.3e^{-0.3x} \Rightarrow y_1'' = 0.09e^{-0.3x} \longrightarrow$ putting this in (1)

$0.09e^{-0.3x} - 0.18e^{-0.3x} + 0.09e^{-0.3x} = 0$ Hence proved (2)

Now

$$y_2' = e^{-0.3x} - 0.3xe^{-0.3x} \Rightarrow y_2'' = \underbrace{-0.3e^{-0.3x} + 0.09e^{-0.3x} \cdot x - 0.3e^{-0.3x}}$$

$$\Rightarrow y_2'' = -0.6e^{-0.3x} + 0.09(e^{-0.3x}) \cdot x$$ putting this in ②

so

$$= 0.6e^{-0.3x} + 0.04e^{-0.3x} + 0.6e^{-0.3x} - 0.18xe^{-0.3x} + 0.09xe^{-0.3x}$$

$= 0$ Hence prove ④

from ③ & ④ it is verified that y_1, y_2 are basis of $f(x)$