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Math 204 – Worksheet 2

Q1. **Solution curves of $y' = g(y/x)$.** Show that any (nonvertical) straight line through the origin of the xy-plane intersects all these curves of a given ODE at the same angle.

Q1) Ans) eqⁿ of line ; $n = y/x$ so now $y' = g(n)$

we know $\tan\theta = \dfrac{x - y}{1 + xy} \Rightarrow \tan\theta = \dfrac{n - g(n)}{1 + g(n)\cdot n}$

$\theta = \tan^{-1}\left|\dfrac{n - g(n)}{1 + g(n)\, n}\right| = \text{Constant}$

Q2. **Rope.** To tie a boat in a harbor, how many times must a rope be wound around a bollard (a vertical rough cylindrical post fixed on the ground) so that a man holding one end of the rope can resist a force exerted by the boat 1000 times greater than the man can exert? First guess. Experiments show that the change ΔS of the force S in a small portion of the rope is proportional to S and to the small angle $\Delta\phi$ in Fig. 16. Take the proportionality constant 0.15. The result should surprise you!

S Small

Q2) Ans) $\dfrac{\Delta S}{\Delta\theta} = 0.15\, S$ [According to question]

$\dfrac{dS}{S} = 0.15\, d\theta \Rightarrow \ln|S| = 0.15\,\phi + C$

$S = Ke^{0.15\phi}$

if $\theta = 0$, $S_0 = S_0$ hence $S = S_0 e^{0.15\phi} \Rightarrow$ if a man force is S_0 then boat's force $1000\, S_0$

$1000\, S_0 = S_0\, e^{0.15\phi} \Rightarrow \phi = \dfrac{100\ln(100)}{15} = 46\ \text{So}$

$\boxed{\text{No. of turn} = \dfrac{46}{2\pi} = 8}$

Q3. **Family of Curves.** A family of curves can often be characterized as the general solution of $y' = f(x, y)$.

(a) Show that for the circles with center at the origin we get $y' = -x/y$.

(b) Graph some of the hyperbolas $xy = c$. Find an ODE for them.

(c) Find an ODE for the straight lines through the origin.

(d) You will see that the product of the right sides of the ODEs in (a) and (c) equals -1. Do you recognize this as the condition for the two families to be orthogonal (i.e., to intersect at right angles)? Do your graphs confirm this?

(e) Sketch families of curves of your own choice and find their ODEs. Can every family of curves be given by an ODE?

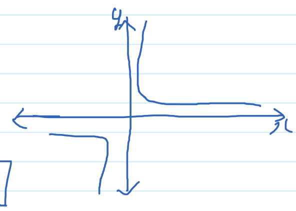
A.T.Q

a) The eqⁿ of the circle with center at origin is $x^2 + y^2 = a^2$ [a is radius]

differentiating both side w.r.t 'x'

$2x + 2yy' = 0 \Rightarrow \boxed{y' = -\dfrac{x}{y}}$

(b) graph of hyperbola $xy = c$



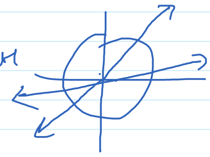
differentiate both side w.r.t x

$xy' + y = 0 \Rightarrow \boxed{y' = -x/y}$

(c) equation of ST $\Rightarrow y = mx$ [where m is slope]

$\boxed{y' = m} = \dfrac{y}{x}$

(d) yes the condition is statisfied as every line passing center of circle is ⊥ to the curve



(e)

Q4. Solve the IVP :

$y' = (x+y-2)^2,\ y(0) = 2.$

Ans) [let $x + y - 2 = u \Rightarrow 1 + y' = \dfrac{du}{dx} \Rightarrow y' = \dfrac{du}{dx} - 1$]

putting value in $y' = (x+y-2)^2 \Rightarrow \dfrac{du}{dx} - 1 = u^2 \Rightarrow \dfrac{du}{dx} = u^2 + 1 \Rightarrow \displaystyle\int \frac{du}{u^2+1} = \int dx \Rightarrow \tan^{-1}(u) = x + C$

$\tan^{-1}(x + y - 2) = x + C$

$\tan^{-1}(0) = 0 + C$

$\boxed{C = 0}$

particular form

$\tan^{-1}(x + y - 2) = x$

particular form

$$\tan^{-1}(x+y-2) = x$$

$$\boxed{y = \tan(x) - x + 2}$$