W5

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Q1) a) $y'' + ay' + by = 0$

let $y = e^{\lambda x}$

$\lambda^2 + a\lambda + b = 0 \rightarrow$ sol^n is λ_1 & λ_2

$\lambda_1 = \dfrac{-a + \sqrt{a^2 - 4b}}{2}$, $\lambda_2 = \dfrac{-a - \sqrt{a^2 - 4b}}{2}$

after solving $e^{\lambda_1 x}$ & $e^{\lambda_2 x}$

if $e^{(\lambda_1 - \lambda_2)x} \neq$ constant

so general soln

$y = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x}$

$$\boxed{\begin{aligned} a &= -(\lambda_1 + \lambda_2) \\ b &= \lambda_1 \lambda_2 \end{aligned}}$$

(b) (i) $y'' + 4y' = 0$

$\lambda^2 + 4\lambda = 0 \Rightarrow \lambda(\lambda + 4) = 0 \Rightarrow \lambda = 0$ or -4

$\therefore y_1 = 1 ; y_2 = e^{-4x}$

sol^n general : $y = C_1 + C_2 e^{-4x}$ (2)

2nd $y'' + 4y' = 0$ let $y' = v$

$v' + 4v = 0 \Rightarrow \dfrac{dv}{v} = -4dx \Rightarrow \ln V = -4x + C$

Now

$\dfrac{dy}{dx} = C' \cdot e^{-4x}$

$y = Ke^{-4x} + C_1$ (1)

both way getting same from (1) & (2) hence confirm

so general sol^n $y = C_1 e^{-ax} + C_2$

(c)

(d) $e^{-\sqrt{5}x}$, $xe^{-\sqrt{5}x}$

$-\sqrt{5}$ is a double root

for $\lambda_1 = \lambda_2 = -\sqrt{5}$

$$a = -(\lambda_1 + \lambda_2) = 2\sqrt{5}$$

$$b = \lambda_1 \lambda_2 = 5$$

Now putting values of a & b

$$\boxed{y'' + 2\sqrt{5}\,y' + 5 = 0}$$

18) <u>Using direct formula</u>

$$y = e^{-a/2\,x}(C_1 \cos \omega x + C_2 \sin \omega x) \quad \left[\text{where } \omega = \sqrt{\frac{4b - a^2}{2}}\right]$$

$$y = e^{0/2\,(x)}(C_1 \cos 2\pi x + C_2 \sin 2\pi x) \quad [\text{here } a = 0]$$

$$\omega = 2\pi = \sqrt{\frac{4b - a^2}{2}}$$

$$4\pi = \sqrt{4b}$$

$$16\pi^2 = 4b$$

$$\boxed{\cancel{4}\,\pi^2 = b} \quad \text{Now} \quad \boxed{y'' + 4\pi^2 y = 0}$$