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Q.1. Solve the ODE by integration
(4 pts) a. $y' + xe^{-\frac{x^2}{2}} = 0$

b. $y' = 4e^{-x}\cos x$

c. $y'' = -y$

d. $y''' = e^{-0.2x}$

Ans a) $y' + xe^{-\frac{x^2}{2}} = 0 \Rightarrow y' = -xe^{-x^2/2} \Rightarrow dy = -xe^{-x^2/2}dx$

putting $x^2/2 = t$ & differentiate
$[x^2/2 = t \Rightarrow x\,dx = dt]$

$\int dy = -\int e^{-t}dt \Rightarrow y = e^{-t} + c \Rightarrow \boxed{y = e^{-x^2/2} + c}$

Ans b) $\int dy = \int (4e^{-x}\cos x)\,dx$ (1)

applying ILATE

$y = 4(e^{-x}\sin x + \int e^{-x}\sin x\,dx) + c_1 \Rightarrow y = 4e^{-x}\sin x + 4(-e^{-x}\cos x - \int e^{-x}\cos x\,dx) + c_1$

$y = 4e^{-x}\sin x + (-4e^{-x}\cos x + \int 4e^{-x}\cos x\,dx) + c_1 \Rightarrow$ from (1) $\cancel{2}y = \cancel{4}^{2}(e^{-x}\sin x - e^{-x}\cos x) + c_1$

$\boxed{y = 2(e^{-x}\sin x - e^{-x}\cos x) + c_1}$

(d) $y''' = e^{-0.2x}$

$\frac{d^3y}{dx^3} = e^{-0.2x} \Rightarrow \int d\left(\frac{d^2y}{dx^2}\right) = \underbrace{\int e^{-0.2x}\cdot dx}_{I}$

$I_1 = \int e^{-0.2x}\,dx$ $\quad\left[\begin{array}{l}-0.2x = t\\ -0.2dx = dt\end{array}\right]$ $[+dx = -5dt]$

$= \int e^{t} - 5dt$

$\boxed{I_1 = -5e^{-0.2x} + c_1}$

Now $\frac{d^2y}{dx^2} = -5e^{-0.2x} + c_1 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = -5e^{-0.2x} + c_1 \Rightarrow \int d\left(\frac{dy}{dx}\right) = \underbrace{\int(-5e^{-0.2x} + c_1)dx}_{I_2}$

$I_2 = \int(-5e^{-0.2x} + c_1)dx \Rightarrow -5(-5e^{-0.2x}) + c_1x + c_2 \Rightarrow \boxed{25e^{-0.2x} + c_1x + c_2}$

Now $\frac{dy}{dx} = 25e^{-0.2x} + c_1x + c_2 \Rightarrow \int dy = (25e^{-0.25x} + c_1x + c_2)dx$

similarly after doing Integration we will get

$\boxed{y = 125e^{-0.2x} + \frac{c_1}{2}x^2 + c_2x + c_3}$

Ans (c) $y'' = -y$

let consider $y = e^{cx}$ where c is constant

$y' = ce^{cx} \Rightarrow y'' = c^2e^{cx}$ [we know $y = e^{cx}$ so replacing $y'' = -y \Rightarrow y'' = -e^{cx}$]

$\Rightarrow -e^{cx} = c^2e^{cx} \Rightarrow e^{cx}(c^2+1) = 0$ [$e^{cx} \neq 0$]

so $c^2 + 1 = 0 \Rightarrow c^2 = -1 \Rightarrow \boxed{c = \pm i}$

so $\boxed{y = e^{\pm ix}}$

$\boxed{\text{Changing to Normal form } y = \cos x \pm i\sin}$

Q2 Ans a) $y' + 5xy = 0$ And $y(0) = \pi$, $y = ce^{-2.5x^2}$ [given]

at $y(0)=\pi$

$\pi = Ce^{-2.5(0)^2} \Rightarrow \pi = C\times 1 \Rightarrow \boxed{C=1}$

Now $y=\pi e^{-2.5x^2}$ ① [diff. both side w.r.t x]

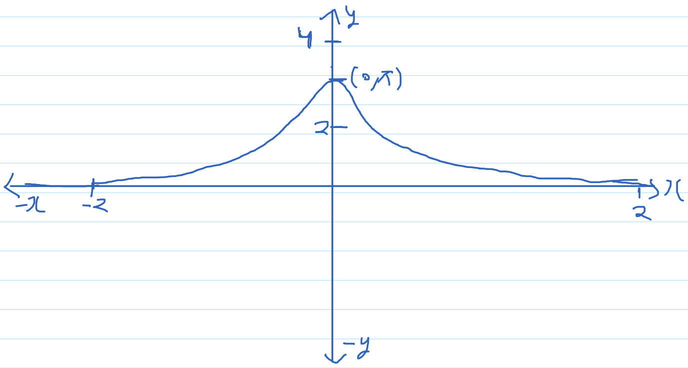
$y' = \pi(-2.5\times 2\times x) \Rightarrow \boxed{y' = -5\pi x e^{-2.5x^2}}$ ②

putting ① & ② in $y'+5xy=0$

$-5\cancel{\pi x e^{-2.5x^2}} + 5x(\cancel{\pi e^{-2.5x^2}}) = 0 \Rightarrow 0=0$

L.H.S = R.H.S Hence, given y is the solution of given ODE

particular sol^n $\Rightarrow \boxed{y=e^{-2.5x^2}}$



Q(B) Given: $yy'=4x$, $y^2-4x^2=C$ $(y>0)$, $y(1)=4$

$y=\pm\sqrt{C-4x^2}$ [given $y>0$] so y is only +ve value

[$y(1)=4$ so $4=+\sqrt{C-4}$; $\boxed{C=12}$]

Now Differe.. both side

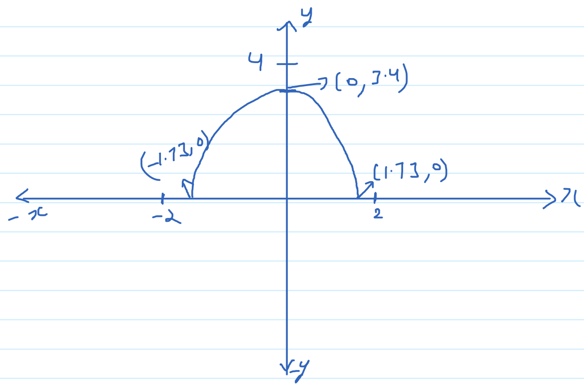
$y' = \frac{8x}{2y} \Rightarrow \boxed{\frac{4x}{y}=y'}$ ①

$\boxed{y=\sqrt{12-4x^2}}$ ②

putting ① & ② in $yy'=4x$

$\cancel{\sqrt{12-4x^2}}\left(\frac{4x}{\cancel{\sqrt{12-4x^2}}}\right) = 4x = 0$

hence given y is a sol^n of given ODE and particular sol^n is $\boxed{y=\sqrt{12-4x^2}}$



Q4) $y' = -5x^4y^2$, $x_0=0$, $y_0=1$, $h=0.2$ (Given)

Now

$x_1 = 0.2$ | $x_2 = 0.4$ [$x_1+0.2$]

$y_1 = 1+(0.2)(f(0,1)) = 1$ | $y_2 = 1+(0.2)(f(0.2,1)) = 0.9984$

$x_3 = 0.6$ [$x_2+0.2$]

$y_3 = y_2 + h\,f(x_2, y_2) \Rightarrow 0.99 + 0.2[-5(0.4)^3(0.99)] = 0.97$

$x_4 = 0.6 \quad [x_3 + 0.2]$

$y_4 = y_3 + h\,f(x_3, y_3) \Rightarrow 0.97 + 0.2[-5(0.6)^4(0.97)] = 0.84$

Q5) b) A) $y^3 y' + x^3 = 0$

A.T.O

$y^3 y' = -x^3 \Rightarrow y^3 \frac{dy}{dx} = -x^3 \Rightarrow y^3 \, dy = -x^3 dx$

integration both sides =

$\int y^3 \, dy = -\int x^3 dx \Rightarrow \frac{y^4}{4} = -\frac{x^4}{4} + c \Rightarrow \boxed{x^4 + y^4 = c}$

B) $y' = e^{2x-1} y^2$

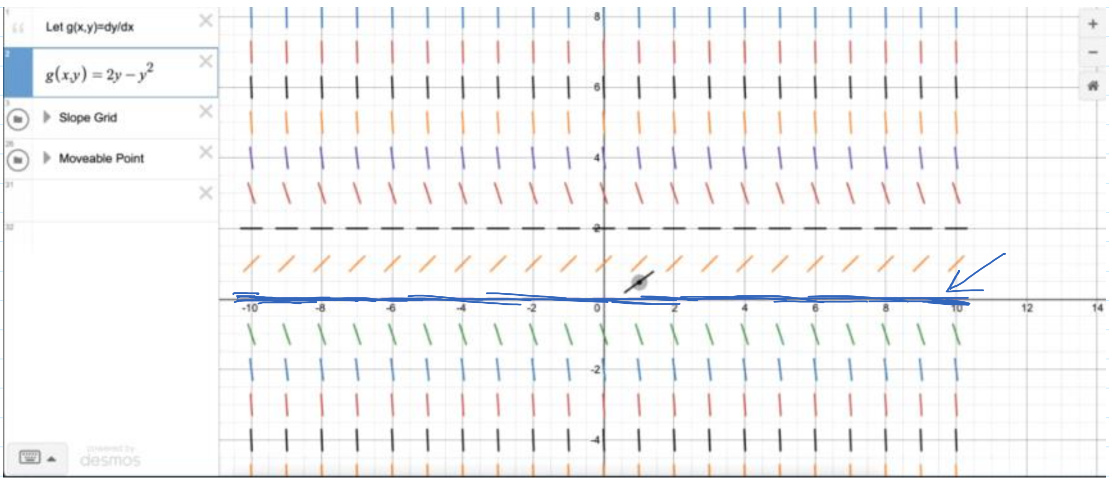
$\frac{dy}{dx} = e^{2x-1} y^2 \Rightarrow \frac{dy}{y^2} = e^{2x-1} dx$

integration both side

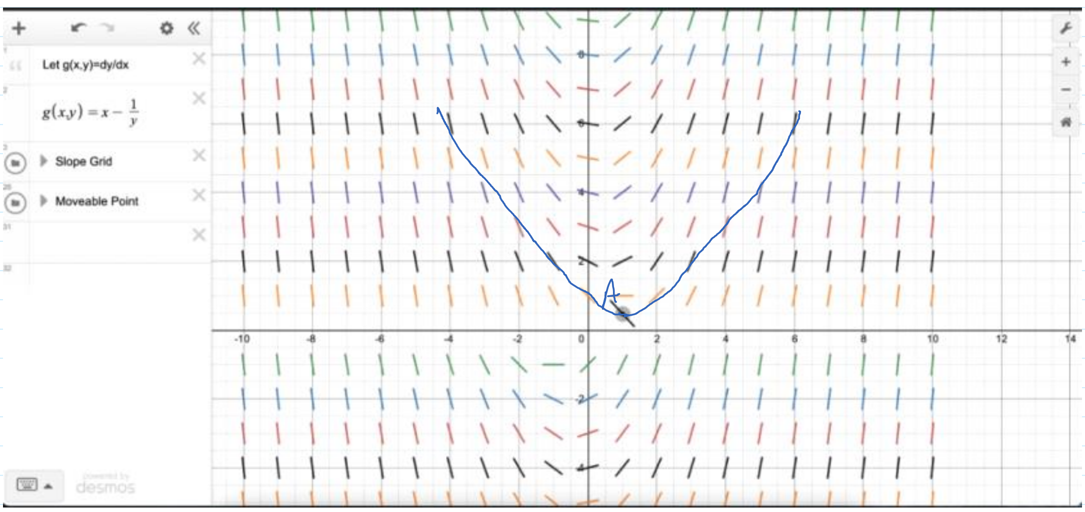
[Let $2x - 1 = t$
$2dx = dt \Rightarrow dx = \frac{dt}{2}$]

$-\frac{1}{y} = \frac{1}{2}\int e^t \, dt \Rightarrow \boxed{-\frac{1}{y} = \frac{1}{2} e^{2x-1} + c}$

Q3) a)



(b)



$A = (1, 0.5)$