W3

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Q1) a)

$(ax+by)\,dx + (kx+ly)\,dx = 0$

$\downarrow$ M $\qquad \downarrow$ N

$\frac{dM}{dy} = b$, $\frac{dN}{dx} = k$ $\qquad \boxed{\text{so to be exact ODE } b=k \text{ is must}}$ (1)

also $\frac{du}{dx} = M \Rightarrow \int du = \int dx\, M \Rightarrow U = \int (ax+by)\,dx \Rightarrow U = \frac{ax^2}{2} + bxy + g(y)$

$\frac{du}{dy} = N \Rightarrow bx + g'(y) = kx + ly \Rightarrow g'(y) = x(k-b) + ly$ from (1)

$g'(y) = ly \Rightarrow g(y) = \frac{ly^2}{2}$

so solution is

$\frac{ax^2}{2} + bxy + \frac{ly^2}{2} + C = 0$

Q2)

3. Drug injection. Find and solve the model for drug injection into the bloodstream if, beginning at $t = 0$, a constant amount A g/min is injected and the drug is simultaneously removed at a rate proportional to the amount of the drug present at time t.

According to question: $y(t) + (y(0) = a$

$\frac{dy}{dt}(t) = A - ky \Rightarrow \int \frac{dy}{A-ky} = dt$

[let $A - ky = u$,]

so $\ln(u) = -kt + C_1 \Rightarrow \ln(A-ky) = -kt + C_1$ $[y(0) = a]$

$\ln(A - ak) = C_1$, $A - ak = C$

$\ln\left(\frac{A-ky}{A-ka}\right) = kt \Rightarrow \frac{A-ka}{k-ky} = e^{kt} \Rightarrow \boxed{y = \frac{A}{k}(1 - e^{-kt})}$

3. Epidemics. A model for the spread of contagious diseases is obtained by assuming that the rate of spread is proportional to the number of contacts between infected and noninfected persons, who are assumed to move freely among each other. Set up the model. Find the equilibrium solutions and indicate their stability or instability. Solve the ODE. Find the limit of the proportion of infected persons as $t \to \infty$ and explain what it means.

A-T-Q

Let total population be $P_T = 1$ so infected + non infected $= 1$ (y, x)

$\boxed{x + y = 1}$

$x = 1 - y \Rightarrow \frac{dy}{dt} = \alpha y(1-y) \Rightarrow \int \frac{dy}{y - y^2} = \int \alpha\, dt$

$y(t) = \frac{\alpha}{\alpha + \alpha C e^{-\alpha t}} = \frac{1}{1 + Ce^{-\alpha t}}$

then let $y(0) \to y(0)$

$y = \frac{y_0}{y_0 + (1 + y_0)e^{-\alpha t}}$ $\qquad$ [at $t \to \infty$

4) **Harvesting renewable resources. Fishing.** Suppose that the population $y(t)$ of a certain kind of fish is given by the logistic equation (11), and fish are caught at a rate Hy proportional to y. Solve this so-called *Schaefer model*. Find the equilibrium solutions y_1 and $y_2 \ (> 0)$ when $H < A$. The expression $Y = Hy_2$ is called the **equilibrium harvest** or **sustainable yield** *corresponding to H.* Why?

11) $y' = Ay - By^2$

$y' = Ay - By^2 - Hy$ [as fishes are getting caught by Hy here]

$y' - (A-H)y = By^2$

$$y(x) = \frac{A-H}{B + (A-H)Ce^{-(A-H)x}}$$

For equilibrium sol^n

$(A-H)y - By^2 = 0 \Rightarrow y = 0,$

$y = \frac{(A-H)}{B}$ [it is +ve value]