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Mathematical model of the Guanay II AUV

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***Abstract*—This work presents a mathematical study to obtain the coefficients that define the dynamics of an autonomous underwater vehicle (AUV), the Guanay II vehicle, with 3 degrees of freedom. This dynamics is given by several forces and moments (hydrostatic, hydrodynamic, added masses and propellers), which largely depend on the vehicle geometry. The Guanay II AUV has been designed following a Myring profile, which improves the hydrodynamics of the vehicle but also yields a tractable mathematical model. The model describes the geometric characteristics of the vehicle and the disposition of the thrusters and has backward movement capacity. The model has been tested by means of simulations and a prototype has been validated in real navigation.**

Keywords—auv; mathematical model

I. INTRODUCTION

The recording of climatic changes and the mapping of the subsoil are important issues in the study of the sea and oceans, for which different infrastructures, like oceanographic vessels, underwater vehicles and aerospace technologies, are available.

Aerospace technologies are useful for global studies or when large portions of the Earth surface are involved, but they do not provide good results at smaller scales. Oceanographic vessels can solve this problem since they are in contact with water, but their operation requires a detailed planning, with growing up costs.

In response to these needs, autonomous vehicles like Gliders, AUV (autonomous underwater vehicles) and ASV (autonomous surface vehicles) are increasingly becoming more popular, since they are able to perform particular missions automatically, taking data with a good spatial-temporal resolution and with reduced costs.

Guanay II AUV [1] [2] [3] is a vehicle developed by the SARTI group of UPC with the objective to have an oceanographic platform able to measure several marine variables, like temperature and salinity of a water column, with a simultaneous high spatial and temporal resolution.

The construction of this vehicle has been done following the Myring profiles [4], which helps to optimize the vehicle hydrodynamics and at the same time, since they have been used in several vehicles [5], [6], [7], have a well-established methodology that allows order the computation of the hydrodynamic coefficients [8].

This work uses that knowledge to obtain each coefficient of the Guanay II and to build a mathematical model following the model proposed by Fossen [9]. However, given the characteristics of the vehicle's own geometry, and its ability to navigate backwards, a number of additional steps are proposed in order to adjust the hydrodynamic coefficients and the dynamic model.

Fig. 1. Guanay II-AUV

II. VEHICLE STRUCTURE

The outer hull of the vehicle has two fundamental hydrodynamic characteristics, namely the torpedo shape and the use of fins for stabilization. Regarding its movement, it is characterized by the use of a main propeller and 2 lateral ones for turning. That allows backward movement and rotation on its axis.

A. Myring Profiles

The torpedo shape follows the Myring profiles, which are defined using the following equations for bow and stern, respectively:

$$r(\Xi)=\frac{1}{2}d\left[1-\left(\frac{\Xi-a}{a}\right)^{2}\right]^{\frac{1}{n}} \tag{1}$$

$$\begin{aligned} r(\Xi)=\frac{1}{2}d-\left[\frac{3d}{2c^{2}}-\frac{\tan\theta}{c}\right](\Xi-a-b)^{2}+ \\ +\left[\frac{d}{c^{3}}-\frac{\tan\theta}{c^{2}}\right](\Xi-a-b)^{3} \end{aligned} \tag{2}$$

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where r(Ξ) is the radius of the torpedo at each point, *a* the length of the bow section, *b* the length of the center, *c* the length of the stern section, *d* the maximum diameter, 2θ the tail angle, and *n* the exponential coefficient of the bow section.

Guanay II-AUV is based on the Myring B hull contour, which classifies body type by the following code form [8]:

$$a/b/n/\theta/\frac{1}{2}d = 15/55/1.25/0.4363/5 \tag{3}$$

Table I gives the dimensions of the Guanay II according to Myring parameters.

TABLE I. MEASURES OF GUANAY II

Parameter	Value	Units
a	0.325	m
b	1.116	m
c	0.924	m
c_{offset}	0.070	m
n	1.25	n/a
θ	0.4363	rad
d	0.326	m
l	2.365	m

B. Fins

Additionally, the vehicle has 5 fins, two pairs in a horizontal position and one in a vertical position, to provide stability and thus minimize rolling. Each fin has been designed as a NACA63-012a profile as shown the Fig. 2.

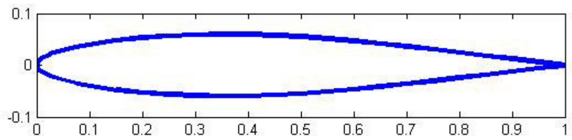

Fig. 2. Proportions of NACA63-012a profile

Table II shows the positions of each pair of fins referenced from the forward end and their dimensions.

TABLE II. POSITIONS OF THE FINS

Parameter	Value	Units
Front central fins	1.204	m
End central fins	1.434	m
Front tail section fins	2.026	m
End tail section fins	2.256	m
Front vertical fin	2.026	m
End vertical fin	2.256	m
Width of fin	0.23	m
Length of fin	0.40	m

III. HIDRODYNAMIC COEFFICIENTS

The vehicle is subjected to several hydrodynamic forces and moments which determine its dynamics, and these forces and moments are linked to constants related to the structure and geometry of the vehicle.

This section shows the coefficients of Guanay II and the way to calculate some of them following the methods of Prestero [8]. The SolidWorks software was also used to model and extract parameters, like the center of buoyancy, the center of gravity and inertial moments.

The center of gravity and buoyancy are shown in Table III (measures referenced from the forward end). The center of buoyancy was taken as the origin of the vehicle because this simplifies some calculations.

TABLE III. CENTER OF GRAVITY AND BUOYANCY

axis	Center of buoyancy	Center of gravity
x	1.21 m	1.25 m
y	0.00 m	0.00 m
z	-0.033 m	-0.041 m

The inertial moments are shown in Table IV. As the inertial products I_{xy}, I_{xz} and I_{yz} are very small compared with the moments I_{xx}, I_{yy} and I_{zz} they were neglected and taken as zero. These values were calculated using the weight of each component and their distribution along the vehicle.

TABLE IV. INERTIAL MOMENTS

Parameter	Value	Units
Masa	96.4	kg
Ixx	1.82	kg.m^2
Iyy	28	kg.m^2
Izz	28.2	kg.m^2

A. Axial drag

When the vehicle is in motion it suffers a force from the fluid it traverses, mainly in the form of underwater friction at the nose of the vehicle. The drag coefficient is calculated using

$$X_{u|u|} = -\frac{1}{2}\rho c_{\mathrm{d}} A_f \tag{4}$$

where ρ is the fluid density, c_d is an empiric coefficient with value 0.3 [8, p. 26], and A_f is the vehicle frontal area.

B. Crossed drag

These forces affect the lateral walls of the AUV. They are calculated using the radii along the vehicle, taking the center of buoyancy as origin. Then the following integrals and some coefficients can be calculated due the simmetry of the vehicle:

$$\begin{aligned}
Y_{v|v|} &= Z_{w|w|} = -\gamma \int_x 2R(x)dx - 2\vartheta \\
M_{w|w|} &= -N_{v|v|} = \gamma \int_x 2xR(x)dx - 2x_{\mathrm{fin}}\vartheta \\
Y_{r|r|} &= -Z_{q|q|} = -\gamma \int_x 2x|x|R(x)dx - 2x_{\mathrm{fin}}|x_{\mathrm{fin}}|\vartheta \\
M_{q|q|} &= N_{r|r|} = -\gamma \int_x 2x^3 R(x)dx - 2x_{\mathrm{fin}}^3\vartheta
\end{aligned} \tag{5}$$

where

$$\begin{aligned}\gamma &= \frac{1}{2}\rho c_{dc} \\ \vartheta &= \frac{1}{2}\rho S_{\text{fin}} c_{df}\end{aligned}$$

C_{dc} is the damping coefficient of a cylinder (approx. 1.1), C_{df} the damping coefficient cross flow (approx. 0.56), R(x) the radii described in (1) and (2) taking the origin in the center of buoyancy, and S_{fin} the fin area of the vehicle.

C. Axial added mass

The added masses represent a measure of the inertia added to the system due to the fact that the vehicle moves a quantity of volume of fluid when it accelerates or decelerates, and this type of movement of fluid yields another term to the force acting on the vehicle.

The added mass referred to the x axis can be calculated by approximating the hull of the vehicle by an ellipsoid of length l and width d. Blevins [10] proposed the following empirical formulas for calculation:

$$\begin{aligned}X_{\dot{u}} &= -\frac{4\alpha\rho\pi}{3}\left(\frac{l}{2}\right)\left(\frac{d}{2}\right)^2 \\ X_{\dot{u}} &= -\frac{4\beta\rho\pi}{3}\left(\frac{d}{2}\right)^3\end{aligned} \tag{6}$$

where α and β are empiric parameters which depend on the dimensions of the approximated ellipsoid [10]. In the case of Guanay II, these coefficients are α=0.0349, β=0.2473.

D. Crossflow added mass

For this calculation we use slender body theory, which states that, as the length of the vehicle is longer than its diameter, it is possible to approximate the cross flow added mass as a sum of slices along the body.

For this purpose a variable m_a , which represents the added mass per length unit, is defined. m_a must be calculated in two different ways, depending on whether the transversal section of the vehicle is a circle or it corresponds to a section with fins, as is shown in the equation (7).

$$m_a(x) = \begin{cases} \pi\rho R(x)^2 \quad \text{for } x \text{ cylinder} \\ \pi\rho\left(a_f^2 - R(x)^2 + \frac{R(x)^4}{a_f^2}\right) \\ \quad \text{for } x \text{ fin}\end{cases} \tag{7}$$

From these, the coefficients of cross flow added masses are calculated using integrals along the x axis as shown in the following equations

$$\begin{aligned} Y_{\dot{v}} = Z_{\dot{w}} &= -\int_x m_a(x)dx \\ M_{\dot{w}} = -N_{\dot{v}} = -Y_{\dot{r}} = Z_{\dot{q}} &= -\int_x x m_a(x)dx \\ M_{\dot{q}} = N_{\dot{r}} &= -\int_x x^2 m_a(x)dx \end{aligned} \tag{8}$$

E. Added masses in roll

To compute this added mass one considers into that the roll generated in smooth sections is minimal and is concentrated in the fin areas. Blevins [10] proposed the empiric equation

$$K_{\dot{p}} = -\int_{x_f} \frac{2}{\pi}\rho a^4 dx \tag{9}$$

where a is the length of the fin taken from the central axis, and x_f are the sectors along the x axis where there are fins.

F. Propulsion from the lateral motors

Unlike conventional AUVs, the Guanay II implements its turning using two thrusters located in the extremes of two fins. Each thruster generates a torque respect to the center of rotation, and the moment generated by each thruster can be expressed as

$$\begin{aligned} N_l &= a_f X_{\text{l}} \\ N_r &= -a_f X_{\text{r}} \end{aligned} \tag{10}$$

where a_f is the perpendicular distance from the central axis to the extreme where the thruster is located, and X_l and X_r are the propulsion forces of the left and right thrusters, respectively-

G. Guanay II theoretical coefficients

Implementing all the equations described above, the coefficients of Table V were obtained for the dynamics of the Guanay II AUV.

IV. Mathematical Model in 3 DoF

The principle of motion of the vehicle is to sail on the sea surface following established routes, and at certain points to dive in a completely vertical form. Therefore, the dynamic model considers only three degrees of freedom: forward (x), lateral (y) and yaw (ψ).

The following equations summarize the model proposed by Fossen [9], but reduced to above 3 degrees of freedom:

$$m\dot{u} - mvr = X_{\dot{u}}\dot{u} - Y_{\dot{v}}vr - Y_{\dot{r}}r^2 + X_{u|u|}u|u| + \\ + X_{prop} + X_l + X_r \quad (11)$$

$$m\dot{v} + mur = Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + X_{\dot{u}}ur + Y_{v|v|}v|v| + \\ Y_{r|r|}r|r| + Y_{uvf}uv + Y_{urf}ur \quad (12)$$

$$I_z\dot{r} = N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + Y_{\dot{r}}ur - (X_{\dot{u}} - Y_{\dot{v}})uv + \\ + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{uvf}uv + \\ + N_{urf}ur + a_{fin}(X_r - X_l) \quad (13)$$

TABLE V. CALCULUS OF GUANAY II COEFFICIENTS

Parameter	Value	Units
$X_{u\|u\|}$	-12.8334	kg/m
$Y_{v\|v\|}$	-425.6532	kg/m
$Z_{w\|w\|}$	-425.6532	kg/m
$M_{w\|w\|}$	42.8456	kg
$N_{v\|v\|}$	-42.8456	kg
$Y_{r\|r\|}$	-43.2557	kg m/rad²
$Z_{q\|q\|}$	-43.2557	kg m/rad²
$M_{q\|q\|}$	-43.6539	kg m²/rad²
$N_{r\|r\|}$	-43.6539	kg m²/rad²
$X_{\dot{u}}$	-4.5983	kg
$Y_{\dot{v}}$	-509.7220	kg
$Z_{\dot{w}}$	-509.7220	kg
$M_{\dot{w}}$	176.3057	kg m
$N_{\dot{v}}$	-176.3057	kg m
$Y_{\dot{r}}$	-176.3057	kg m/rad
$Z_{\dot{q}}$	176.3057	kg m/rad
$M_{\dot{q}}$	-224.5006	kg m²/rad
$N_{\dot{r}}$	-224.5006	kg m²/rad
$K_{\dot{p}}$	24.9127	kg m²/rad²

This model includes the mass and inertial moments of the rigid body, the contributions of centripetal and Coriolis forces, the forces due to added masses, the hydrodynamic drag, and forces due to propellers.

V. MODEL CORRECTIONS

Simulation of the system described above displayed two problems: instability for particular conditions because of the coefficients, and instability when using negative forward speeds. The corrections made to the model to solve these are described next.

A. Coefficient correction

The model displayed strange behavior when it was excited by the lateral thrusters. Detailed studies showed that the system was unstable in the transfer function $r(s)/X_{lateral}(s)$ because the poles were in the right half-plane. To correct this, it was necessary to set the cross damping coefficient to the value

$$Y_{\dot{r}} = -320 kg \cdot m / rad \quad (14)$$

B. Model equations correction

On the other hand, Guanay II is designed to make various types of movements, including movements in reverse, which are not covered by the original dynamic model, resulting in simulations with unbounded values of some variables. To address this, the model equations were adapted as follows:

- The terms with the product *uv* become $|u|v$, in order to preserve the sign of crossover friction in *v* when it goes in reverse.
- The terms that have the product *ur* become $|u|r$, in order to preserve the sign of crossover friction in *r* when it goes in reverse.

Note that these changes compromise the calculation of hydrodynamic coefficients, but as will be discussed in Section VII, these changes are very satisfactory for a first approximation of the backwards movement.

Introducing these changes in equations (11–13), and solving for the accelerations, the following equations are obtained

$$\dot{u} = c_{u1p}X_{prop} + c_{u1r}X_r + c_{u1l}X_l + \\ + c_{u2}vr + c_{u3}r^2 + c_{u4}u|u| \quad (15)$$

$$\dot{v} = c_{v1r}X_r - c_{v1l}X_l + c_{v2}|u|v + \\ + c_{v3}|u|r + c_{v4}v|v| + c_{v5}r|r| \quad (16)$$

$$\dot{r} = c_{r1r}X_r - c_{r1l}X_l + c_{r2}|u|v + \\ + c_{r3}|u|r + c_{r4}v|v| + c_{r5}r|r| \quad (17)$$

VI. FIELD TESTS AND SECOND CORRECTIONS

Various tests were performed in a saltwater canal in order to compare real paths with simulation paths made by applying specific propulsion to the thrusters.

The travel in water took 2½ hours, yielding a considerable amount of data of main propulsion, right propulsion, left propulsion and positions with different types of movements.

After making several simulations and comparing them with the real values some constants were modified, in particular those involving the thrusters, as shown in (18):

$$c_{u1p}=7.9283e\text{-}04; \quad c_{u1r}=4.5455e\text{-}04; \\ c_{u1l}=4.2284e\text{-}04; \quad c_{v1r}=-9.7660e\text{-}05; \\ c_{v1l}=-6.5107e\text{-}05; \quad c_{r1r}=1.8303e\text{-}04; \\ c_{r1l}=1.2202e\text{-}04; \quad (18)$$

A. Guanay II - final coefficients

Table VI shows the final coefficients for the Guanay II using the model proposed in equations (15–17).

TABLE VI. GUANAY II - FINAL COEFFICIENTS

Parameter	Value	Units
c_{u1p}	7.9283e-04	kg^{-1}
c_{u1r}	4.5455e-04	kg^{-1}
c_{u1l}	4.2284e-04	kg^{-1}
c_{u2}	6.3397	n/a
c_{u3}	3.3827	m
c_{u4}	-0.1357	m^{-1}
c_{v1r}	-9.7660e-05	kg^{-1}
c_{v1l}	-6.5107e-05	kg^{-1}
c_{v2}	2.4774	m^{-1}
c_{v3}	1.5808	n/a
c_{v4}	-1.9095	m^{-1}
c_{v5}	0.0618	m
c_{r1r}	1.8303e-04	kg^{-1}
c_{r1l}	1.2202e-04	kg^{-1}
c_{r2}	-5.1488	m^{-1}
c_{r3}	-3.2684	n/a
c_{r4}	2.2485	m^{-1}
c_{r5}	-0.2510	m

VII. RESULTS

This section presents the comparison between the field tests and the simulations. Figure 3 shows the results for the velocity r when a negative propulsion is consigned to the lateral thrusters.

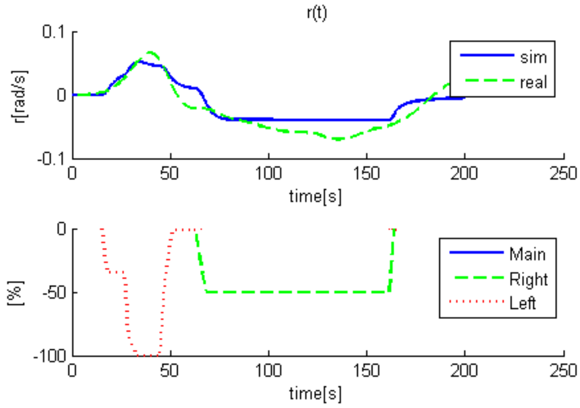

Fig. 3. Simulation and results for a turns with negative propulsions.

Figure 4 shows the results for the velocities u and r for different propulsions in the main thruster, right thruster and left thruster, and it can be seen that the data from the simulations is quite similar to the one obtained from the field tests. This Figure also shows a complex path, consisting in a movement in straight line, next a turn and next another movement in straight line, for both the simulation of the model and the experiment with the vehicle. Although the results are not really accurate, they are quite similar and, moreover, it must be taken into account that the waves that were present in the channel may have affected some movements.

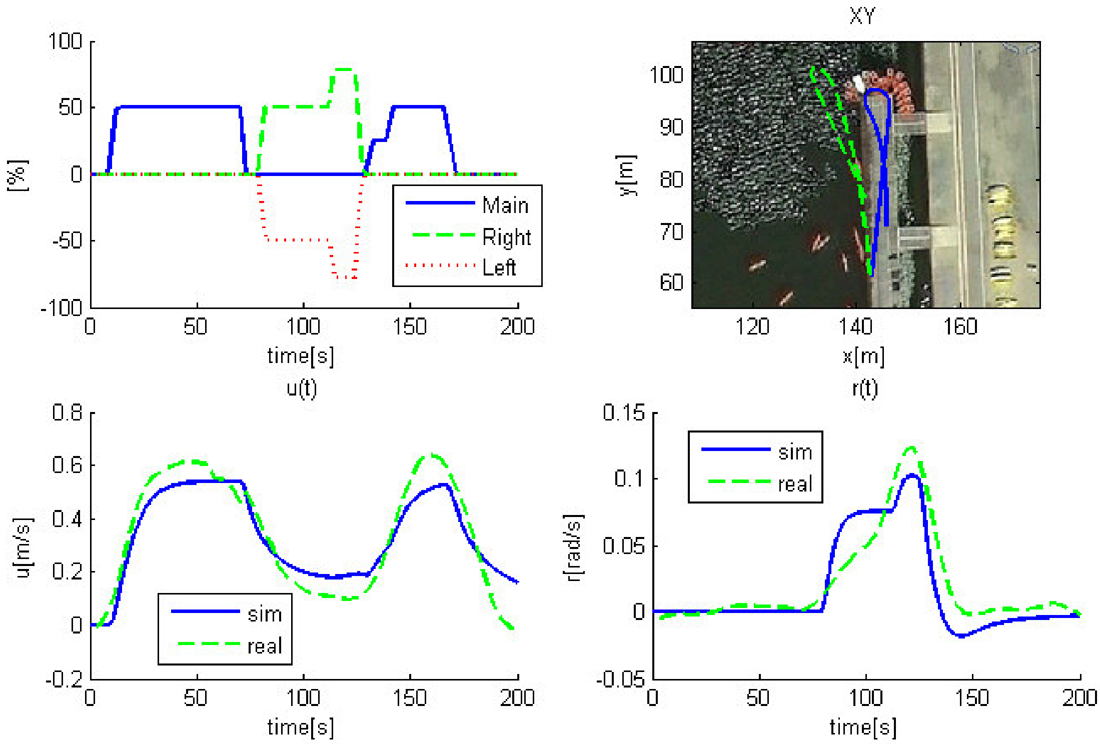

Fig. 4. Simulation and field test with several propulsions including all thrusters.

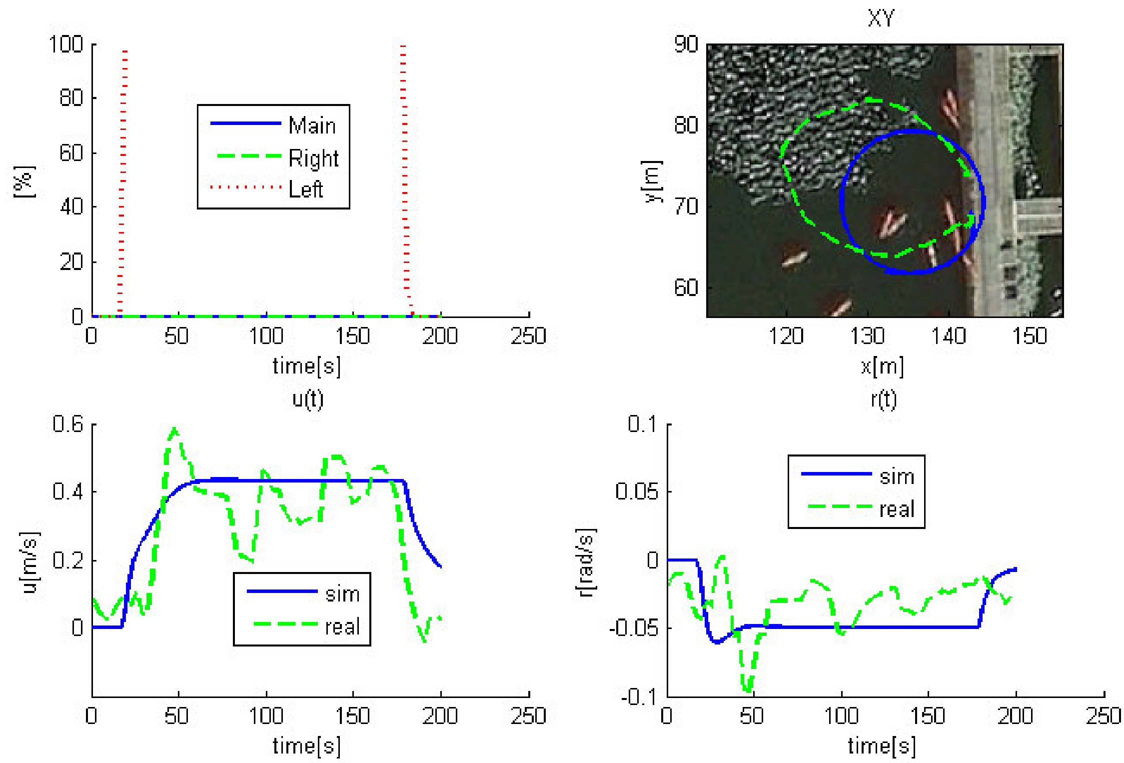

Fig. 5. Simulation and field test with only one lateral thruster, resulting in a circle in the XY plane.

Figure 5 shows the results using only the left thruster. In this case the path described is a circle and has a radius of curvature of about 10 m. Similar to what was said in the previous graph, the results are very similar.

VIII. Conclusions and Future Work

This paper presents a mathematical model that describes the behavior of the 3 degrees of freedom Guanay II AUV. The coefficients have been calculated following a well-established methodology, using its hydrodynamic geometry.

The model has been improved in certain aspects in order to allow negative velocities, and has been validated experimentally, obtaining satisfactory results.

Future work will include a more detailed characterization of the thrusters, as well as more marine trials in order to further adjust the hydrodynamic coefficients.

Acknowledgment

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