Mathematical Model of the Surface Mini Vessel

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ABSTRACT

Problems of analysis and movement simulation of mobile robots (surface mini vessel in our case) require complete nonlinear multivariable mathematical model that allows to consider nonlinear cross-relationships between various components of rotary and forward movement. Such system is necessary for synthesis of robot control system and its development is presented in the paper. Besides mathematical model, accurate estimation of aerial and / or hydrodynamic influence from the continuous medium is also required for the synthesis of adequate control system. However, calculation of these influences is a very time-consuming task from the computational standpoint. The solution to this problem is in development of such calculation procedures that take in to account specific characteristics of the interaction between a carrier and a solid medium - single-phase or multi – that greatly accelerates the process of calculation at the algorithmic level. Such procedure for the surface mini vessel is presented in the paper. Mathematical model and calculation procedure are followed by the numerical simulation of movement for the vessel, managed by position-path regulator. Experiment was conducted at small angles of heel and with the presence of sea waves on the basis of proposed mathematical model and algorithms.

CCS Concepts

• **Computer systems organization~Robotic control**

Keywords

Mathematical model, mini vessel, dynamics forces.

1. INTRODUCTION

In recent years, robots got widely spread in all areas of human interactions with environment. As for water vessels, mobile robots help with ecology control, climate monitoring, sea exploraion and so on. Not to mention emergency and military applications.

One of the main questions for robot is efficiency. Semi-automatic unit can be backed up by a remote operator, but this increase operational cost. Fully automatic models are very attractive but need smart control system in order to perform complex tasks.

Southern Federal University in Russia is developing fully automatic surface and underwater vessels. Methods for control of different subsystems [1-3] as well as implementation of control unit [4,5] were already presented and proved to be efficient in simulation and real experiments. In order to step further to construction of robot, a mathematical model of vessel motion is required. Development of this model is presented in next sections.

2. THE COMPLETE MULTIVARIABLE MODEL OF THE VESSEL MOTION

A distinctive feature of the surface vessel dynamics is its movement on the boundary of two media, air and water, which increases the number of arguments in the functional relationships of forces and moments generated by the continuous medium. The presence of significant wind loads and / or underwater currents requires differentiated consideration of these phenomena, which, in the simplest case a steady flow, requires consideration of two pairs of angles of attack and slip. In addition, surge of the sea is a separate disturbance that is very difficult to process. All this together results in a substantial (orders of magnitude) increase of calculation time.

Let's consider a complete mathematical model of the vessel's motion. We use the following related OXYZ coordinate system: its origin O is the intersection point of the normal dropped from the geometric center of the vessel perpendicular to the media boundary in a static position and the keel line; X axis is directed in parallel to a diametral plane of vessel media interface in its static position; OY axis is oriented along said normal; OZ axis forms a right-handed with OX and OY (see. Figure 1 associated OXYZ coordinate system is highlighted in orange). The base coordinate system is chosen so that its coordinate plane Og Xg Zg coincided with the undisturbed free surface (see. Figure 2).

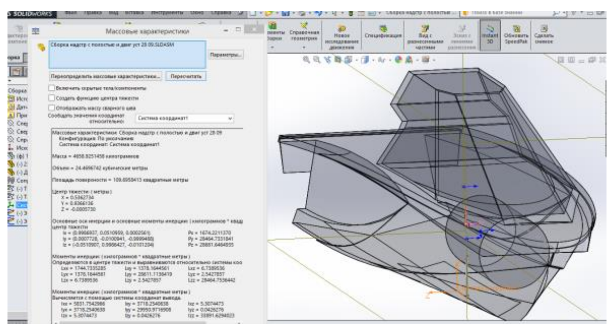

Figure 1. On the definition of the coordinate system related to boats

Complete nonlinear multivariable dynamics model can be represented in matrix form [6]:

$$\dot{\mathbf{Y}} = \left[\hat{\mathbf{A}}\right]\cdot\mathbf{X}, \quad \frac{d\mathbf{X}}{dt} = \left[\mathbf{M}\right]^{-1}\left(\tilde{\mathbf{F}}_{упр} + \tilde{\mathbf{F}}_{дин} + \tilde{\mathbf{F}}_{в}\right) \quad (1)$$

Where $\tilde{\mathbf{A}}, \tilde{\mathbf{G}}, \tilde{\mathbf{F}}_{WA}$, $\tilde{\mathbf{F}}_{в} = \tilde{\mathbf{A}} + \tilde{\mathbf{G}} + \tilde{\mathbf{F}}_{AW}$ - vectors of Archimedes generalized forces, gravity, hydro and aerodynamic effects of full

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strength, respectively; $\tilde{\mathbf{F}}_{дин}$ - generalized vector of the nonlinear dynamics of the elements; $\tilde{\mathbf{F}}_{упр}$ - generalized vector control actions; $\left[\tilde{\mathbf{M}}\right]$ - matrix of mass-inertial characteristics; $\bar{Y} = \left[\mathbf{r}(x_0, y_0, z_0), \mathbf{\Theta}(\varphi, \psi, \gamma)\right]^T$ - vector of the external coordinates describing the position (The radius vector $\mathbf{r}(x_0, y_0, z_0)$) and orientation (vector $\mathbf{\Theta}(\varphi, \psi, \gamma)$) coupled system relative to the base; $\mathbf{X} = (\omega_x, \omega_y, \omega_z, V_x, V_y, V_z)^T$ - vector of internal origin - projections on the associated linear axis vectors $\mathbf{V}(V_x, V_y, V_z)$ and the corner $\boldsymbol{\omega}(\omega_x, \omega_y, \omega_z)$ velocity; $\left[\hat{\mathbf{A}}\right]$ - full matrix kinematics.

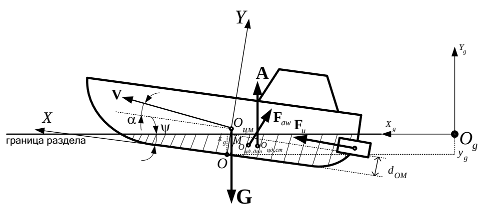

Figure 2.-Visualization of the parameters that define the position of the free surface in the contact coordinate system and the external force

Now it is possible to proceed to the next step: definition of generalized hydro-aero static / dynamic forces $\tilde{\mathbf{F}}_{AW} = (\mathbf{F}_{AW}, \mathbf{M}_{AW})$.

3. METHODS FOR ESTIMATION OF FUNCTIONAL RELATIONSHIPS BETWEEN HYDRO-AERO STATIC AND DYNAMIC FORCES

As we work with the continuous medium, forces and moments can be presented as superposition of the corresponding effects on the calm water $\mathbf{F}_{aw}, \mathbf{M}_{aw}$ and the contribution of sea waves $\mathbf{F}_{волн}, \mathbf{M}_{волн}$:

$$\mathbf{F}_{AW} = \mathbf{F}_{aw} + \mathbf{F}_{волн}, \mathbf{M}_{AW} = \mathbf{M}_{aw} + \mathbf{M}_{волн}. \tag{2}$$

Let's consider the components $\mathbf{F}_{aw}, \mathbf{M}_{aw}$. Angles of attack α and sliding β characterize the orientation of the vector of linear velocity $\mathbf{V}$ ship movement relative to water and air environments. However, to set the orientation of the spacecraft with respect to the free surface section requires three additional values: banking angle γ, trim angle ψ and displacement $U_{подв}$ or any value, clearly defined through $U_{подв}$ and said corners γ, ψ. Thus, each of the projections $\mathbf{F}_{aw}(\xi), \mathbf{M}_{aw}(\xi)$ It depends on nine quantities $\left(V, \alpha, \beta, \omega_x, \omega_y, \omega_z, \psi, \gamma, U_{подв}\right) \equiv \xi$.

A large number of the arguments of these dependences greatly complicates the analysis and motion simulation of the surface between the media. Therefore, it seems appropriate to develop this approach of evaluation of these dependencies, which would be adequate and at the same time will significantly reduce the time of the identification of hydro- aerodynamics of the model parameters. Now we'll show that in order to determine dependencies $\mathbf{F}_{aw}(\xi), \mathbf{M}_{aw}(\xi)$ in the first approximation it is enough to simulate hydro and aerodynamic for fixed displacement $U_{подв,0}$.

Forces and moments $\mathbf{F}_{aw}, \mathbf{M}_{aw}$ can always be represented as a superposition of the corresponding effects on the underwater $\mathbf{F}_w, \mathbf{M}_w$ and aerial coverage $\mathbf{F}_a, \mathbf{M}_a$ lapped surface of the mini vessels. Aerodynamic effect is neglected for simplicity.

Let $\mathbf{F}_w^0, \mathbf{M}_w^0$ be value of vectors $\mathbf{F}_w, \mathbf{M}_w$ with a displacement $U_{подв,0}$.

As it is well known [7-10], hydro aerodynamic effects at a constant speed are proportional to the wetted surface area and a corresponding hydraulic drag coefficient, which takes into account, first of all, the shape of the surface. If we neglect the change in the form of the submerged part if the vessel at a variation of displacement, but with fixed angles of heel and trim γ, ψ we can approximately assume that the vectors $\mathbf{F}_w, \mathbf{M}_w$ are proportional to vectors $\mathbf{F}_w^0, \mathbf{M}_w^0$ and to the ratio function of wetted surface areas $f_S(\gamma, \psi, U_{подв})$ for the given displacement $U_{подв}$ and etalon $U_{подв,0}$:

$$\mathbf{F}_w = \mathbf{F}_w^0 \cdot f_S(\gamma, \psi, U_{подв}), \mathbf{M}_w = \mathbf{M}_w^0 \cdot f_S(\gamma, \psi, U_{подв}) \tag{3}$$

where

$$f_S(\gamma, \psi, U_{подв}) = \frac{S_{подв}(\gamma, \psi, U_{подв})}{S_{подв,0}(\gamma, \psi)} \tag{4}$$

$S_{подв}(\gamma, \psi, U_{подв})$ - wetted surface area at angles of heel γ, trim ψ and displacement $U_{подв}$, $S_{подв,0}(\gamma, \psi) = S_{подв}(\gamma, \psi, U_{подв,0})$ - wetted surface area at the reference displacement $U_{подв,0}$ and the same angles γ, ψ. Let d_{OM} be a distance from the origin of the system related to M OY axis point of intersection with the free surface (Figure 2). The value d_{OM} with the angles γ, ψ completely determines the orientation of the underwater part of the ship relative to the free surface, so $U_{подв} = U_{подв}(d_{OM}, \gamma, \psi)$ and according to (3) instead of argument can be used d_{OM}.

Hydrostatic influences are calculated according to common formulas, including functional dependencies of coordinates where Archimedes force is applied $x_A y_A, z_A$ and the volume of the underwater part of the machine $U_{подв}$ from d_{OM}, γ, ψ [6-9].

The peculiarity of the proposed approach to the determination of hydrodynamic effects is to effect an underwater $\mathbf{F}_w^0, \mathbf{M}_w^0$ part of the device for a fixed displacement $U_{подв,0}$, and then the approximate formula (3) to assess the impact of other relevant $U_{подв}$.

This approximation is very accurate, if the change of the ship's displacement during movement is negligable, since the latter can not lead to a notable change in the shape of its underwater part with the same angles of heel and trim. For higher speeds, this method allows only to approximately assess the impact of the continuous medium.

The components of the sea surge $\mathbf{F}_{волн}, \mathbf{M}_{волн}$ can be estimated from empirical data given, for example, in [9]. For their projections on the axis associated with the boat coordinate system after the conversion of the high-speed system the following approximation formulas were obtained:

$$F_{\text{волн},x} = Ck_\lambda\left(\frac{\lambda}{L}\right) f_\beta(\beta)\zeta_A{}^2 f_V(V), \text{Н} \qquad (5)$$

$$F_{\text{волн},y} = Ck_\lambda\left(\frac{\lambda}{L}\right) *$$

$$* \frac{\left[f_{1\gamma\beta\psi}(\gamma,\beta,\psi) - 15.56 f_\beta(\beta) sin\psi\right]}{cos\psi} f_V(V), \text{Н} \qquad (6)$$

$$F_{\text{волн},z} = Ck_\lambda\left(\frac{\lambda}{L}\right) f_{1\gamma\beta}(\gamma,\beta)\zeta_A{}^2 f_V(V), \text{Н} \qquad (7)$$

$$M_{A\text{волн},x} = \text{C}\, k_\lambda\left(\frac{\lambda}{L}\right)\, f_{2\gamma\beta}(\gamma,\beta)\, \zeta_A{}^2 f_V(V), \ \ \text{Нм} \qquad (8)$$

$$M_{\text{волн},y} = Ck_\lambda\left(\frac{\lambda}{L}\right) *$$

$$* \frac{\left[0{,}46\gamma - 0{,}049 f_{2\gamma\beta}(\gamma,\beta) sin\psi\right]}{cos\psi} \zeta_A{}^2 f_V(V), \text{Нм} \qquad (9)$$

$$M_{\text{волн},z} = Ck_\lambda\left(\frac{\lambda}{L}\right) f_{2\gamma\beta\psi}(\gamma,\beta,\psi)\zeta_A{}^2 f_V(V), \ \ \text{Нм} \qquad (10)$$

where $C = \rho g(B^2/L)$ and included in the expression function of the rate of waves β angles, vessel trim and roll ψ and γ are the following:

$$f_V(V) = 0{,}12 + 0{,}25V - 0{,}004V^2,\ f_\beta(\beta) = 4.835\text{e-}007\ \beta^2|\beta| - 4.63\text{e-}005\ \beta^2 - 0.01871\ |\beta| + 2.609,$$

$$f_{1\gamma\beta}(\gamma,\beta) = -(7{,}73\gamma + 5{,}50\beta),\ f_{2\gamma\beta}(\gamma,\beta) = 5{,}43\gamma - 0.0121\gamma|\gamma| + sign(\beta)(6.222\text{e} - 011\beta^6|\beta| - 5.169\text{e} - 008\beta^6 + 1.615\text{e} - 005\beta^4|\beta| - 0.00238\beta^4 + 0.169\beta^2|\beta| - 5.607\ \beta^2 + 117.2\ |\beta| - 21.31),$$

$$f_{1\gamma\beta\psi}(\gamma,\beta,\psi) = 4{,}24 + 1{,}625\psi + 0.0167\gamma + 0.0194|\beta| - 5.81410^{-4}\beta^2,\ f_{2\gamma\beta\psi}(\gamma,\beta,\psi) = 136{,}5 + 1{,}274\psi - 0{,}0063|\gamma|\psi - 0{,}00402|\beta|\psi - 0{,}00024V\psi^2,$$

$$k_\lambda\left(\frac{\lambda}{L}\right) = -29.95\ \left(\frac{\lambda}{L}\right)^6 + 213\ \left(\frac{\lambda}{L}\right)^5 - 592.7\ \left(\frac{\lambda}{L}\right)^4 + 814.3\ \left(\frac{\lambda}{L}\right)^3 - 573.8\left(\frac{\lambda}{L}\right)^2 + 195.4\ \left(\frac{\lambda}{L}\right) - 24.6.$$

In these expressions, V - vessel speed (varies in the range from 0 to 20 m/s), L - length of the ship, B - its width at the normal waterline, ζ_A – the amplitude of the wave, λ – length wave, β - waves course angle (in degrees): this angle is zero when overrunning wave counter is positive - when the waves run to the left side, and is equal to 180 degrees, when the overrunning wave - towards the stern; ψ - trim angle, γ - roll angle (given in degrees). Formula (12) are sufficiently accurate for the amplitudes of the waves $\frac{2\zeta_A}{L} < 1/15$.

4. ESTIMATION OF MASS-INERTIA AND DAMPING PARAMETERS OF THE VESSEL

Simulation was provided for a surface mini vessel (see its 3d model and the related coordinate system in Fig. 1). Parameters with the normal displacement of submerged part: the maximum length - $L = 9{,}5$ m; width - $B = 2{,}3\ m$; immersion depth - $T = 0{,}46\ m$, and the following values of weight and inertia characteristics:

$$m = 4658{,}9\ kg, \quad x_T = -1{,}305\ m, y_T = 0{,}936\ m,$$
$$z_T = 0\ m,$$
$$J_{xx} = 5831.75\ \text{kh}\cdot m, \quad J_{yy} = 29950.97\ \text{kg}\cdot m^2,$$
$$J_{zz} = 33891.63\ \text{kg}\cdot m^2,$$
$$J_{xy} = 3718.25\ kg\cdot m^2, \quad J_{xz} = J_{yz} = 0. \quad (11)$$

We will assume that attached mass and damping coefficients are independent of the displacement and calculate their values for $U_{подв} = U_{подв,0}$. For the calculation of added mass tensor components $\lambda_{11}, \lambda_{33}, \lambda_{55}, \lambda_{15}$ approximate formulas (11.177) will be used from the handbook [11]:

$$\lambda_{11} = 0{,}67\rho VF\ /\ S \quad , \quad \lambda_{33} = 0{,}44\rho VF\ /\ S \quad ,$$
$$\lambda_{55} = 0{,}028\rho V^2 S\left[1\text{–}3{,}6(F/S)^2\right] / \ F^3 ,$$
$$\lambda_{15} = 0{,}125\rho V^2 \left[1\text{–}3{,}6(F/S)^2\right] / \ F \ , \qquad (12)$$

where V - the volume of the submerged part, F – space of the center plane of a submerged vessel, S - water line area. Equations (12) describe the lateral vessel descent, behavior during flurry, etc. To determine the $\lambda_{22}, \lambda_{66}$ components let's use the approximate formula [11], obtained by Bloch E.L. for the semi-submersible ellipsoid of revolution for the case where a circular mid-section perpendicular to the free surface of the ellipsoid:

$$\lambda_{22} = k_{22}\, 2\pi\rho ab^2\ /\ 3,$$
$$\lambda_{26} = k_{26}\, 2\pi\rho ab^2\ \left(a^2 + b^2\right) / \ 15, \qquad (13)$$

where the dimensionless coefficients k_{22}, k_{26} are believed to coincide with the coefficients $k_{33.0}, k_{35,0}$. Calculation by formulas (12), (13) in our case gives the following values of nonzero elements of the tensor $\left\{\lambda_{ij}\right\}$:

$$\lambda_{11} = 846{,}78\ kg, \lambda_{33} = 556{,}1\ kg, \lambda_{55} = 107{,}2\ kg\ m^2,$$

$$\lambda_{15} = 7341{,}00\ kg,\ \lambda_{22} = 9545\ kg,$$

$$\lambda_{26} = 70370\ kg \qquad (14)$$

Damping moment M_(wy,демпф) relative to middle-ship frame plane can be approximately calculated using the formula (2.160) [11]:

$$M_{wy,\text{демпф}}(V,\omega_y) = -C_{My}^{\omega}\left(\frac{\rho S_{\text{ДП},0}L^2}{2}\right)V\omega_y \quad (15)$$

Where $C_{My}^{\omega} = \left(0{,}739 + \frac{8{,}7T_0}{L_0}\right)(1{,}611\sigma^2 - 2{,}873\sigma + 1{,}33)$; $L_0, T_0, S_{\text{ДП},0}, \sigma = \frac{S_{\text{ДП},0}}{L_0T_0}$ – the maximum length, width and area of the center plane and the coefficient of fullness of the underwater part of the normal level for a given waterline.

Damping moment $M_{wx,\text{демпф}}$ with respect to the center plane can be approximately calculated by the approximation empirical formulas (3.22) - (3.25) in [11] obtained by Shmurun A.N.:

$$M_{wx,\text{демпф}}(V,\omega_y) = [0{,}75\pi(\mu_{\theta,1} + \mu_{\theta,2})/\theta_0]\omega_x|\omega_x| \qquad (16)$$

where

$$\mu_{\theta,1} = 10^{-2}(1{,}78 - 0{,}078\bar{\tau}_0)(0{,}5 + 0{,}005\theta_0) \times \times \left[0{,}00125(B/T_0)^2 + 0{,}044 + \left(0{,}262 - 0{,}484(T_0/B_0)\right)\bar{S}_k\right](B_0/h_0),$$

$$\mu_{\theta,2} = 8(T_0/L_0h_0\delta\bar{\tau}_0)\bar{r}_m\left(z_g - 0{,}67T_0\right)\sqrt{L_0/B_0}\,Fr,$$

$$\bar{r}_m = (1/\pi)\left\{(0{,}887 + 0{,}145\delta)[1{,}7(T_0/B_0) + \delta] - 2\left(T - z_g\right)/B_0\right\},$$

$r_0 = \frac{\left(\frac{2}{3}\right)\int_a^b y^3(x)dx}{U_{\text{подв}}}$, $h_0 = r_0 + z_{C,0} - z_g$, $\bar{S}_k = 100\,S_k/S_{\text{ВЛ}}$, $Fr = V/\sqrt{gL_0}$, $\bar{\tau}_0 = \tau_0\sqrt{g/B_0}$; θ_0 - pitching amplitude, rad; τ_0- own rolling period; z_g and $z_{C,0}$ - vertical coordinates of the center of gravity and center of buoyancy underwater at zero bank angle; - $\delta = S_{\text{погр}}/L_0T_0$ – coefficient general fullness; S_k- the total area of the zygomatic keels; $S_k -$ the main part of the plane area in the normal displacement limiting the waterline; r_0 - metacentric radius at small angles of heel, $y(x)$ - equation profile normal waterline based on the longitudinal coordinate x ($x \in (a,b)$), V – velocity of the vessel.

5. CONCLUSIONS

Spesial aspects of kinematics and dynamics of surface mini vessel are examined on the basis of complete nonlinear multivariable mathematical model. Results provided data for development of computational procedure for both hydrodynamic and static forces and moments, acting on the vessel. This procedure significantly accelerates identification of the corresponding functional relations in mathematical model. To check the procedure, the functional relations between static and dynamic influences of the continuous medium were analytically defined for the stated vessel. Next step of the research will be CFD simulation (using Ansys Fluent and Fine Hexa software) on the basis of the obtained data.

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