**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Solution:**

We have a normal distribution with *μ* = 45 minutes and *σ* = 8 minutes.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour, you must have X ≤ 50

So, the question is to find P (X > 50)

P(X > 50) = 1 - P(X ≤ 50)

Z = (X -*μ* )/ *σ* = (X - 45)/8.0

Thus, the question can be answered by using the normal table to find

P (X ≤ 50) = P (Z ≤ (50 - 45)/8.0) = P (Z ≤ 0.625)=73.4%

Therefore, Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Solution:**

We have a normal distribution with *μ* = 38 and Standard deviation *σ* =6.

Let X be the number of employees.

So according to question

1. **Probability of employees greater than age of 44= P(X>44)**

P (X > 44) = 1 - P (X ≤ 44)

Z = (X - *μ*)/*σ* = (X - 38)/6

Thus, the question can be answered by using the normal table to find

P(X ≤ 44) = P(Z ≤ (44 - 38)/6) = P(Z ≤ 1)=84.1345%

So, Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So, the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-

0.5= 34.1345%

Therefore, the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

**B) Probability of employees less than age of 30 = Pr(X<30).**

Z = (X - *μ* )/ *σ* = (30 - 38)/6

Thus, the question can be answered by using the normal table to find

P (X ≤ 30) = P (Z ≤ (30 - 38)/6) = P (Z ≤ -1.333) =9.12%

So, the number of employees with probability 0.912 of them being under age 30 =

0.0912\*400=36.48 (or 36 employees).

Therefore, the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Solution:**

Ans- As we know that if X ∼ N (µ1, σ1^2), and Y ∼ N (µ2, σ2^2) are two independent random variables then X + Y ∼ N (µ1 + µ2, σ1^2 + σ2^2), and X − Y ∼ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = ax + by, where X and Y are as defined above, i.e. Z is linear combination of X and Y, then Z ∼ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question 2X1~ N (2 u,4 σ^2) and X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

**Difference = 2X1-(X1+X2) = N (4µ,6 σ^2)**

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Solution:** Identify symmetric values for the standard normal distribution such that the area enclosed is .99

From the above details,we have to excluded area of .005 in each of the left and right tails. Hence, we want to find the 0.5th and the 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

                                         Z (0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z (99.5) = stats.norm.ppf(0.995) = 2.576

Z = (x - 100)/20 = > x = 20z+100

      a = -(20\*2.576) + 100= 48.5

      b = (20\*2.576) +100= 151.5

Two values symmetric about mean for the given standard normal distribution are [48.5,151.5]

So, the option D is correct

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Solution:**

(Code attached in Mail)

Mean profit in Rs 540 million

Standard deviation in Rs 225.0 Million

Range in RS (99.00810347848784, 980.9918965215122) millions

5th percentile of profit (in Million Rupees) is 170.0