(121910319020)

Assignment - I

A. Dheeraj Kumar

(Q1) Find the maximum of 'xy' Subject to $x+y=7$

$f(x) = xy$ $\qquad$ $x+y=7$

$y = 7-x$ —(1)

$f(x) = x(7-x)$

$f'(x) = 7-2x$

$f''(x) = -2 < 0$ There occurs a minima and it is concave

$f'(x) = 0$

$7-2x = 0$

$7 = 2x$ $\qquad$ from (1) we get $y = \frac{7}{2}$

$x = \frac{7}{2}$

The maximum Value is $\frac{49}{4}$

(2) Find the ~~maximum~~ minimum Value of $(x-3)^2-4$

$f(x) = (x-3)^2 - 4$

$f'(x) = 2(x-3)$

$f''(x) = 2 > 0$ (function is convex and has only minima)

To find minima $\quad f'(x) = 0$

$2(x-3) = 0$

$x = 3$, Minimum Value $(3-3)^2 - 4 = -4$

(3) find the minimum Value of x^2-5x+6

$f(x) = x^2-5x+6$ $\qquad$ $f'(x) = 0$ $\qquad$ $\frac{25}{4} - \frac{25}{2} + 6$

$f'(x) = 2x-5$ $\qquad$ $2x-5=0$ $\qquad$ $-\frac{25}{4} + 6 = -\frac{1}{4}$

$f''(x) = 2 > 0$ has a minimum $\qquad$ $x = 5/2$

(121910319020)

(4) write karush - khun - Tucker conditions for solving the two Variable and 1 constraint type problems

$$L = f(x) - \lambda_1 g_1(x) - \lambda_2 g_2(x)$$

$\frac{\partial L}{\partial x_1} = 0$ - (1) $\quad \lambda(h) = 0$ - (3)

$h \leq 0$

$\frac{\partial L}{\partial x_2} = 0$ - (2)

(5) write the kuhn - Tucker conditions for given set of model

$$Z = 2x_1^2 - 7x_2^2 - 16x_1 + 2x_2 + 12x_1x_2$$

S.T to $2x_1 + 5x_2 \leq 105$; $x_1 x_2 \geq 0$

$\frac{\partial L}{\partial x_1} = 0 \quad \lambda(h) = 0$

$\lambda \geq 0$

$h \leq 0$

$\frac{\partial L}{\partial x_2} = 0 \quad x_1 x_2 \geq 0$

$L(x_1, x_2, \lambda)$

$= (2x_1^2 - 7x_2^2 - 16x_1 + 2x_2 + 12x_1x_2)$

$- \lambda(2x_1 + 5x_2 - 105)$

$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 4x_1 - 16 + 12x_1 - 2\lambda = 0$

$\frac{\partial L}{\partial x_2} = 0 \Rightarrow -14x_2 + 2 + 12x_2 - \lambda(5)$

$\lambda(2x_1 + 5x_2 - 105) = 0$ - (3) , $2x_1 + 5x_2 - 105 \leq 0$ - (4)

$x_1 x_2 \geq 0$, $\lambda \geq 0$ - (5)

(1219103190 20)

(Q6) Mention 6 applications of optimization techniques

Ans: (1) Design and fitting

(2) Engineering Design

(3) Industrial Engineering, manufacturing design

(4) Transportation

(5) Decision making

(Q7) Determine concavity of the function(s) $x^4 + 2x^2 - 5x$; $3x^5 - 5x^3$

Ans: (b) $f(x) = 3x^5 - 5x^3$

$f'(x) = 0$

$\frac{\partial}{\partial x}[3x^5 - 5x^3] = 0$

$15x^4 - 15x^2 = 0$

$x = 0$ (or) $x = \pm 1$

conditions

$f''(x) = 15(4x^3 - 2x)$

At $x = 0$, It is point of inflection

At $x = 1$ $f''(1) = 30 > 0$

point of minimum

At $x = -1$

It is a maximum point

$(f''(x) < 0)$

(a) $f(x) = x^4 + 2x^2 - 5x$

$f'(x) = 4x^3 + 4x - 5$ (slope)

$f''(x) = 12x^2 + 4$

This can be Either convex (or) Concave

(Q8) Write the Lagrangian Equation for the given set of Equations

$f(x) = 3e^{2x+1} + 2e^{y+5}$; Subjected to $x + y = 7$; $x, y \geq 0$

Ans:

$L = (3e^{2x+1} + 2e^{y+5}) - \lambda(x + y - 7)$

(C12190312 90 20)

(9) Given $f(x,y) = 2x_1^2 - 7x_2^2 - 16x_1 + 2x_2 + 12x_1x_2$ $x_1^2 - 2x_1x_2 - 4x_1 + 2x_2 + 2x_2^2$

Let $x = x_1,\ y = x_2$ $\Rightarrow$ $2x^2 - 7y^2$

$$x^2 - 2xy - 4x + 2x + 2y^2$$

$$\frac{\partial f}{\partial x} = 2x - 2y - 4 = 0 \quad -①$$

from ① & ② $x = 3,\ y = 1$

$$\frac{\partial f}{\partial y} = -2x + 2 + 4y = 0 \quad -②$$

so $(3,1)$ is Extreme point

$$\frac{\partial^2 f}{\partial x^2} = 2,\quad \frac{\partial^2}{\partial y^2} = 4 \quad \frac{\partial^2 f}{\partial x \cdot \partial y} = -2 \quad = 2 \times 4 - 4 = 4$$

i.e > 0

so the Extreme point $(3,1)$ is a minimum point

min value of function $= f(x,y) = 3^2 - 2 \times 3 \times 1 - 4 \times 3 + 2 = -5$

(10) If jacobian's $J1 < 0$ and $J2 > 0$ then the nature of decision Variable will be ___ and function has ___ Solution

Positive definite

minimum / Relative minimum

Ans: negative definite, relative maximum

(11) If jacobians $J1 > 0$ and $J2 > 0$, then the nature of decision Variable will be ___ and the function has ___ Solution

Ans: positive definite, relative minimum

(12) If jacobians $J1 < 0$ and $J2 < 0$ then nature of decision variable will be ___ and function has ___ Solution

Ans: indefinite saddle point

(1219103 19020)

(13) write the design Vector matrix for the given function

$x_1^2 - 2x_1x_2 - 4x_1 + 2x_2 + 2x_2^2$

Solution $\frac{\partial f(x)}{\partial x_1} = 2x_1 - 2x_2 - 4 = 0$ -①

$\frac{\partial f(x)}{\partial x_2} = -2x_1 + 2 + 4x_2 = 0$ -②

Solve ① & ② $x_1 = 3,\ x_2 = 1$

$\frac{\partial^2 f(x)}{\partial x_1} = 2 \quad \left| \frac{\partial^2 f(x)}{\partial x_2} = 4 \right|$

Vector matrix : $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$\frac{\partial^2 f(x)}{\partial x_1 \partial x_2} = -2$

(14) what is the difference between design Variables and pre assigned Variables

Ans: In a set of quantities defined in any Engineering system (or) component, Some of term are Viewed as Variables during design process but other are fixed, former are Design Variables and present are pre assigned Variables

(15) what is the difference between bound point and free point in the design space

Ans: A design Point that lies on one (or) more than one point constraint Surface is called Bound point

and the design point which does not lie on any constraint Surface is known as free point

(121910319020)

16) what will be the order of Hessian matrix, if a function has 3 decision variables and 2 constraint Equations.

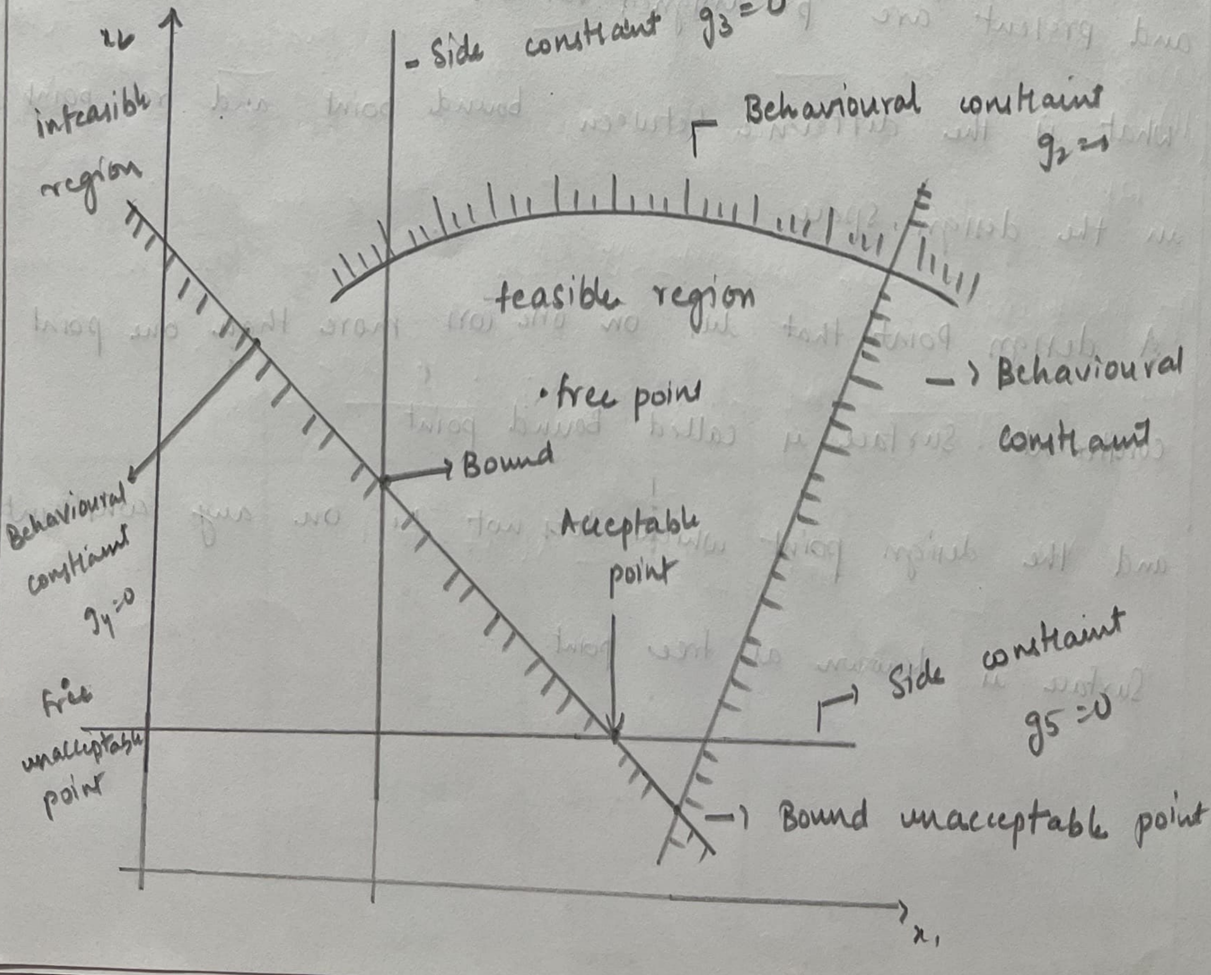
Ans: order = $(x+y) \times (x+y)$ $\qquad$ m = no. of constraints = 2

$(2+3) \times (2+3)$ $\qquad$ n = Variables = 3

5×5

$= 5 \times 5$ order matrix

(17) Draw constraint surface in 2D Space



(12191031902)

(18) Find the maximum and the minimum Value of the function

$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

Ans: $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

$\frac{\partial f}{\partial x} = f(x) = 3x^2 + 3y^2 + 72 - 30x = 0$ - ①

$\frac{\partial f}{\partial y} = 6xy - 30y = 0$ - ② (f_y)

$f_x = 0$ & $f_y = 0$ To find points

$6xy - 30y = 0$

$xy - 5y = 0$

$y(x-5) = 0$

$y = 0$ & $x = 5$

Putting $y = 0$ in eq ①

$3x^2 - 30x + 72 = 0$

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$

$x = 4$ & $x = 6$ $\therefore (4,0), (6,0)$

Stationary point

Putting $x = 5$

$75 + 3y^2 - 150 + 72 = 0$

$3y^2 - 3 = 0$

$y = \pm 1$ $(5,1), (5-1)$ are stationary points

$A = f_{xx} = \frac{\partial^2 f}{\partial x^2}$ $B = f_{yx} = \frac{\partial^2 f}{\partial x \partial y}$ $C = f_{yy} = \frac{\partial^2 f}{\partial y^2}$

	$(4,0)$	$(6,0)$	$(5,1)$	$(5,-1)$
$A = 6x - 30$	$-6 < 0$	$6 > 0$	0	0
$B = 6y$	0	0	6	-6
$C = 6x - 30$	-6	6	0	0
$AC - B^2$	$36 > 0$	$36 > 0$	$-36 < 0$	$-36 < 0$
	max point	min point	saddle point	Saddle point

(121910319020)

(19) (Q) Min Value of $2e^{x} + e^{-x}$

$f(x) = 2e^{x} + e^{-x}$

$f'(x) = 2e^{x} - e^{-x} = 0$

$2e^{x} = e^{-x}$

$2 \cdot e^{2x} = 1$

$e^{2x} = \frac{1}{2}$

$2x = \log_e\left(\frac{1}{2}\right)$

$x = \frac{1}{2}\log_e\left(\frac{1}{2}\right)$

$x = \log_e\left(\frac{1}{\sqrt{2}}\right)$

$f''(x) = 2e^{x} + e^{-x}$ which is positive

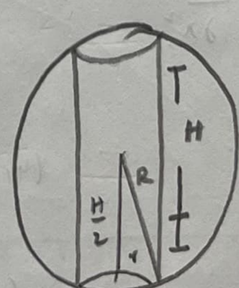
$f\left[\log_e\left(\frac{1}{\sqrt{2}}\right)\right] = 2e^{\log_e\left(\frac{1}{\sqrt{2}}\right)} + e^{\log\frac{1}{\sqrt{2}}}$

$= 2 \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

(20) If the curved surface of a right circular cylinder inscribed in a sphere of radius 'R' is max show that height is $\sqrt{2}R$

Let 'r' be the radius and height of cylinder inscribed in a sphere of radius R, the according to Pythagoras theorem



$(R)^2 = r^2 + \left(\frac{H}{2}\right)^2$

$R^2 - \frac{H^2}{4} = r^2$

CSA of cylinder $= 2\pi r H$

(let CSA be c)

$c^2 = (2\pi)^2 r^2 H^2$ (square on Both sides

$f(H) = 4\pi^2\left[R^2 - \frac{H^2}{4}\right] = 4\pi^2\left[R^2H^2 - \frac{H^4}{4}\right]$

differentiating $f'(H) = 4\pi^2\left[2R^2H - H^3\right]$

for min (or max (1219103190200)

$f'(H) = 0$

$4\pi^2 [2R^2 H - H^3] = 0$

$H [2R^2 - H^2] = 0$

$H = 0$ not possible

$2R^2 - H^2 = 0$

$\boxed{H = \sqrt{2} R}$

At $H = \sqrt{2} R$

$f''(H) = < 0$, maximum at $H = \sqrt{2} R$

25) Solve $f(x) = 2x^2 - 24x + 2y^2 - 8y + 2z^2 - 8y + 2z^2 - 12z + 200$

w.r.t $x + y + z = 1$, $x, y, z >= 0$

Ans:

$L(x, \lambda) = (2x^2 - 24x + 2y^2 - 8y + 2z^3 + 200 - 12z)$

$+ \lambda (x + y + z - 1)$

necessary conditions

$\frac{\partial L}{\partial x_1} = 0 = 4x - 24 + \lambda = 0$ — 1 $\qquad x = \frac{24 - \lambda}{4} \qquad z = \frac{12 - \lambda}{4}$

$\frac{\partial L}{\partial y} = 0 = 4y - 8 + \lambda = 0$ — 2 $\qquad y = \frac{8 - \lambda}{4}$

$\frac{\partial L}{\partial z} = 0 \quad 4z - 12 + \lambda = 0$ — 3 $\qquad$ from 4

$\frac{\partial L}{\partial \lambda} = 0 = x + y + z - 1 = 0$ — 4 $\qquad \frac{24 - \lambda + 12 - \lambda + 8 - \lambda + 4}{4} = 0$

$x = 2, \; y = -1, \; y = -2$ $\qquad 48 - 3\lambda = 0, \; \lambda = -16$

stationary points are : $(2, -2, -1, -16)$ $(\lambda = -16)$

(12191031 9020)

(26) Solve $f(x) = x_1^2 + 3x_2^2 + 5x_3^2$ subjected to $x_1 + x_2 + 3x_2 = 21$, $5x_1 + 2x_2 + x_3 = 5$; $x_i \geq 0$ using Lagrangian method

Point is $x_0\ (2, -2, -1)$

$$\Delta = \begin{vmatrix} 0 & \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial z} \\ \frac{\partial h}{\partial x} & \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x \partial y} & \frac{\partial^2 L}{\partial x \partial z} \\ \frac{\partial h}{\partial y} & \frac{\partial^2 L}{\partial y \partial x} & \frac{\partial^2 L}{\partial y^2} & \frac{\partial^2 L}{\partial y \partial z} \\ \frac{\partial h}{\partial z} & \frac{\partial^2 L}{\partial z \partial x} & \frac{\partial^2 L}{\partial x \partial y} & \frac{\partial^2 L}{\partial z^2} \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 0 \\ 1 & 0 & 0 & 4 \end{vmatrix}$$

$0 - 16 - 16 - 16$

$= -48$

x_0 is minima

$Z_{min} = 2(2)^2 - 24(2) + 2(-2)^2 + 8(-2) + 2(-1)^2$

$= 2(-1) + 200$

$= 198$

(121910319020)

(23) Optimize : $f(x) = x_1^2 + 12x_3 + 56 - 8x_2 - 4x_1 + x_2^2 + x_3^2$

Ans. $\frac{\partial f(x)}{\partial(x_1)} = +2x_1 - 4 = 0$ — (1)

$\frac{\partial f(x)}{\partial(x_2)} = 2x_2 - 8 = 0$ — (2)

$\frac{\partial f(x)}{\partial(x_3)} = 2x_3 + 12 = 0$ — (3)

Solving (1) & (2) & (3)

$x_1 = 2$

$x_2 = 4$

$x_3 = -6$

$\therefore (2, 4, -6)$ are Extreme point

(24) Optime $z = 3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$ Subject to $2x_1 + x_2 = 4$, $x_1, x_2 \geq 0$ using Lagrangian Method.

Ans: Let $L = z - \lambda h$

$L = [3x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2] - \lambda[2x_1 + x_2 - 4]$

$\frac{\partial L}{\partial x_1} = 0 \Rightarrow 6x_1 + 2x_2 + 6 - 2\lambda = 0$ — (1)

$\frac{\partial L}{\partial x_2} = 0 \Rightarrow 2x_2 + 2x_1 + 2 - \lambda = 0$ — (2)

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow -[2x_1 + x_2 - 4] = 0$ — (3)

(1) − (2) = $x_1 - x_2 = -1$ — (4)

Solve (3) and (4) we get $x_1 = 1$, $x_2 = 2$

Putting in ②

$$\lambda = 2x_2 + 2x_1 + 2 = 4+2+2$$

$$\lambda = 8$$

$\therefore$ point is $x_0(1,2)$

$$\Delta = \begin{vmatrix} 0 & \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \\ \frac{\partial h}{\partial x_1} & \frac{\partial^2 h}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} \\ \frac{\partial L}{\partial x_2} & \frac{\partial^2 L}{\partial x_2 x_1} & \frac{\partial^2 L}{\partial x_2^2} \end{vmatrix} = \begin{vmatrix} 0 & 2 & 1 \\ 2 & 6 & 2 \\ 1 & 2 & 2 \end{vmatrix}$$

$\Delta = -6 < 0$

$\therefore$ x_0 is minima

$$Z_{min} = 3+4+4+6+4 = 21$$

(26) Solve $f(x) = x_1^2 + 3x_2^2 + 5x_3^2$ Subjected to $x_1 + x_2 + 3x_3 = 2$,

$5x_1 + 2x_2 + x_3 = 5$; $x \geq 0$ by using Lagrangian method

$$L = f - \lambda_1 h_1 - \lambda_2 h_2$$

$$L = (x_1^2 + 3x_2^2 + 5x_3^2) - \lambda_1(x_1 + x_2 + 3x_3 - 2)$$
$$- \lambda_2(5x_1 + 2x_2 + x_3 - 5)$$

$$\frac{\partial L}{\partial x_1} = 2x_1 - \lambda_1 - 5\lambda_2 = 0 \qquad \frac{\partial L}{\partial x_2} = 6x_2 - \lambda_1 - 2\lambda_2 = 0 \text{ ---②}$$

$$\frac{\partial L}{\partial x_3} = 10x_3 - 3\lambda_1 - \lambda_2 = 0 \quad - (3)$$

(12191031 9020)

$$\frac{\partial L}{\partial \lambda_1} = x_1 + x_2 + 3x_3 = 2 \quad - (4)$$

$$\frac{\partial L}{\partial \lambda_2} = 5x_1 + 2x_2 + x_3 = 5 \quad - (5)$$

from (1), (2), (3) we get

$$x_1 = \frac{\lambda_1 + 5\lambda_2}{2}$$

$$x_2 = \frac{\lambda_1 + 2\lambda_2}{6}$$

$$x_3 = \frac{3\lambda_1 + \lambda_2}{10}$$

Substitute in (4),

$$\frac{\lambda_1 + 5\lambda_2}{2} + \frac{\lambda_1 + 2\lambda_2}{6} + \frac{3(3\lambda_1 + \lambda_2)}{10} = 2$$

$$44\lambda_1 + 94\lambda_2 = 60 \quad - (6)$$

$$5\left(\frac{\lambda_1 + 5\lambda_2}{2}\right) + 2\left(\frac{\lambda_1 + 2\lambda_2}{3}\right) + \left(\frac{3\lambda_1 + \lambda_2}{10}\right) = 5$$

$$47\lambda_1 + 342\lambda_2 = 75 \quad - (7)$$

Equate (6) & (7)

$$\lambda_2 = \frac{3}{46} \qquad \lambda_1 = \frac{1251}{989}$$

$x_1 = 0.796$

$x_2 = 0.235$

$x_3 = 0.386$

(22) optimize $f(x) = -x_1^2 - x_2^2 + x_1 + 2x_2 + x_1x_2$

(121910319020)

Sol:

$$\frac{\partial f(x)}{\partial x_1} = -2x_1 + x_2 + 1 = 0 \quad -①$$

$$\frac{\partial f(x)}{\partial x_2} = x_1 - 2x_2 + 2 = 0 \quad -②$$

Equate ① and ②,

$$-2x_1 + x_2 + 1 = 0$$

$$2x_2 - 4x_2 + 4 = 0$$

$$-3x_2 + 5 = 0$$

$$x_2 = \frac{2}{3}$$

$$-2x_1 + \frac{5}{3} + 1 = 0$$

$$x_1 = \frac{4}{3}$$

$\therefore \left(\frac{4}{3}, \frac{5}{3}\right)$ is extreme point.

$A = -2$

$B = 1$

$C = -2$

$AC - B^2 = 4 - 1 = 3 > 0$

min Value $= -\left(\frac{4}{3}\right)^2 - \left(\frac{5}{3}\right)^2 + \frac{4}{3} + \frac{10}{3} + \left(\frac{4}{3}\right)\left(\frac{10}{3}\right)$

$$= \frac{-16}{9} - \frac{25}{9} + \frac{4}{3} + \frac{10}{3} + \frac{40}{9}$$

$$= \frac{41}{9}$$