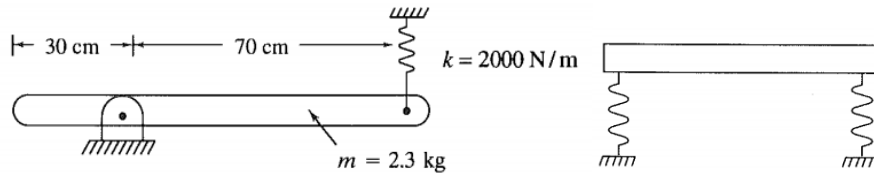




Physics 3 : Series 2

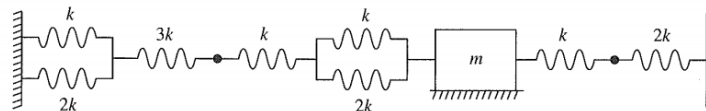
Exercise 1

Determine the number of **degrees of freedom** to be used in the vibrations analysis in each system, and specify a set of **generalized coordinates** can be used in each vibration system.



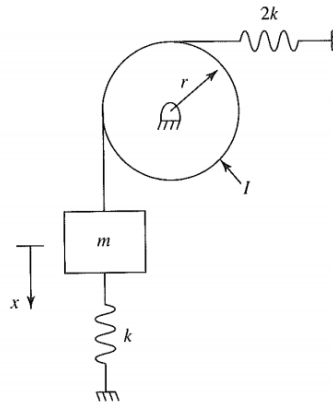
Exercise 2

Model the system shown in figure by a block attached to a single spring of an equivalent stiffness.



Exercise 3

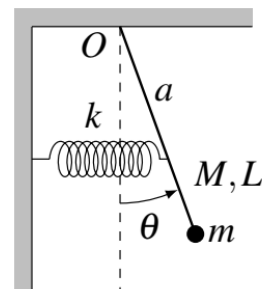
Determine m_{eq} and k_{eq} for the system of the figure when x , the downward displacement of the block, measured from the system's equilibrium position, is used as the generalized coordinate.



Exercise 4

A rod of mass M and length L oscillates without friction in a vertical plane Oxy around a fixed axis passing through O . A point-like mass m is fixed at its other end. At a distance a from the point O , a spring of stiffness k connects the rod to a fixed frame.

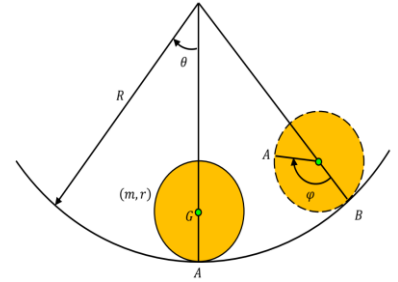
- Find the expressions for the **kinetic energy** and the **potential energy** of the system. What is the deformation of the spring at equilibrium ($\theta = 0$) ?
- Using the Lagrangian, establish the differential equation of motion in the case of small oscillations.
- Is there a condition on the natural pulsation of the system so that it can oscillate?



Exercise 5

A cylinder, of mass m and radius r rolls without slipping on a circular surface of radius R .

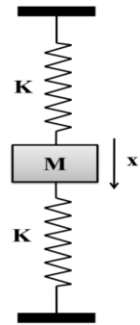
1. Find the expressions for the kinetic energy and the potential energy of the system.
2. Using the Lagrangian method, establish the differential equation of motion and give the expression for the angular frequency.



Exercise 6

Consider a system consisting of a mass $m=500\text{g}$ and two identical springs of stiffness $k=100\text{ N/m}$.

1. Find the kinetic energy and the potential energy.
2. Establish the differential equation as a function of x using the Lagrange method.
3. Give the final solution if: $x(0) = 1\text{ cm}$; $\dot{x}(0) = 0$.
4. Calculate the total energy



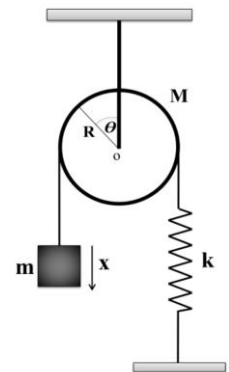
Exercise 7

A spring of stiffness $k = 60\text{ N/m}$, and a point-like mass $m = 0.5\text{ kg}$ are fixed to the ends of the pulley by an inextensible wire of negligible mass. We also neglect the mass of the spring and the friction around the axis of the pulley.

We give: $M = 2\text{ kg}$, $R = 0.2\text{m}$, $I_o = \frac{1}{2}MR^2$.

If x is the vertical displacement of the mass m .

1. Give the kinetic energy and the potential energy of the system as a function of θ .
2. Find the Lagrangian L and deduce the equation of motion.
3. Find the natural pulsation, the period T_0 and the frequency f_0 .
4. Find the final solution by taking as initial conditions: $\theta(0) = \frac{\pi}{25}$ et $\dot{\theta}(0) = 0$.



Exercise 8

Consider the rod-spring system.

At equilibrium, the rod is in a vertical position and the spring is at rest.

1. Applying Huygens' Theorem, give the moment of inertia of the system.
2. Give the expression for the kinetic energy and the potential energy of the system.
3. Find the differential equation of motion as a function of θ .
4. Find the final solution. Initial conditions : $\theta(0) = \frac{\pi}{22}$ et $\dot{\theta}(0) = 0$

