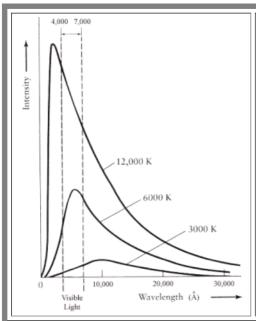
## **Graduate Course: Quantum Physics**

David Spetzler, Dept of Mathematics, and John Venables, Dept of Physics and Astronomy, Arizona State University, Tempe, Arizona

## **Blackbody Radiation**

One of the observations that was unexplained at the end of the 19th century was the spectrum of light emitted by hot objects. It was known that that objects emit radiation whose total intensity is proportional to the the fourth power of the temperature in Kelvin. The spectrum of light emitted by an ideal hot dense object, called a blackbody, is shown in figure 1.



**Figure 1: Blackbody curves for T = 3000, 6000 and 12,000 K.** Note that the three curves are not drawn at the same (vertical) scale, but the horizontal scale is as given. These curves were calculated with a simple MatLab program given <a href="here">here</a>.

The wavelength,  $\lambda$ , at the peak of the curve is related to the Kelvin temperature of the object by:

$$\lambda T = 2.90 \times 10^{-3} \, m \cdot K$$

This is known as Wien's displacement law. Note that the peak of the blackbody spectrum for  $6000 \, \text{K}$ , the surface temperature of the sun, falls at the blue end of the visible range.

In the 1890's one of the major problems in physics was trying to explain blackbody radiation. Maxwell's electromagnetic theory predicted that oscillating electromagnetic charges would produce electromagnetic waves, and the radiation emitted by a hot object could be due to the oscillations of electric charges of the molecules in the object. This explained where the radiation came from, but it did not correctly predict the observed spectrum of emitted light. Two important curves based upon the classical ideas were introduced by Wien in 1896 and Rayleigh in 1900. Rayleigh's curve was later modified by Jeans and is now known as the Rayleigh-Jean's formula.

$$u_{\mathit{W}}(\lambda) = \frac{W(\lambda T)}{\lambda^5}$$
 Wien's Formula:

Rayleigh-Jeans: 
$$u_{RJ}(v) = \frac{8\pi v^2}{c^3} k_B T$$

where 
$$k_B$$
 is the Boltzmann constant  $k_b = 1.381 \times 10^{-23} \text{ J/K}$ 

However, neither of these curves fit the entire range of observed results. Wien's formula was accurate at short wavelengths but deviated at longer wavelengths, while the opposite was true for the Rayleigh-Jean's theory. The resolution to this problem came in late 1900 when Max Planck proposed multiplying the Rayleigh-Jean's formula by the factor:

$$\frac{\binom{hv/k_BT}}{e^{\binom{hv/k_BT}{2}}-1}$$

which resulted in the Planck formula.

$$u_{P}(v) = \frac{8\pi v^{3}}{c^{3}} \frac{h}{e^{\left(\frac{kv}{k_{B}T}\right)} - 1}$$

The constant h is referred to as Planck's constant, which has subsequently been determined to be:

$$h = 6.6260755(40) \times 10^{-34} J \cdot s$$

Note that as  $\nu$  goes to zero, Planck's formula converges to the Rayliegh-Jean's formula, and as  $\nu$  goes to infinity it agrees with Wien's law.



After having determined an empirical formula to fit the observed light spectrum, Planck sought a theory that would justify the formula. He came up with an explanation that required making an assumption that defied classical theory. Planck proposed that the energy distribution among the (molecular) oscillators is not continuous, but instead is consists of a finite number of very small discrete amounts. Planck's suggested that the energy of any molecular vibration could only be some whole number multiple of *hv*:

E = nhv

where *n* is a natural number. This is referred to as Planck's quantum hypothesis. Later, the formula for the factor that Planck had introduced became the classic example of the Bose-Einstein distribution applied to integral spin particles (photons).

**References** Any book on modern physics will have a section on Blackbody radiation, and any book on Statistical Mechanics will treat the Planck law, and the various statements of Wien's law in some detail. A concise account is given by F. Mandl "Statistical Physics" (John Wiley, 2nd Edition) pages 250-256.

```
% David Spetzler
% Black Body Radiation
function blackbody
h = 6.6261*10^{-34}; % Planck's constant J s
c = 2.9979*10^8; % speed of light m/s
k = 1.3807*10^{-23}; % Boltzmann's constant J/K
lambda = 1e-9:10e-9:3000e-9;
T1 = 4500;
T2 = 6000;
T3 = 7500;
A1=(h.*c)./(k.*T1.*lambda);
A2=(h.*c)./(k.*T2.*lambda);
A3=(h.*c)./(k.*T3.*lambda);
%energy density function:
B=(8.*pi.*h.*c)./lambda.^5;
BB1=B.*(1./(exp(A1)-1));
BB2=B.*(1./(\exp(A2)-1));
BB3=B.*(1./(\exp(A3)-1));
figure(1),clf
plot(lambda,BB1,lambda,BB2,lambda,BB3)
```

The MatLab code for Figure 1 looks like:

Return to **Background information** page.

Latest version of this document: 18 February 2004.

