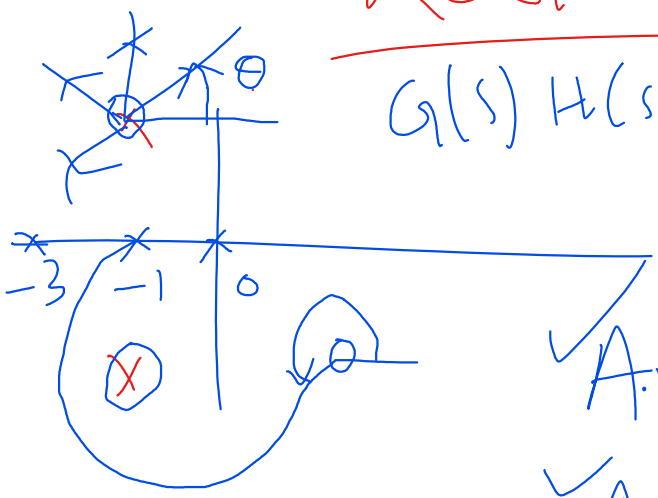
Root Locus

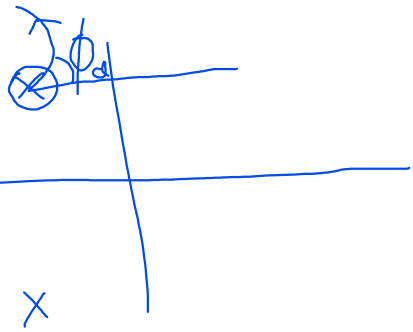
$$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$$



① Angle of departure
Angle of arrival

*) Angle of dep.

$$\phi_d = 180^\circ - \phi$$

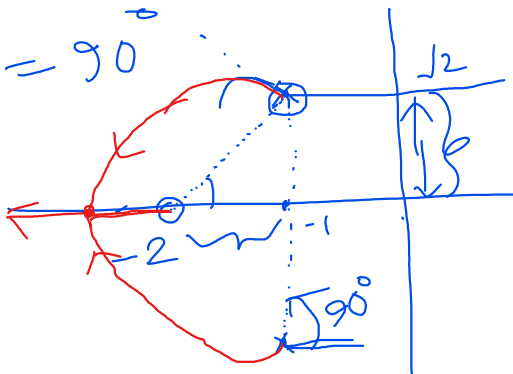


$$\phi = \sum \phi_p - \sum \phi_z$$

Ex

$$G(s)H(s) = \frac{K(s+2)}{s^2+2s+3}$$

$\sum \phi_p = 90^\circ$



poles $= -1 \pm j\sqrt{2}$

$\sum \phi_z = \tan^{-1}(\sqrt{2})$

$s^2+2s+3=0$

$$s = \frac{-2 \pm \sqrt{4-12}}{2}$$

$(= -1 \pm j\sqrt{2})$

$$\phi = 90^\circ - \tan^{-1}(\sqrt{2})$$

$$=$$

$$\phi_{d_1} = 180^\circ - (90^\circ - \tan^{-1}\sqrt{2}) = 145^\circ$$

$$\phi_{d_2} = -145^\circ$$

Break in point :-

$$1 + \frac{K(s+2)}{s^2+2s+3} = 0 \qquad \frac{dK}{ds} = 0$$

$$\Rightarrow s^2 + 2s + 3 + K \cdot s + 2K = 0$$

$$\Rightarrow 2s + 2 + s\frac{dK}{ds} + K + 2 = 0$$

$s = ?$

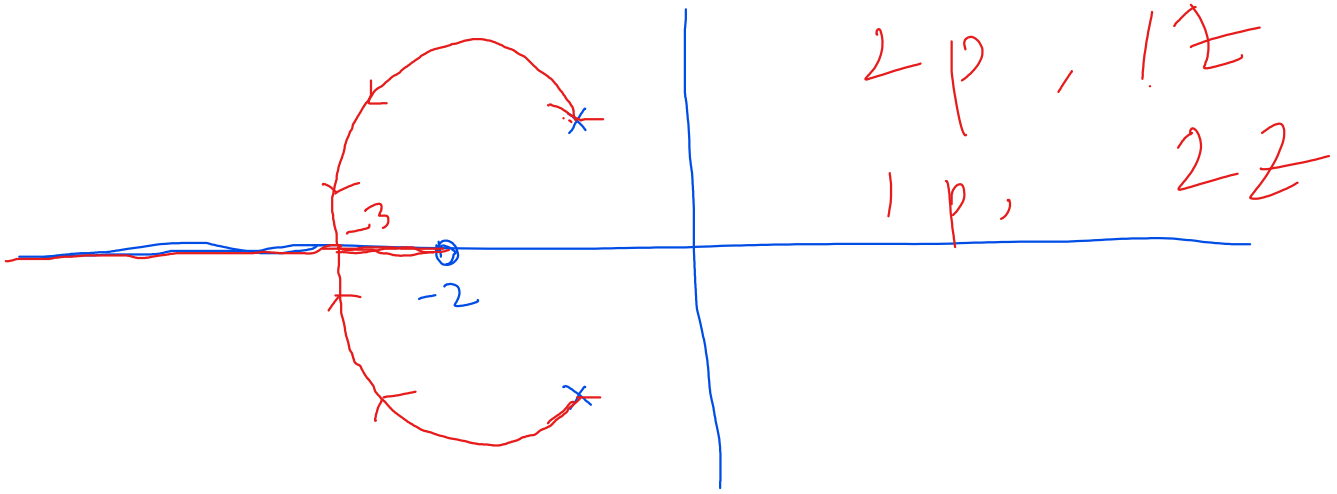
$$K = -\frac{s^2+2s+3}{s+2}$$

$$\frac{dK}{ds} = 0, \quad \boxed{s = -3.73} \text{ or } -0.268$$

angle of asymptote :-

$$\theta = \frac{(2k+1)180^\circ}{p-z}, \quad k=0$$

$$\theta = 180^\circ$$



2p, 1z

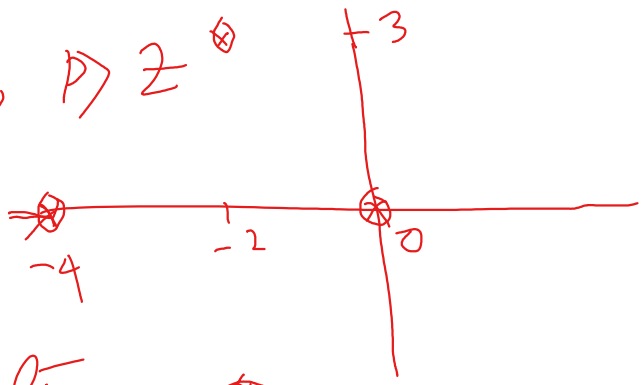
1p, 2z

*) $$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+13)}$$

1) $P = 4$

$z = 0$

, $N = P = 4$, $P > Z$



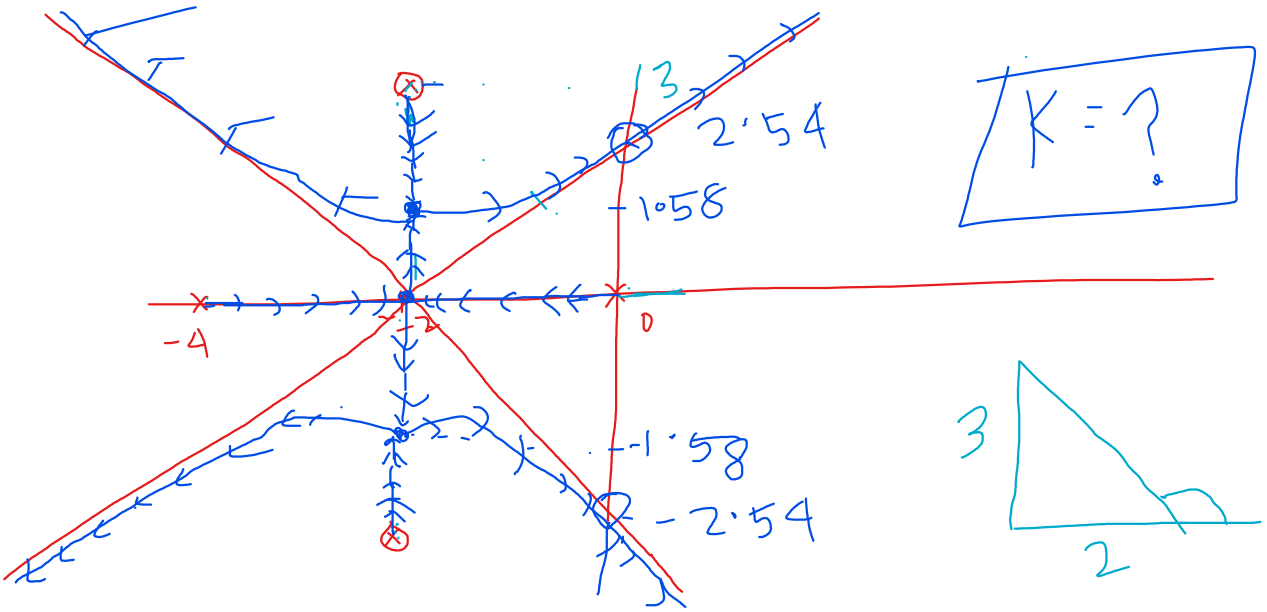
2) Angle of asymptote

$$\theta = \frac{(2k+1)180^\circ}{p-z}, \quad k = 0, 1, 2, 3$$

$$\theta = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

Centroid $= \dfrac{\sum p - \sum z}{p - z}$

$= \dfrac{(0 - 4 - 2 + j\sqrt{3} - 2 - j\sqrt{3})}{4} = -2.$



K = ?

A) Angle of departure: $180^\circ - \phi$

$\phi = \sum \phi_1 - \sum \phi_2$

$124^\circ + 90^\circ + 56^\circ = 270^\circ$

$180^\circ - 270^\circ = -90^\circ$

$-90^\circ, 90^\circ$

break away or in point

$s(s+4)(s^2+4s+13) + k = 0$

$$\frac{dk}{ds} = 0, \quad 4s^3 + 24s^2 + 58s + 52 = 0$$

$$\Rightarrow s = -2, \boxed{-2 \pm j1.58}$$

*) **Routh Table**

$$s^4 + 8s^3 + 29s^2 + 52s + k = 0$$

s^4	1	29	k
s^3	8	52	0
s^2	22.5	k	
s^1	( )		
s^0	k		

$$52 \times 22.5 - 8k = 0$$

$$\boxed{k = 146.25}$$

$$\boxed{k < 146.25}$$

$$22.5 s^2 + k = 0$$

$$\Rightarrow 22.5 s^2 + 146.25 = 0$$

$$s = \pm j\, 2.54$$

num = k[ ]

den = [1 4 + 2]

sys (num, den)

rlocus (sys)