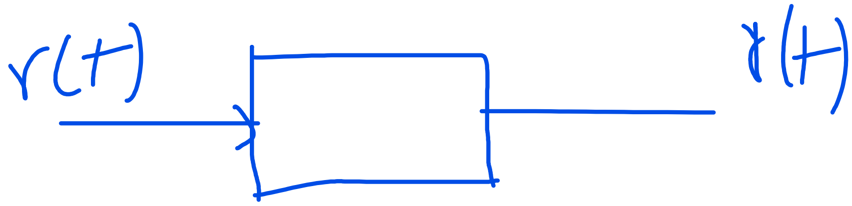
Time Response Analysis



o/p vs time

Order of the system

Test signal
- impulse
- step
- ramp

no. of poles at origin

1) Transient Res.
2) Steady state Res.

initial state
final state } Transient

if $t \to \infty$, o/p → steady state

Stability

Absolute

Relative

1st Order system

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{sT+1}$$

location of the pole $-1/T$

$$r(t) = u(t)$$

$$r(t) \longrightarrow \boxed{1/(sT+1)} \longrightarrow c(t)$$

$$R(s) = 1/s$$

$$G(s) = \frac{1}{sT+1} \times \frac{1}{s} = C(s)$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{T}{Ts+1}$$

$$= \frac{1}{s} - \frac{1}{s + 1/T}$$

$$c(t) = 1 - e^{-t/T} \qquad \boxed{t \geq 0}$$

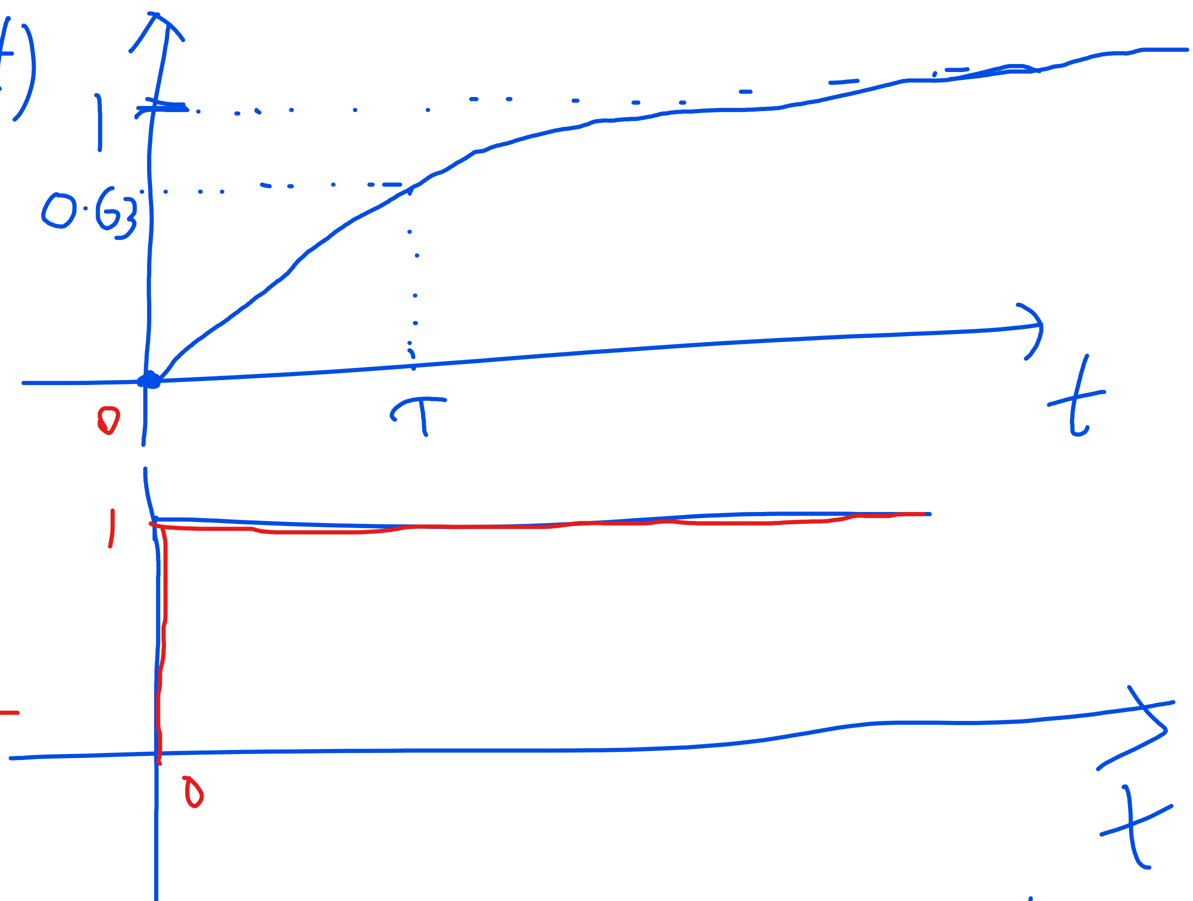
$t = 0$

$c(t) = 0$

$t (\infty)$

$c(t) = 1$

$t \geq 0$



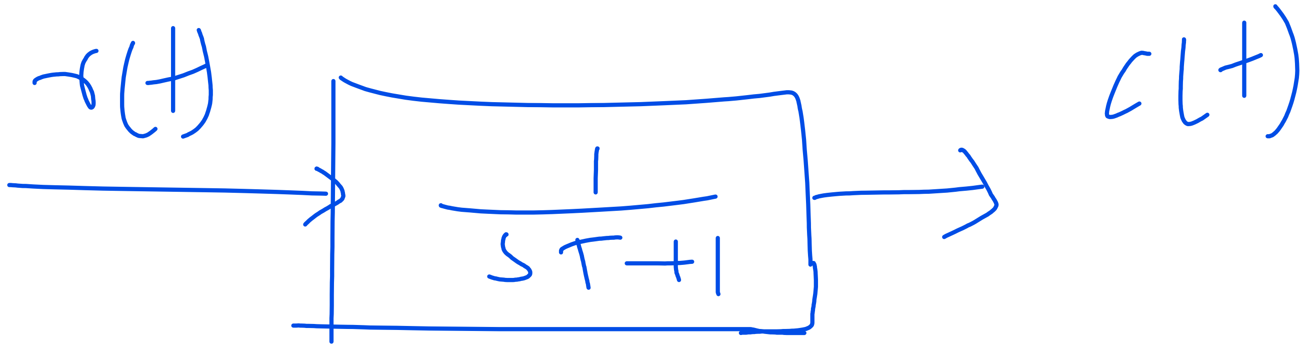
$t = T$

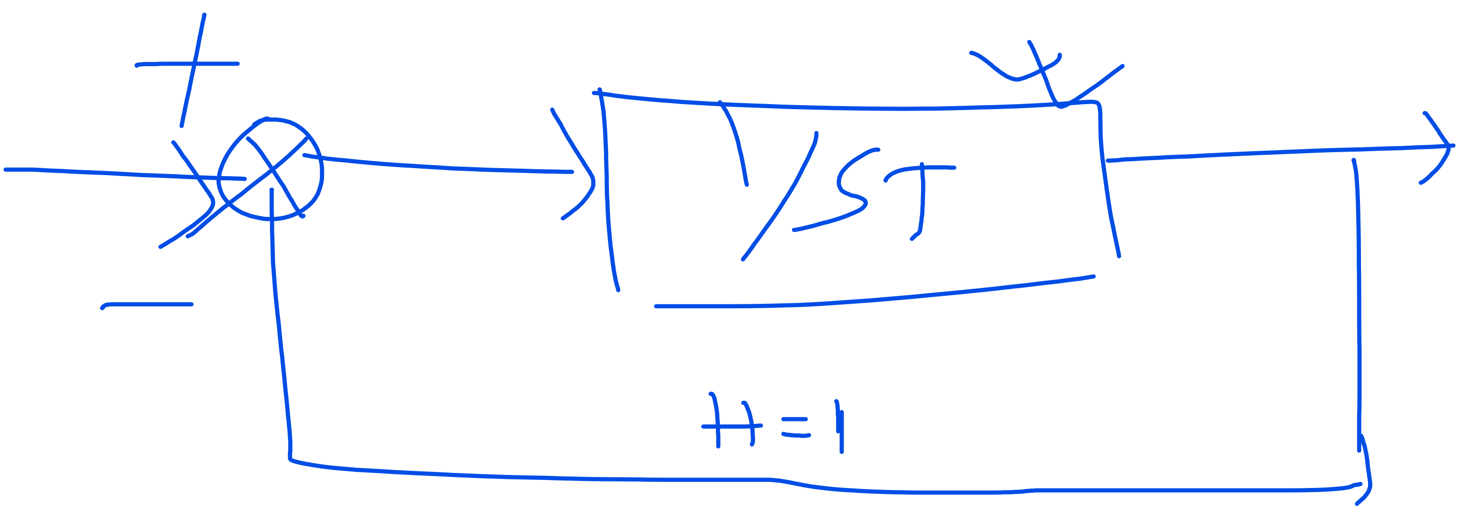
$c(t) = 1 - e^{-1}$

$= 0.632$

$t = T$, $c(t)$ 63%

time constant





$$\frac{1/sT}{1 + 1/sT} \qquad \frac{G}{1+GH}$$

$$\Rightarrow \frac{1}{1+sT}$$

$c(t) = 1 - e^{-t/T}$

$t = T$, $c(t) = 63\%$

$t = 2T$, $c(t) = 86.4\%$

$t = 3T$, $c(t) = 95\%$

$t = 4T$, $c(t) = 98\%$

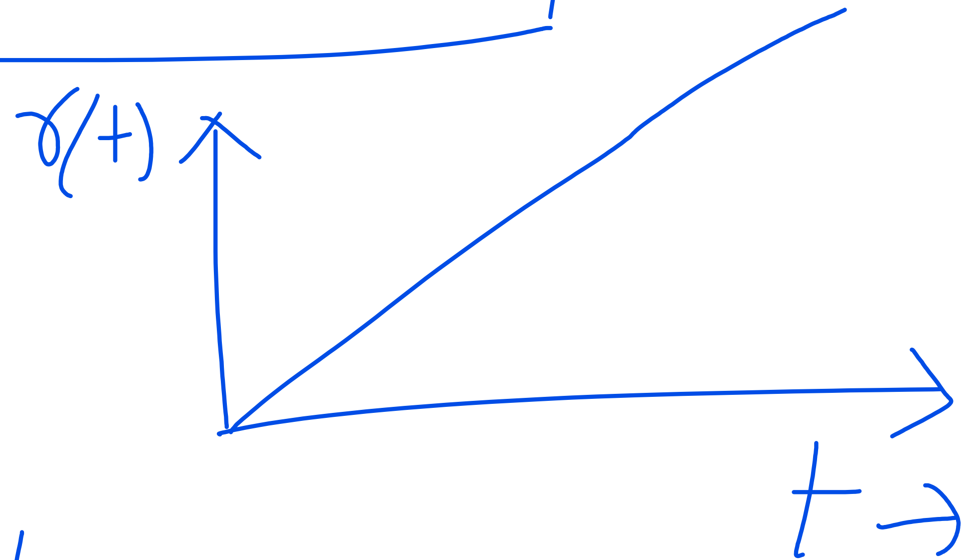
$t = 5T$, $c(t) = 99\%$

$\boxed{2\% \rightarrow \text{error}}$ $t = 4T$

steady state Res.

$1\% \longrightarrow t = 5T$

*) unit ramp i/p



$r(t) = t \cdot u(t)$

$R(s) = \frac{1}{s^2}$

$$C(s) = \frac{1}{sT+1} \times \frac{1}{s^2}$$

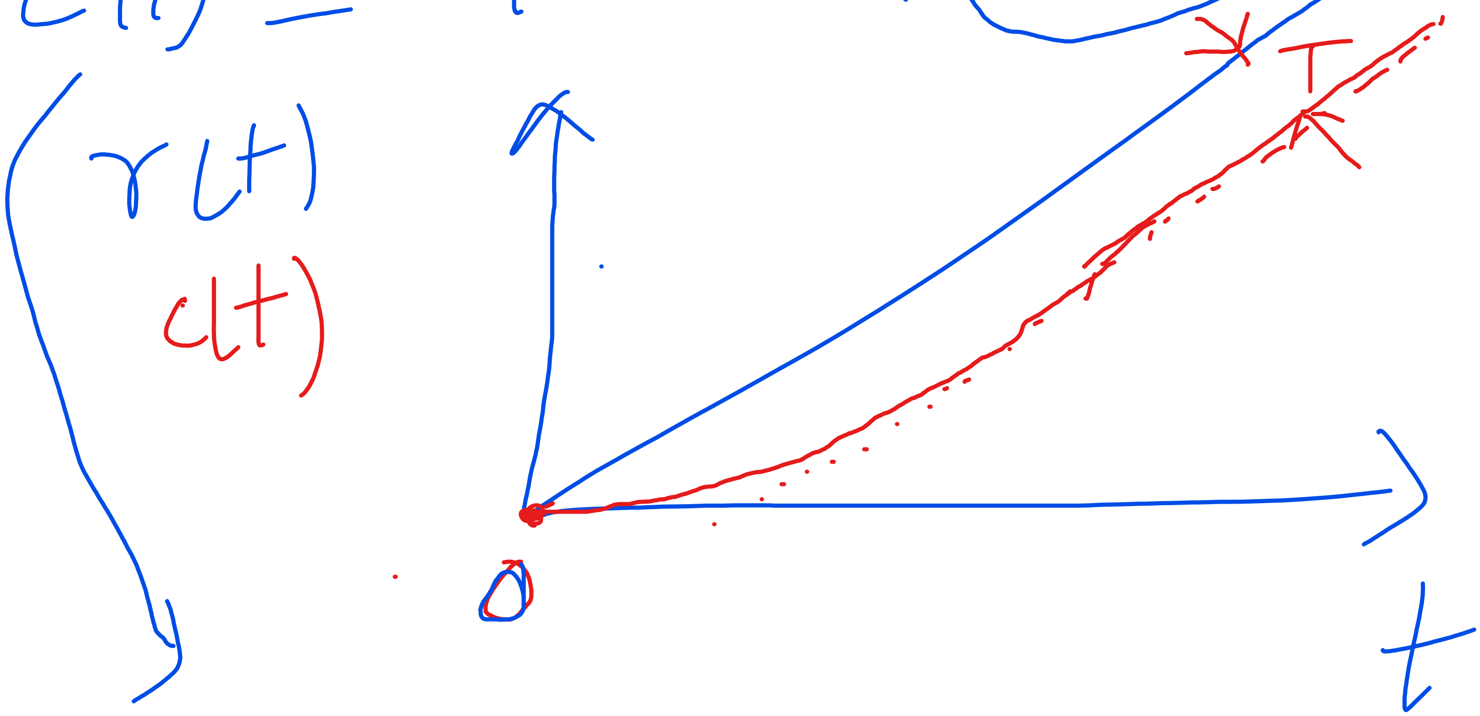
$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{sT+1}$$

$$= -\frac{T}{s} + \frac{1}{s^2} + \frac{T^2}{sT+1}$$

$$c(t) = -T + t + T e^{-t/T}$$

$t \geq 0$

$$c(t) = t - T + \boxed{T e^{-t/T}}$$



$$\text{error} = r(t) - c(t)$$

$$= T\left(1 - e^{-t/T}\right)$$

$t \to \infty$, error $= T$

$t = 0$, " $= 0$

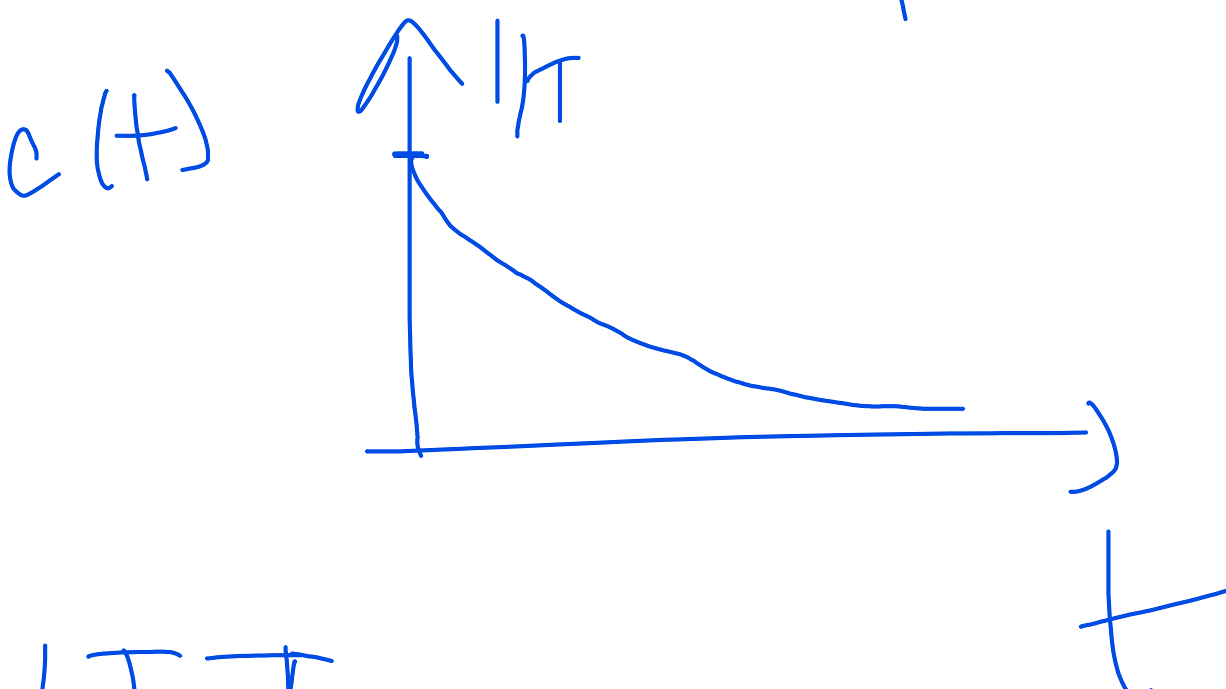
* impulse as i/p

$$r(t) = \delta(t)$$

$$R(s) = 1$$

$$C(s) = \frac{1}{s\tau + 1}$$

$$\Rightarrow c(t) = \frac{1}{\tau} e^{-t/\tau}, \quad t > 0$$



LTI

$t \cdot u(t)$ —— $t - T + T \cdot e^{-t/T}$

d/dt

$u(t)$ —— $1 - e^{-t/T}$

d/dt

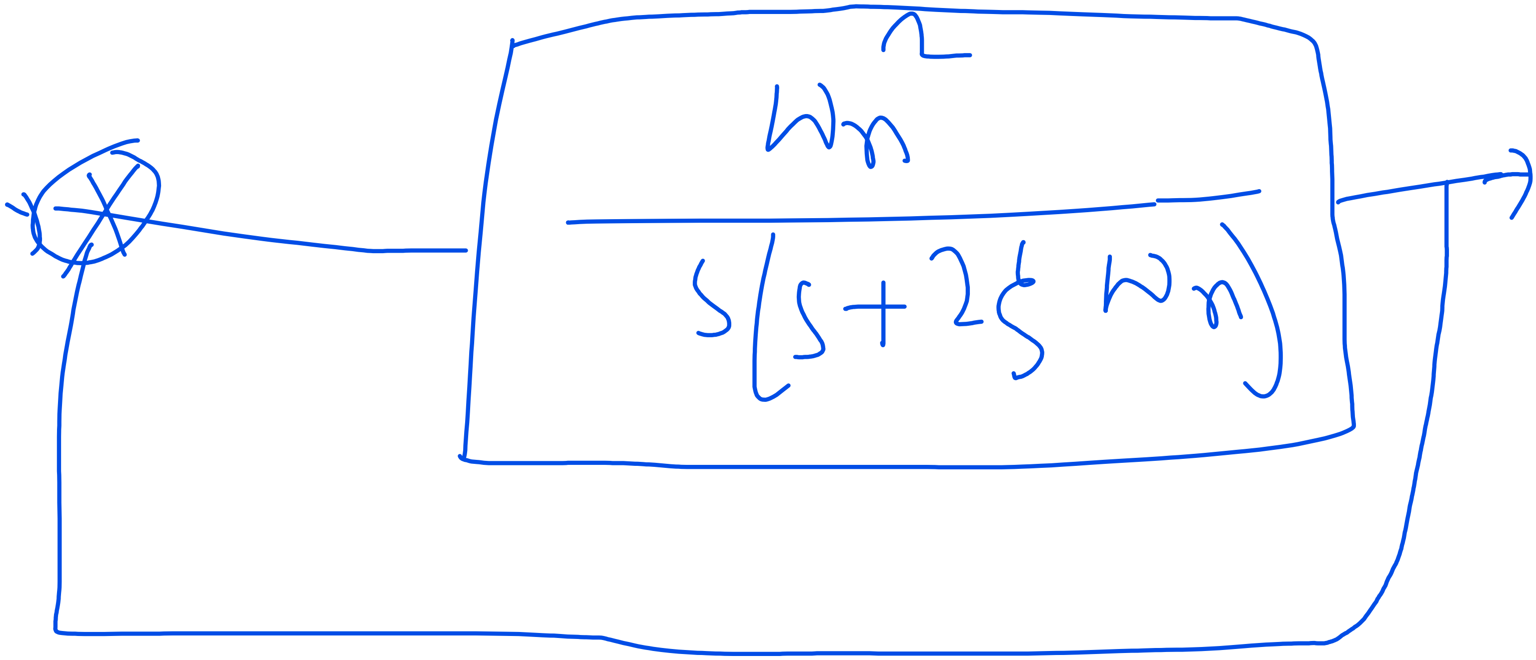
$\delta(t)$ —— $\frac{1}{T} e^{-t/T}$

2nd order system :-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2s\xi\omega_n + \omega_n^2}$$

ω_n = natural freq.

ξ = damping factor.



$$s^2+2\zeta\omega_n s+\omega_n^2=0$$

ch. eqn =

$$s=-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$

if $\zeta=0$, imaginary

$\zeta=1$, , real

$\xi > 1$, real and distinct

$0 < \xi < 1$, complex conj

$$\boxed{\xi,\ \omega_n}$$

$\xi = 1$, critically damped

$\xi = 0$, undamped

$\xi > 1$, overdamped

$0 < \xi < 1$, underdamped

i/p → unit step

1) underdamped

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$= -\xi\omega_n \pm j\omega_d$$

damped freq.

$$C(s) = \frac{\omega_n^2}{(s+\xi\omega_n+j\omega_d)(s+\xi\omega_n-j\omega_d)} \times \frac{1}{s}$$

$$= \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n+\omega_n^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(\quad)}$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos(\omega_d t) - e^{-\xi\omega_n t}\sin\omega_d t \times \frac{\xi}{\sqrt{1-\xi^2}}$$

$$\cos\omega t \leftrightarrow \frac{s}{s^2+\omega^2}$$

$$e^{-at}\cos\omega t \leftrightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

$$e^{-at}\sin\omega t = \frac{\omega}{(s+a)^2+\omega^2}$$

$$\frac{\xi\omega_n\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}\times\left((s+\xi\omega_n)^2+\omega_d^2\right)}$$

$$=\frac{\xi}{\sqrt{1-\xi^2}}\times\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\frac{}{}\times\frac{}{}$$

$$\sqrt{1-\xi^2}=\sin\theta$$

$$\xi=\cos\theta$$

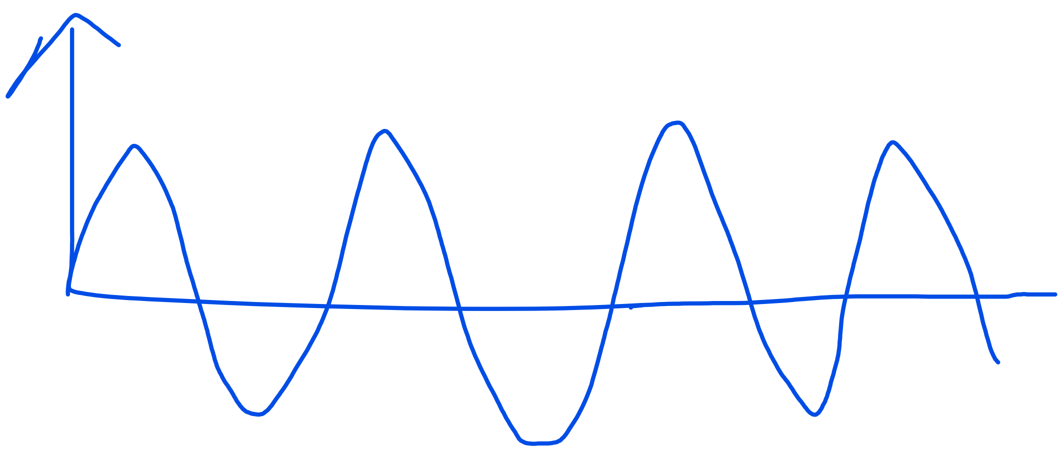
$$\cancel{c(t)=1-e^{-\xi\omega_n t}\cos\omega_d t-\cot\theta}$$

$$\cancel{e^{-\xi\omega_n t}\sin\omega_d t}$$

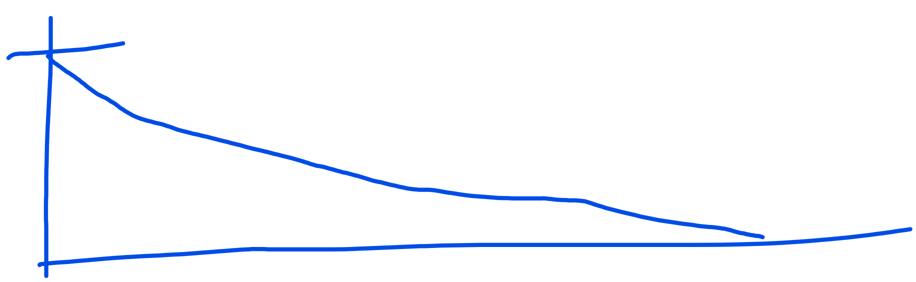
$$1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}\cos\omega_d t + \zeta\sin\omega_d t\right]$$

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \theta)$$

$A = \sin$



$B = e^{-x}$



$A \times B$