T.F $\rightarrow$ D.E $\rightarrow$ state variables

$\downarrow$

A, B, C, D $\leftarrow$ state eqn

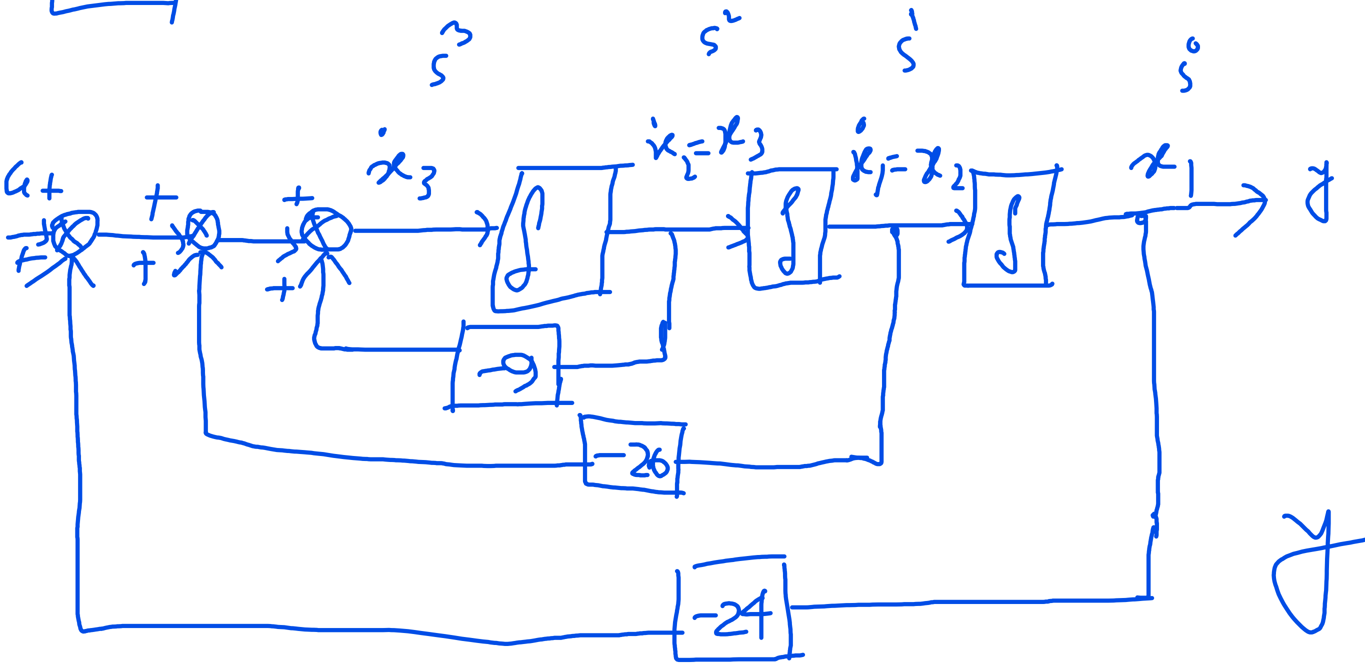
T.F

Direct decomposition :-

$$G(s) = \left(\frac{1}{s^3 + 9s^2 + 26s^1 + 24s^0} \right) \quad (1)$$

$\boxed{3}$ 3rd order

$y = x_1$



$y = x_1$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + u$$

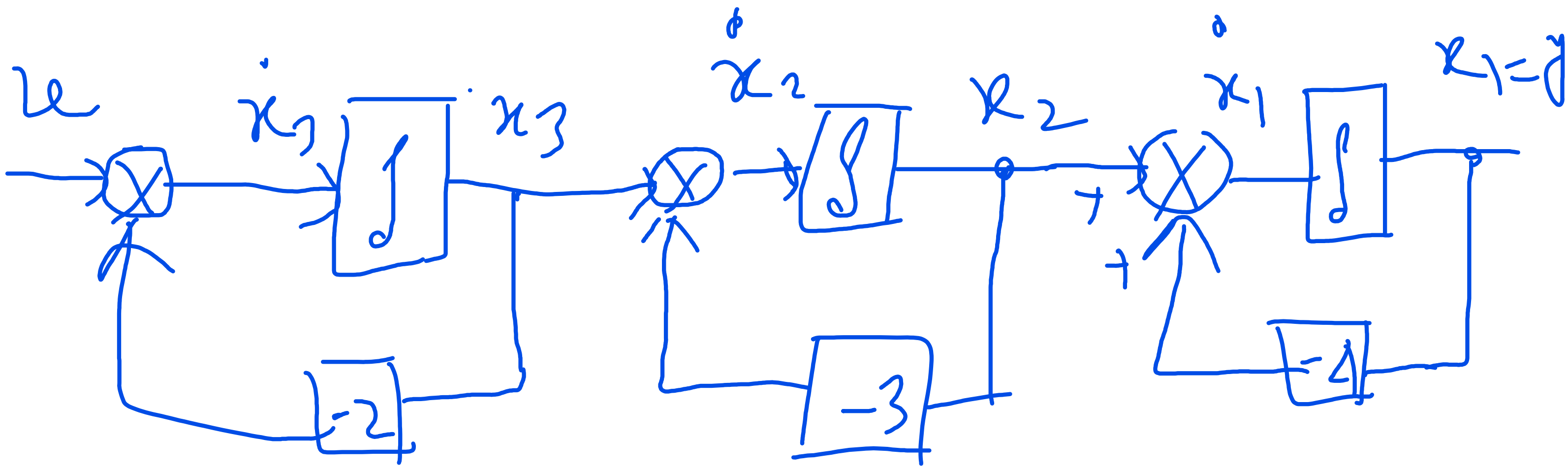
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$G(s) = \frac{2s+1}{s^2+2s+3}$$

$$? = \frac{2s}{s^2+2s+3} + \frac{1}{s^2+2s+3}$$

$$\left(\frac{1}{s+2}\right)\times\left(\frac{1}{s+3}\right)\times\left(\frac{1}{s+4}\right) - 2$$



$y = x_1$

$$\dot{x}_1 = -4x_1 + x_2$$

$$\dot{x}_2 = -3x_2 + x_3$$

$$\dot{x}_3 = -2x_3 + u$$

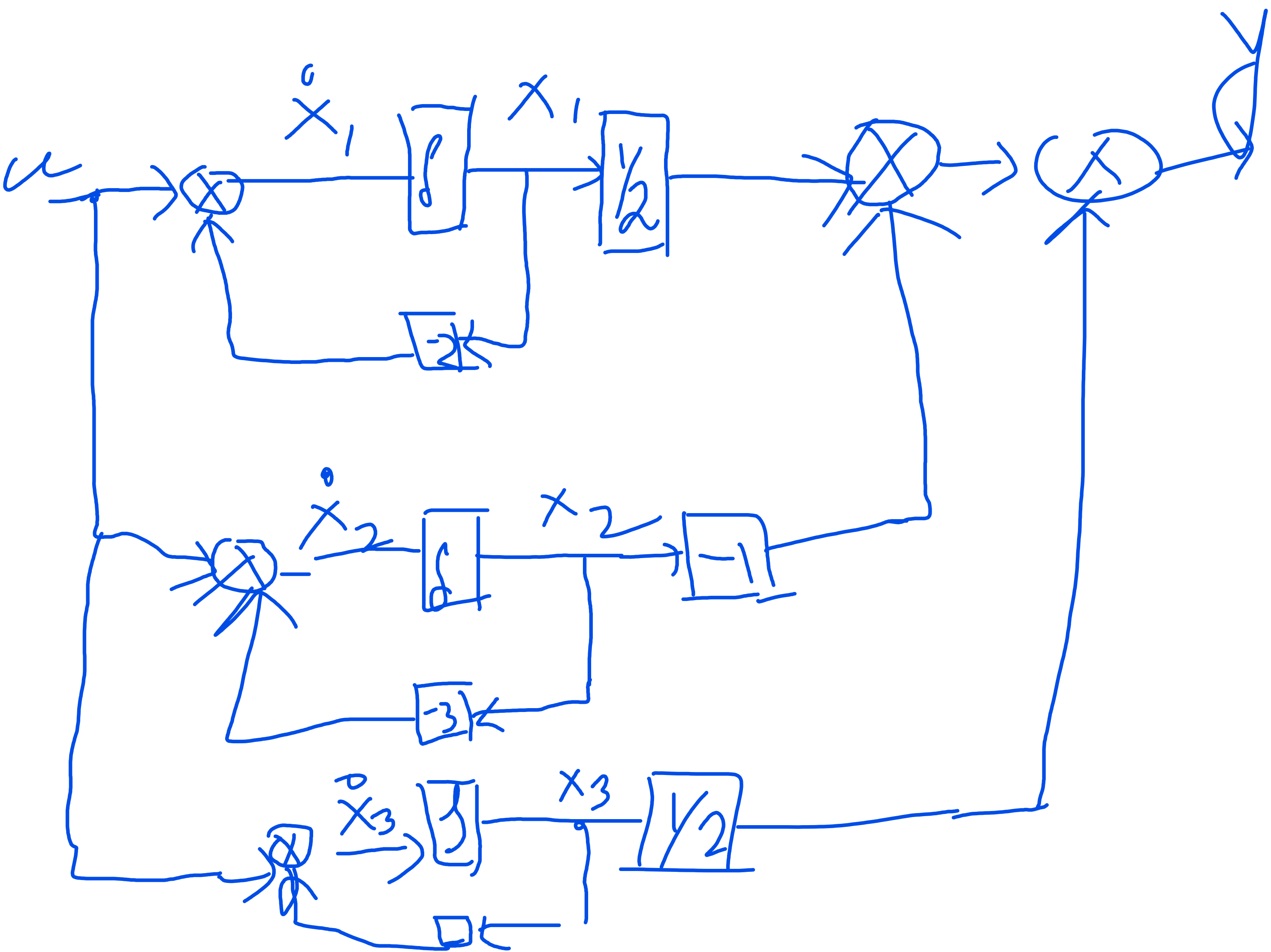
$$[\dot{X}] = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}[X] + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

the location of the poles

are the diagonal elements.

$$\frac{1}{(s+2)(s+3)(s+4)} \text{ — } ③$$

$$= \frac{1/2}{(s+2)} + \left(\frac{-1}{s+3}\right) + \frac{1/2}{s+4}$$



$$\dot{x}_1 = -2x_1 + u$$
$$\dot{x}_2 = -3x_2 + u$$
$$\dot{x}_3 = -4x_3 + u$$

$$y = \frac{1}{2}x_1 - x_2 + \frac{1}{2}x_3$$

$$[\dot{x}] = \overset{A}{\begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix}} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1/2 & -1 & 1/2 \end{bmatrix} [x]$$

$\cancel{A}$

$$\frac{1}{s+1} + \frac{1}{s+2} + \frac{1}{(s+3)^2}$$

$$[A] = \ ?$$

$$\dot{x} = Ax + Bu$$

$$\Rightarrow \frac{dx}{dt} = Ax + Bu$$

natural response :-

$$\frac{dx(t)}{dt} = Ax(t)$$

Homogeneous D.E.

$$x(t) = e^{At} x(0)$$

$$e^{At} = \phi(t) = \text{state transition matrix}$$

$$\Rightarrow \frac{dx}{dt} = Ax + Bu$$

$$\Rightarrow sX(s) - x(0) = AX(s) + BU(s)$$

$$\Rightarrow X(s)\left[sI - A\right] = x(0) + BU(s)$$

$$\Rightarrow X(s) = \underbrace{(sI-A)^{-1}}_{\phi(s)} x(0) + \underbrace{(sI-A)^{-1}} BU(s)$$

$$x(t) = \phi(t)\, x(0) + \int_0^t \phi(t-\tau) B\, U(\tau)\, d\tau$$

$$\Rightarrow \frac{dx(t)}{dt} - Ax(t) = Bu(t)$$

$$\Rightarrow e^{-At}\{\dot{x} - Ax\} = e^{-At} Bu$$

$$\Rightarrow \frac{d}{dt}\{e^{-At} x(t)\} = e^{-At} Bu(t)$$

$$\Rightarrow e^{-At} x(t) = x(0) + \int_0^t e^{-A\tau} Bu(\tau)\, d\tau$$

$$\Rightarrow x(t) = e^{At} x(0) + \int (\qquad)$$

$$= \underbrace{\phi(t)\, x(0)} + \underbrace{\int_0^{\tau} \phi(t-\tau) B u(\tau)\, d\tau}$$

natural Response

forced Response

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \dots$$

$$L\{e^{At}\} = \frac{1}{[sI - A]}$$

$$= (sI - A)^{-1}$$

$$(sI - A)^{-1} = \frac{\mathrm{Adj}\,|sI - A|}{|sI - A|}$$

$$\boxed{G(s) = C(sI - A)^{-1}B + D}$$

$$e^{At} = \phi(t)$$

1) $t = 0$, $\phi(0) = e^{A \times 0}$

$$= I$$

2) $\phi e^{-At} = e^{At} \cdot e^{-At} = I$

$$\phi(t) e^{-At} = I$$

$$\Rightarrow \phi^{-1}(t)\, \phi(t) e^{-At} = \phi^{-1}(t) \cdot I$$

$$\therefore\ e^{-At} = \phi(-t)$$

$$\therefore\ \phi^{-1}(t)\,\phi(t)\,\phi(-t) = \phi^{-1}(t)$$

$$\Rightarrow \boxed{\phi^{-1}(t) = \phi(-t)}$$

Controllability

Observability