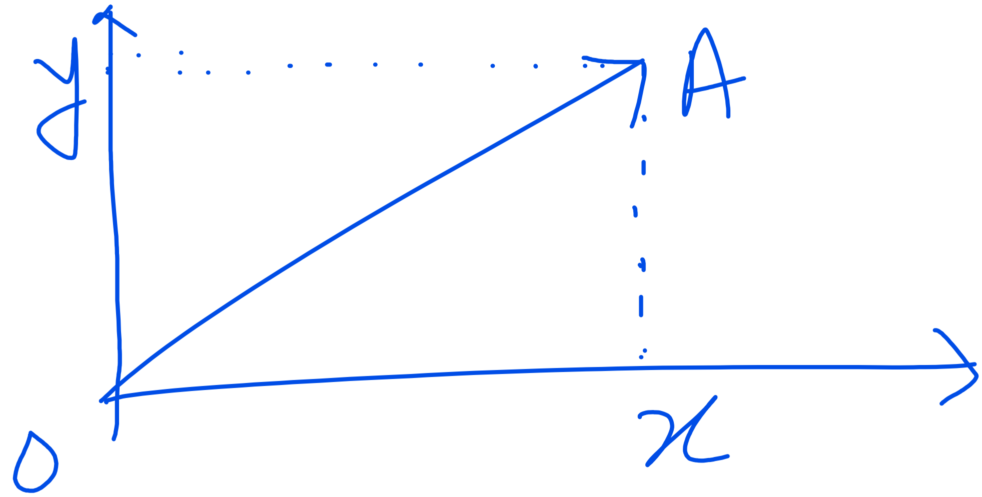


$$A = OX + OY$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 6 \\ 1 & 0 & 3 \end{bmatrix}$$ rank = 2

$\boxed{\text{rank} \leq 3}$ 3x3 | rank 3 ✗

How to Find rank?

Echelon Matrix

* squear matrix
[3 x 3]

rank $\leq$ 3

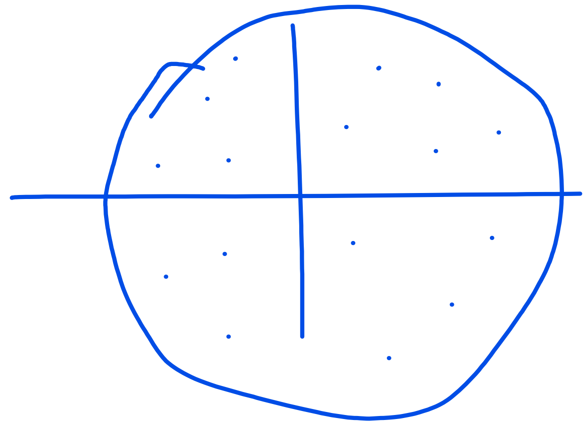
$|A| = 0$ | rank $\neq 3$

$|A| \neq 0$, rank $= 3$

Controllability

$$\dot{x} = Ax + Bu$$

$x(0)$



$\boxed{u}$

$x_1(t)$ (or) $x_2(t)$ ... $x_n(t)$

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$\dot{x}_1 = 2x_1 + u$$

$$\dot{x}_2 = -2x_2$$

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u$$

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$\dot{x}_2 = x_1 - 2x_2$$

$$\dot{x} = \check{A} x + \check{B} u$$

$$x(1) = A x(0) + B u(0)$$

$$x(2) = A x(1) + B u(1)$$

$$= A\left[A x(0) + B u(0)\right] + B u(1)$$

$$= A^2 x(0) + AB u(0) + B u(1)$$

$$x(n) = A^n x(0) + A^{n-1} B u(0) + \cdots$$

$$\cdots Bu(n-1)$$

$$x(n) - A^n x(0) = \Big(\qquad \Big)$$

$$\underbrace{\begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix}}_{\text{Controllability}} \begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(1) \\ u(0) \end{bmatrix}$$

$$\downarrow$$

$$\mathcal{C}$$

Controllability

rank $(\mathcal{C})$ = no. of column

2×2

$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$[B \quad AB]$

$AB = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\mathcal{C} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

$\boxed{\text{rank} = 2}$

$\mathcal{C} = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} \neq 0$

controllable

CVD

GaN

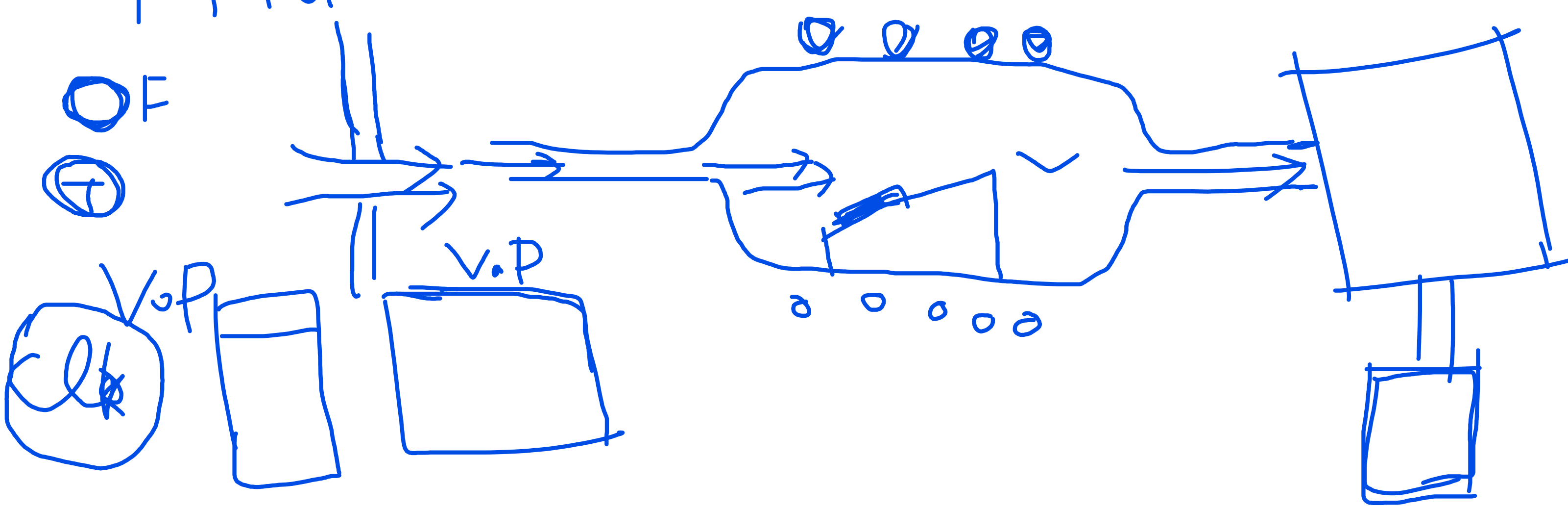
Si

HfO_2

GaAs

AsH_3

TMGa



MATLAB

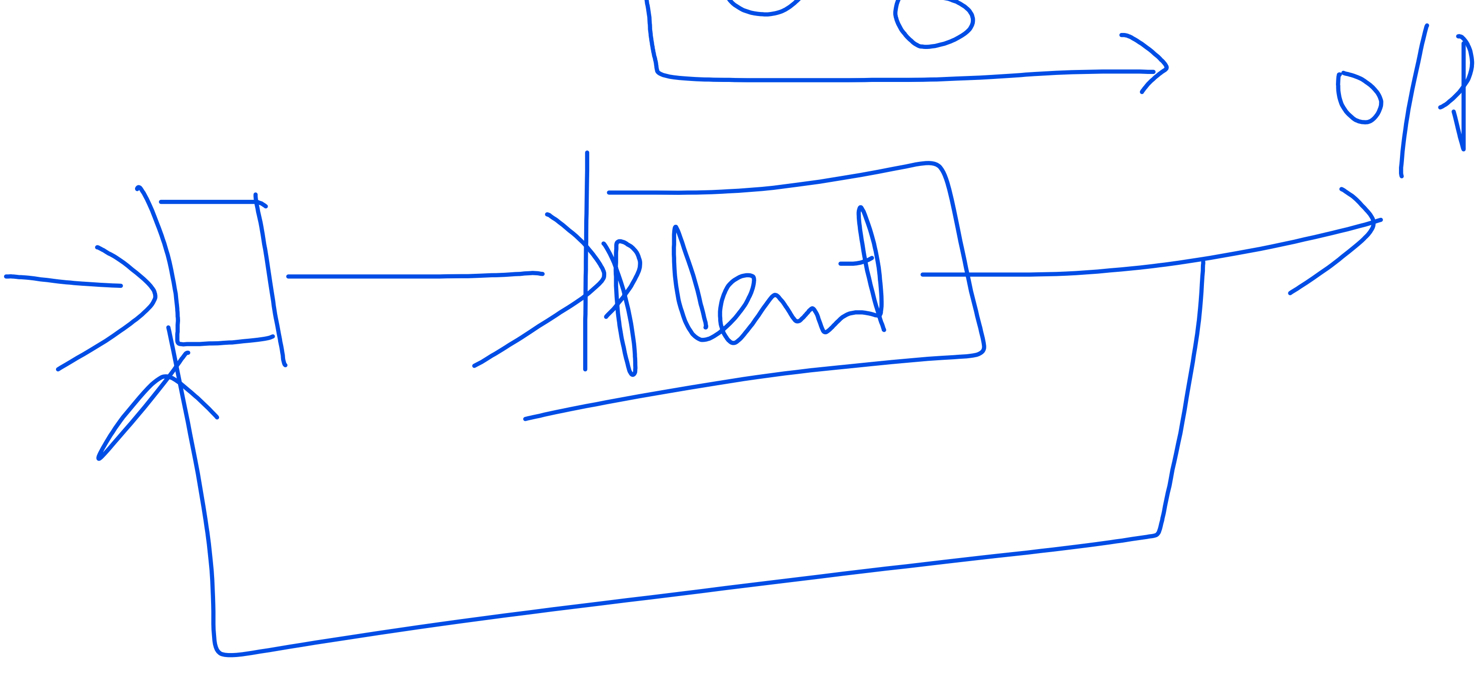
A, B

```
>> ctrb(A, B);
C = [        ]
```

>> rank(e)

>> 2

Observability



$$\boxed{Y = CX}$$

$$\begin{bmatrix} - \; C \\ - \; CA \\ - \; CA^2 \\ \vdots \\ - \; CA^{n-1} \end{bmatrix} = O, \quad \text{rank}(O)$$

$$y(0) = Cx(0)$$

$$y(1)$$

$$y(2)$$

$$y(n-1)$$

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(n-1) \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} x(0)$$

$*$ $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

$$\boxed{\text{rank} < 2}$$

unobservable

*) $A = \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

observable (or)

unobservable

$$\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

$\boxed{\text{rank} = 2}$

$$\text{Obsv}(A, C)$$

$$[\quad]$$

$$\text{rank} = (\quad)$$

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$