EC601: Assignment 1

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CONTENTS

I Problem 1 2

II Problem 2 4

III Problem 3 5

IV Problem 4 8

References 9

I. Problem 1

1) Consider the system defined by

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Except for an obvious choice of $c_1 = c_2 = c_3 = 0$, find an example of a set of c_1, c_2, c_3 that will make the system unobservable.

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Lets say, $y = C \cdot x$ where, $C = [c_1\ c_2\ c_3]$

We have $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$

$$\begin{aligned} \therefore CA &= \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \\ &= \begin{bmatrix} -6c_3 & c_1 - 11c_3 & c_2 - 6c_3 \end{bmatrix} \end{aligned} \tag{1}$$

$$\begin{aligned} \therefore CA^2 &= \begin{bmatrix} -6c_3 & c_1 - 11c_3 & c_2 - 6c_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \\ &= \begin{bmatrix} -6c_2 + 36c_3 & -11c_2 + 60c_3 & c_1 - 6c_2 + 25c_3 \end{bmatrix} \end{aligned} \tag{2}$$

$$\begin{aligned} \therefore \text{Observability Matrix, } Q_O &= \begin{bmatrix} A \\ CA \\ CA^2 \end{bmatrix} \\ &= \begin{bmatrix} c_1 & c_2 & c_3 \\ -6c_3 & c_1 - 11c_3 & c_2 - 6c_3 \\ -6c_2 + 36c_3 & -11c_2 + 60c_3 & c_1 - 6c_2 + 25c_3 \end{bmatrix} \quad (3) \end{aligned} \tag{4}$$

$$\therefore |Q_O| = \begin{vmatrix} c_1 & c_2 & c_3 \\ -6c_3 & c_1 - 11c_3 & c_2 - 6c_3 \\ -6c_2 + 36c_3 & -11c_2 + 60c_3 & c_1 - 6c_2 + 25c_3 \end{vmatrix} \tag{5}$$

$$\begin{aligned} &= 6c_3((c_2(c_1 - 6c_2 + 25c_3c_1 - 6c_2 + 25c_3) - c_3(-11c_2 + 60c_3)) \\ &\quad +(c_1 - 11c_3)(c_1(c_1 - 6c_2 + 25c_3c_1 - 6c_2 + 25c_3) - c_3(-6c_2 + 36c_3)) \\ &\quad -(c_2 - 6c_3)(c_1(-11c_2 + 60c_3) - c_2(-6c_2 + 36c_3)) \end{aligned} \tag{6}$$

$$\begin{aligned} &= c_1^3 - 6c_2c_1^2 + 14c_3c_1^2 + 11c_2^2c_1 + 49c_3^2c_1 \\ &\quad -48c_2c_3c_1 - 6c_2^3 + 36c_3^3 - 66c_2c_3^2 + 36c_2^2c_3 \text{ [1]} \end{aligned} \tag{7}$$

The question asked to find the non-trivial solutions of c_1, c_2, c_3 (i.e. $c_1 \neq c_2 \neq c_3 \neq 0$) such that the system is unobservable (i.e. $|Q_O| = 0$)

$\therefore$ to eliminate the trivial solutions, the following relations are chosen : $|Q_O| = 0$

from the first, second and third part of Eq.6 we get respectively,

$$(c_2(c_1 - 6c_2 + 25c_3c_1 - 6c_2 + 25c_3) - c_3(-11c_2 + 60c_3) = 0 \tag{8}$$

$$(c_1 - 11c_3) = 0 \tag{9}$$

$$(c_2 - 6c_3) = 0 \tag{10}$$

Now solving Eq.8, Eq.9, Eq.10 we get,

$$c_1 = \frac{7}{5}\ ,\ c_2 = \frac{42}{55}\ ,\ c_3 = \frac{7}{55}\ \text{[2]}$$

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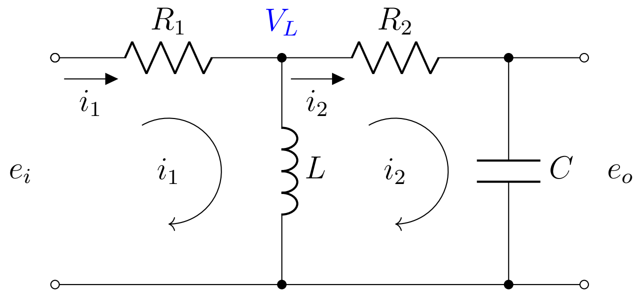
II. Problem 2

2) Obtain the transfer function $\frac{e_i}{e_o}$ of the electrical circuit shown in the figure.



■ Lets say the node voltage of L is V_L

$$\therefore V_L = e_i \times \frac{G_1}{G_1 + Y_2} \tag{11}$$

Where,

$$\begin{aligned} G_1 &= \frac{1}{R_1} \\ Y_2 &= \frac{1}{sL} + \frac{1}{R_2 + \frac{1}{sC}} \end{aligned}$$

$$\begin{aligned} \therefore e_o &= V_L \times \frac{\frac{1}{sC}}{R_2 + \frac{1}{sC}} \qquad (12) \\ \implies e_o &= e_i \times \frac{G_1}{G_1 + Y_2} \times \frac{\frac{1}{sC}}{R_2 + \frac{1}{sC}} \\ \implies \frac{e_o}{e_i} &= H(s) = \frac{G_1}{G_1 + Y_2} \times \frac{\frac{1}{sC}}{R_2 + \frac{1}{sC}} \\ \implies H(s) &= \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{sL} + \frac{1}{R_2 + \frac{1}{sC}}} \times \frac{\frac{1}{sC}}{R_2 + \frac{1}{sC}} \\ \implies H(s) &= \frac{sL}{LC(R_1 + R_2)s^2 + (R_1 R_2 C + L)s + R_1} \qquad (13) \end{aligned}$$

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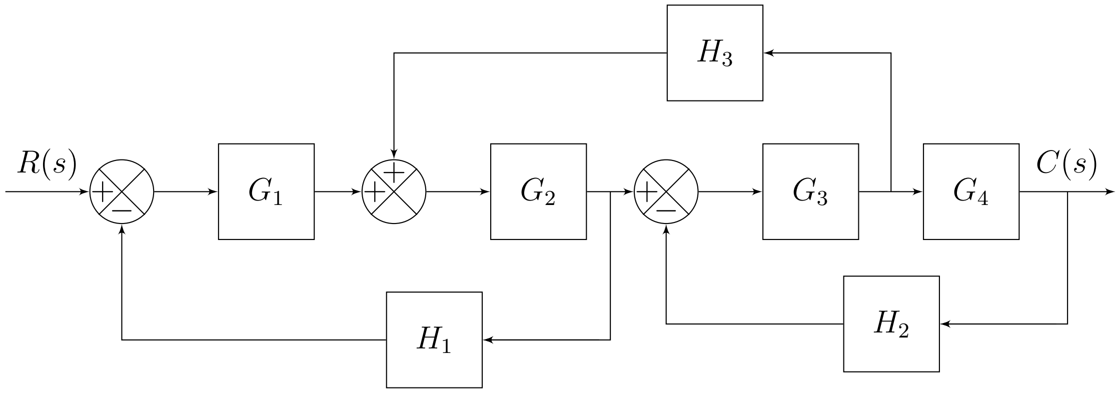
III. Problem 3

3) Simplify the block diagram shown in Figure. Then obtain the closed-loop transfer function $\frac{C(s)}{R(s)}$.



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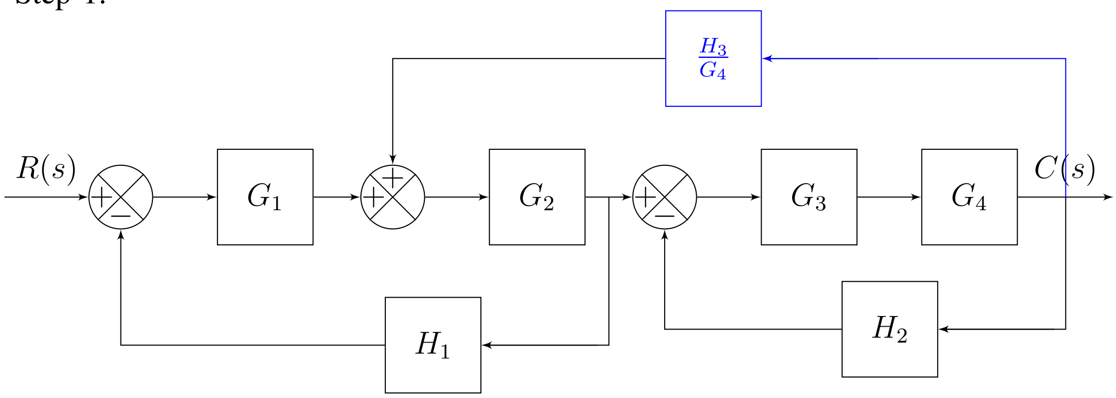
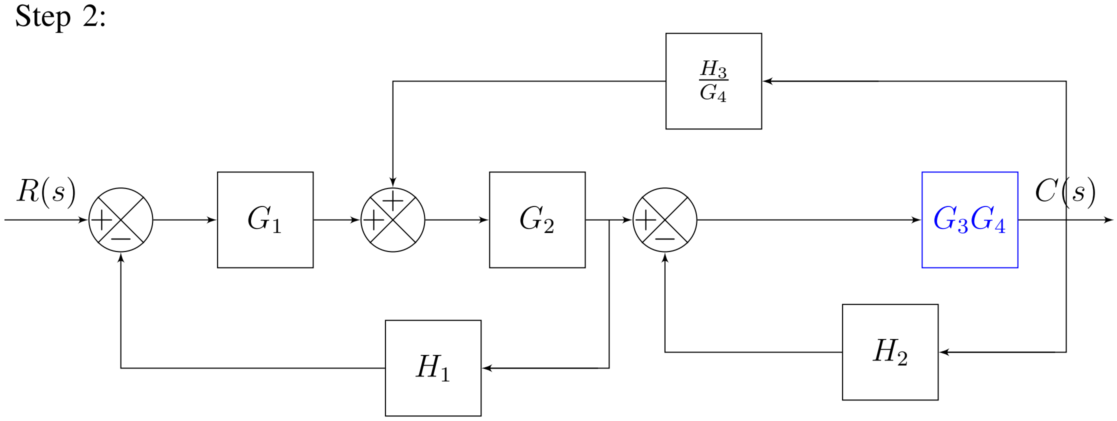
Step 1:



Step 2:



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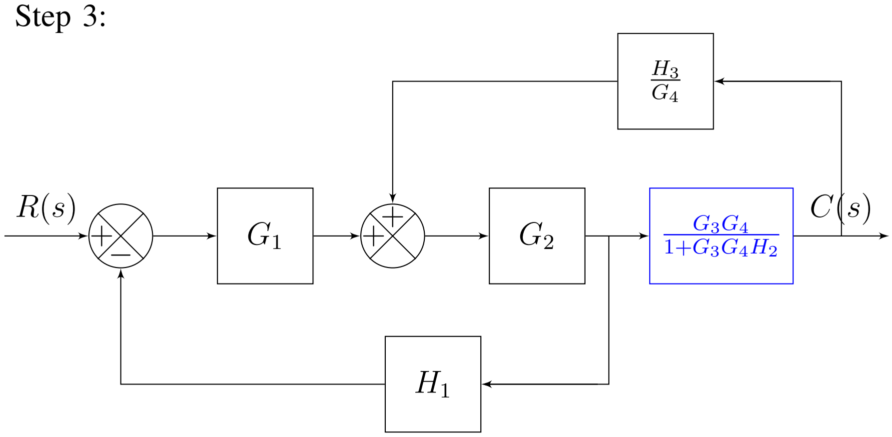

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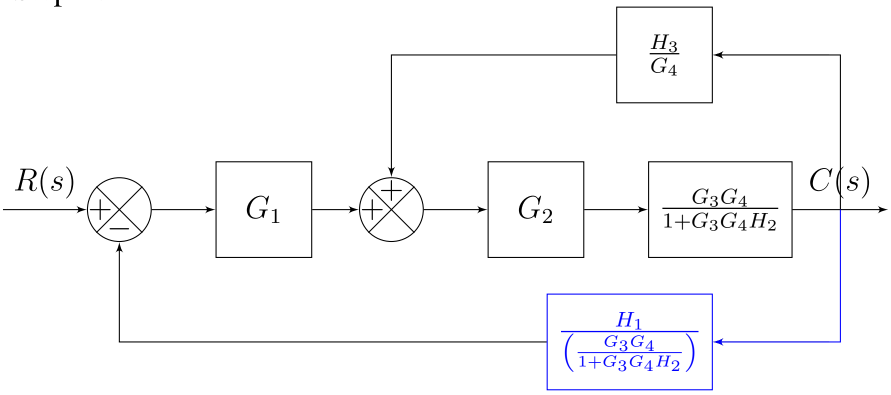

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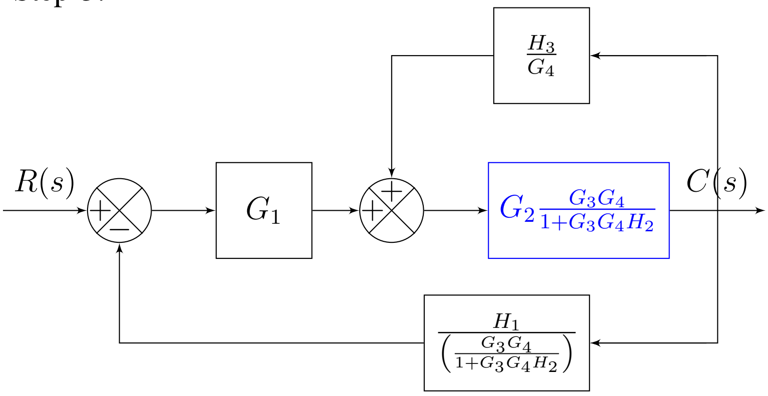

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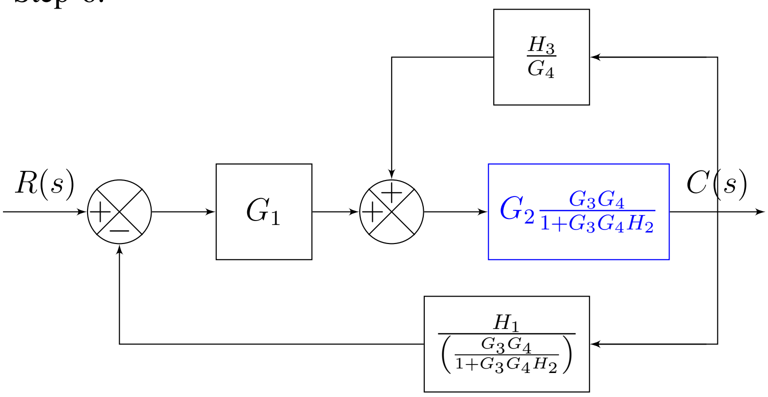

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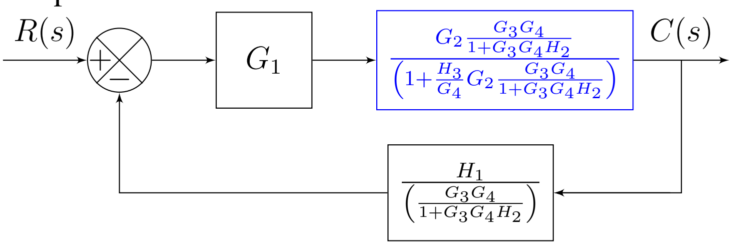
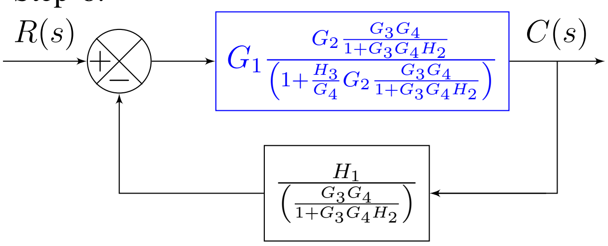


Step 8:



Step 9:

$$R(s) \longrightarrow \boxed{\frac{G_1 \frac{G_2 \frac{G_3G_4}{1+G_3G_4H_2}}{\left(1+\frac{H_3}{G_4}G_2\frac{G_3G_4}{1+G_3G_4H_2}\right)}}{\left(1+\left(G_1 \frac{G_2 \frac{G_3G_4}{1+G_3G_4H_2}}{\left(1+\frac{H_3}{G_4}G_2\frac{G_3G_4}{1+G_3G_4H_2}\right)}\right)\left(\frac{H_1}{\left(\frac{G_3G_4}{1+G_3G_4H_2}\right)}\right)\right)}} \longrightarrow C(s)$$

$$\therefore \text{Transfer Function}\frac{C(s)}{R(s)} = \frac{G_1 \frac{G_2 \frac{G_3G_4}{1+G_3G_4H_2}}{\left(1+\frac{H_3}{G_4}G_2\frac{G_3G_4}{1+G_3G_4H_2}\right)}}{\left(1+\left(G_1 \frac{G_2 \frac{G_3G_4}{1+G_3G_4H_2}}{\left(1+\frac{H_3}{G_4}G_2\frac{G_3G_4}{1+G_3G_4H_2}\right)}\right)\left(\frac{H_1}{\left(\frac{G_3G_4}{1+G_3G_4H_2}\right)}\right)\right)} \tag{14}$$

$$= \tag{15}$$

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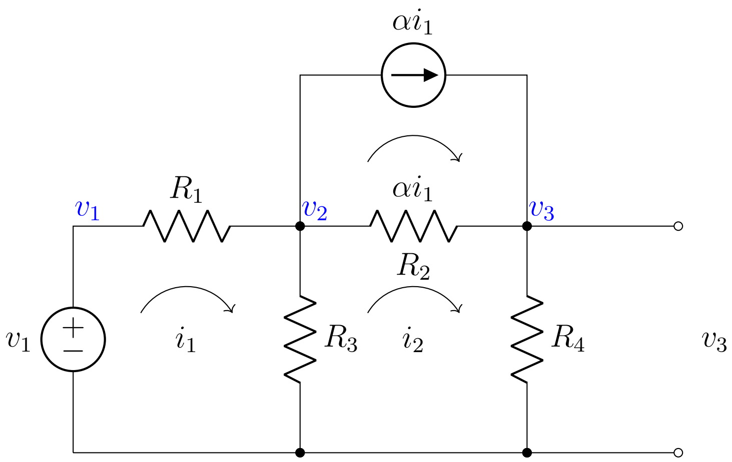
IV. Problem 4

4) Draw the signal low graph of the following electrical circuit and find its transfer function using Mason's gain formula.



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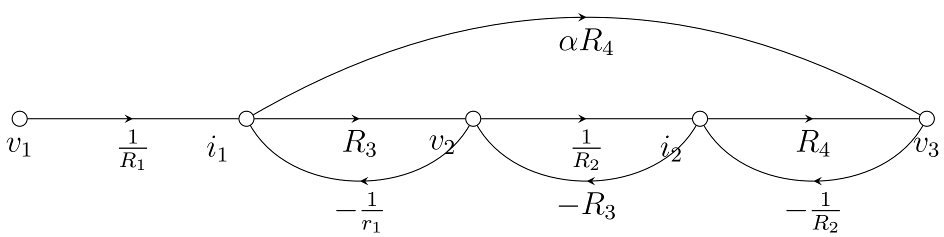
Custom Nodes has been marked in blue.

$$
\begin{aligned}
v_1 &= \text{Input Voltage} && (16)\\
v_2 &= (i_1 - i_2)(R_3) \\
&= (R_3)\, i_1 + (-R_3)\, i_2 && (17)\\
v_3 &= (\alpha i_1 + i_2) R_4 \\
&= (\alpha R_4) i_1 + (R_4) i_2 && (18)\\
i_1 &= \frac{v_1 - v_2}{R_1} \\
&= \left(\frac{1}{R_1}\right) v_1 + \left(-\frac{1}{R_1}\right) v_2 && (19)\\
i_2 &= \frac{v_2 - v_3}{R_2} \\
&= \left(\frac{1}{R_2}\right) v_2 + \left(-\frac{1}{R_2}\right) v_3 && (20)
\end{aligned}
$$

Signal Flow Graph of the given circuit:



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REFERENCES

[1] W. A. LLC, “Wolfram—alpha,” Web Link, accessed on 10/06/2021.
[2] ——, “Wolfram—alpha,” Web Link, accessed on 10/06/2021.