

$$\frac{d\theta}{dt} \propto T$$

Heat

Hg ↑

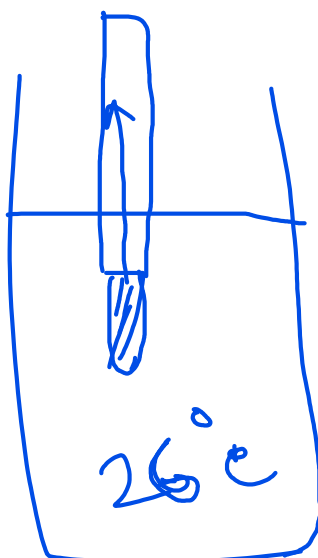
$$\frac{d\theta}{dt} \propto \frac{1}{t_0}$$

$\propto \frac{1}{t}$

1 + ST

26°C

$$\frac{d\theta}{dt} \propto T$$



2nd Order sys

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) u(t)$$

$$\xi = \cos \theta$$

$$\frac{\omega_d}{\omega_n}$$

$$\sqrt{1-\xi^2} = \sin \theta$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{1-\xi^2}}{\xi} \right)$$

$0 < \xi < 1 \rightarrow$ under damped.

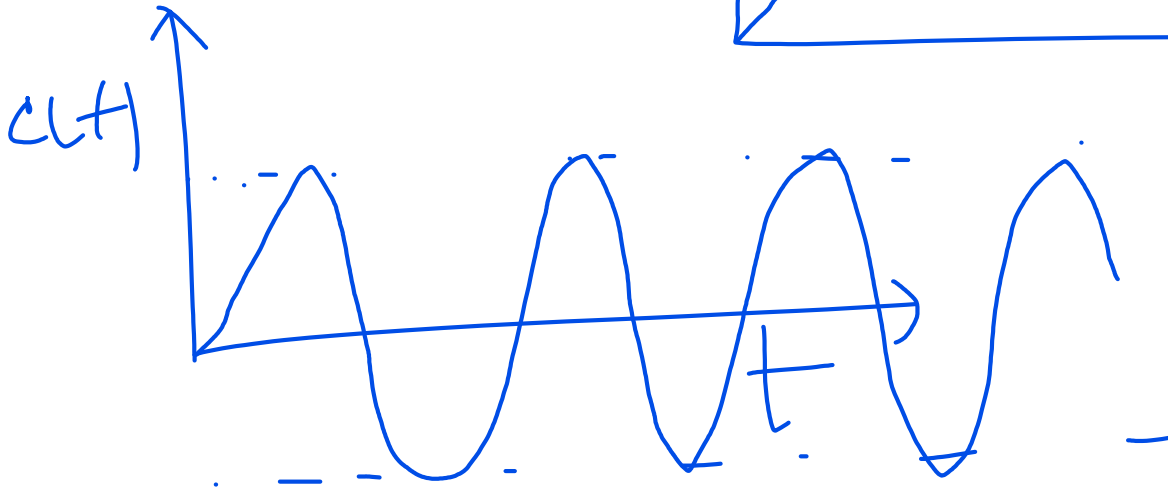
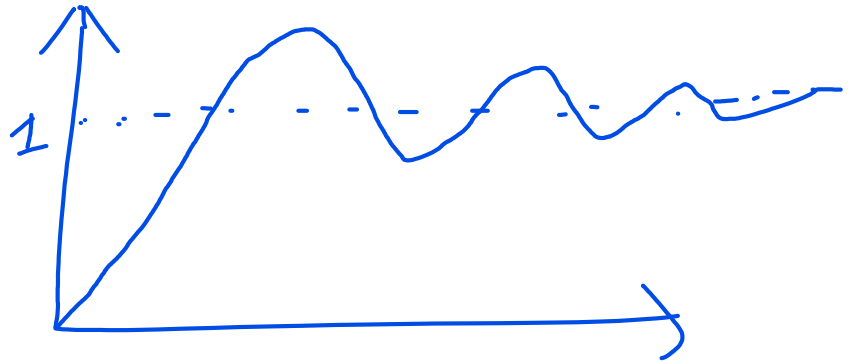
$\xi = 0 \rightarrow$ undamped.

$$c(t) = (1 - \cos \omega_d t), \quad t > 0$$

$$1 - e^{-\xi \omega_n t} \cos \omega_d t - \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_d t \quad // \quad c(t)$$

$$c(t) = 1 - \text{exp} \times \sin$$

$$1 - \cos \omega_d t$$



⑧ $\xi = 1$, critically damped.

$$I(s) = \frac{\omega_n^2}{(s + \mu_n)^2} \times \frac{1}{s}$$

$$c(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$$

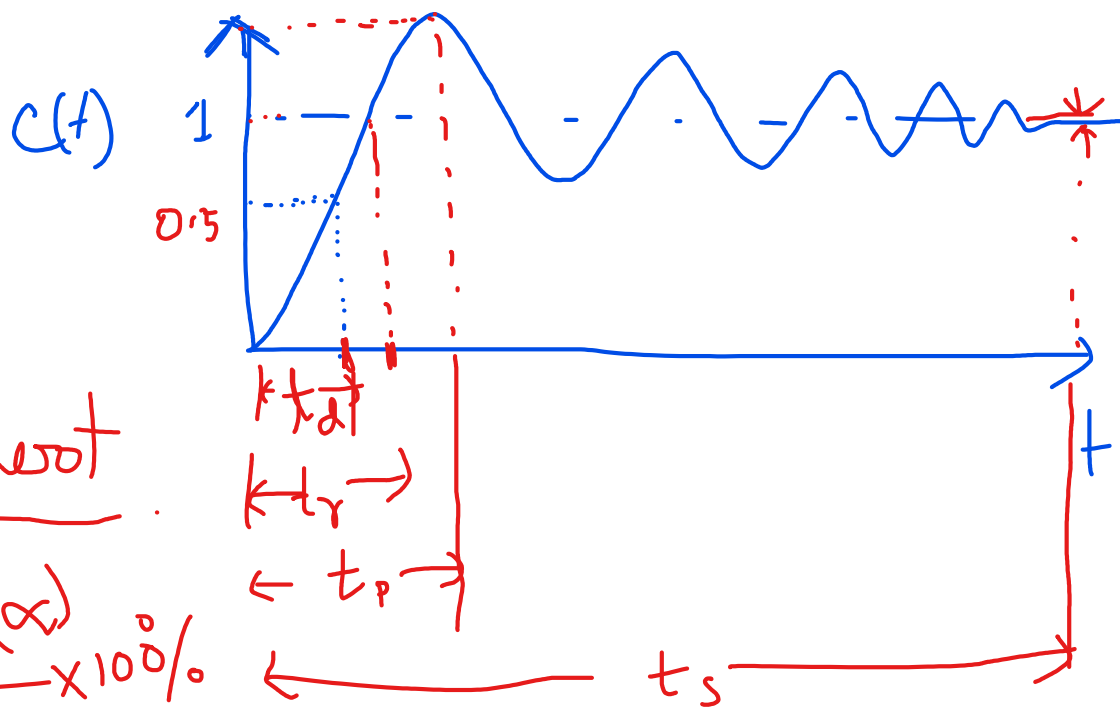
⑨ $\xi > 1$ overdamped.

$$c(t) = 1 + \frac{\omega_n}{2\sqrt{\xi^2 - 1}} \left(\frac{e^{s_1 t}}{s_1} - \frac{e^{s_2 t}}{s_2} \right)$$

$$s_1 = (\zeta + \sqrt{\zeta^2 - 1}) \omega_n$$

$$s_2 = (\zeta - \sqrt{\zeta^2 - 1}) \omega_n$$

(*)



$(0.02 - 0.01)\%$

Max. Overshoot

$$\frac{c(p) - c(\infty)}{c(\infty)} \times 100\%$$

Rise time:-

$$c(t) = 1 - e^{-\zeta \omega_n t_r} \left(\cos \omega_d t_r + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \omega_d t_r \right)$$

$$\left(t = t_r, c(t) = 1 \right)$$

$$1 = 1$$

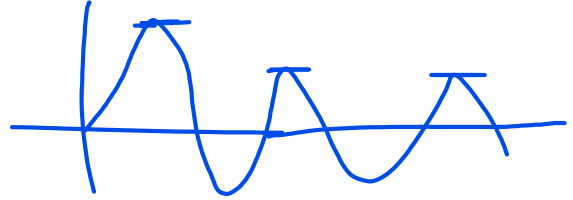
$$\begin{pmatrix} \text{cap} \\ \# \\ 0 \end{pmatrix} \left(\cos \omega_d t_r + \frac{\xi}{\sqrt{1-\xi^2}} \sin \omega_d t_r \right) = 0$$

$$\tan \omega_d t_r = - \frac{\sqrt{1-\xi^2}}{\xi}$$

$$t_r = \frac{1}{\omega_d} \tan^{-1} \left(\frac{-\omega_n \sqrt{1-\xi^2}}{\omega_n \xi} \right)$$

$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

$$t_r = \frac{1}{\omega_d} \tan^{-1} \left(\frac{-\omega_d}{\xi} \right)$$

2) Peak time $t_p =$ 

$$\boxed{\frac{dc}{dt} = 0} \quad t = t_p$$

$$\sin \omega_d t_p = 0 \quad \sin(n\pi)$$

$n = 1, 2, 3, \dots$

$$\omega_d t_p = n\pi$$

$$\Rightarrow \boxed{t_p = \frac{n\pi}{\omega_d}}$$

3) Max Ov. :-

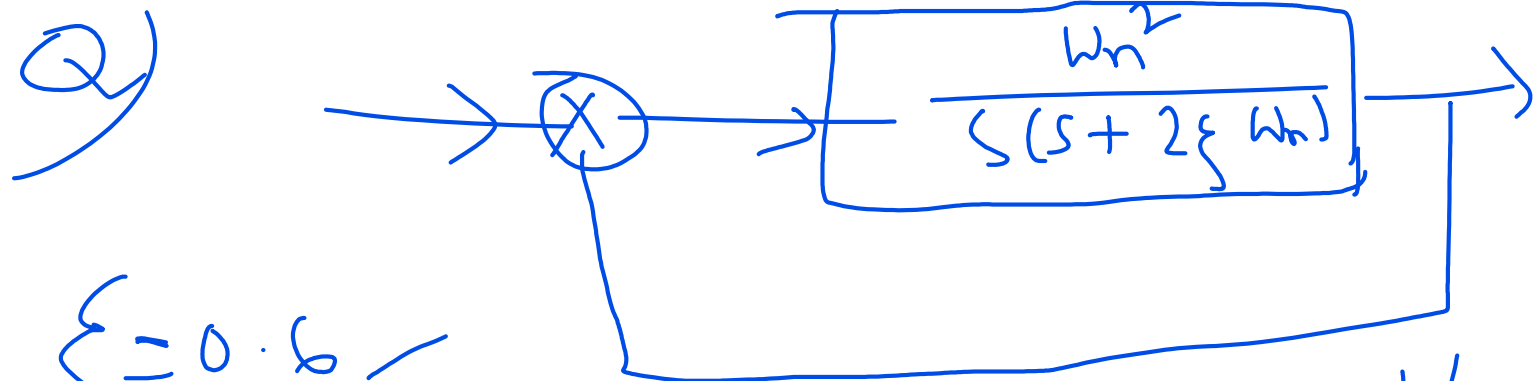
$$\frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

$$\Rightarrow [c(t_p) - 1] \times 100\%$$

$$M_p\% = e^{-(\xi/\sqrt{1-\xi^2})\pi} \times 100\%$$

4) settling time (2%)

$$t_s = 4T = \frac{4}{\xi\omega_n}$$



$$\xi = 0.6$$

$$\omega_n = 5 \text{ rad/s} \quad R(s) = 1/s$$

$$t_s = \frac{1}{\omega_d} \tan^{-1} \left(\frac{\sqrt{1-\xi^2}}{\xi} \right) = s.$$

$$t_p = \frac{\pi}{\omega_d} s.$$

$$M_p = e^{-\left(\xi / \sqrt{1-\xi^2}\right) \pi} \times 100 \%$$

$$t_s = 4T = \frac{4}{\xi \omega_n}$$

2nd. $T(s) \times \frac{1}{s^2} \checkmark$

$$\boxed{T(s) \times \frac{1}{s}} \times \frac{1}{s}$$