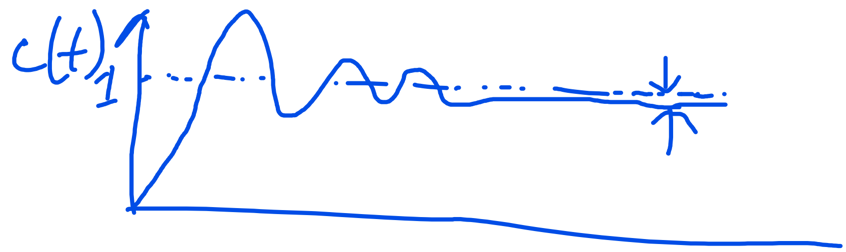
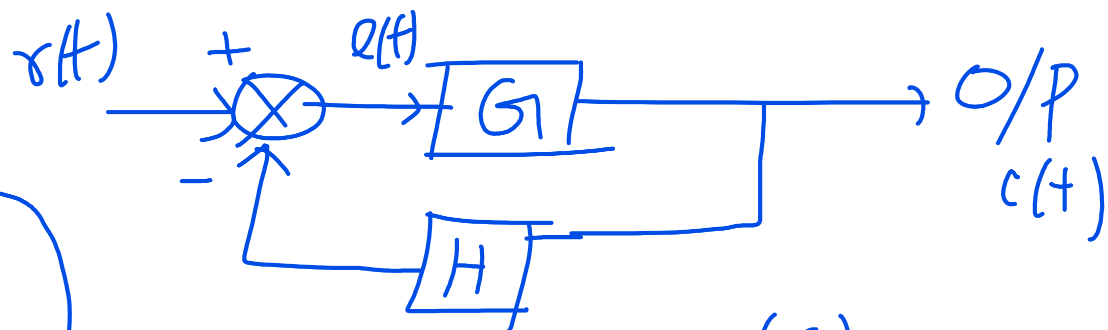
Steady state error -

o/p follow the i/p at steady state

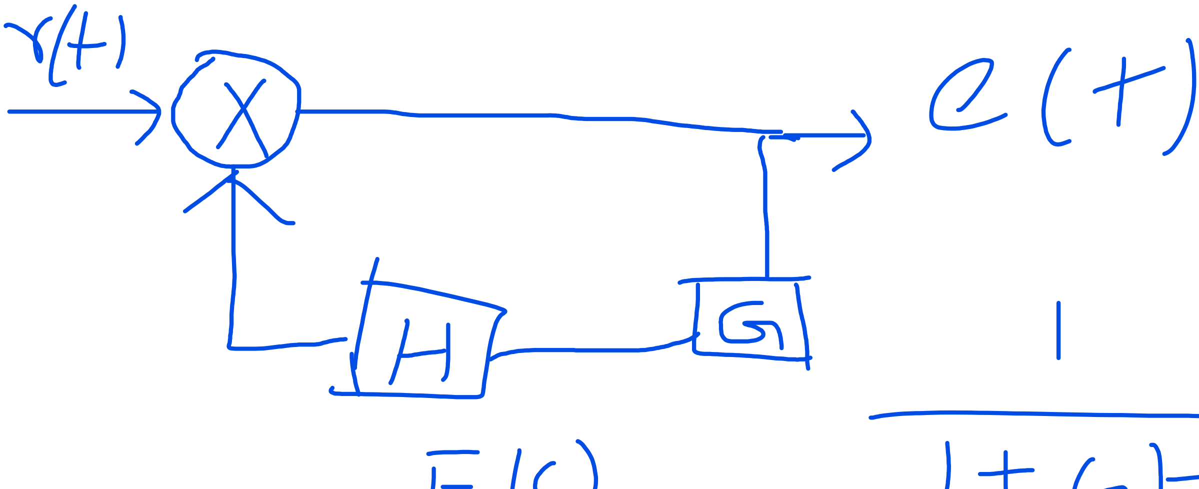


$t \to \infty$, i/p − o/p = error



lt e(t)
$t \to \infty$

$$\frac{E(s)}{R(s)}$$



$$\frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

$$E(s) = \frac{1}{1+GH} \times R(s)$$

$$\boxed{\lim_{t \to \infty} e(t)} = \lim_{s \to 0} sE(s) \quad \text{(F.V.T.)}$$

$$= \lim_{s \to 0} sR(s) \times \frac{1}{1+GH}$$

Why error occurs?

⇒ Ref. i/p changes

⇒ Imperfection in system Component (aging)

⇒ system is incapable to

follow some i/p.

#) SSE for a given type of i/p depends on the OL. T.F.

Type of <u>system.</u>

O.L. T.F $\rightarrow$ poles

$\downarrow$

poles at the origin $=N$

$N=0$, Type 0 system

$N=1$, " 1 "

$N=2$, " 2 "

position or displacement $\qquad e$

$\downarrow d/dt$

velocity $\qquad \dfrac{de}{dt}$

$\downarrow d^2/dt^2$

Acceleration $\qquad \dfrac{d^2e}{dt^2}$

Static error coeff.

$$e_{ss} = \lim_{s \to 0} sR(s) \times \frac{1}{1+GH}$$

$$R(s) = \frac{1}{s} \Big| \quad \lim_{s \to 0} \frac{1}{1+GH} \quad [G(s)H(s)]$$

$$= \frac{1}{1 + \lim_{s \to 0} GH}$$

$$K_p = \lim_{s \to 0} G(s)H(s) = \text{positional error const.}$$

$$\therefore e_{ss} = \frac{1}{1 + K_p}$$

<u>Type '0'</u>:

$$K_p = \lim_{s \to 0} \frac{K(T_a s+1)(T_b s+1)}{(T_1 s+1)(T_2 s+1)} = K.$$

$$\boxed{e_{ss} = \frac{1}{1+K}}$$

<u>Type '1' or higher</u>:

$$K_p = \lim_{s \to 0} \frac{K(\;)(\;)}{s^N(\;)(\;)} = \infty \quad , N \geq 1$$

$$\boxed{\therefore e_{ss} = \frac{1}{1+\infty} = 0.}$$

2) velocity error const. or K_v

$$r(t) = t \cdot u(t), \quad R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \to 0} s \times \frac{1}{s^2} \times \frac{1}{1+GH}$$

$$= \lim_{s \to 0} \frac{1}{s + sG(s)H(s)}$$

$$\lim_{s \to 0} sG(s)H(s) = K_v$$

$$e_{ss} = \frac{1}{K_v}$$

Type '0',

$$K_v = \lim_{s \to 0} \frac{s \cdot K(\,)(\,)}{(\,)(\,)} = 0$$

$$e_{ss} = \infty$$

Type-1, $K_v = K$.

$$e_{ss} = \frac{1}{K}.$$

Type-2 or higher, $K_v = \infty$

$$e_{ss} = 0.$$

3) acceleration error const.

i/p $= \frac{t^2}{2} u(t)$. , $R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2} \times \frac{1}{1+GH}$$

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

$$e_{ss} = \frac{1}{K_a}$$

Type '0' : $K_a = 0$

$e_{ss} = \infty$

Type '1' : $K_a = 0$

$e_{ss} = \infty$

Type 2 : $K_a = K$

$e_{ss} = \frac{1}{K}$

Type '3' or higher : $K_a = \infty$

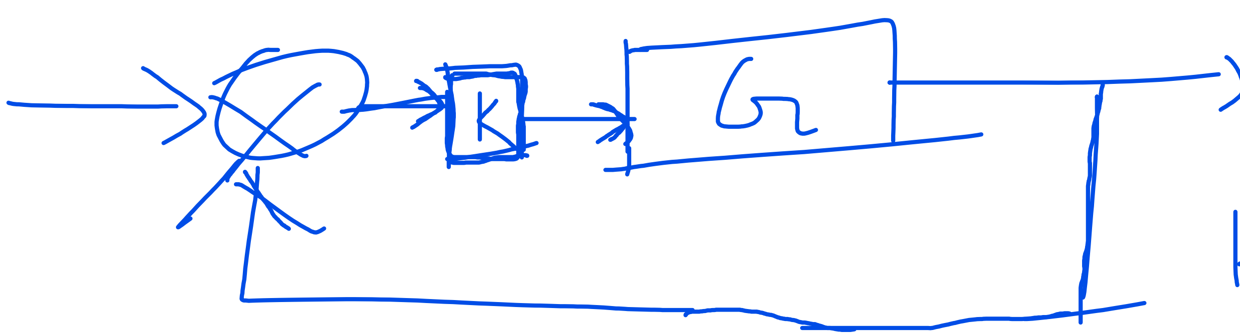
$e_{ss} = 0$

i/p	Type 0		Type 1		Type 2	
	const.	error	const.	error	const.	error
$1/s$	$K_p = K$	$\frac{1}{1+K}$	$K_p = \infty$	0	∞	0
$1/s^2$	$K_v = 0$	∞	$K_v = K$	$1/K$	$K_v = \infty$	0
$1/s^3$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = K$	$\frac{1}{K}$

$$e_{ss} \propto \frac{1}{K}$$

fast & min ss error

Controller



$H(s) = 1$

Proportional controller

$$G(s) = \frac{K}{sT+1}$$

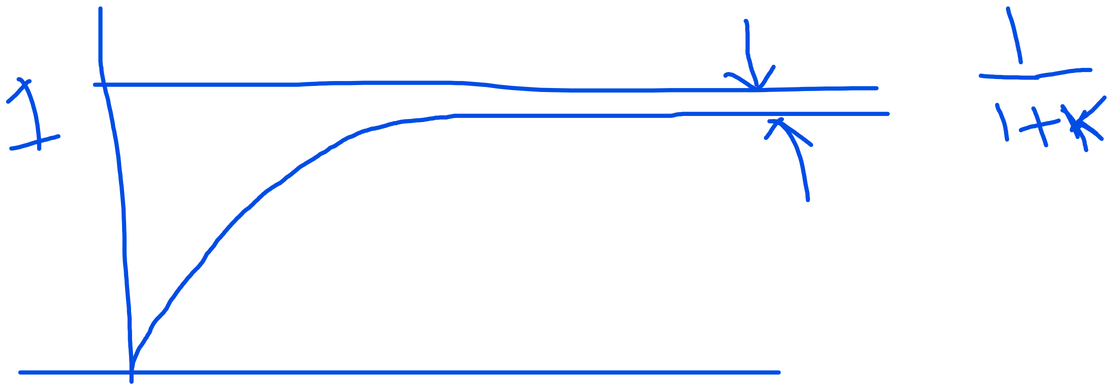
$$e_{ss} = \lim_{s \to 0} s\,E(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$E(s) = \frac{R(s)}{1+G(s)}$$

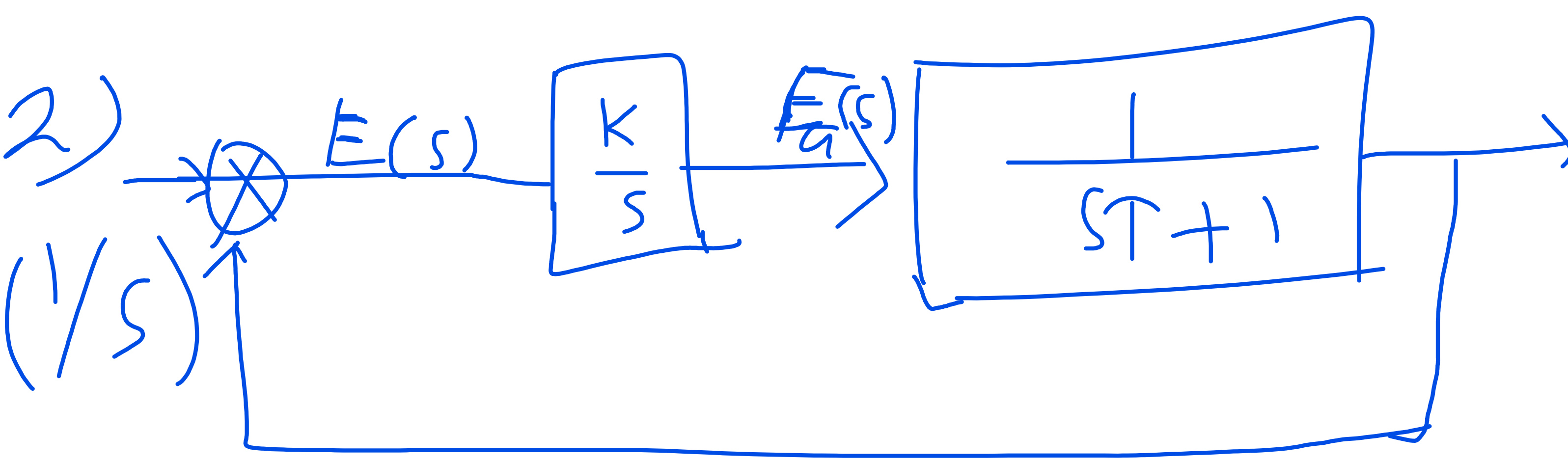
$R = \frac{1}{s}$ | $E(s) = \frac{Ts+1}{Ts+1+K} \times \frac{1}{s}$

$$e_{ss} = \frac{1}{1+K}$$



$M_p\% \uparrow$, $K \uparrow$

e_{ss}



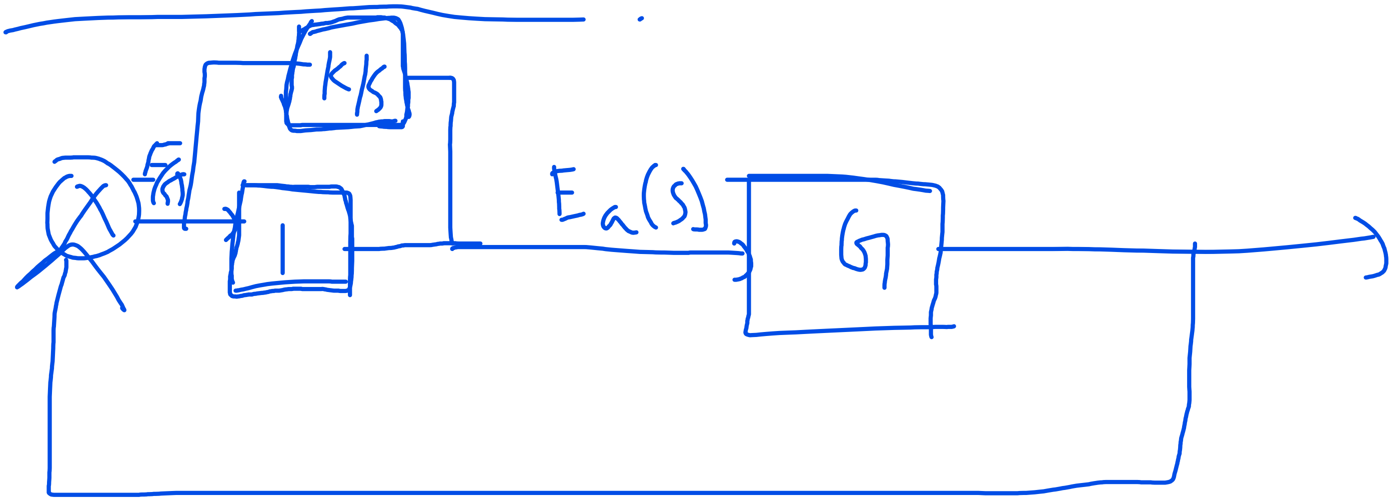
$e_{ss} = 0$ Integral controller

It is used to reduce the ss error:

$$E_a(s) = E(s) + K \cdot \frac{E(s)}{s}$$

$$= E(s)\left[1 + K/s\right].$$

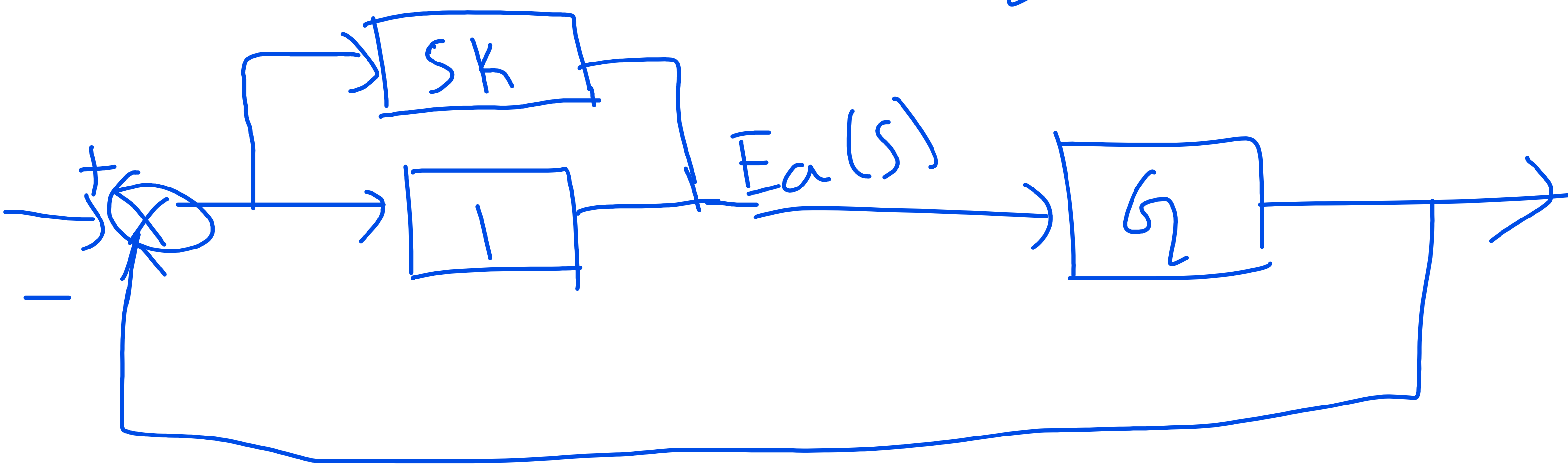
P-I controller



* $1/s^2$, $e_{ss} = 0$

P-D

$$E_a(s) = E(s) + s \cdot K \cdot E(s)$$
$$= E(s)\left[1 + sK\right]$$



P-D Controller

$\xi \uparrow$, $M_p \downarrow$

e_{ss} will not be effected

$C(s)/R(s)$, $s^2 + 2\xi\omega_n s + \omega_n^2$

P-I-D Controller

$$E_a(s) = E(s)\left[1 + T_d s + \frac{K_i}{s}\right]$$