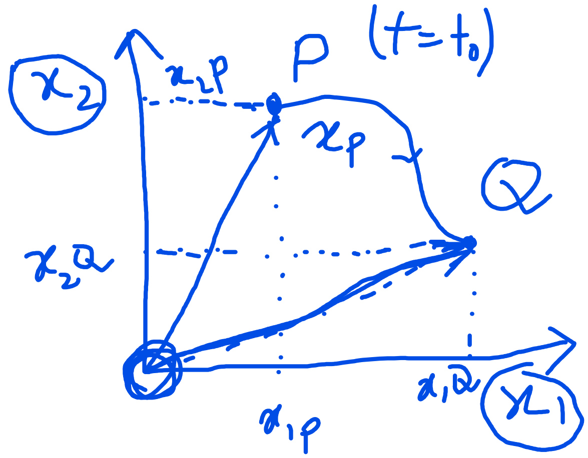
State variable:-

State:-

State vector

state space



$\vec{x_P} = \vec{x_{1P}} + \vec{x_{2P}}$

$(x_1, x_2 \cdots\cdots x_n)$, $t = t_0$

$u(t)$, $t \geq t_0$ $y(t)$

Vivek

Morning	Afternoon
Lazy	Jolly
Procastinating	
Cranky	

$$\frac{\text{Evening}}{\text{Party}} \qquad \frac{\text{Night}}{\text{Romantic}}$$

n^{th} Order D.E.

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y = u(t)$$

$t = t_0$

$y(0)$, $\frac{dy(0)}{dt}$,

let, $y = x_1$

$\dot{y} = x_2$

$\vdots$

$y^{n-1} = x_n$

$$y = x_1$$

n 1st D.F

$$\dot{x}_1 = x_2 \checkmark$$

$$\dot{x}_2 = x_3 \checkmark$$

$$\dot{x}_3 = x_4 \checkmark$$

$$\vdots$$

$$\dot{x}_{n-1} = x_n$$

$$\Rightarrow \dot{x}_n = -a_n x_1 - a_{n-1} x_2 \cdots - a_1 x_n + u$$

$$\frac{d^n y}{dt^n} = -a_1 \frac{d^{n-1} y}{dt^{n-1}} - \cdots - a_n y + u$$

$$= -a_1 x_n - \cdots - a_n x_1 + u$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}; \quad A = \begin{bmatrix} 0 & 1 \cdots & 0 & 0 \\ 0 & 0 \cdots & 0 & 0 \\ 0 & & & \\ \vdots & & & \\ -a_n & -a_{n-1} & \cdots & a_1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\boxed{\dot{x} = Ax + Bu}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$y = Cx$$

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}}$$

$$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = u(t)$$

$t = t_0$, $y(t)$, $\frac{dy(t)}{dt} = 0$

$$\frac{d^2y(t)}{dt^2} = \frac{1}{a}u(t) - \frac{b}{a}\frac{dy(t)}{dt} - \frac{c}{a}y(t)$$

let, $y(t) = x_1$

$$\frac{dy(t)}{dt} = x_2$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{a}u(t) - \frac{b}{a}x_2 - \frac{c}{a}x_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -c/a & -b/a\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1/a\end{bmatrix}u(t)$$

$$\dot{X} = Ax + Bu$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Cx \checkmark$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

A, B, C

A = state Matrix

B = I/P ″

C = O/P ″

* 2nd order D.E. T.F

$$= \frac{1}{s^2 + 2s + 1}$$

$[A \; B \; C \; D] = tf2ss(num, den) \leftarrow$

A =

B =

C =

D = ?

x, u, y

x = state variables | $\dot{x} =$

u = input 〃

y = o/p 〃

linear D.E. i/p having derivative terms

$$y^{n} + a_1 y^{n-1} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 u^{n} + b_1 u^{n-1} + \cdots + b_{n-1}\dot{u} + b_n u$$

Let, $x_1 = y - \beta_0 u$

$$x_2 = \dot{y} - \beta_0 \dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u$$

$$x_3 = \ddot{y} - \beta_0 \ddot{u} - \beta_1 \dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u$$

$\vdots$

$$x_n = y^{n-1} - \beta_0 u^{n-1} - \beta_1 u^{n-2} - \cdots - \beta_{n-1} u$$

$$= \dot{x}_{n-1} - \beta_{n-1} u$$

where,

$$\beta_0 = b_0$$

$$\beta_1 = b_1 - a_1 \beta_0$$

$$\beta_2 = b_2 - a_1 \beta_1 - a_2 \beta_0$$

$$\beta_{n-1} = b_{n-1} - a_1\beta_{n-2} - \cdots - a_{n-2}\beta_1 - a_{n-1}\beta_0$$

$$\Rightarrow \quad \dot{x}_1 = x_2 + \beta_1 u$$

$$\dot{x}_2 = x_3 + \beta_2 u$$

$$\vdots$$

$$\dot{x}_{n-1} = x_n + \beta_{n-1} u$$

$$\dot{x}_n = -a_n x_1 - a_{n-1} x_2 - \cdots - a_1 x_n + \beta_n u$$

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ -a_n & -a_{n-1} & \cdots & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} u$$

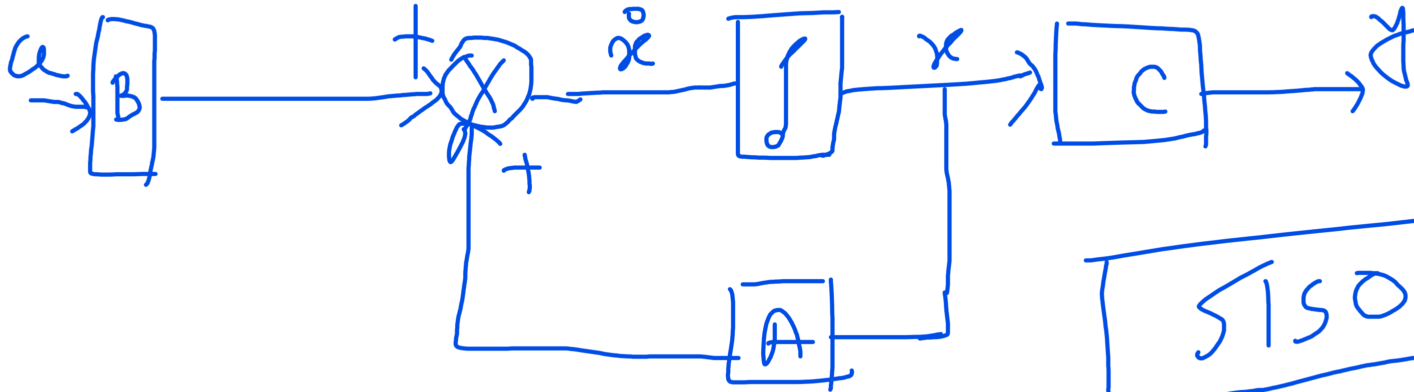
$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \beta_0 u$$

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

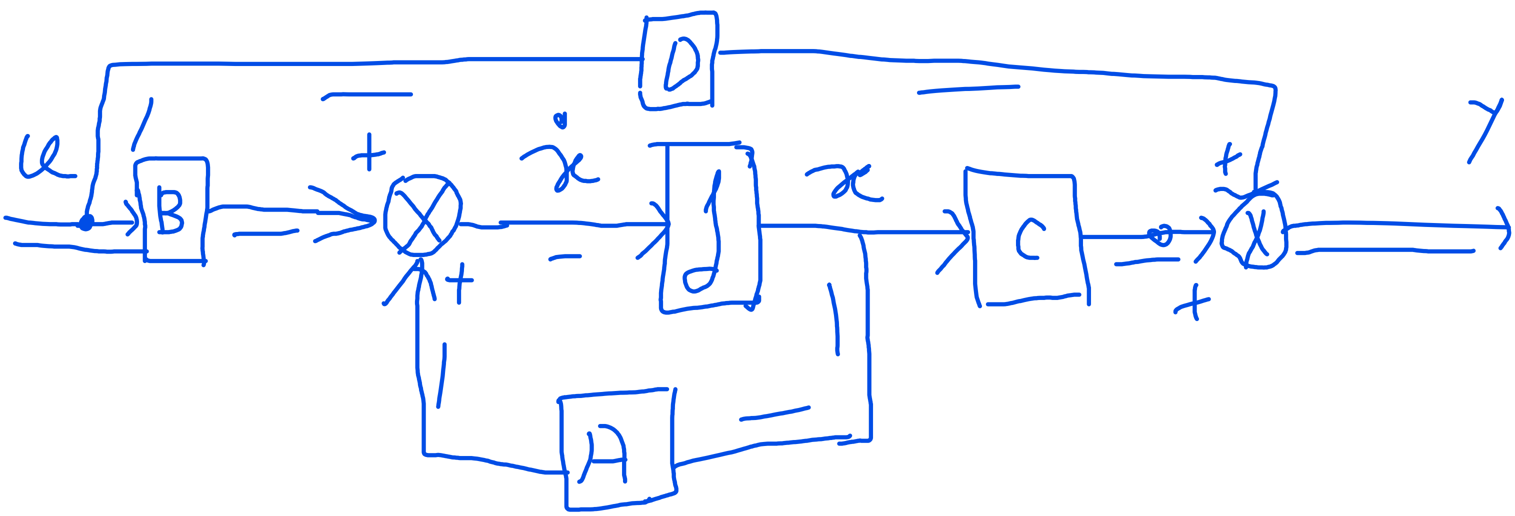
D = transmission matrix

$\dot{x} = Ax + Bu \quad , \quad y = Cx$



$\boxed{\text{SISO}}$

$\dot{X} = Ax + Bu \quad , \quad Y = Cx + Du$ (MIMO)



T.F & S.S eqn

$$G(s) = \frac{Y(s)}{U(s)}$$

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

$\mathcal{L}$

$$sX(s) - x(0) = AX(s) + BU(s)$$

$$\Rightarrow X(s) = (sI - A)^{-1} BU(s)$$

$\mathcal{L}$ $Y(s) = CX(s) + DU(s)$

$$\Rightarrow Y(s) = C(sI - A)^{-1} BU(s) + DU(s)$$

$$\Rightarrow \frac{Y(s)}{U(s)} = \left(C(sI - A)^{-1} B + D\right) = G(s)$$

$$\boxed{G(s) = \frac{Q(s)}{(|sI - A|)}}$$

$|sI - A| =$ char. polynomial of $G(s) = 0$

$\{$ poles $\}$

A matrix

$|SI - A| =$ Eigen Value (λ)

state matrix A

= poles position