Wave

Theory and Numericals

$$x = x_m \cos(\omega t + \phi)$$ Equation of SHM

$$y = A \sin(\omega t - kx)$$ Equation of Wave

A complete understanding of SHM

will help you to understand wave completely.

Q: So what is a wave?

It is a disturbance that travels or propagates without the transport of matter.

Wave motion travels through a medium because of the periodic motion of the particles of the medium about their mean position and at the same time, the disturbance passes over from one particle to the other.

Examples:

- The ripple on a pond
- Visible light
- The sound we hear
- Earthquake (seismic waves)
- tsunami
- Radio signals

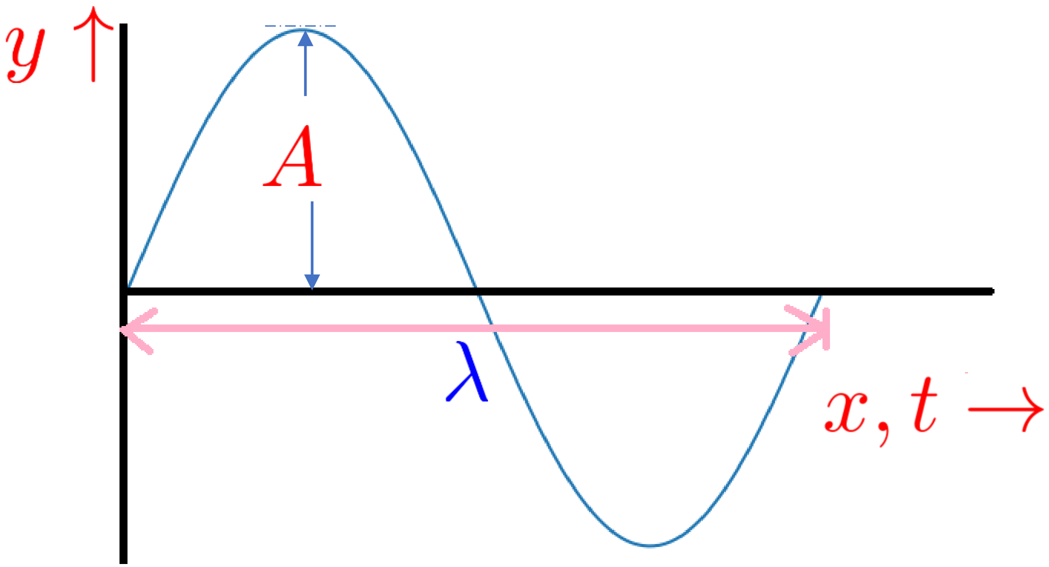
Wave motion is a disturbance produced in a medium by repeated periodic motion of the particles of the medium.

The wave travels forward but the particles in the medium vibrate about their mean positions.

There is a regular phase change between various particles of the medium. This is because the neighboring particles start vibration at different times.

Equation of a plane progressive wave (travelling wave):

$$y = A\sin(\omega t - kx)$$



This represents the equation of a plane progressive wave traveling along the positive X-direction.

where y is the particle displacement

A is the amplitude (maximum particle displacement)

$\omega = 2\pi f$ is the angular frequency

$k = \dfrac{2\pi}{\lambda}$ is the propagation constant or the wave number

Wave velocity or phase velocity (v):

For the wave traveling in the X-direction, the wave velocity or the phase velocity is denoted by v and is given by

$$v = \frac{dx}{dt}$$

Equation of the wave is $y = A\sin(\omega t - kx)$

If $\omega t - kx$ is constant,

$$\frac{d}{dt}(\omega t - kx) = 0$$

Or, $\omega - k\frac{dx}{dt} = 0$

So,

$$\omega - kv = 0$$

$$\boxed{v = \frac{\omega}{k}}$$

Also, $\boxed{v = \lambda f}$

Numerical/Assignment:

(1)

The equation of a transverse wave traveling in a rope is given by $y = 10 \sin \pi (0.01x - 2.00t)$

Where x and y are expressed in cm and t in seconds.

(i) Find the amplitude, frequency, velocity and wavelength of the wave,

(ii) Find the maximum transverse speed of a particle in the rope.

2. Write the equation for a wave traveling in the negative x-direction and having amplitude of 0.01 m, a frequency of 550 Hz and a speed of 330 m/s.

3. A wave of frequency 500 Hz has a phase velocity of 350 m/s. (i) How far apart are two points 60^0 ($\frac{\pi}{3}$ radian) out of phase? (ii) What is the phase difference between two displacements at a certain point at times 10^{-3} sec $apart$?

(Ans: 0.12 m, π radian)

Relation between wave velocity (v) and particle velocity (u):

Equation of a plane progressive (traveling) wave is given by

$$y = A\sin(\omega t - kx) \qquad \text{(1)}$$

This is also the particle displacement

Now the particle velocity is given by

$$u = \frac{dy}{dt} \qquad \text{(2)}$$

Relation between wave velocity (v) and particle velocity (u):

Equation of a plane progressive (traveling) wave is given by

$$y = A\sin(\omega t - kx) \quad \text{———} \quad (1)$$

This is also the particle displacement

Now the particle velocity is given by

$$u = \frac{dy}{dt} \quad \text{———} \quad (2)$$

$$u = \frac{d}{dt}[A\sin(\omega t - kx)]$$

$$= A\cos(\omega t - kx)\omega$$

$$= A\omega\cos(\omega t - kx) \quad \text{———} \quad (3)$$

Since $v = \frac{\omega}{k}$

$\Rightarrow \omega = vk$

So, equation (2) becomes

$$u = (Ak)v\cos(\omega t - kx) \quad \text{———} \quad (4)$$

Again,

The slope is given by

$$\frac{dy}{dx} = \frac{d}{dx}[A\sin(\omega t - kx)]$$

$$= A\cos(\omega t - kx)(-k)$$

$$= (-Ak)\cos(\omega t - kx) \quad \text{———} \quad (5)$$

Equation (4) can be written as

$$u = (-v)(-Ak)\cos(\omega t - kx) \quad \text{———} \quad (6)$$

From (5) & (6)

$$\boxed{u = (-v)\frac{dy}{dx}}$$

$$\boxed{u = (-v)\frac{dy}{dx}}$$

Or, $\frac{dy}{dt} = (-v)\frac{dy}{dx}$

i.e. Particle velocity at a point = (-Wave velocity) x (slope of displacement curve at that point)

This is the required relation between the particle velocity(u) and the wave velocity(v)

Differential equation of wave:

Recall:

Differential equation of SHM:

?

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

It's a two-way traffic

$$x = x_m \cos(\omega t + \phi)$$

Goal is to arrive at

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

Starting from

$$y = A\sin(\omega t - kx)$$

Now, let's start

$$y = A\sin(\omega t - kx)$$

$$u = \frac{dy}{dt}$$

$$= A\omega\cos(\omega t - kx)$$

Again, differentiating w.r.t. 't'

$$a = \frac{d^2y}{dt^2} = \frac{du}{dt}$$

$$a = \frac{d}{dt}[A\cos(\omega t - kx)]$$

$$= A\omega[-\sin(\omega t - k)].\omega$$

$$= -A\omega^2\sin(\omega t - kx)$$

$$= -Ak^2v^2\sin(\omega t - kx)$$

$$= v^2(-Ak^2)\sin(\omega t - kx)$$

$$= v^2[-Ak^2\sin(\omega t - kx)] \quad \text{—— (i)}$$

Again, differentiating y twice w.r.t. 'x'

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\frac{dy}{dx})$$

$$= \frac{d}{dx}[-Ak\cos(\omega t - kx)]$$

$$= (-Ak)[-\sin(\omega t - kx)](-k)$$

$$= -Ak^2\sin(\omega t - kx) \text{ ——— (ii)}$$

Again, differentiating w.r.t. 't'

$$a = \frac{d^2y}{dt^2} = \frac{du}{dt}$$

$$a = \frac{d}{dt}[A\cos(\omega t - kx)]$$

$$= A\omega[-\sin(\omega t - k)].\omega$$

$$= -A\omega^2\sin(\omega t - kx)$$

$$= -Ak^2v^2\sin(\omega t - kx)$$

$$= v^2(-Ak^2)\sin(\omega t - kx)$$

$$= v^2[-Ak^2\sin(\omega t - kx)] \text{ ——— (i)}$$

Previous Slide

From (i) & (ii)

$$a = v^2\frac{d^2y}{dx^2}$$

i.e. $\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$

This is the required differential equation of Wave i.e. $$\boxed{\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}}$$

This is the required differential equation of Wave

This tells that

Particle acceleration at a point

= Square of the wave velocity times

Curvature of the displacement curve at that point at that time

Energy, **intensity** And **Power** of a **Wave**:

The energy carried by a wave is, in fact, due to the vibrating particles of the medium. Since the particles are vibrating in SHM, the energy of a single particle is given by

$$E = K.E. + P.E.,$$
$$= \frac{1}{2}mu^2 + \frac{1}{2}Ky^2$$

where $y = A\sin(\omega t - kx)$ is the particle displacement

$$u = \frac{dy}{dt} = \frac{d}{dt}[A\sin(\omega t - kx)] = \omega A\cos(\omega t - kx)$$

is the particle velocity

Energy, intensity And Power of a Wave:

The energy carried by a wave is, in fact, due to the vibrating particles of the medium. Since the particles are vibrating in SHM, the energy of a single particle is given by

$$E = K.E. + P.E.,$$
$$= \frac{1}{2}mu^2 + \frac{1}{2}Ky^2$$

where $y = A\sin(\omega t - kx)$ is the particle displacement

$$u = \frac{dy}{dt} = \frac{d}{dt}[A\sin(\omega t - kx)] = \omega A\cos(\omega t - kx)$$

is the particle velocity

Note: K is force constant (as in SHM) and k is propagation in wave

So $E = \frac{1}{2}m\omega^2 A^2\cos^2(\omega t - kx)$ + $\frac{1}{2}m\omega^2 A^2\sin^2(\omega t - kx)$

$$\boxed{E = \frac{1}{2}m\omega^2 A^2}$$

Let n be the number of particles per unit volume of the medium, then

Energy per unit volume,

$$U = nE$$
$$= \frac{1}{2} m n \omega^2 A^2$$

Recall:

$$\boxed{E = \frac{1}{2} m \omega^2 A^2}$$

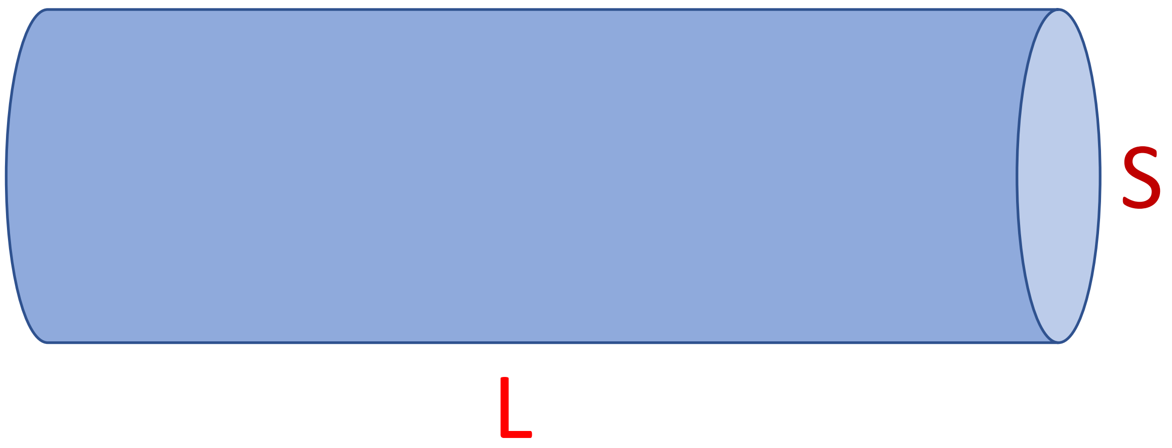
But $mn = \rho$ (Density of the medium)

So,

$$\boxed{U = \frac{1}{2} \rho \omega^2 A^2}$$

→ Energy per unit volume

Consider a cylinder of cross-section S and length L.



Then, volume of the cylinder

= S L

Now, the total energy of the particles within the volume of the cylinder is

$$W = UV = \frac{1}{2}\rho\omega^2 A^2 S.L$$

Now, the intensity of the wave is defined as the energy carried per unit area per unit time.

It is denoted by I and given as

$$I = \frac{Energy}{Area.time}$$

$$I = \frac{W}{S.t}$$

$$I = \frac{1}{2}\frac{\rho\omega^2 A^2 S.L}{S.t}$$

$$I = \frac{1}{2}\rho\omega^2 A^2 . \frac{L}{t}$$

$$\boxed{I = \frac{1}{2}\rho v\omega^2 A^2}$$

where $v = \frac{L}{t}$

is the wave velocity

Hence the required expression for the **intensity** of the **wave** is given by

$$\boxed{I = \frac{1}{2}\rho v \omega^2 A^2}$$

Note:

$$I \propto \rho$$

$$I \propto v$$

$$I \propto \omega^2$$

$$I \propto A^2$$

For a plane progressive wave,

Amplitude is constant,

Hence the intensity is constant

Numerical:

1. A source of sound has a frequency 2.56 Hz and an amplitude of 0.25 cm. Calculate the flow of energy across a square centimeter in one second, if the velocity of sound in air is 340 m/s and the density of air is 0.00129 gm/c.c.

(Ans: 0.354 W/sq m)

2. Calculate the minimum intensity of audibility in watts per square cm from a note of 1000 Hz if the amplitude of vibration is 10^{-9} cm. Given, density of air is 0.0013 gm/cc and velocity of sound in air is 340 m/s. (Ans: 8.715×10^{-13} W/m^2)

(3) Spherical waves are emitted from a 1 W source in an isotropic non-absorbing medium. What is the wave intensity 1m from the source? (Ans: 0.0795 W/m^2)

(4) Calculate the amplitude of vibration of air particle in a plane progressive wave of frequency 500 Hz and intensity 10^-14 W/cm^2 , velocity of sound in air is 330 m/s and density of air is 0.001293 gm/c.c.(Ans: 2.18x10^-10 m)

Theory(contd….)

Hence the required expression for the **intensity** of the **wave** is given by

$$\boxed{I = \frac{1}{2}\rho v \omega^2 A^2}$$

For a plane progressive wave,

Amplitude is constant,

Hence the intensity is constant

Note:

$$I \propto \rho$$

$$I \propto v$$

$$I \propto \omega^2$$

$$I \propto A^2$$

created by _Arun Devkota 21

Previous Slide

For a spherical wave,

intensity at a distance r from the source is given by

$$\boxed{I = \frac{P}{4\pi r^2}}$$

Also for a plane progressive wave,

$$\boxed{I = \frac{1}{2}\rho v \omega^2 A^2}$$

Combining these two, we get

$$\frac{1}{2}\rho v \omega^2 A^2 = \frac{P}{4\pi r^2}$$

$$\boxed{A = \frac{1}{\omega r}\sqrt{\frac{P}{2\pi \rho v}}}$$

For a spherical wave,

intensity at a distance r from the source is given by

$$I = \frac{P}{4\pi r^2}$$

Also for a plane progressive wave,

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

Combining these two, we get

$$\frac{1}{2}\rho v \omega^2 A^2 = \frac{P}{4\pi r^2}$$

$$A = \frac{1}{\omega r}\sqrt{\frac{P}{2\pi \rho v}}$$

$$A \propto \frac{1}{r}$$

$$I \propto \frac{1}{r^2}$$

For a spherical wave

Recall:

For a plane progressive wave,

$$A = constant$$

$$I = constant$$

Hence the required expression for the **intensity** of the **wave** is given by

$$I = \frac{1}{2}\rho v \omega^2 A^2$$

Note:

$I \propto \rho$

$I \propto v$

$I \propto \omega^2$

$I \propto A^2$

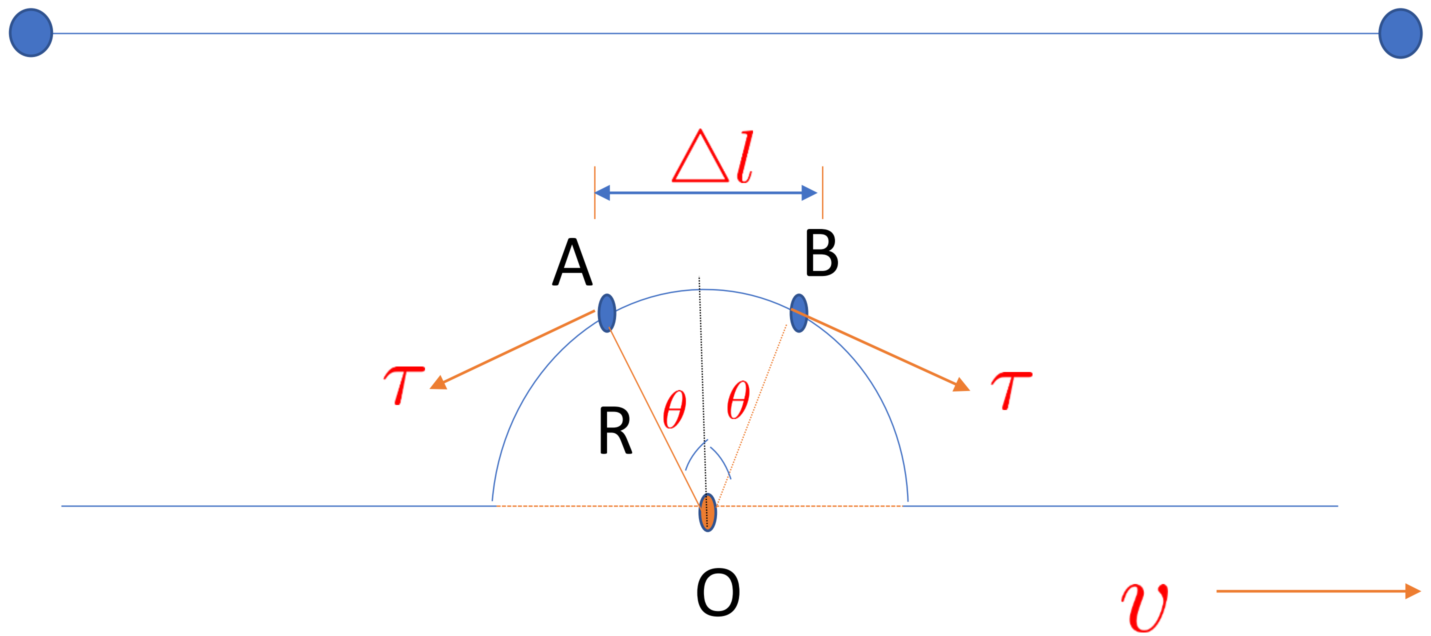
For a plane progressive wave,

Amplitude is constant,

Hence the intensity is constant

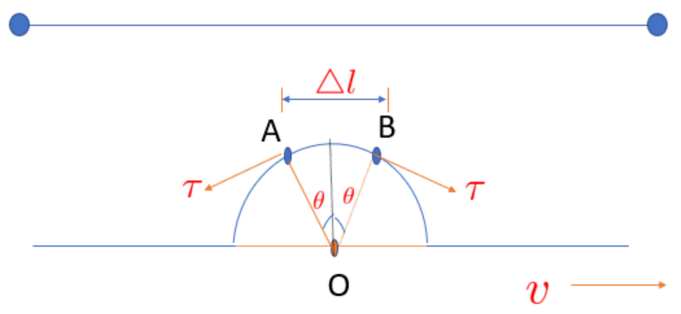
Previous slide

Velocity or speed of wave generated along a stretched string:



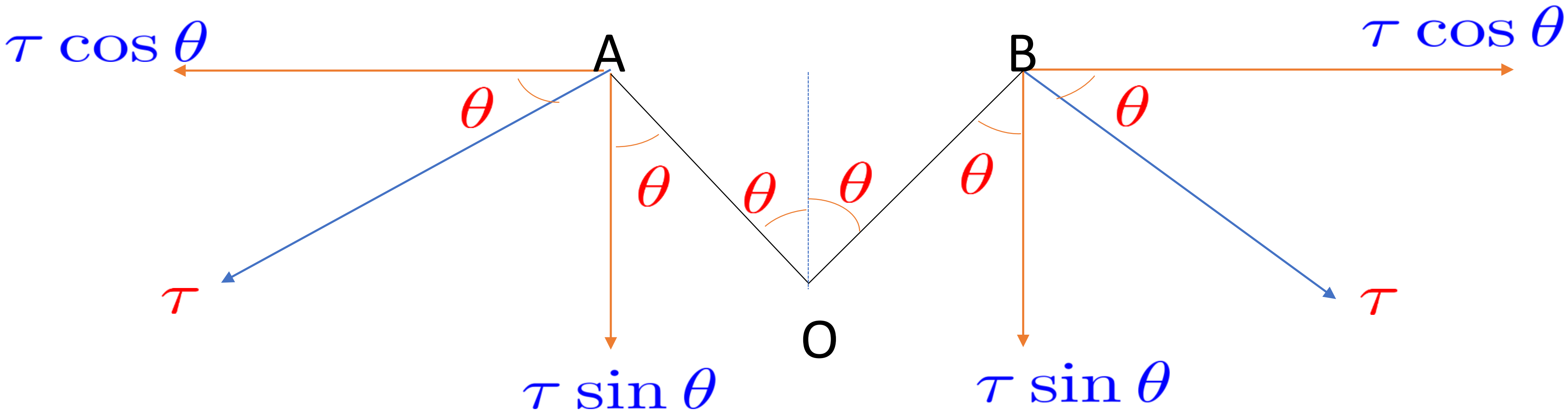
Let a wave or a pulse be generated along a stretched string of linear mass density μ. The wave travels from left to right, as shown in the figure.

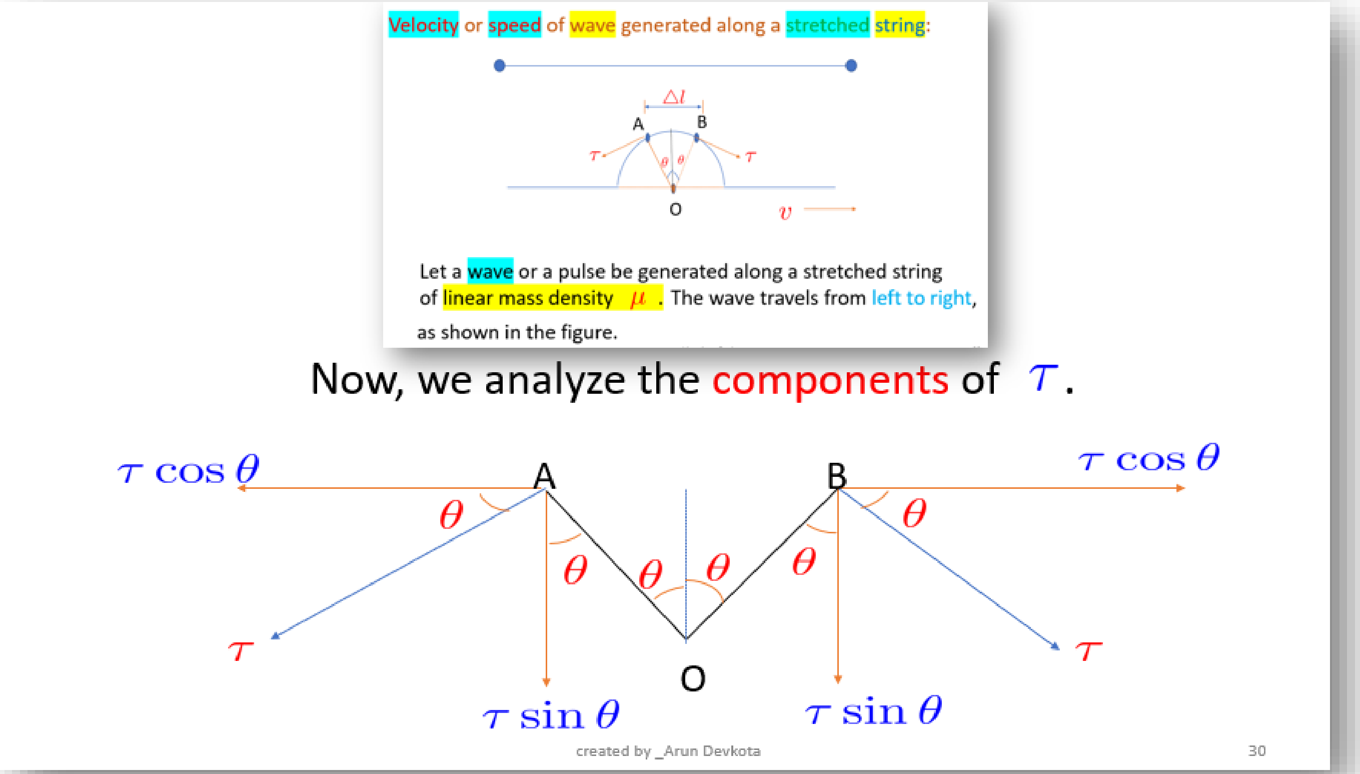
Velocity or speed of wave generated along a stretched string:



Let a wave or a pulse be generated along a stretched string of linear mass density μ . The wave travels from left to right, as shown in the figure.

Now, we analyze the components of τ.





From figure, we see that the horizontal components of tension τ cancel each other, while the vertical components add up to provide the necessary centripetal force, so as to sustain the circular motion of arc AB.

Then,

$$\tau \sin\theta + \tau \sin\theta = \frac{(\Delta m)v^2}{R}$$

where $\Delta m = \mu \Delta l$

is the mass of the string element of AB.

$$\tau.2\sin\theta = \frac{(\mu\Delta l)v^2}{R}$$

For small angle θ,

$$\sin\theta \approx \theta$$

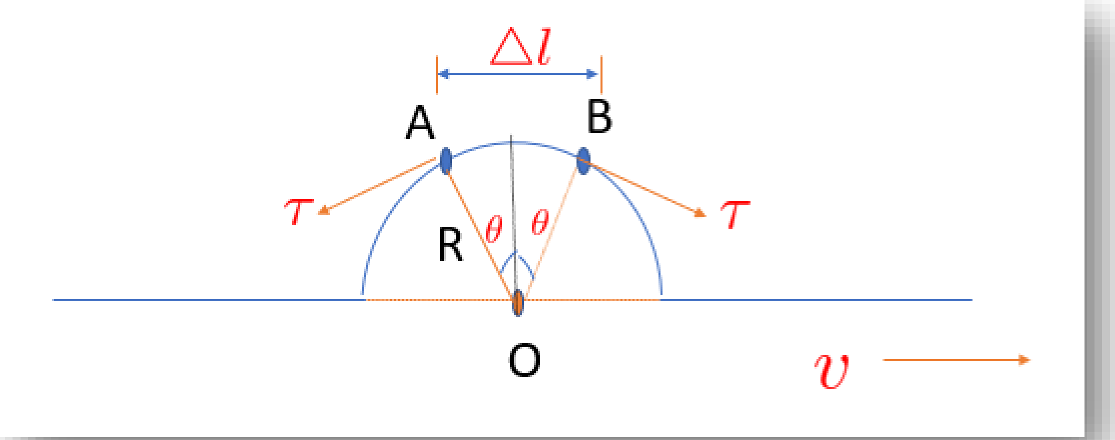
So the previous equation

$$\tau.2\sin\theta = \frac{(\mu\triangle l)v^2}{R}$$

becomes

$$\tau.2\theta = \frac{(\mu\triangle l)v^2}{R}$$

Here, $2\theta = \frac{\triangle l}{R}$



So, $\tau.\frac{\triangle l}{R} = \frac{(\mu\triangle l)v^2}{R}$

$$\tau = \mu v^2$$

$$\boxed{v = \sqrt{\frac{\tau}{\mu}}}$$

V. Important

Rate of transfer of energy along a stretched string:

The kinetic energy associated with an string element of length dx is given by

$$dK = \frac{1}{2}(dm)u^2$$

where dm = mass of the string element of length dx

u = particle velocity associated with the string

Since $u = \frac{dy}{dt}$ where $y = A\sin(\omega t - kx)$ is particle displacement

Rate of transfer of energy along a stretched string:

The kinetic energy associated with an string element of length dx is given by

$$dK = \frac{1}{2}(dm)u^2$$

where dm = mass of the string element of length dx

u = particle velocity associated with the string

Since $u = \frac{dy}{dt}$ where $y = A\sin(\omega t - kx)$ is particle displacement

Then, $u = \omega A \cos(\omega t - kx)$

$$dK = \frac{1}{2}\mu dx \omega^2 A^2 \cos^2(\omega t - kx)$$

[Here, we have used $dm = \mu dx$]

μ = linear mass density

So,

The (time)rate of change of kinetic energy,

$$\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 A^2 \cos^2(\omega t - kx)$$

where, $v = \frac{dx}{dt}$

is the wave or phase velocity

Rate of transfer of energy along a stretched string:

The kinetic energy associated with an string element of length dx is given by

$$dK = \frac{1}{2}(dm)u^2$$

where dm = mass of the string element of length dx

u = particle velocity associated with the string

Since $u = \frac{dy}{dt}$ where $y = A\sin(\omega t - kx)$ is particle displacement

Then, $u = \omega A\cos(\omega t - kx)$

$$dK = \frac{1}{2}\mu dx\omega^2 A^2\cos^2(\omega t - kx)$$

[Here, we have used $dm = \mu dx$]

μ = linear mass density

So,

The (time)rate of change of kinetic energy,

$$\frac{dK}{dt} = \frac{1}{2}\mu v\omega^2 A^2\cos^2(\omega t - kx)$$

where, $v = \frac{dx}{dt}$

is the wave or phase velocity

Now the average rate of transfer of kinetic energy is given by,

$$[\frac{dK}{dt}]_{avg} = \frac{1}{2}\mu v\omega^2 A^2[\cos^2(\omega t - kx)]_{avg}$$

But

$$[\cos^2(\omega t - kx)]_{avg} = \frac{1}{2}$$

So, $[\frac{dK}{dt}]_{avg} = \frac{1}{4}\mu v\omega^2 A^2$

Again, the average rate of transfer of potential energy is given by

$$[\frac{dU}{dt}]_{avg} = \frac{1}{2}\mu v\omega^2 A^2[\sin^2(\omega t - kx)]_{avg}$$

$$[\frac{dU}{dt}]_{avg} = \frac{1}{4}\mu v\omega^2 A^2$$

Rate of transfer of energy along a stretched string:

The kinetic energy associated with an string element of length dx is given by

$$dK = \frac{1}{2}(dm)u^2$$

where dm = mass of the string element of length dx

u = particle velocity associated with the string

Since $u = \frac{dy}{dt}$ where $y = A\sin(\omega t - kx)$ is particle displacement

Then, $u = \omega A\cos(\omega t - kx)$

$$dK = \frac{1}{2}\mu dx \omega^2 A^2 \cos^2(\omega t - kx)$$

[Here, we have used $dm = \mu dx$]

μ = linear mass density

So,

The (time)rate of change of kinetic energy,

$$\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 A^2 \cos^2(\omega t - kx)$$

where, $v = \frac{dx}{dt}$

is the wave or phase velocity

Now the average rate of transfer of kinetic energy is given by,

$$[\frac{dK}{dt}]_{avg} = \frac{1}{2}\mu v \omega^2 A^2 [\cos^2(\omega t - kx)]_{avg}$$

But

$$[\cos^2(\omega t - kx)]_{avg} = \frac{1}{2}$$

So, $[\frac{dK}{dt}]_{avg} = \frac{1}{4}\mu v \omega^2 A^2$

Again, the average rate of transfer of potential energy is given by

$$[\frac{dU}{dt}]_{avg} = \frac{1}{2}\mu v \omega^2 A^2 [\sin^2(\omega t - kx)]_{avg}$$

$$[\frac{dU}{dt}]_{avg} = \frac{1}{4}\mu v \omega^2 A^2$$

Hence the average rate of transfer of total energy is given by the sum of the average rate of transfer of Kinetic & Potential energies i.e. $[\frac{dE}{dt}]_{avg} = \frac{1}{4}\mu v \omega^2 A^2 + \frac{1}{4}\mu v \omega^2 A^2$

$$\boxed{[\frac{dE}{dt}]_{avg} = \frac{1}{2}\mu v \omega^2 A^2}$$

or, $\boxed{P_{avg} = \frac{1}{2}\mu v \omega^2 A^2}$

Stationary or Standing wave:

The wave limited in a certain region is called standing or stationary wave.

It results due to the superposition of two waves traveling in the opposite directions and with the same amplitude and same frequency.

It is produced due to the interaction of a plane progressive wave and its reflection or reflected wave.

Mathematical treatment:

Let the two waves traveling in the opposite directions be represented by

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

Then the resultant wave is given by the principle of superposition

$$y_1 = A\sin(kx - \omega t)$$

$$y_2 = A\sin(kx + \omega t)$$

$$y = y_1 + y_2$$

$$= A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$$= A[2\sin(\frac{kx - \omega t + kx + \omega t}{2})cos(\frac{kx - \omega t - kx - \omega t}{2})]$$

$$= A[2\sin kx \cos \omega t]$$

$$\boxed{y = (2A\sin kx)\cos \omega t}$$

This represents the standing wave, not the progressive wave.

Then the resultant wave is given by the principle of superposition

$y_1 = A\sin(kx - \omega t)$

$y_2 = A\sin(kx + \omega t)$

$$y = y_1 + y_2$$

$$= A[\sin(kx - \omega t) + \sin(kx + \omega t)]$$

$$= A[2\sin(\frac{kx - \omega t + kx + \omega t}{2})cos(\frac{kx - \omega t - kx - \omega t}{2})]$$

$$= A[2\sin kx \cos \omega t]$$

$$\boxed{y = (2A\sin kx)\cos \omega t}$$

This represents the standing wave, not the progressive wave.

Let $|2A\sin kx| = y_m$

(the amplitude of the standing wave)

Then, $\boxed{y = y_m \cos \omega t}$

Cases:

(i) For y to be maximum,

$$\sin kx = 1 = maximum$$

$$\Rightarrow \sin kx = 1 = \sin(n + \frac{1}{2})\pi \qquad [n = 0, 1, 2, ...]$$

$$\Rightarrow kx = (n + \frac{1}{2})\pi$$

$$\Rightarrow \quad kx = (n + \frac{1}{2})\pi$$

$$\Rightarrow \quad x = (n + \frac{1}{2})\frac{\pi}{k}$$

$$\Rightarrow \quad x = (n + \frac{1}{2})\frac{\pi}{\frac{2\pi}{\lambda}}$$

$$\Rightarrow \quad \boxed{x = (n + \frac{1}{2})\frac{\lambda}{2}}$$

where $n = 0, 1, 2, ...$

The values of x give the positions of antinodes (maximum displacement)

If $n = 0, x_0 = \frac{\lambda}{4}$

If $n = 1, x_1 = \frac{3\lambda}{4}$

If $n = 2, x_2 = \frac{5\lambda}{4}$ & so on

Therefore, the separation between the consecutive antinodes

$$= x_1 - x_0 = x_2 - x_1 = \frac{\lambda}{2}$$

Hence the consecutive antinodes are separated by $\frac{\lambda}{2}$.

(ii) For y to be minimum,

$$y_m = 0$$

$$\Rightarrow \sin kx = 0$$

$$\Rightarrow \sin kx = 0 = \sin n\pi$$

where $n = 0, 1, 2, \ldots$

$$\Rightarrow kx = n\pi$$

$$\Rightarrow x = \frac{n\pi}{k}$$

$$\Rightarrow x = \frac{n\pi}{\frac{2\pi}{\lambda}}$$

$$|2A \sin kx| = y_m$$

$$\Rightarrow \boxed{x = \frac{n\lambda}{2}}$$

$$\boxed{y = y_m \cos \omega t}$$

The values of x give the positions of nodes or minimum displacement.

For $n = 0, x_0 = 0$

For $n = 1, x_1 = \frac{\lambda}{2}$

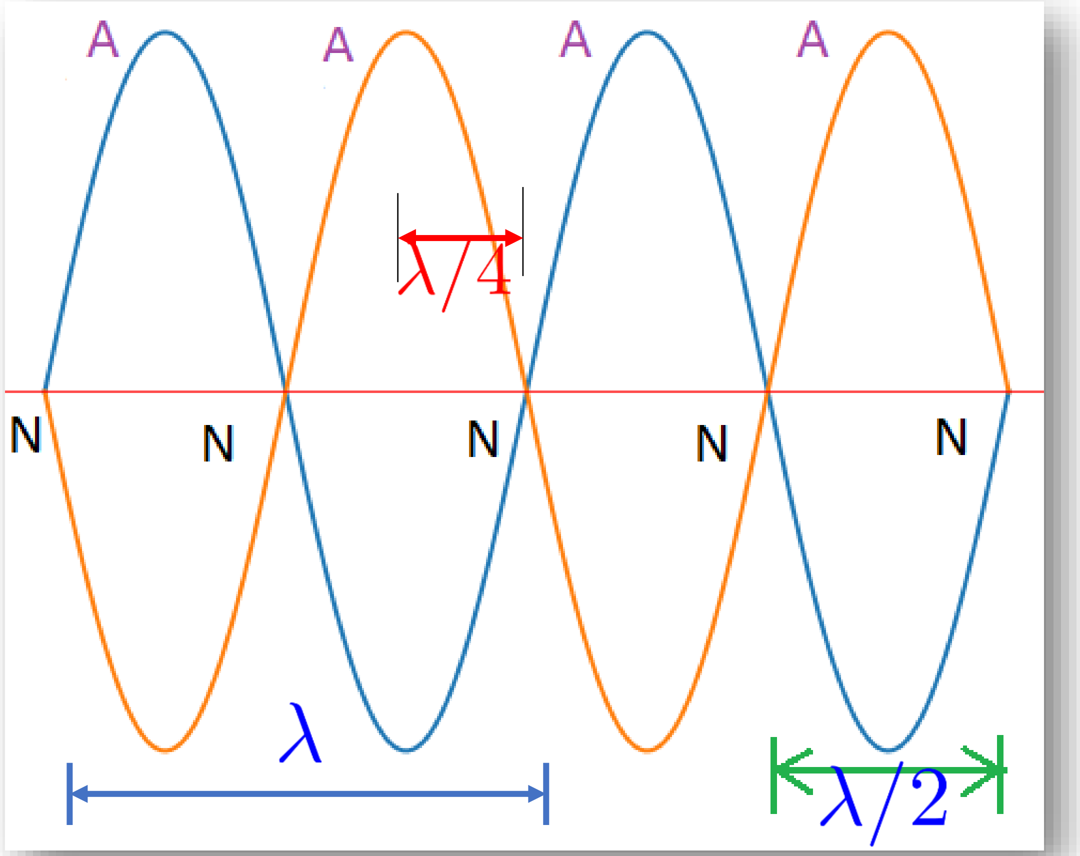
For $n = 2, x_2 = \lambda$

& so on.

Therefore, the separation between two consecutive nodes

$$= x_1 - x_o = x_2 - x_1 = \frac{\lambda}{2}$$

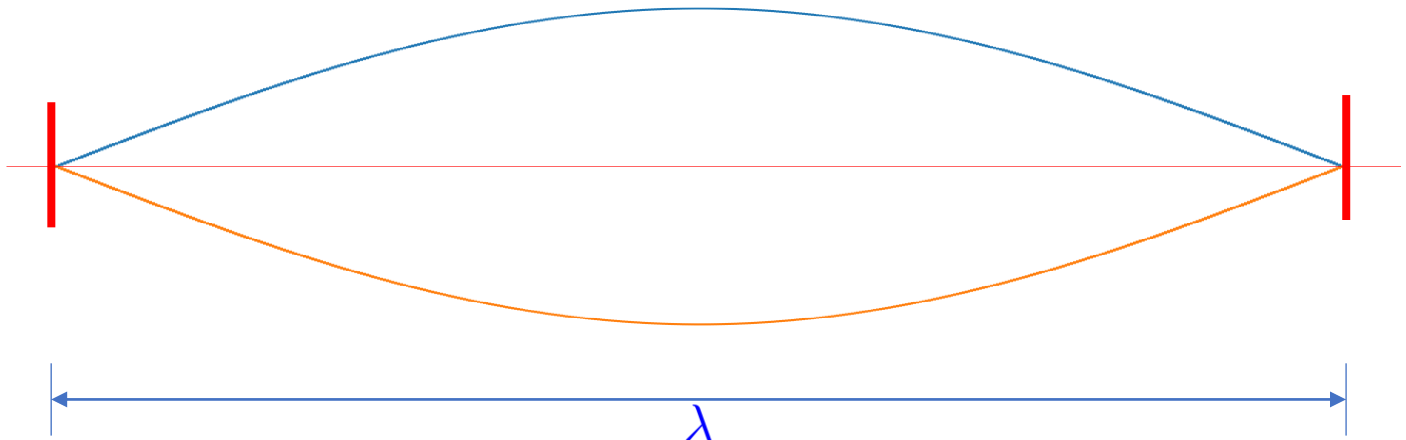
Hence, nodes and antinodes are separated by a distance of half wavelength.



Resonance:

- Resonance occurs due to the continuous reflections of wave within the limited region.
- There are certain frequencies which are discussed below.

1. Fundamental frequency or Frequency of First Harmonic

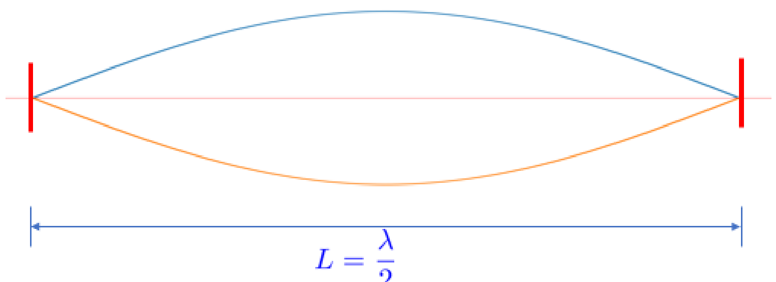


$$L = \frac{\lambda}{2}$$

Resonance:

- Resonance occurs due to the continuous reflections of wave within the limited region.
- There are certain frequencies which are discussed below.

1. Fundamental frequency or Frequency of First Harmonic



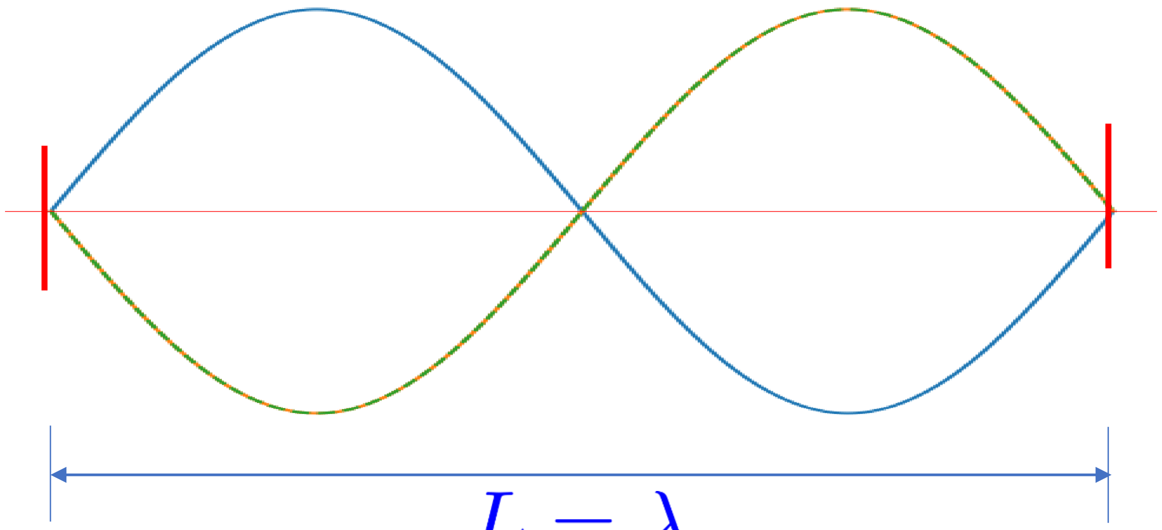
created by _Arun Devkota

The frequency in this mode of vibration is given by

$$f_0 = \frac{v}{\lambda} = \frac{v}{2L}$$

Also, $v = \sqrt{\frac{T}{\mu}} \Rightarrow f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

(ii) Frequency of Second Harmonic or 1st overtone:



$$L = \lambda$$

The frequency in this mode is given by

$$f_1 = \frac{v}{\lambda} = \frac{v}{L} = 2(\frac{v}{2L}) = 2f_0$$

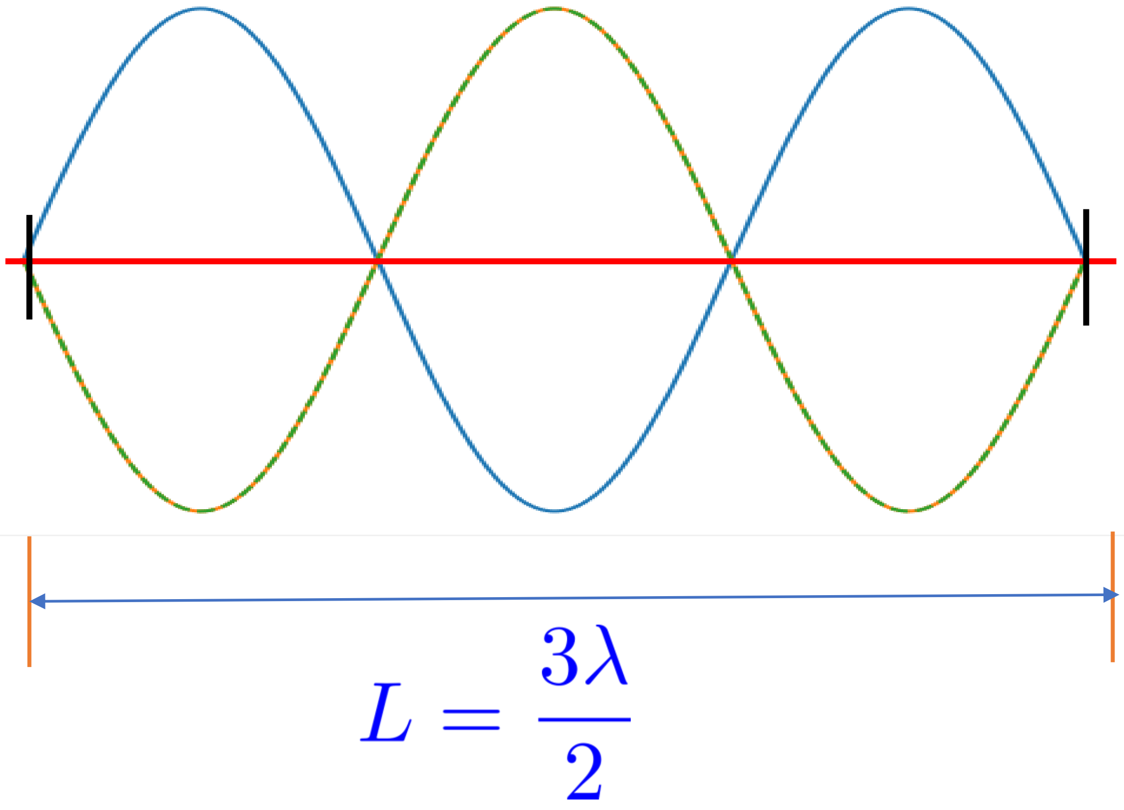
(iii) Frequency of 3rd Harmonic or 2nd overtone:

The frequency in this mode is given by

$$f_2 = \frac{v}{\lambda} = 3(\frac{v}{2L}) = 3f_0$$

In general, $\boxed{f_n = (n+1)f_0}$

where $n = 1, 2, 3, \ldots$ are overtones



Hence, both odd and even harmonics are present.

Q. 1. What is a wave?

Q. 2. What are the types of waves?

Q.3. What are Mechanical, Transverse, Longitudinal, Electromagnetic and de-Broglie (Matter) Wave?

Q.4. What is resonance? What are the conditions of resonance?

Numericals:

1. A sinusoidal wave travels along a stretched string. The time for a particular point to move from maximum displacement to zero is 0.17 s. What are the (a) period and (b) frequency? The wavelength is 1.40m. What is the wave speed?

(Ans: 0.68 s, 1.47 Hz, 2.06 m/s)

2. A stretched string has a linear density 525 g/m and is under tension 45 N. We send a sinusoidal wave with frequency 120 Hz and amplitude 8.5 mm along the string from one end. At what average rate does the wave transport energy?

Ans: 100 W

3. A transverse sinusoidal wave is generated at one end of a long horizontal spring by a bar which moves the end up and down through a distance 15 cm. The motion is continuous and is repeated regularly twice each second. Find the speed, amplitude and wavelength of the wave if the spring has a linear density of 0.45 X 10^-4 kg/m and is kept under a tension of 2 N.

4. A stretched string has a mass per unit length of 5.0 g/cm and a tension of 10 N. A sinusoidal wave on this string has an amplitude of 0.12 mm and a frequency of 100 Hz and is traveling in the –ve x direction. Write an equation for this wave. [Ans: $y(x,t) = 0.12 \sin (141x+628t)$, with y in mm, x in m and t in s]

(5) Here are the three equations of a wave:

$y(x,t) = 2 \sin(4x - 2t)$, $y(x,t) = \sin(3x - 4t)$, $y(x,t) = 2 \sin(3x - 3t)$.

Rank the waves according to their (a) wave speed and (b) maximum transverse speed, greatest first.

6. Show that for a wave on string, the kinetic energy per unit length of string is K.E. = T/2, where T is the tension on the string.

Soln:

Since K.E. $= \frac{1}{2}mv^2$

K.E. per unit length $= \frac{1}{2}\frac{m}{l}v^2$ μ

$= \frac{1}{2}\mu.\frac{T}{\mu}$

$= \frac{T}{2}$

7. The speed of a transverse wave on a string is 170 m/s when the string tension is 120 N. To what value must the tension be changed to raise the wave speed to 180 m/s?

Ans: 135 N

8. Show that maximum angular frequency occurs when

$$x = \frac{l}{\sqrt{12}}$$

9. A block is in SHM on the end of a spring, with position given by $x = x_m \cos(\omega t + \phi)$. If $\phi = \frac{\pi}{5}$ rad, then at t = 0, what percentage of the total energy is P.E.? (Ans: 65.5%)