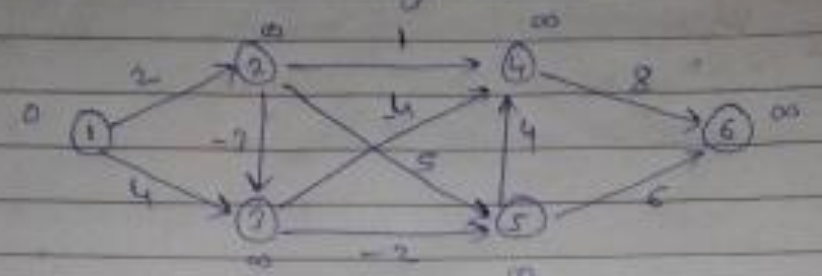
Tutorial - 7

Q. find the shortest path from node 1 to every other node in the graph of fig using the Bellman ford Algorithm.

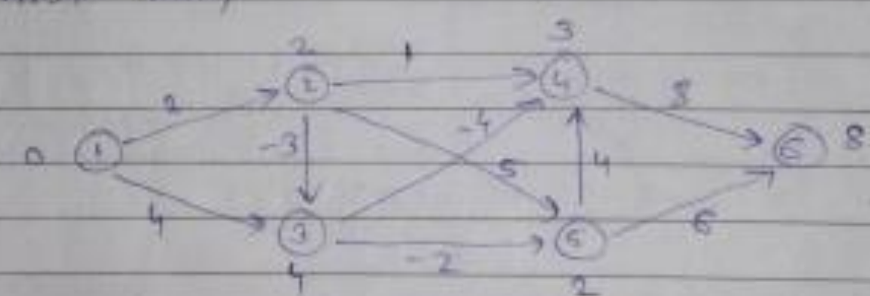


→ In DP, we try all posible solution and pick up the best solution.

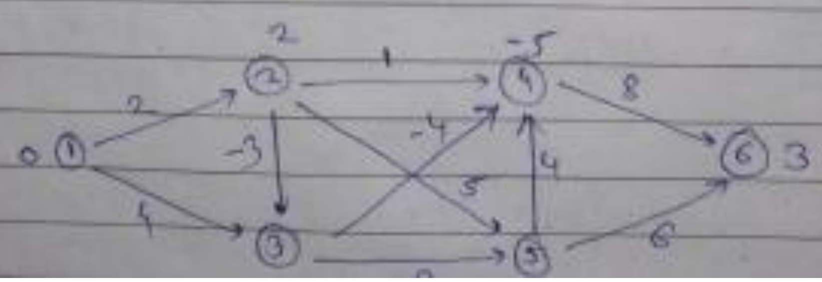
we repeat the relaxation process for $(|V|-1)$ times

$|V| = 6$ ∴ repeatation $= 6 - 1 = 5$ times.

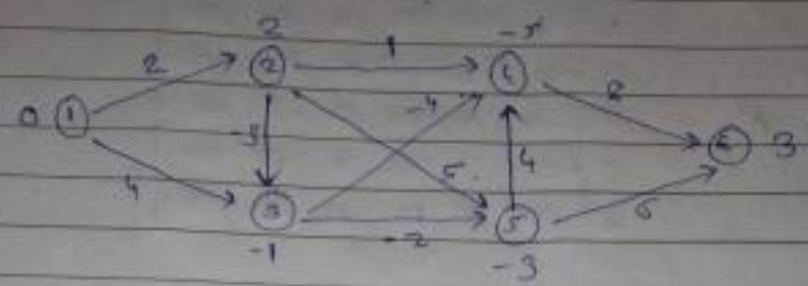
first time,



second time,



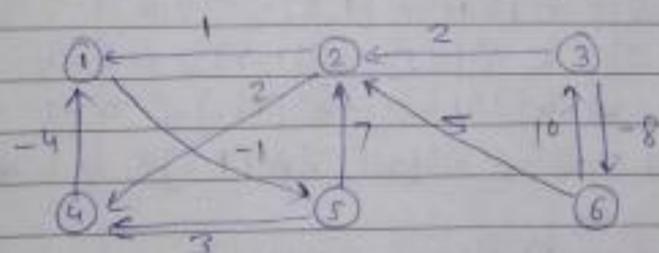
third time,



here we can see that after second step, values are not changing so this is our final solution.

distance	0	2	-1	-5	-3	3
node	1	2	3	4	5	6

Q. find the shortest path using floyd-warshall algo.



→ here we have to find shortest distance from every vertex to other vertex.

$$A^0 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & \infty & \infty & \infty & -1 & \infty \\ 2 & 1 & 0 & \infty & 2 & \infty & \infty \\ 3 & \infty & 2 & 0 & \infty & \infty & -8 \\ 4 & -4 & \infty & \infty & 0 & \infty & \infty \\ 5 & \infty & 7 & \infty & 3 & 0 & \infty \\ 6 & \infty & 5 & 10 & \infty & \infty & 0 \end{array}$$

$$A^1 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & \infty & \infty & \infty & -1 & \infty \\ 2 & 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & \infty & 2 & 0 & \infty & \infty & -8 \\ 4 & -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & \infty & 7 & \infty & 3 & 0 & \infty \\ 6 & \infty & 5 & 10 & \infty & \infty & 0 \end{array}$$

- $A^0[2,3]$ $\quad A^0[2,1] + A^0[1,3]$
  $\infty < 1 + \infty$
- $A^0[2,4]$ $\quad A^0[2,1] + A^0[1,4]$
  $2 < 1 + \infty$
- $A^0[2,5]$ $\quad A^0[2,1] + A^0[1,5]$
  $\infty > 1 + (-1) = 0$
- $A^0[2,6]$ $\quad A^0[2,1] + A^0[1,6]$
  $\infty < 1 + (\infty)$

$A^0[3,2] = 2 < A^0[3,1] + A^0[1,2] = \infty + \infty$
$A^0[3,4] = \infty < A^0[3,1] + A^0[1,4] = \infty + \infty$
$A^0[3,5] = \infty > A^0[3,1] + A^0[1,5] = \infty - 1$
$A^0[3,6] = -8 < A^0[3,1] + A^0[1,6] = \infty + \infty$

$A^0[4,2] = \infty > A^0[4,1] + A^0[1,2] = -4 + \infty$
$A^0[4,3] = \infty > A^0[4,1] + A^0[1,3] = -4 + \infty$
$A^0[4,5] = \infty > A^0[4,1] + A^0[1,5] = -4 - 1$
$A^0[4,6] = \infty > A^0[4,1] + A^0[1,6] = -4 + \infty$

$A^0[5,2] = 7 < A^0[5,1] + A^0[1,2] = \infty + \infty$
$A^0[5,3] = \infty < A^0[5,1] + A^0[1,3] = \infty + \infty$
$A^0[5,4] = 3 < A^0[5,1] + A^0[1,4] = \infty + \infty$
$A^0[5,6] = \infty < A^0[5,1] + A^0[1,6] = \infty + \infty$

$A^0[6,2] = 5 < A^0[6,1] + A^0[1,2] = \infty + \infty$

$A^0[6,3] = 10 < A^0[6,1] + A^0[1,3] = \infty + \infty$

$A^0[6,4] = \infty < A^0[6,1] + A^0[1,4] = \infty + \infty$

$A^0[6,5] = \infty > A^0[6,1] + A^0[1,5] = \infty - 1$

similarly,

$$A^2 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 3 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \end{array}$$

$$A3 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -\infty \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 3 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \end{array}$$

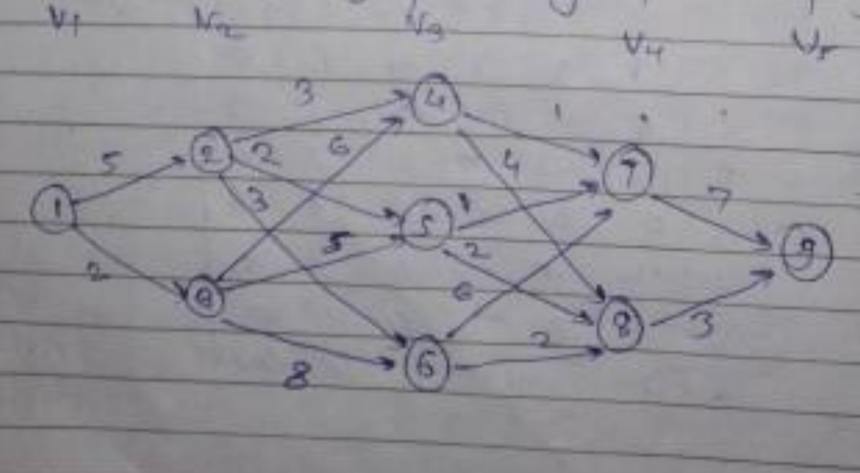
$$A4 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ -1 & 7 & \infty & 3 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 6 & \infty & 2 & -1 & \infty \\ -4 & 0 & \infty & 0 & -3 & \infty \\ 0 & 2 & 0 & 2 & -1 & -8 \\ -6 & 2 & \infty & 0 & -5 & \infty \\ -1 & 7 & \infty & 3 & 0 & \infty \\ 1 & 5 & 10 & 5 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^6 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 6 & \infty & 2 & -1 & \infty \\ -4 & 0 & \infty & 0 & -3 & \infty \\ -7 & -3 & 0 & -3 & -6 & -8 \\ -6 & 2 & \infty & 0 & -5 & \infty \\ -1 & 7 & \infty & 3 & 0 & \infty \\ 1 & 5 & 10 & 5 & 2 & 0 \end{bmatrix} \end{array}$$

This is shortest path from each vertex.

Q. Find the minimum cost path from s to t in multistage graph using dynamic programming.



step 1

v	1	2	3	4	5	6	7	8	9
cost									0
distance									9

step 2

for cost (4,7) = 7

step 3

for cost (4,8) = 3

v	1	2	3	4	5	6	7	8	9
cost							7	3	0
distance							9	9	9

step 4

for cost (3,4)

cost (3,4) = min { c(4,7) + cost (4,7),
c(4,8) + cost (4,8) }

= min { 1+7, 4+3 }

= min { 8, 7 }

cost (3,4) = 7

step 5

for cost (3,5)

cost (3,5) = min { c(5,7) + cost ~~(7,9)~~ (4,7)
c(5,8) + cost (4,8) }

= min { 1+7, 2+3 } = 5

cost (3,5) = 5

v	1	2	3	4	5	6	7	8	9
cost				7	5		7	3	0
dist				8	8		9	9	9

step 6

for cost(3,6) = min{ c(6,7) + cost(4,7),
c(6,8) + cost(4,8) }
= min{ 6+7, 2+3 } = 5

cost (3,6) = 5

step 7

for cost(2,2) = min{ c(2,4) + cost(3,4),
c(2,5) + cost(3,5),
c(2,6) + cost(3,6) }
= min{ 3+7, 2+5, 3+5 }
= min{ 10, 7, 8 }

cost(2,2) = 7

step 8

for cost(2,3) = min{ c(3,4) + cost(3,4),
c(3,5) + cost(3,5),
c(3,6) + cost(3,6) }
= min{ 6+7, 5+5, 8+5 }
= min{ 13, 10, 13 }

cost (2,3) = 10

step 9

for cost(1,1) = min{ c(1,2) + cost(2,2),
c(1,3) + cost(2,3) }
= min{ 5+7, 2+10 }
= min{ 12, 12 }

cost (1,1) = 12

V	1	2	3	4	5	6	7	8	9
~~d~~ cost	12	7	10	7	5	5	7	3	0
distance	2/3	5	5	8	8	8	9	9	9

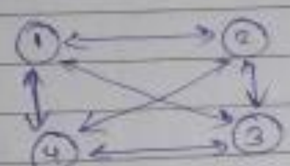
Here we get optimum minimum cost 12 but here we get this value for 2 different path.

$\boxed{cost = 12}$

path 1 :- 1 → 2 → 5 ⟷ 8 → 9

path 2 :- 1 → 3 → 5 → 8 → 9

Q. Consider the directed graph of fig. with edge lengths are given by matrix using TVSP



$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix} \end{array}$$

formula → $g(i,S) = \min_{k \in S} \{ C_{ik} + g(k, S - \{k\}) \}$

$g(1, \{2,3,4\}) = 35$

1

2: $C_{12} + g(2, \{3,4\})$ = $10 + 25$

3: $C_{13} + g(3, \{2,4\})$ = $15 + 25$

4: $C_{14} + g(4, \{2,3\})$ = $20 + 24$

3: $C_{23} + g(3, \{4\})$ = $9 + 20$ → 4: $C_{34} + g(4, \{\phi\})$ = $12 + 8$

4: $C_{24} + g(4, \{3\})$ = $10 + 15$ → 3: $C_{43} + g(3, \{\phi\})$ = $9 + 6$

2: $C_{32} + g(2, \{4\})$ = $13 + 18$ → 4: $C_{24} + g(4, \{\phi\})$ = $10 + 8$

4: $C_{34} + g(4, \{2\})$ = $12 + 13$ → 2: $C_{42} + g(2, \{\phi\})$ = $8 + 5$

2: $C_{42} + g(2, \{3\})$ = $8 + 15$ → 3: $C_{23} + g(3, \{\phi\})$ = $9 + 6$

3: $C_{43} + g(3, \{2\})$ = $9 + 18$ → 2: $C_{32} + g(2, \{\phi\})$ = $13 + 5$

from the tree diagram,

$g(2, \phi) = 5$

$g(3, \phi) = 6$

$g(4, \phi) = 8$

$g(2, \{3\}) = 15$

$g(2, \{4\}) = 18$

$g(3, \{2\}) = 18$

$g(3, \{4\}) = 20$

$g(4, \{2\}) = 13$

$g(4, \{3\}) = 15$

$g(2, \{3, 4\}) = 25$

$g(3, \{2, 4\}) = 25$

$g(4, \{2, 3\}) = 23$

$g(1, \{2, 3, 4\}) = \underline{\underline{35}}$

So minimum cost of TSP = 35