Quadcopter Dynamics

```
% State Variables
syms x y z x_dot y_dot z_dot % position & derivatives
syms phi theta psi phi_dot theta_dot psi_dot % euler angular & derivatives
syms w_b [1 3] % rotational velocity of body
syms T [1 4]   % Thrust due to motor
syms M [1 4]   % Moment due to motor
syms S_T Mx My Mz % sum thrust, Moments
syms w_m [1 4] % motor angular velocities
syms p s r p_dot s_dot r_dot % rotational values
syms U % Input vector of torques and moments

% System Properties
syms I_xx I_xy I_xz I_yy I_yz I_zz   % Body Inertia Matrix
syms k_t k_m     % Propeller Coeff
syms blade_pos_body [4 3] % Rotor positions
syms b_w b_l a_l  % refer to image, body and arm dim
syms g m
```

Defining Rotation Matrices and State vector

```
R3 = [cos(psi) sin(psi) 0; -sin(psi) cos(psi) 0; 0 0 1]
```

R3 =

$$\begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
R2 = [cos(theta ) 0 -sin(theta ); 0 1 0; sin(theta ) 0 cos(theta )]
```

R2 =

$$\begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

```
R1 = [1 0 0;0 cos(phi ) sin(phi );0 -sin(phi ) cos(phi )]
```

R1 =

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}$$

```
% First Rotation R3, then 2 then 1 by definition of yaw pitch roll
Base_2_Body_Rot = R1 * R2 * R3
```

Base_2_Body_Rot =

$$\begin{pmatrix} \cos(\psi)\cos(\theta) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\theta)\sin(\phi) \\ \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

```
Body_2_Base_Rot = Base_2_Body_Rot.'
```

Body_2_Base_Rot =

$$\begin{pmatrix} \cos(\psi)\cos(\theta) & \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) \\ \cos(\theta)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) \\ -\sin(\theta) & \cos(\theta)\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

```
% State vector definition, splitting into logical parts
q1 = [x; y; z;]
```

q1 =

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

```
q2 = [x_dot; y_dot; z_dot ;]
```

q2 =

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

```
q3 = [phi; theta; psi;]
```

q3 =

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix}$$

```
q3_dot = [phi_dot; theta_dot; psi_dot]
```

q3_dot =

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

```
q4 = [p; s; r]
```

q4 =

$$\begin{pmatrix} p \\ s \\ r \end{pmatrix}$$

```
q4_dot = [p_dot; s_dot; r_dot]
```

q4_dot =

$$\begin{pmatrix} \dot{p} \\ \dot{s} \\ \dot{r} \end{pmatrix}$$

```
q = [q1;q2;q3;q4]
```

q =

$$\begin{pmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \phi \\ \theta \\ \psi \\ p \\ s \\ r \end{pmatrix}$$

```
% Euler angle to rotational velocity matrix
w_2_euler = [1 sin(phi)*tan(theta) cos(phi)*tan(theta); 0 cos(phi) -sin(phi); 0 sin(phi)/cos(th
```

w_2_euler =

$$\begin{pmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{pmatrix}$$

```
euler_2_w = inv(w_2_euler)
```

euler_2_w =

$$\begin{pmatrix} 1 & 0 & -\cos(\theta)\tan(\theta) \\ 0 & \dfrac{\cos(\phi)}{\sigma_1} & \dfrac{\cos(\theta)\sin(\phi)}{\sigma_1} \\ 0 & -\dfrac{\sin(\phi)}{\sigma_1} & \dfrac{\cos(\phi)\cos(\theta)}{\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = \cos(\phi)^2 + \sin(\phi)^2$$

Thrust and Moments produced by each motor

```
T1 = k_t * w_m1^2
```

T1 = $k_t \, w_{\mathrm{m1}}{}^2$

```
T2 = k_t * w_m2^2
```

T2 = $k_t \, w_{\mathrm{m2}}{}^2$

```
T3 = k_t * w_m3^2
```

T3 = $k_t \, w_{\mathrm{m3}}{}^2$

```
T4 = k_t * w_m4^2
```

T4 = $k_t \, w_{\mathrm{m4}}{}^2$

```
S_T = T1 + T2 + T3 + T4;

M1 = k_m * w_m1^2
```

M1 = $k_m \, w_{\mathrm{m1}}{}^2$

```
M2 = k_m * w_m2^2
```

M2 = $k_m \, w_{\mathrm{m2}}{}^2$

```
M3 = k_m * w_m3^2
```

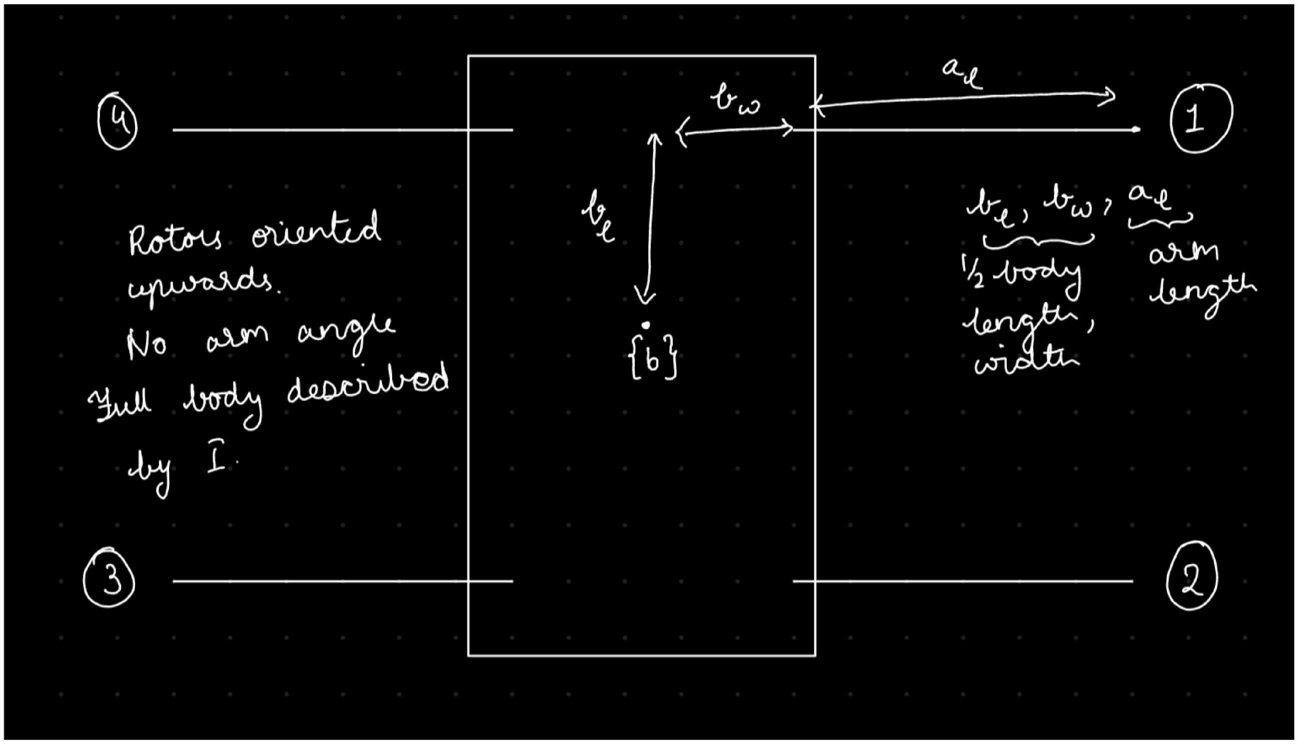
M3 = $k_m \, w_{\mathrm{m3}}{}^2$

```
M4 = k_m * w_m4^2
```

M4 = $k_m \, w_{\mathrm{m4}}{}^2$

Propeller Positions and Condensing to Torque and Moment Vector

```
imshow(imread("drone_body_rough_wtf.png"))
```




```
blade_pos_body = [b_w + a_l b_l 0;b_w + a_l -b_l 0;-b_w - a_l -b_l 0;-b_w - a_l b_l 0;].'
```

blade_pos_body =

$$\begin{pmatrix} a_l + b_w & a_l + b_w & -a_l - b_w & -a_l - b_w \\ b_l & -b_l & -b_l & b_l \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
H1 = cross(blade_pos_body(:, 1), [0; 0; T1])
```

H1 =

$$\begin{pmatrix} b_l\, k_t\, w_{\mathrm{m1}}{}^2 \\ -k_t\, w_{\mathrm{m1}}{}^2\, (a_l + b_w) \\ 0 \end{pmatrix}$$

```
H2 = cross(blade_pos_body(:, 2), [0; 0; T2])
```

H2 =

$$\begin{pmatrix} -b_l\, k_t\, w_{\mathrm{m2}}{}^2 \\ -k_t\, w_{\mathrm{m2}}{}^2\, (a_l + b_w) \\ 0 \end{pmatrix}$$

```
H3 = cross(blade_pos_body(:, 3), [0; 0; T3])
```

H3 =

$$\begin{pmatrix} -b_l\, k_t\, w_{m3}^2 \\ k_t\, w_{m3}^2\, (a_l + b_w) \\ 0 \end{pmatrix}$$

```
H4 = cross(blade_pos_body(:, 4), [0; 0; T4])
```

H4 =

$$\begin{pmatrix} b_l\, k_t\, w_{m4}^2 \\ k_t\, w_{m4}^2\, (a_l + b_w) \\ 0 \end{pmatrix}$$

```
H = H1 + H2 + H3 + H4
```

H =

$$\begin{pmatrix} b_l\, k_t\, w_{m1}^2 - b_l\, k_t\, w_{m2}^2 - b_l\, k_t\, w_{m3}^2 + b_l\, k_t\, w_{m4}^2 \\ -k_t\, (a_l + b_w)\, w_{m1}^2 - k_t\, (a_l + b_w)\, w_{m2}^2 + k_t\, (a_l + b_w)\, w_{m3}^2 + k_t\, (a_l + b_w)\, w_{m4}^2 \\ 0 \end{pmatrix}$$

```
Mx = H(1)
```

Mx = $b_l\, k_t\, w_{m1}^2 - b_l\, k_t\, w_{m2}^2 - b_l\, k_t\, w_{m3}^2 + b_l\, k_t\, w_{m4}^2$

```
My = H(2)
```

My = $-k_t\, (a_l + b_w)\, w_{m1}^2 - k_t\, (a_l + b_w)\, w_{m2}^2 + k_t\, (a_l + b_w)\, w_{m3}^2 + k_t\, (a_l + b_w)\, w_{m4}^2$

```
Mz = M1 + M2 + M3 + M4
```

Mz = $k_m\, w_{m1}^2 + k_m\, w_{m2}^2 + k_m\, w_{m3}^2 + k_m\, w_{m4}^2$

```
U = [S_T; Mx; My; Mz]
```

U =

$$\begin{pmatrix} k_t\, w_{m1}^2 + k_t\, w_{m2}^2 + k_t\, w_{m3}^2 + k_t\, w_{m4}^2 \\ b_l\, k_t\, w_{m1}^2 - b_l\, k_t\, w_{m2}^2 - b_l\, k_t\, w_{m3}^2 + b_l\, k_t\, w_{m4}^2 \\ -k_t\, (a_l + b_w)\, w_{m1}^2 - k_t\, (a_l + b_w)\, w_{m2}^2 + k_t\, (a_l + b_w)\, w_{m3}^2 + k_t\, (a_l + b_w)\, w_{m4}^2 \\ k_m\, w_{m1}^2 + k_m\, w_{m2}^2 + k_m\, w_{m3}^2 + k_m\, w_{m4}^2 \end{pmatrix}$$

```
motorw_2_forces = [k_t k_t k_t k_t;b_l*k_t -b_l*k_t -b_l*k_t b_l*k_t;-k_t*(a_l + b_w) -k_t*(a_l
```

motorw_2_forces =

$$\begin{pmatrix} k_t & k_t & k_t & k_t \\ b_l\, k_t & -b_l\, k_t & -b_l\, k_t & b_l\, k_t \\ -k_t\, (a_l + b_w) & -k_t\, (a_l + b_w) & k_t\, (a_l + b_w) & k_t\, (a_l + b_w) \\ k_m & k_m & k_m & k_m \end{pmatrix}$$

Forces and Moments on the Quadcopter

```
env_gravity_base = [0; 0; -g ]
```

env_gravity_base =

$$\begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

```
q1_ddot = 1/m *(env_gravity_base + Body_2_Base_Rot * ([0; 0; S_T]))
```

q1_ddot =

$$\begin{pmatrix} \dfrac{(\sin(\phi)\,\sin(\psi) + \cos(\phi)\,\cos(\psi)\,\sin(\theta))\,\sigma_1}{m} \\ -\dfrac{(\cos(\psi)\,\sin(\phi) - \cos(\phi)\,\sin(\psi)\,\sin(\theta))\,\sigma_1}{m} \\ -\dfrac{g - \cos(\phi)\,\cos(\theta)\,\sigma_1}{m} \end{pmatrix}$$

where

$$\sigma_1 = k_t\, {w_{m1}}^2 + k_t\, {w_{m2}}^2 + k_t\, {w_{m3}}^2 + k_t\, {w_{m4}}^2$$

```

% Full form inertia matrix should ?
% I_mat = [I_xx -I_xy -I_xz; -I_xy I_yy -I_yz; -I_xz -I_yz I_zz]
I_mat = [I_xx 0 0; 0 I_yy 0; 0 0 I_zz]
```

I_mat =

$$\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

```
q4 = w_2_euler \ q3_dot
```

q4 =

$$\begin{pmatrix} \dot{\phi} - \dot{\psi}\cos(\theta)\tan(\theta) \\ \dfrac{\dot{\theta}\cos(\phi) + \dot{\psi}\cos(\theta)\sin(\phi)}{\cos(\phi)^2 + \sin(\phi)^2} \\ -\dfrac{\dot{\theta}\sin(\phi) - \dot{\psi}\cos(\phi)\cos(\theta)}{\cos(\phi)^2 + \sin(\phi)^2} \end{pmatrix}$$

```
q4 = [p;s;r ]
```

q4 =

$$\begin{pmatrix} p \\ s \\ r \end{pmatrix}$$

```
q4_dot = [p_dot; s_dot; r_dot]
```

q4_dot =

$$\begin{pmatrix} \dot{p} \\ \dot{s} \\ \dot{r} \end{pmatrix}$$

```
q4_dot = inv(I_mat) * (-cross(q4, I_mat * q4) + [Mx; My;Mz])
```

q4_dot =

$$\begin{pmatrix} \dfrac{b_l\, k_t\, w_{\mathrm{m1}}{}^2 - b_l\, k_t\, w_{\mathrm{m2}}{}^2 - b_l\, k_t\, w_{\mathrm{m3}}{}^2 + b_l\, k_t\, w_{\mathrm{m4}}{}^2 + I_{\mathrm{yy}}\, r\, s - I_{\mathrm{zz}}\, r\, s}{I_{\mathrm{xx}}} \\ -\dfrac{k_t\,(a_l + b_w)\, w_{\mathrm{m1}}{}^2 + k_t\,(a_l + b_w)\, w_{\mathrm{m2}}{}^2 - k_t\,(a_l + b_w)\, w_{\mathrm{m3}}{}^2 - k_t\,(a_l + b_w)\, w_{\mathrm{m4}}{}^2 + I_{\mathrm{xx}}\, p\, r - I_{\mathrm{zz}}\, p\, r}{I_{\mathrm{yy}}} \\ \dfrac{k_m\, w_{\mathrm{m1}}{}^2 + k_m\, w_{\mathrm{m2}}{}^2 + k_m\, w_{\mathrm{m3}}{}^2 + k_m\, w_{\mathrm{m4}}{}^2 + I_{\mathrm{xx}}\, p\, s - I_{\mathrm{yy}}\, p\, s}{I_{\mathrm{zz}}} \end{pmatrix}$$