INSTRUMENTATION AND MEASUREMENT

SENSORS AND PRIMARY TRANSDUCERS

This tutorial provides an overview of instrument sensors used in process and automatic control. It is useful to anyone studying measurement systems and instrumentation but it is provided mainly in support of the Measurements and Instrumentations Module 212CNE . This tutorial is mainly descriptive.

Control is a broad concept and the following might apply to an automated system such as a robot or to a process control system such as a pneumatic valve controlling the flow of steam in a pipe.

On completion of this tutorial, you should be able to do the following.

- Explain a basic measurement system.
- Explain the basic working principles of a variety of temperature sensors.
- Explain the basic working principles of a variety of pressure sensors.
- Explain the basic working principles of a variety of speed transducers.
- Explain the basic working principles of a variety of flow meters.
- Explain the basic working principles of a variety of force gauges.
- Explain the basic working principles of a variety of displacement gauges.
- Explain the basic working principles of a variety of level (depth) gauges.
- Explain in some detail the theory and use of strain gauges.

In order to complete the theoretical part of this tutorial, you must be familiar with basic mechanical and electrical science.

1. INTRODUCTION

A basic instrument system consists of three elements:

i SENSOR or INPUT DEVICE
ii SIGNAL PROCESSOR
iii RECEIVER or OUTPUT DEVICE

This tutorial is devoted to input devices but you can never separate it from the rest of the system as in many cases they are all integral (e.g. a mechanical pressure gauge incorporates all of these elements). A block diagram of a basic system is shown but they are usually more complex.

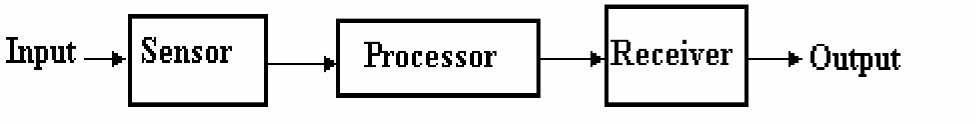


Figure 1

Most modern analogue equipment works on the following standard signal ranges.

- Electric 4 to 20 mA
- Pneumatic 0.2 to 1.0 bar

Older electrical equipment use 0 to 10 V. Increasingly the instruments are digital with a binary digital encoder built in to give a binary digital output. Pneumatic signals are commonly used in process industries for safety especially when there is a risk of fire or explosion.

The advantage of having a standard range or using digital signals is that all equipment may be purchased ready calibrated. For analogue systems the minimum signal (Temperature, speed, force, pressure and so on) is represented by 4 mA or 0.2 bar and the maximum signal is represented by 20 mA or 1.0 bar.

This tutorial is an attempt to familiarise you with the many types of input sensors on the market today. Usually such sensors are called PRIMARY TRANSDUCERS.

Things that we commonly measure are:

Temperature
Speed
Force
Stress and Strain
Mass or Weight
Size or Volume
Pressure
Flow rate
Movement, Velocity and Acceleration
Level or Depth
Density
Acidity/Alkalinity

Sensors may operate simple on/off switches to detect the following:

Objects(Proximity switch)
Hot or cold (thermostat)
Empty or full (level switch)
Pressure high or low (pressure switch)

The block diagram of a sensor is shown below.

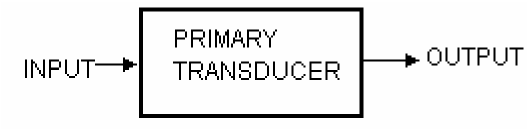


Figure 2

2 TEMPERATURE TRANSDUCERS

2.1 THERMOCOUPLES

When two wires with dissimilar electrical properties are joined at both ends and one junction is made hot and the other cold, a small electric current is produced proportional to the difference in the temperature. Seebeck discovered this effect. It is true no matter how the ends are joined so the cold end may be joined at a sensitive millivolt meter. The hot junction forms the sensor end.

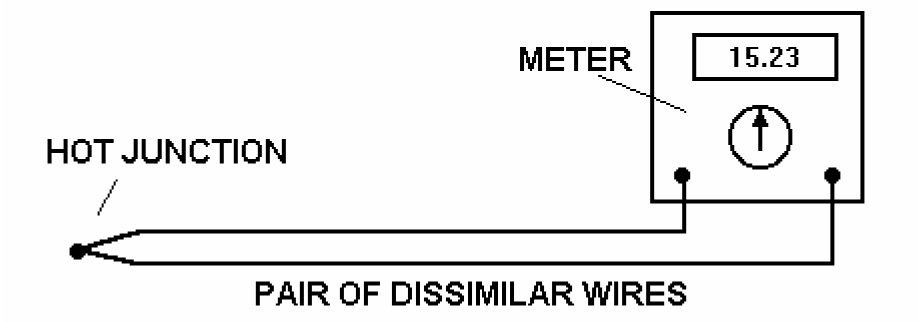


Figure 3

The picture shows a typical industrial probe with a flexible extension and standard plug.

Figure 4

Peltier showed that heat is absorbed at the hot end and rejected at the cold end. Thompson showed that part of the e.m.f. is due to the temperature gradient in the wire as well as the temperature difference between the junctions. Most thermocouple metals produce a relationship between the two temperatures and the e.m.f as follows.

$$e = \alpha(\theta_1 - \theta_2) + \beta(\theta_1^2 - \theta_2^2$$

α and β are constants for the type of thermocouple. The relationship is nearly linear over the operating range. The actual characteristic and suitable operating temperatures depends upon the metals used in the wires. The various types are designated in international and national standards. Typical linear operating ranges are shown for standard types.

It is important that thermocouples are standard so that the same e.m.f will always represent the same temperature.

Type J	0 to 800°C
Type K	0 to 1200°C
Type T	-199 to 250°C
Type E	0 to 600°C
Type R/S	0 to 1600°C
Type B	500 to 1800°C
Type N	0 to 1200°C
Type L	0 to 800°C

Thermocouples come in several forms. They may be wires insulated from each other with plastic or glass fibre materials. For high temperature work, the wire pairs are put inside a tube with mineral insulation. For industrial uses the sensor comes in a metal enclosure such as stainless steel.

2.2 RESISTANCE TYPE SENSORS

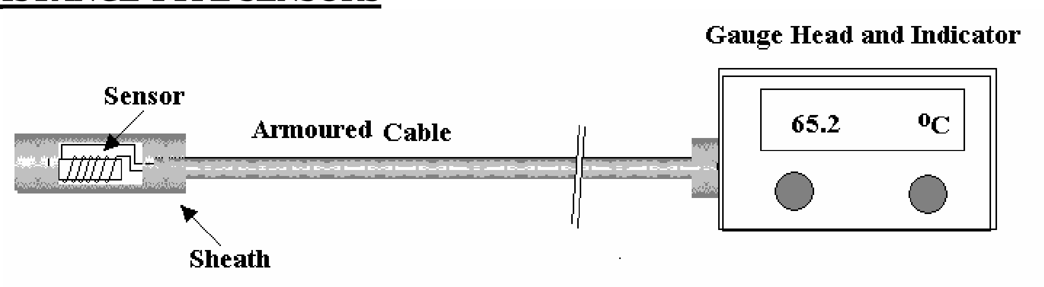


Figure 5

These work on the principle that the electrical resistance of a conductor change with temperature. If a constant voltage is applied to the conductor then the current flowing through it will change with temperature. The resistivity of the conductor change with temperature. This usually means the resistance gets bigger as the conductor gets hotter. The following law relates the resistance and temperature.

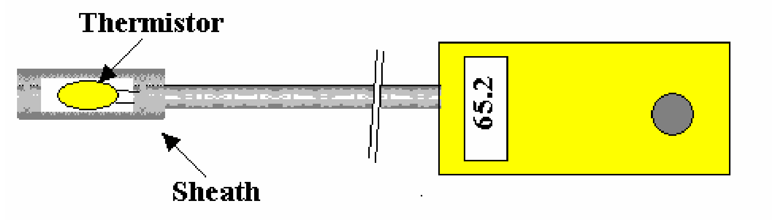
$$R = R_o(1 + \alpha\theta)$$

α is the temperature coefficient of resistance. R_o is the resistance at 0°C. Sometimes the equation is given as

$$R = R_o(1 \;\; \alpha\theta - \beta\theta^2)$$

A basic temperature sensor is made by winding a thin resistance wire into a small sensor head. The resistance of the wire then represents the temperature. This has an advantage over a thermocouple in that it is unaffected by the temperature of the gauge end. The main type of wire used is ***PLATINUM.*** The sensors are usually manufactured to have a resistance of 100 Ω at 0°C and the value of α is 0.00385 to 0.00390. A typical operating range is -200 to 400°C.

A special type of resistance sensor is called a **THERMISTOR.** They are made from a small piece of semi-conductor material. The material is special because the resistance changes a lot for a small change in temperature and so can be made into a small sensor and it costs less than platinum wire. The temperature range is limited. They are only used for a typical range of -20 to 120°C and are commonly used in small hand held thermometers for every day use. The relationship between resistance and temperature is of the form $R = Ae^{B/\theta}$



WORKED EXAMPLE No.1

A Platinum resistance thermometer has a resistance of 100 Ω at 0^oC and the value of α is 0.00385. In operation the resistance is 101 Ω. Calculate the temperature.

SOLUTION

Rearrange the formula to make θ the subject and evaluate.

$$? = \frac{\frac{R}{R_o} - 1}{a} = \frac{\frac{105}{100} - 1}{0.00385} = 12.987^o C$$

WORKED EXAMPLE No.2

A thermocouple produces an e.m.f. in mV according to the temperature difference between the sensor tip θ_1 and the gauge head θ_2 such that

$$e = \alpha(\theta_1 - \theta_2) + \beta(\theta_1^2 - \theta_2^2)$$

$\alpha = 3.5 \times 10^{-2}$ and $\beta = 8.2 \times 10^{-6}$ The gauge head is at 20^oC. The mV output is 12 mV. Calculate the temperature at the sensor.

SOLUTION

$$10 = 0.035(?_1 - 20) + 8.2 \times 10^{-6}(?_1^2 - 20^2)$$

$$10 = 0.035?_1 - 0.7 + 8.2 \times 10^{-6}?_1^2 - 0.00328$$

$$10 = 8.2x10^{-6}?_1^2 + 0.035?_1 - 0.69672$$

$$8.2 \times 10^{-6}?_1^2 + 0.035?_1 - 9.30328 = 0$$

Solving the quadratic equation yields $\theta_1 = 251^oC$

SELF ASSESSMENT EXERCISE No.1

1. A thermocouple produces an e.m.f. in mV according to the temperature difference between the sensor tip θ_1 and the gauge head θ_2 such that $e = \alpha(\theta_1 - \theta_2) + \beta(\theta_1^2 - \theta_2^2)$
   Given $\alpha = 3.5 \times 10^{-2}$ and $\beta = 8.2 \times 10^{-6}$ determine the mV output when the tip is at 220^oC and the gauge head at 20^oC.
   (Answer 7.394 mV)

2. Describe the basic construction of a resistance type temperature sensor and state the reason why it is unaffected by the temperature of the gauge head.

3. State two reasons why instrument systems use standard transmission signal of either 4 - 20 mA or 0.2 - 1 bar.

2.3 LIQUID EXPANSION and VAPOUR PRESSURE SENSORS

These are thermometers filled with either a liquid such as mercury or an evaporating fluid such as used in refrigerators. In both cases the inside of the sensor head and the connecting tube are completely full. Any rise in temperature produces expansion or evaporation of the liquid so the sensor becomes pressurised. The pressure is related to the temperature and it may be indicated on a simple pressure gauge.

Ways and means exist to convert the pressure into an electrical signal. The movement may also directly operate a thermostat. These instruments are robust and used over a wide range. They can be fitted with electric switches to set off alarms.

Figure 6

2.4 BIMETALLIC TYPES

It is a well-known principle that if two metals are rigidly joined together as a two-layer strip and heated, the difference in the expansion rate causes the strip to bend.

Figure 7

In the industrial type, the strip is twisted into a long thin coil inside a tube. One end is fixed at the bottom of the tube and the other turns and moves a pointer on a dial. The outward appearance is very similar to the pressure type. They can be made to operate limit switches and set off alarms or act as a thermostat. (e.g. on a boiler).

2.5 GLASS THERMOMETER

The ordinary glass thermometer is also a complete system. Again the bulb is the sensor but the column of liquid and the scale on the glass is the processor and indicator. Mercury is used for hot temperatures and coloured alcohol for cold temperatures.

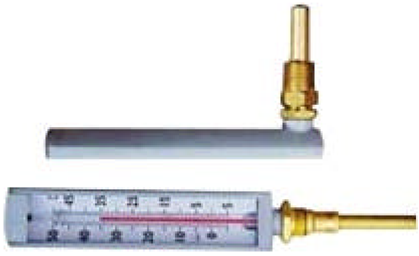


Figure 8

The problems with glass thermometers are that they are

- Brittle
- Mercury solidifies at -40^{o}C.
- Alcohol boils at around 120 ^{o}C.
- Accurate manufacture is needed and this makes accurate ones expensive.
- It is easy for people to make mistakes reading them.

Glass thermometers are not used much now in industry but if they are, they are usually protected by a shield from accidental breakage. In order to measure the temperature of something inside a pipe they are placed in thermometer pockets.

3. PRESSURE TRANSDUCERS

Pressure sensors either convert the pressure into mechanical movement or into an electrical output. Complete gauges not only sense the pressure but indicate them on a dial or scale.

Mechanical movement is produced with the following elements.

- Bourdon Tube.
- Spring and Piston.
- Bellows and capsules.
- Diaphragm.

3.1. BOURDON TUBE

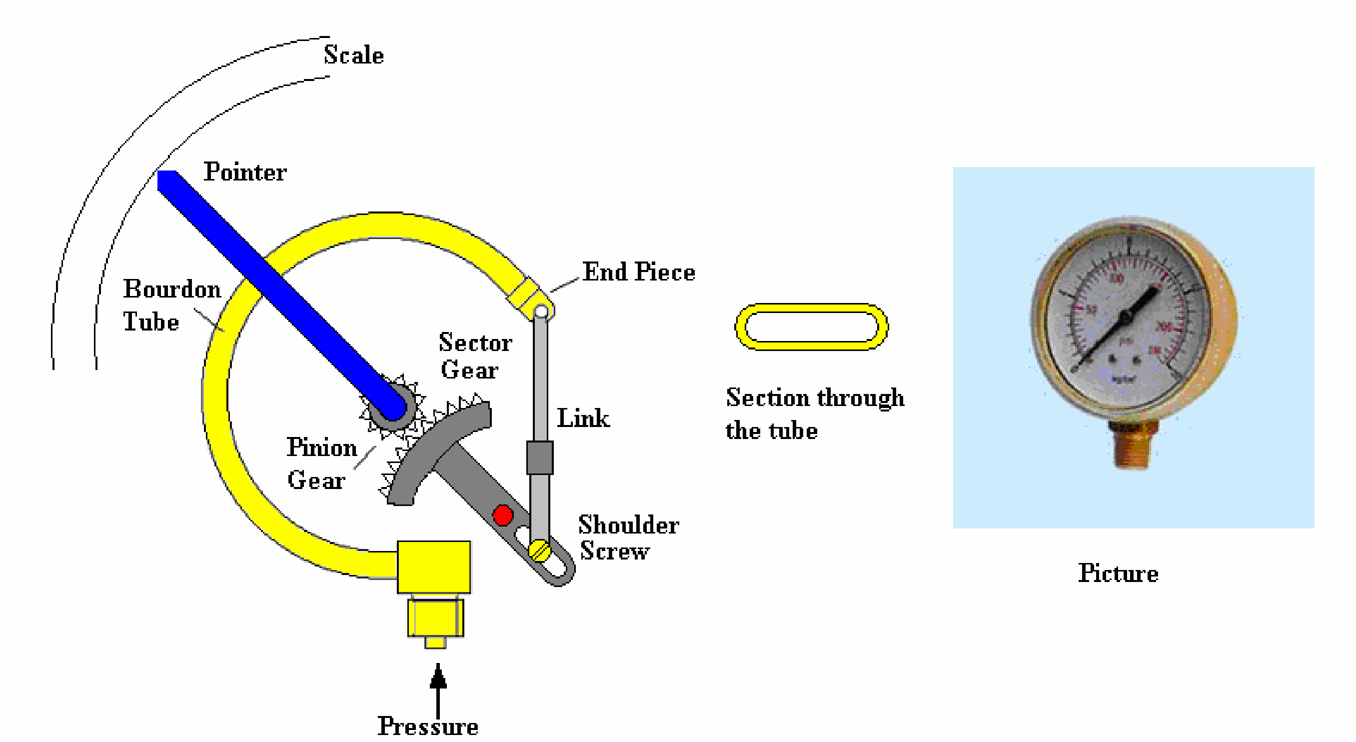


Figure 9

The Bourdon tube is a hollow tube with an elliptical cross section. When a pressure difference exists between the inside and outside, the tube tends to straighten out and the end moves. The movement is usually coupled to a needle on a dial to make a complete gauge. It can also be connected to a secondary device such as an air nozzle to control air pressure or to a suitable transducer to convert it into an electric signal. This type can be used for measuring pressure difference.

3.2 PISTON TYPE

The pressure acts directly on the piston and compresses the spring. The position of the piston is directly related to the pressure. A window in the outer case allows the pressure to be indicated. This type is usually used in hydraulics where the ability to withstand shock, vibration and sudden pressure changes is needed (shock proof gauge). The piston movement may be connected to a secondary device to convert movement into an electrical signal.

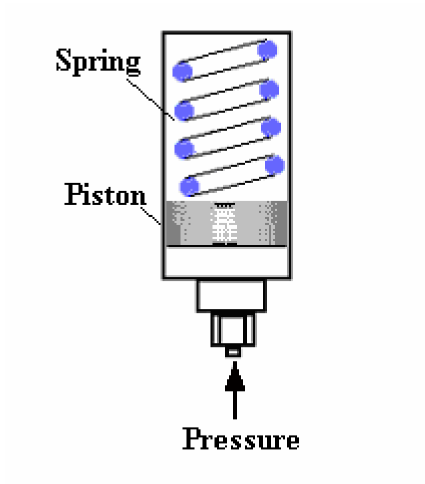


Figure 10

3.3. CAPSULES AND BELLOWS

A bellows is made of several capsules. These are hollow flattened structures made from thin metal plate. When pressurised the bellows expand and produce mechanical movement. If the bellows is encapsulated inside an outer container, then the movement is proportional to the difference between the pressure on the inside and outside. Bellows and single capsules are used in many instruments. They are very useful for measuring small pressures.

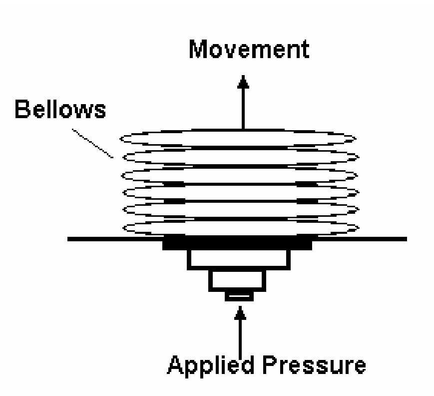


Figure 11

3.4 DIAPHRAGMS

These are similar in principle to the capsule but the diaphragm is usually very thin and perhaps made of rubber. The diaphragm expands when very small pressures are applied. The movement is transmitted to a pointer on a dial through a fine mechanical linkage.

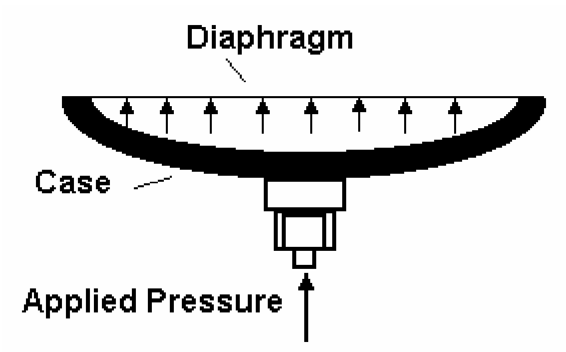


Figure 12

3.5 ELECTRICAL PRESSURE TRANSDUCERS

There are various ways of converting the mechanical movement of the preceding types into an electric signal. Th following are types that directly produce an electric signal.

- Strain Gauge types.
- Piezo electric types.
- Other electric effects.

3.5.1 STRAIN GAUGE TYPES

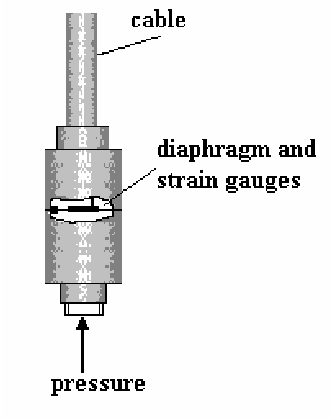


Figure 13

The principles of electric strain gauges are covered later. Strain gauges are small elements that are fixed to a surface that is strained. The change in length of the element produces changes in the electrical resistance. This is processed and converted into a voltage. A typical pressure transducer would contain a metal diaphragm which bends under pressure.

3.5.2. PIEZO ELECTRIC TYPES

The element used here is a piece of crystalline material that produces an electric charge on its surface when it is mechanically stressed. The electric charge may be converted into voltage. This principle is used in the pick up crystal of a record player, in microphones and even to generate a spark in a gas ignitor. When placed inside a pressure transducer, the pressure is converted into an electric signal.

3.5.3. OTHER ELECTRIC EFFECTS

Other electric effects commonly used in transducers are CAPACITIVE and INDUCTIVE. In these cases, the pressure produces a change in the capacitance or inductance of an electronic component in the transducer. Both these effects are commonly used in an electronic oscillator and one way they may be used is to change the frequency of the oscillation. The frequency may be converted into a voltage representing the pressure.

4. SPEED TRANSDUCERS

Speed transducers are widely used for measuring the output speed of a rotating object. There are many types using different principles and most of them produce an electrical output.

4.1 OPTICAL TYPES

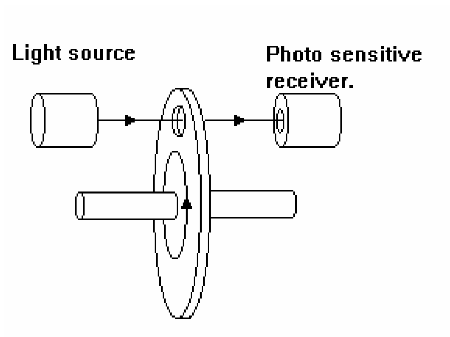


Figure 14

These use a light beam and a light sensitive cell. The beam is either reflected or interrupted so that pulses are produced for each revolution. The pulses are then counted over a fixed time and the speed obtained. Electronic processing is required to time the pulses and turn the result into an analogue or digital signal.

4.2 MAGNETIC PICK UPS

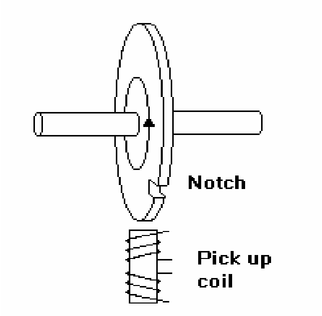


Figure 15

These use an inductive coil placed near to the rotating body. A small magnet on the body generates a pulse every time it passes the coil. If the body is made of ferrous material, it will work without a magnet. A discontinuity in the surface such as a notch will cause a change in the magnetic field and generate a pulse. The pulses must be processed to produce an analogue or digital output.

4.3 TACHOMETERS

There are two types, A.C. and D.C. The A.C. type generates a sinusoidal output. The frequency of the voltage represents the speed of rotation. The frequency must be counted and processed. The D.C. type generates a voltage directly proportional to the speed. Both types must be coupled to the rotating body. very often the tachometer is built into electric motors to measure their speed.

Figure 16

5. FLOW METERS

There are many hundreds of types of flow meters depending on the make and application. They may be classified roughly as follows.

- POSITIVE DISPLACEMENT TYPES
- INFERENTIAL TYPES
- VARIABLE AREA TYPES
- DIFFERENTIAL PRESSURE TYPES

5.1. POSITIVE DISPLACEMENT TYPES

These types have a mechanical element that makes the shaft of the meter rotate once for an exact known quantity of fluid. The quantity of fluid hence depends on the number of revolutions of the meter shaft and the flow rate depends upon the speed of rotation. Both the revolutions and speed may be measured with mechanical or electronic devices. Some of the most common listed below.

- Rotary piston type.
- Vane type.
- Lobe type or meshing rotor.
- Reciprocating piston type
- Fluted spiral gear.

5.1.1 MESHING ROTOR

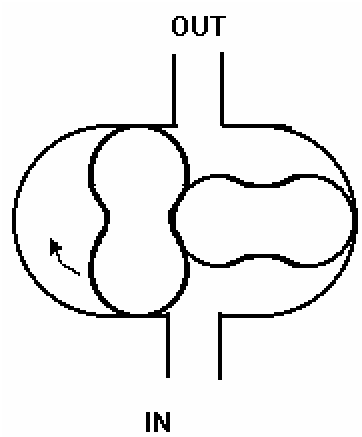


Figure 17

The MESHING ROTOR type consists of two rotors with lobes. When fluid is forced in, the rotors turn and operate the indicating system.

5.2. INFERENTIAL TYPE METERS

The flow of the fluid is inferred from some effect produced by the flow. Usually this is a rotor which is made to spin and the speed of the rotor is sensed mechanically or electronically. The main types are :

- Turbine rotor types
- Rotary shunt types
- Rotating vane types
- Helical turbine types

5.2.1 TURBINE TYPE

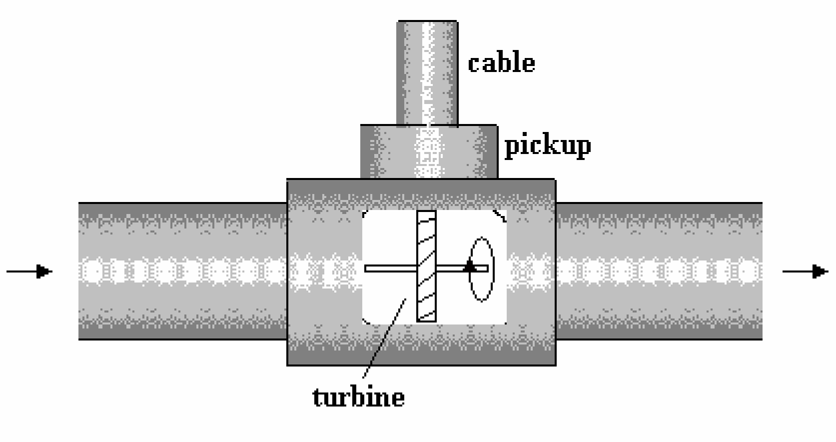


Figure 18

The pictures show two industrial flow meters.

Figure 19

The turbine type shown has an axial rotor which is made to spin by the fluid and the speed represents the flow rate. This may be sensed electrically by coupling the shaft to a small electric tachometer. Often this consists of a magnetic slug on the rotor which generates a pulse of electricity each time it passes the sensor.

5.2.2 ROTATING VANE TYPE

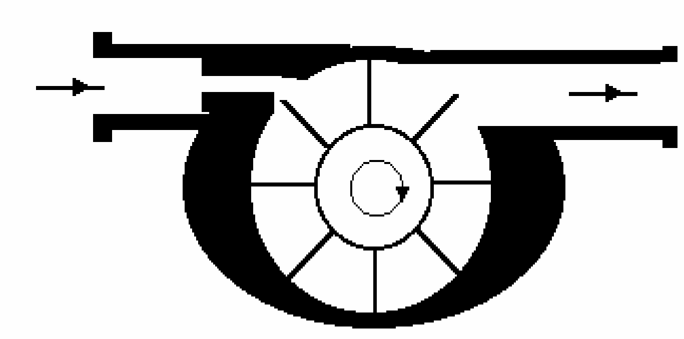


Figure 20

The jet of fluid spins around the rotating vane and the speed of the rotor is measured mechanically or electronically.

5.3.3. VARIABLE AREA TYPES

There are two main types of this meter

- Float type (Rotameter)
- Tapered plug type.

5.3.3.1 FLOAT TYPE

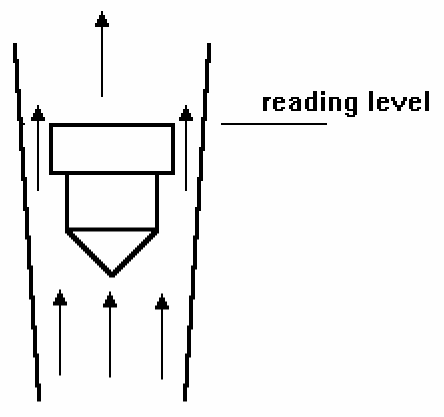


Figure 21

The float is inside a tapered tube. The fluid flows through the annular gap around the edge of the float. The restriction causes a pressure drop over the float and the pressure forces the float upwards. Because the tube is tapered, the restriction is decreased as the float moves up. Eventually a level is reached where the restriction is just right to produce a pressure force that counteracts the weight of the float. The level of the float indicates the flow rate. If the flow changes the float moves up or down to find a new balance position.

When dangerous fluids are used, protection is needed against the tube fracturing. The tube may be made of a non-magnetic metal. The float has a magnet on it. As it moves up and down, the magnet moves a follower and pointer on the outside. The position of the float may be measured electrically by building a movement transducer into the float.

5.3.3.2 TAPERED PLUG TYPE.

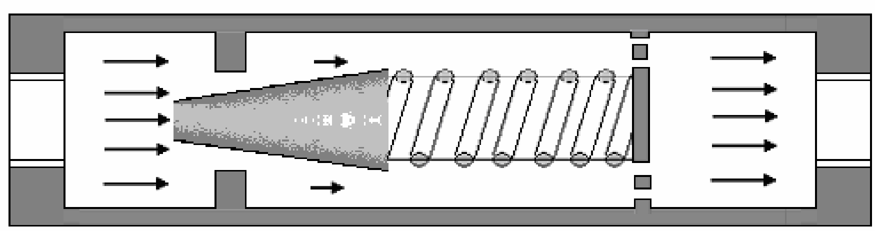


Figure 22

In this meter, a tapered plug is aligned inside a hole or orifice. A spring holds it in place. The flow is restricted as it passes through the gap and a force is produced which moves the plug. Because it is tapered the restriction changes and the plug takes up a position where the pressure force just balances the spring force. The movement of the plug is transmitted with a magnet to an indicator on the outside.

5.4 DIFFERENTIAL PRESSURE FLOW METERS

These are a range of meters that convert flow rate into a differential pressure. The important types conform to BS 1042 and are

- ORIFICE METERS.
- VENTURI METERS
- NOZZLE METERS
- PITOT TUBES.

The diagram shows a cross section through the four types of d.p. meters.

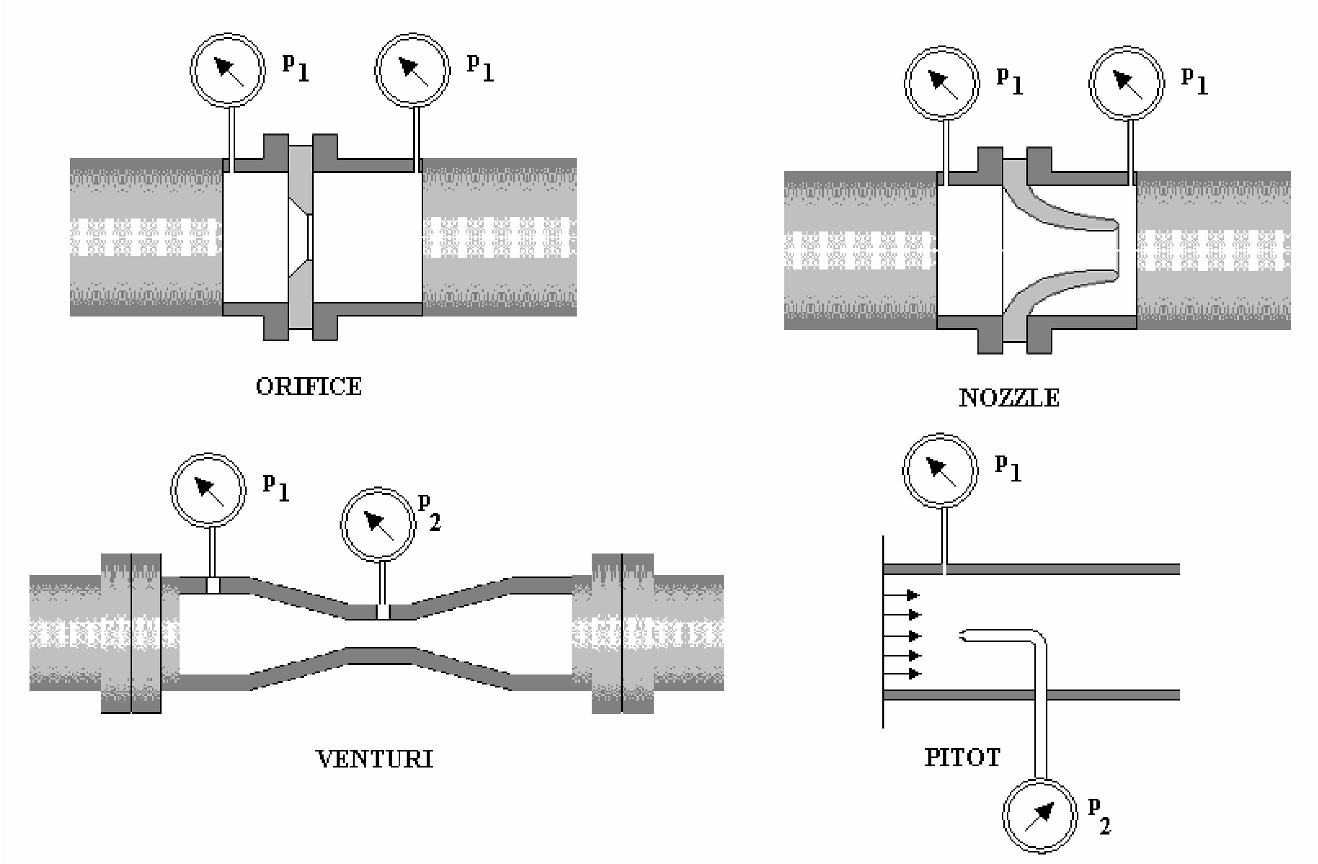


Figure 23

The working principle for all these is that something makes the velocity of the fluid change and this produces a change in the pressure so that a difference $\Delta p = p_2 - p_1$ is created. It can be shown for all these meters that the volume flow rate Q is related to Δp by the following formula.

$$\mathbf{Q = K(\Delta p)^{0.5}}$$

Figure 24

K is the meter constant. A full explanation of these meters is covered in the tutorials on fluid mechanics. The picture shows an industrial d.p.meter. Extra instrumentation heads can be fitted to produce an electrical output (4 – 20 mA) or a pneumatic output (0.2 – 1 bar).

WORKED EXAMPLE No.3

A Venturi meter has a meter constant of 0.008 $m^4\ N^{-0.5}\ s^{-1}$. Calculate the flow rate when Δp = 180 Pa

SOLUTION

$Q = K(\Delta p)^{0.5} = 0.008\ m^4\ N^{-0.5}\ s^{-1}(180)^{0.5} = 0.1073\ (m^4\ N^{-0.5}\ s^{-1})(N^{0.5}\ m^{-1})$ or m^3/s

SELF ASSESSMENT EXERCISE No.2

An Orifice meter has a meter constant of 0.004 $m^4\ N^{-0.5}\ s^{-1}$. Calculate the flow rate when a differential pressure of 200 Pa is obtained.

(Answer 0.0566 m^3/s)

6. FORCE SENSORS

The main types of force sensors are

- Mechanical types.
- Hydraulic types.
- Electrical strain gauge types.

6.1. MECHANICAL TYPES

Mechanical types are usually complete measuring systems involving some form of spring such as in a simple spring balance or bathroom scale. It is a basic mechanical principle that the deflection of a spring is directly proportional to the applied force so if the movement is shown on a scale, the scale represents force.

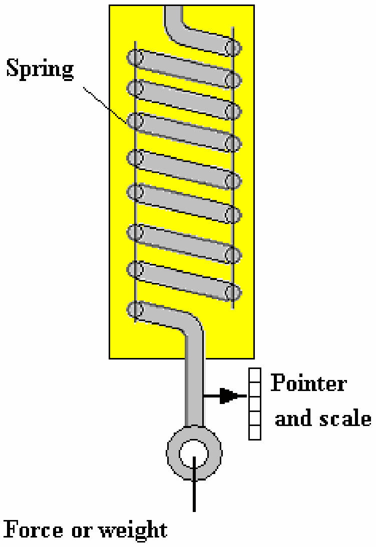


Figure 25

6.2. HYDRAULIC TYPES

Hydraulic types are often referred to as hydraulic load cells. The cell is a capsule filled with liquid. When the capsule is squeezed, the liquid becomes pressurised. The pressure represents the force and may be indicated with a calibrated pressure gauge. The capsule is often a short cylinder with a piston and the pressure produced is given by $p = F/A$ where F is the force and A the piston area.

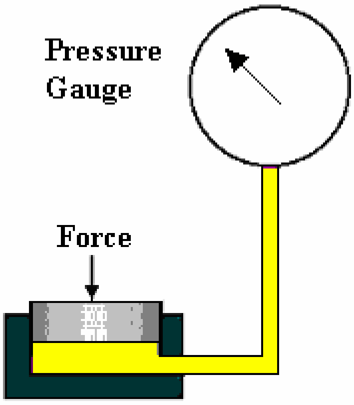


Figure 26

6.3 STRAIN GAUGE TYPE

A typical load cell consists of a metal cylinder with strain gauges fixed to it. When the cylinder is stretched or compressed, the strain gauges convert the force into a change in resistance and hence voltage. Since the elements require a supply voltage, the cell usually has 4 wires, two for the supply and two for the output.

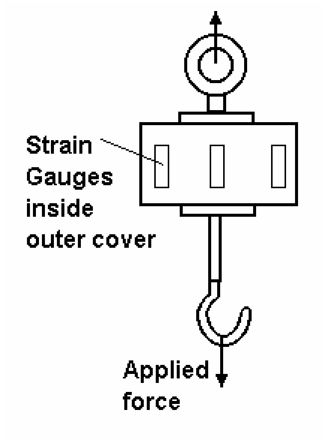


Figure 27

7. POSITION SENSORS

Position sensors are essential elements in the control of actuators. The position of both linear and rotary actuators is needed in robotic type mechanisms. There are three principle types.

- RESISTIVE
- OPTICAL
- INDUCTIVE

7.1. RESISTIVE TYPES

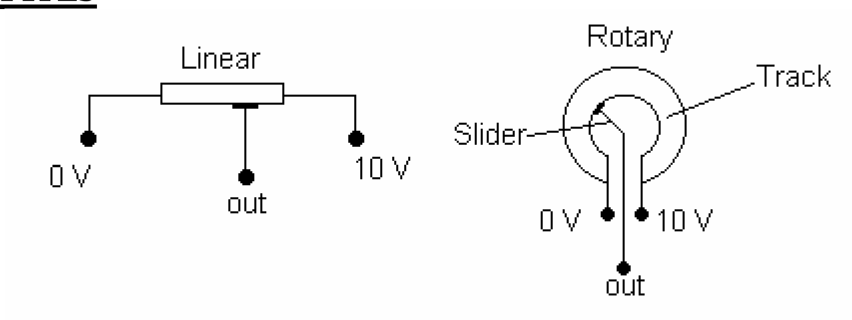


Figure 28

A potentiometer is a variable electrical resistance. A length of resistance material has a voltage applied over its ends. A slider moves along it (either linear or rotary) and picks off the voltage at its position or angle. The tracks may be made from carbon , resistance wire or piezo resistive material. The latter is the best because it gives a good analogue output. The wire wound type produces small step changes in the output depending on how fine the wire is and how closely it is coiled on the track.

7.2 OPTICAL TYPES

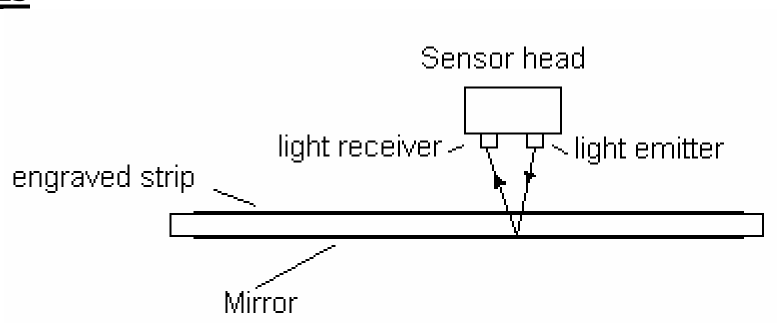


Figure 29

Optical types are mainly used for producing digital outputs. A common example is found on machine tools where they measure the position of the work table and display it in digits on the gauge head. Digital micrometers and verniers also use this idea. The basic principle is as follows. Light is emitted through a transparent strip or disc onto a photo electric cell. Often reflected light is used as shown. The strip or disc has very fine lines engraved on it which interrupt the beam. The number of interruptions are counted electronically and this represents the position or angle. This is very much over simplified and you should refer to more advanced text to find out how very accurate measurements are obtained and also the direction of movement.

7.3. INDUCTIVE TYPES

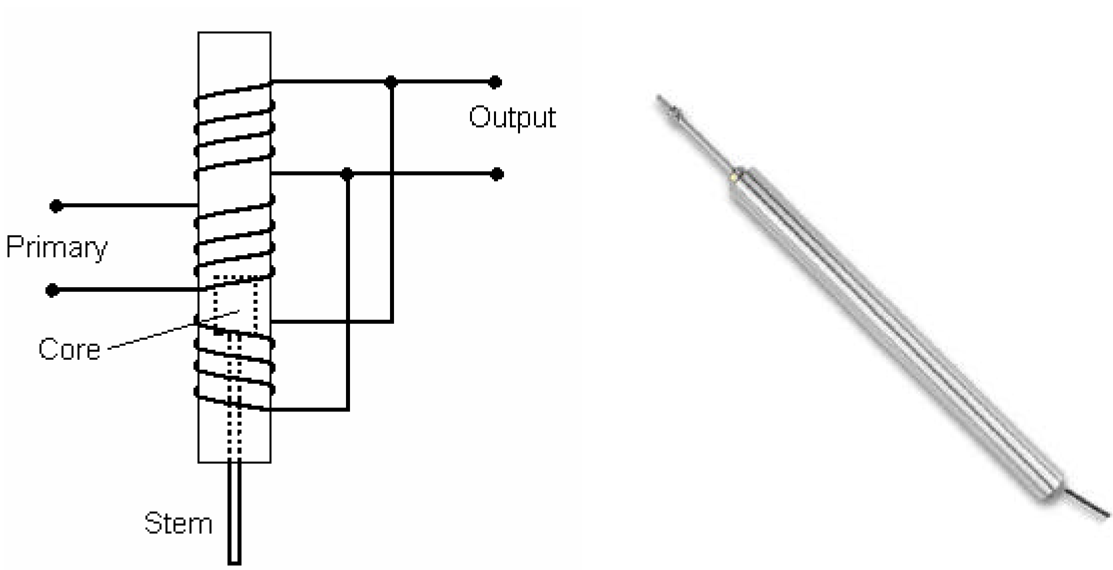


Figure 30

The most common of these is the Linear Variable Differential transformer or LVDT. The transformer is made with one primary coil and two secondary coils, one placed above and the other below the primary. The coils are formed into a long narrow hollow tube. A magnetic core slides in the tube and is attached to the mechanism being monitored with a non magnetic stem (e.g. brass). A constant alternating voltage is applied to the primary coil. This induces a voltage in both secondary coils. When the core is exactly in the middle, equal voltages are induced and when connected as shown, they cancel each other out. When the core moves, the voltage in one secondary coil grows but reduces in the other. The result is an output voltage which represents the position of the core and hence the mechanism to which it is attached. The output voltage is usually converted into D.C. With suitable electronic equipment for phase detection, it is possible to detect which direction the core moves and to switch the DC voltage from plus to minus as the core passes the centre position. These can be very accurate and are widely used for gauging the dimensions of machined components.

8. DEPTH GAUGES

Depth gauges measure the depth of liquids and powder in tanks. They use a variety of principles and produce outputs in electrical and pneumatic forms. The type to use depends on the substance in the tank. Here are a few.

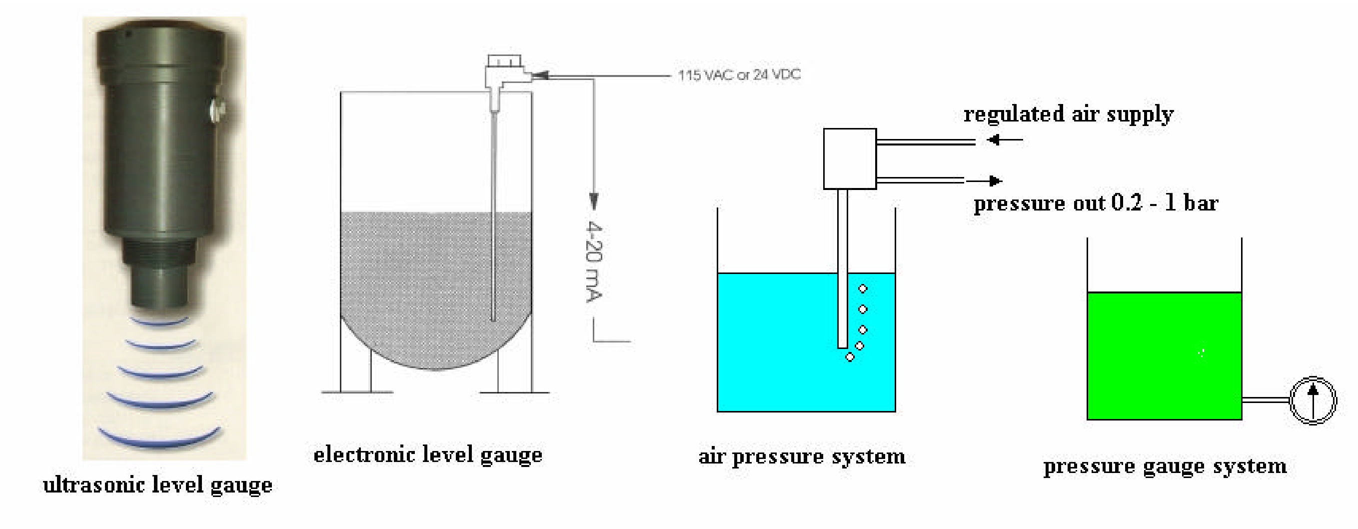


Figure 31

The ultrasonic system reflects sound waves from the surface and determines the depth from the time taken to receive the reflected sound. The electronic version uses a variety of electrical affects including conduction of the fluid and capacitance. The pneumatic version bubbles air through the liquid and the pressure of the air is related to the depth. A simple pressure gauge attached to a tank is also indicates the depth since depth is proportional to pressure.

9. STRAIN GAUGES

Strain gauges are used in many instruments that produce mechanical strain because of the affect being measured. In their own right, they are used to measure the strain in a structure being stretched or compressed.

The strain gauge element is a very thin wire that is formed into the shape shown. This produces a long wire all in one direction but on a small surface area. The element is often formed by etching a thin foil on a plastic backing. The completed element is then glued to the surface of the material or component that will be strained. The axis of the strain gauge is aligned with the direction of the strain. When the component is stretched or compressed, the length of the resistance wire is changed. This produces a corresponding change in the electrical resistance.

Let the length of the gauge be L and the change in length be ΔL.
The mechanical strain $\varepsilon = \Delta L/L$
Let the resistance of the gauge be R (typically 120 Ω) and the change in resistance be ΔR.
The electrical strain $\xi = \Delta R/R$.
The electrical and mechanical strain are directly proportional and the constant relating them is called the gauge factor (typically 2).

Gauge Factor = Electrical Strain/Mechanical strain = $\xi/\varepsilon = L\ \Delta R/R\ \Delta L$

WORKED EXAMPLE No.4

A strain gauge is glued to a structure. It has a gauge factor of 2.1 and a resistance of 120.2 Ω. The structure is stressed and the resistance changes to 120.25 Ω. Calculate the strain and convert this into stress. Take E = 205 GPa

SOLUTION

$\Delta R = 120.25 - 120.2 = 0.05\Omega$ $\quad \xi = \Delta R/R_1 = 0.05/120.2 = 4.16 \times 10^{-4}$

$\varepsilon = \xi/G = 4.16 \times 10^{-4}/2.1 = 1.981 \times 10^{-4}$ $\quad \sigma = \varepsilon E = 1.981 \times 10^{-4} \times 205 \times 10^{9} = 40.61$ MPa

STRAIN GAUGE ARRANGEMENTS

A strain gauge is of little use unless we can convert the change in resistance into a voltage. This is best done with a Wheatstone bridge.

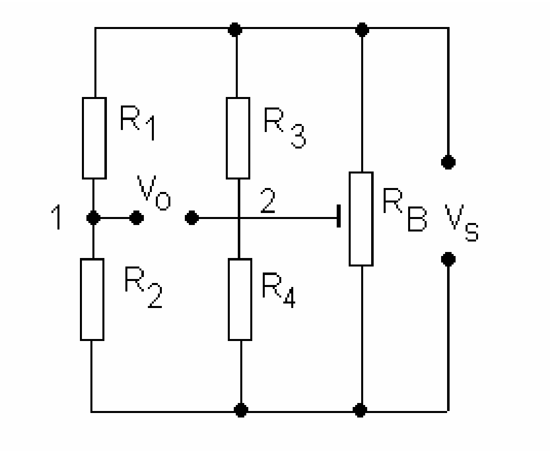


Figure 32

If only one active gauge is used, this would be R_1 or R_2. R_1 and R_2 must be equal, so must R_3 and R_4. In this case, the voltage at points 1 and 2 are equal to $V_s/2$ and so the output V_o is zero. In order to ensure this, the balancing resistor R_B is adjusted to make the output zero with no strain applied to the gauge. Suppose that R_1 is the active gauge. If the bridge is balanced then the voltage at points 1 and 2 is half the supply voltage. $V_1 = V_2 = V_s/2$

When R_1 changes its resistance by ΔR the voltage at point 1 becomes:

	$V_sR/(2R + \Delta R)$ (using ratio of resistances)
The output becomes	$V_o = V_2 - V_1 = V_s/2 - V_sR/(2R + \Delta R)$
	$V_o = V_s\,\Delta R/\{(4R + 2\Delta R)\}$
Dividing top and bottom by R we get	$V_o = V_s\,(\Delta R/R)/\{4 + 2\Delta R/R\}$
The gauge factor is defined as	G = electrical strain/mechanical strain
	$G = (\Delta R/R)/\varepsilon$ so $(\Delta R/R) = G\varepsilon$
Substituting we get	$V_o = V_s\,G\varepsilon/\{4 + 2G\varepsilon\}$

WORKED EXAMPLE No.5

Four strain gauges are formed into bridge with only one active gauge. The nominal resistance of all of them is 120 Ω. The gauge factor is 2.1 and the supply voltage is 10 V. Calculate the strain when the output from the bridge is 20 mV.

SOLUTION

$V_o = V_s\,G\varepsilon/\{4 + 2G\varepsilon\}$ $\quad \varepsilon = 4V_o \div G(V_s - 2V_o) = (4 \times 0.02) \div \{2.1(10 - 0.04)\} = 3.825 \times 10^{-3}$

TEMPERATURE EFFECTS

One of the problems with strain gauges is that the resistance also changes with temperature and so it is vital that each pair of resistors is maintained at the same temperature.

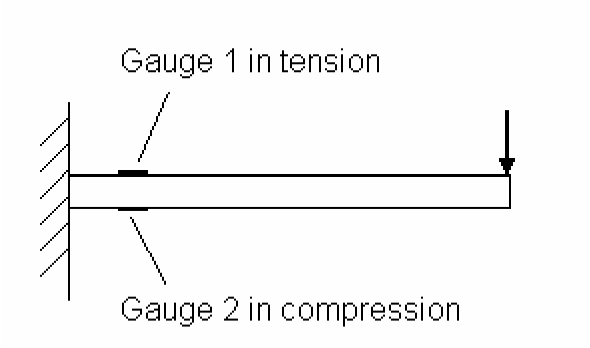


Figure 33

If one active gauge is used, say R_1, then the other resistor R_2 must be placed near to it and this is best done by using a DUMMY GAUGE fixed close to the active gauge but in a position where it is unstrained. Better still, make R_2 another active gauge and so double the output from the bridge. For example, if a beam is used to produce the strain, one gauge is placed on top and the other on the bottom as shown. Let R_1 increase and R_2 decrease by ΔR. The voltage at point 1 becomes

$V_s(R - \Delta R)/2R$ (using ratio of resistances)

The output becomes $V_o = V_2 - V_1 = V_s/2 - V_s(R - \Delta R)/2R$

$V_o = V_s \Delta R/\{2(2R + \Delta R)\}$

Dividing top and bottom by R we get $V_o = V_s \Delta R/2R$

$V_o = V_s G\varepsilon/2$ which is almost double the output.

If the load cell only produces tension or compression, the active gauges are R_1 and R_4 with R_2 and R_3 being dummy gauges. All 4 gauges are then at the same temperature. This is shown in the diagram.

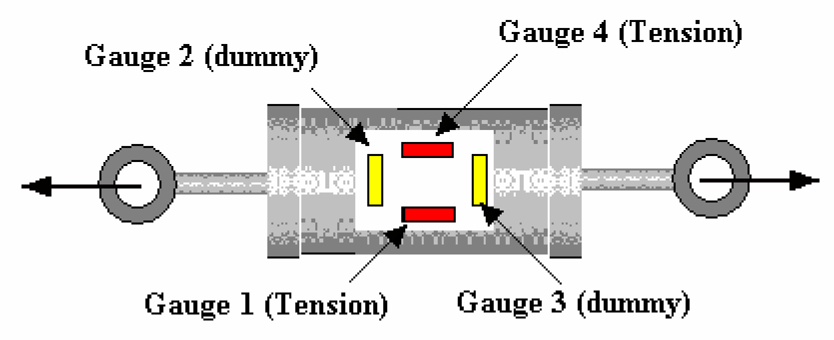


Figure 34

The voltage at point 1 becomes $V_s R/(2R + \Delta R)$

and at point 2 becomes $V_s(R + \Delta R)/(2R + \Delta R)$

The output becomes $V_o = V_2 - V_1 = V_s \Delta R/(2R + \Delta R)$

Dividing top and bottom by R we get $V_o = V_s (\Delta R/R)/\{2 + \Delta R/R\}$

$V_o = V_s G\varepsilon/(2+G\varepsilon)$

This is double the output of a single active gauge and fully temperature stable.

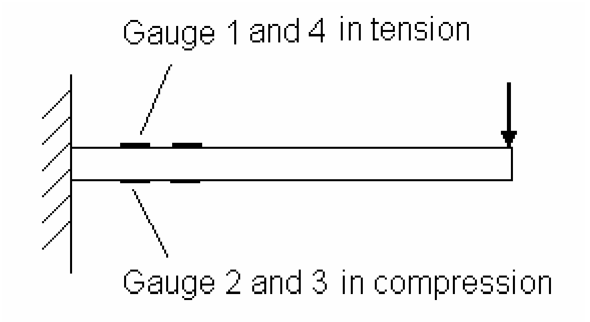


Figure 35

If a beam is used in the load cell, all 4 gauges may be made active as shown.

The output at point 1 becomes

$V_1 = V_s(R-\Delta R)/2R$

and at point 2 becomes

$V_2 = V_s(R + \Delta R)/2R$

The output becomes $V_o = V_2 - V_1 = V_s\Delta R/R$ $\quad V_o = V_s G\varepsilon$

This is 4 times the output of a single active gauge and fully temperature stable.

SELF ASSESSMENT EXERCISE No.3

1. A strain gauge is glued to a structure. It has a gauge factor of 2.1 and a resistance of 120.2 Ω. The structure is stressed and the resistance changes to 120.25 Ω. Calculate the strain and convert this into stress.

   Take E = 205 GPa

   (Answer 40.6 MPa)

2. A strain gauge has a resistance of 120.6 Ohms at 20^{o}C. Calculate its resistance at 30^{o}C.
   $\alpha = 8 \times 10^{-6}$ Ω/Ω^{o}C.

   (Answer 120.61 Ω)

3. Describe how to eliminate temperature error in a strain gauge bridge when it has

   a. one active gauge.
   b. two active gauges.

4. A STRAIN GAUGE has a gauge factor of 2.2. It is glued to tensile test piece and the resistance before straining is 119.8 Ω. The test piece is stretched and the resistance goes up to 120 Ω. Calculate the following. The modulus of elasticity E for the test piece is 200 GPa.

   i. The strain in the test piece. (7.588×10^{-4})

   ii. The stress in the test piece. (15.18 MPa)

SELF ASSESSMENT EXERCISE No.4

1. State what each of the sensors below measures (flow, temperature and so on)

a. Thermocouple.
b. Potentiometer.
c. Thermistor.
d. Optical fringes.
e. Venturi meter.
f. Pitot tube.
g. Bimetallic type.
h. Platinum resistance probe.
i. D.C. type generator.
j. L.V.D.T.
k. Bourdon tube.
l. Orifice meter.
m. Piezo electric.

2. State two types of sensors that could be used to measure each of the following.

a. Speed of revolution.
b. Flow rate of liquids.
c. Pressure.
d. Temperature.

Module 5: Sensors and signal processing
Lecture 6
Signal Conditioning Devices

Signal Conditioning Operations

In previous lectures we have studied various sensors and transducers used in a mechatronics system. Transducers sense physical phenomenon such as rise in temperature and convert the measurand into an electrical signal viz. voltage or current. However these signals may not be in their appropriate forms to employ them to control a mechatronics system. Figure 2.6.1 shows various signal conditioning operations which are being carried out in controlling a mechatronics based system. The signals given by a transducer may be nonlinear in nature or may contain noise. Thus before sending these signals to the mechatronics control unit it is essential to remove the noise, nonlinearity associated with the raw output from a sensor or a transducer. It is also needed to modify the amplitude (low/high) and form (analogue/digital) of the output signals into respective acceptable limits and form which will be suitable to the control system. These activities are carried out by using signal conditioning devices and the process is termed as ‘signal conditioning’.

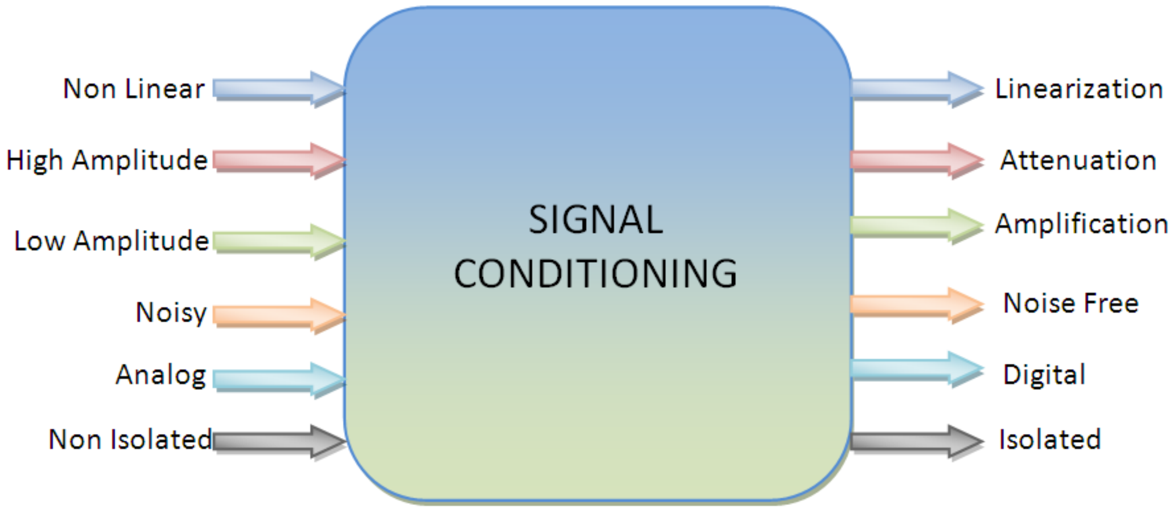


Figure 2.6.1 Signal conditioning operations

Signal conditioning system enhances the quality of signal coming from a sensor in terms of:

1. Protection
To protect the damage to the next element of mechatronics system such microprocessors from the high current or voltage signals.

2. Right type of signal
To convert the output signal from a transducer into the desired form i.e. voltage / current.

3. Right level of the signal
To amplify or attenuate the signals to a right /acceptable level for the next element.

4. Noise
To eliminate noise from a signal.

5. Manipulation
To manipulate the signal from its nonlinear form to the linear form.

1. Amplification/Attenuation

Various applications of Mechatronics system such as machine tool control unit of a CNC machine tool accept voltage amplitudes in range of 0 to 10 Volts. However many sensors produce signals of the order of milli volts. This low level input signals from sensors must be amplified to use them for further control action. Operational amplifiers (op-amp) are widely used for amplification of input signals. The details are as follows.

1.1 Operational amplifier (op-amp)

Operational Amplifier is a basic and an important part of a signal conditioning system. It is often abbreviated as op-amp. Op-amp is a high gain voltage amplifier with a differential input. The gain is of the order of 100000 or more. Differential input is a method of transmitting information with two different electronic signals which are generally complementary to each other. Figure 2.6.2 shows the block diagram of an op-amp. It has five terminals. Two voltages are applied at two input terminals. The output terminal provides the amplified value of difference between two input voltages. Op-amp works by using the external power supplied at *Vs+* and *Vs-* terminals.

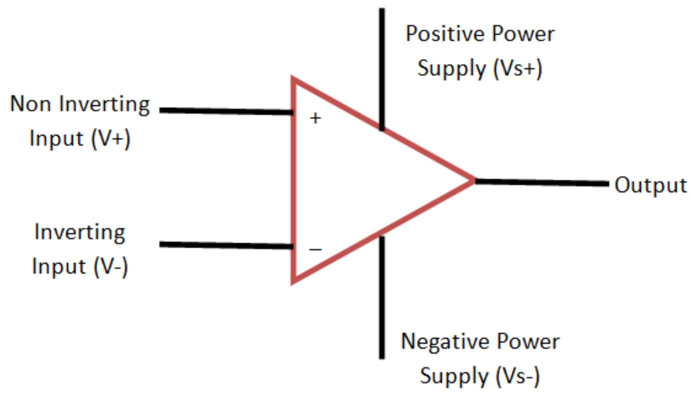


Figure 2.6.2 circuit diagram of an Op-amp

In general op-amp amplifies the difference between input voltages (V+ and V-). The output of an operational amplifier can be written as

$$V_{out} = G * (V+ - V-) \qquad (2.6.1)$$

where G is Op-amp Gain.

Figure 2.6.3 shows the inverting configuration of an op-amp. The input signal is applied at the inverting terminal of the op-amp through the input resistance $R_{in.}$ The non-inverting terminal is grounded. The output voltage (V_{out}) is connected back to the inverting input terminal through resistive network of R_{in} and feedback resistor R_f. Now at node a, we can write,

$$I_1 = V_{in}/R_1 \qquad (2.6.2)$$

The current flowing through R_f is also I_1, because the op-amp is not drawing any current. Therefore the output voltage is given by,

$$V_{out} = -I_1 R_f = -V_{in} R_f/R_1 \qquad (2.6.3)$$

Thus the closed loop gain of op-amp can be given as,

$$G = V_{out}/V_{in} = -R_f/R_1 \qquad (2.6.4)$$

The negative sign indicates a phase shift between V_{in} and V_{out}.

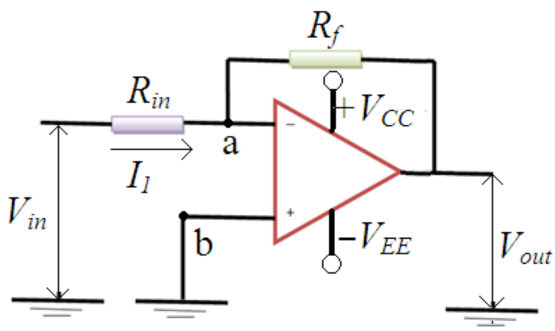


Figure 2.6.3 Inverting op-amp

1.2 Amplification of input signal by using Op-amp

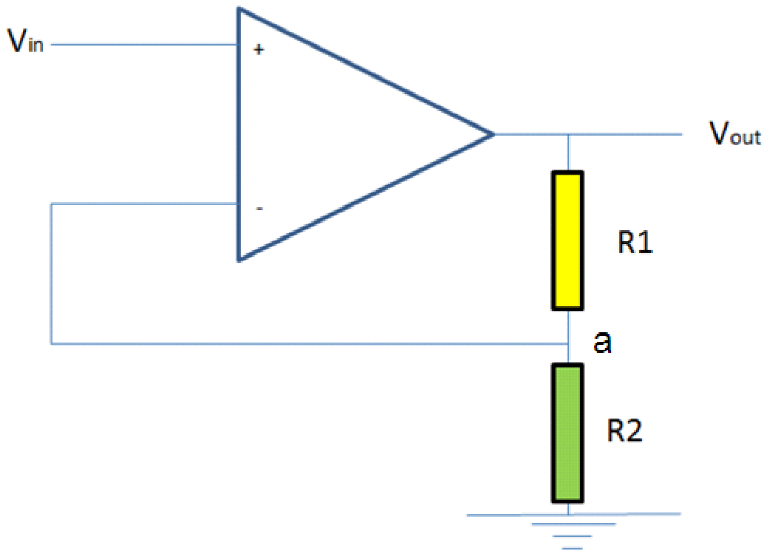


Figure 2.6.4 Amplification using an Op-amp

Figure 2.6.4 shows a configuration to amplify an input voltage signal. It has two registers connected at node a. If we consider that the voltage at positive terminal is equal to voltage at negative terminal then the circuit can be treated as two resistances in series. In series connection of resistances, the current flowing through circuit is same. Therefore we can write,

$$\frac{V_{out}-V_{in}}{R_1} = \frac{V_{in}-0}{R_2} \qquad (2.6.5)$$

$$\frac{V_{out}-V_{in}}{R_1} = \frac{V_{in}}{R_2} \qquad (2.6.6)$$

Thus by selecting suitable values of resistances, we can obtain the desired (amplified/attenuated) output voltage for known input voltage.

There are other configurations such as Non-inverting amplifier, Summing amplifier, Subtractor, Logarithmic amplifier are being used in mechatronics applications. The detail study of all these is out of scope of the present course. Readers can refer Bolton for more details.

2. Filtering

Output signals from sensors contain noise due to various external factors like improper hardware connections, environment etc. Noise gives an error in the final output of system. Therefore it must be removed. In practice, change in desired frequency level of output signal is a commonly noted noise. This can be rectified by suing filters. Following types of filters are used in practice:

1. Low Pass Filter
2. High Pass Filter
3. Band Pass Filter
4. Band Reject Filter

2.1 Low Pass Filter

Low pass filter is used to allow low frequency content and to reject high frequency content of an input signal. Its configuration is shown in Figure 2.6.5

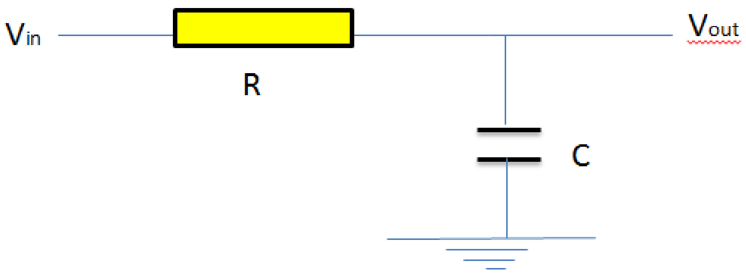


Figure 2.6.5 Circuitry of Low Pass Filter

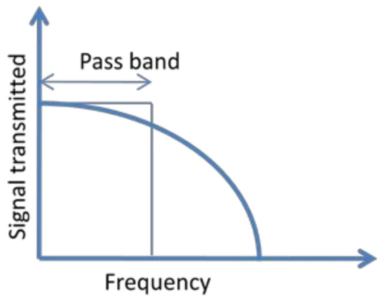


Figure 2.6.6 Pass band for low pass filter

In the circuit shown in Figure 2.6.5, resistance and capacitance are in series with voltage at resistance terminal is input voltage and voltage at capacitance terminal is output voltage. Then by applying the Ohm's Law, we can write,

$$V_{out} = \frac{\frac{1}{j\omega C}}{R + \left(\frac{1}{j\omega C}\right)} V_{in} \qquad (2.6.7)$$

$$V_{out} = \frac{1}{1 + j\omega CR} V_{in} \qquad (2.6.8)$$

From equation 2.6.8 we can say that if frequency of Input signal is low then $j\omega CR$ would be low. Thus $\frac{1}{1+j\omega CR}$ would be nearly equal to 1. However at higher frequency $j\omega CR$ would be higher, then $\frac{1}{1+j\omega CR}$ would be nearly equal to 0. Thus above circuit will act as Low Pass Filter. It selects frequencies below a breakpoint frequency ω = 1/RC as shown in Figure 2.6.6. By selecting suitable values of R and C we can obtain desired values of frequency to pass in.

2.2 High Pass Filter

These types of filters allow high frequencies to pass through it and block the lower frequencies. The figure 2.6.7 shows circuitry for high pass filter.

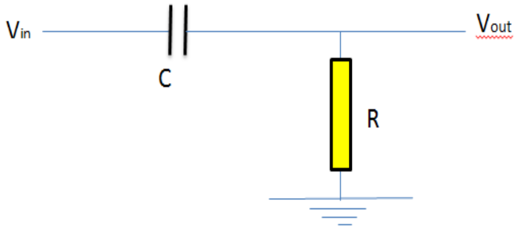


Figure 2.6.7 Circuitry of High Pass Filter

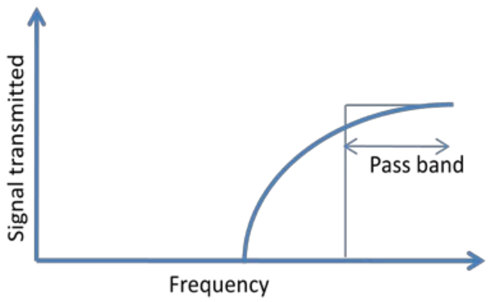


Figure 2.6.8 Pass band for high pass filter

$$V_{out} = \frac{R}{R + \left(\frac{1}{j\omega C}\right)} V_{in} \qquad (2.6.9)$$

$$V_{out} = \frac{j\omega CR}{1 + j\omega CR} V_{in} \qquad (2.6.10)$$

From equation 2.6.10, we can say that if frequency of input signal is low then $\frac{1}{j\omega C}$ would be high and thus $\frac{R}{R+\left(\frac{1}{j\omega C}\right)}$ would be nearly equal to 0. For high frequency signal, $\frac{1}{j\omega C}$ would be low and $\frac{R}{R+\left(\frac{1}{j\omega C}\right)}$ would be nearly equal to 1. Thus above circuit will act as High Pass Filter. It selects frequencies above a breakpoint frequency ω = 1/RC as shown in Figure 2.6.8. By selecting suitable values of R and C we can allow desired (high) frequency level to pass through.

2.3 Band Pass Filter

In some applications, we need to filter a particular band of frequencies from a wider range of mixed signals. For this purpose, the properties of low-pass and high-pass filters circuits can be combined to design a filter which is called as band pass filter. Band pass filter can be developed by connecting a low-pass and a high-pass filter in series as shown in figure 2.6.9.

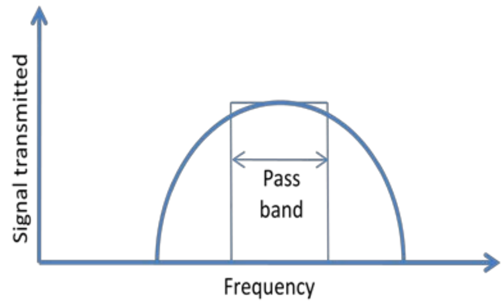


Figure 2.6.9 Band pass filter

2.4 Band Reject Filter

These filters pass all frequencies above and below a particular range set by the operator/manufacturer. They are also known as band stop filters or notch filters. They are constructed by connecting a low-pass and a high-pass filter in parallel as shown in Figure 2.6.10.

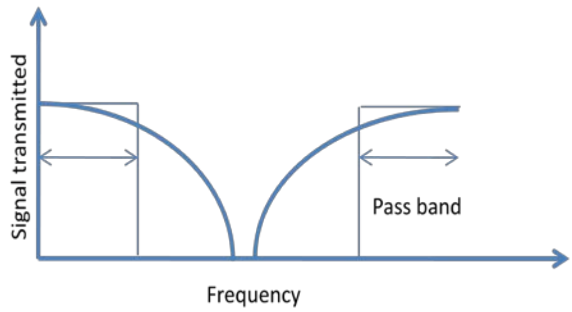


Figure 2.6.10 Band reject filter

Quiz

1. Explain the principle of working of op-amp as an inverting amplifier.
2. What kind of signal conditioning operations will be required to develop a table top CNC turning center for small job works?

Module 5: Sensors and signal processing
Lecture 7
Protection, conversion and pulse width modulation

1. Protection

In many situations sensors or transducers provide very high output signals such as high current or high voltage which may damage the next element of the control system such as microprocessor.

1.1 Protection from high current

The high current to flow in a sensitive control system can be limited by:

1. Using a series of resistors
2. Using fuse to break the circuit if current value exceeds a preset or safe value

1.2 Protection from high voltage

Zener diode circuits are widely used to protect a mechatronics control system from high values of voltages and wrong polarity. Figure 2.7.1 shows a typical Zener diode circuit.

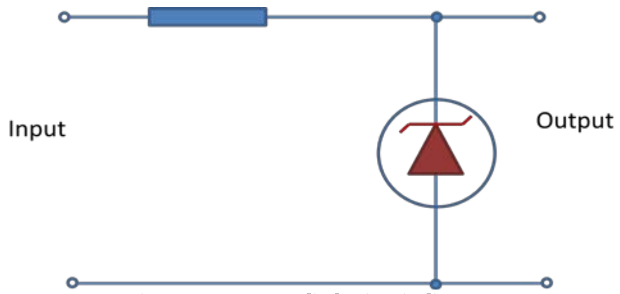


Figure 2.7.1 Zener diode circuit diagram

Zener diode acts as ordinary or regular diodes upto certain breakdown voltage level when they are conducting. When the voltage rises to the breakdown voltage level, Zener diode breaks down and stops the voltage to pass to the next circuit.

Zener diode as being a diode has low resistance for current to flow in one direction through it and high resistance for the opposite direction. When connected in correct polarity, a high resistance produces high voltage drop. If the polarity reverses, the diode will have less resistance and therefore results in less voltage drop.

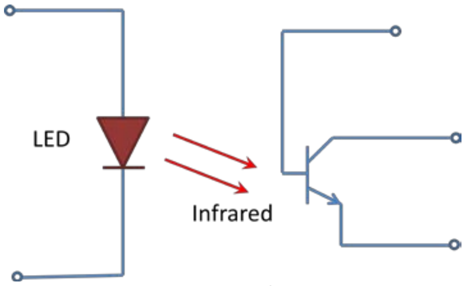


Figure 2.7.2 Schematic of an Optoisolator

In many high voltage scenarios, it is required to isolate the control circuit completely from the input high voltages to avoid the possible damage. This can be achieved by Optoisolators. Figure 2.7.2 shows the typical circuit of an Optoisolator. It comprises of a Light emitting diode (LED) and a photo transistor. LED irradiates infra red due to the voltage supplied to it from a microprocessor circuit. The transistor detects irradiation and produces a current in proportion to the voltage applied. In case of high voltages, output current from Optoisolator is utilized for disconnecting the power supply to the circuit and thus the circuit gets protected.

2. Wheatstone bridge

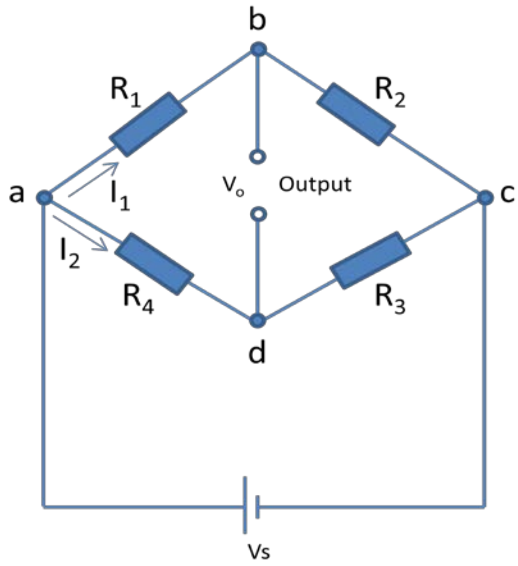


Figure 2.7.3 Configuration of a Wheatstone bridge

Wheatstone bridge is used to convert a resistance change detected by a transducer to a voltage change. Figure 2.7.3 shows the basic configuration of a Wheatstone bridge. When the output voltage *Vout* is zero then the potential at B must be equal to D and we can say that,

$$Vab = Vad, \qquad (2.7.1)$$
$$I1\,R\,1 = I2\,R2 \qquad (2.7.2)$$

Also,

$Vbc = Vdc,$ (2.7.3)

$I1\,R2 = I2\,R4$ (2.7.4)

Dividing equation 2.7.2 by 2.7.4,

$R1/R2 = R3/R4$ (2.7.5)

The bridge is thus balanced.

The potential drop across $R1$ due to supply voltage Vs,

$Vab = Vs\,R1/(R1 + R2)$ (2.7.6)

Similarly,

$Vad = VsR3/(R3 + R4)$ (2.7.7)

Thus the output voltage Vo is given by,

$Vo = Vab - Vad$ (2.7.8)

$Vo = Vs\,\{(R1/[R1 + R2]) - (R3/[R3 + R4])\}$ (2.7.9)

When $Vo = 0$, above equation gives balanced condition.

Assume that a transducer produces a resistance change from $R1$ to $R1 + \delta R1$ which gives a change in output from $Vo + \delta Vo$,

From equation 2.7.9 we can write,

$$Vo + \delta Vo = Vs\left(\frac{R1+\delta R1}{R1+\delta R1+R2} - \frac{R3}{R3+R4}\right) \qquad (2.7.10)$$

Hence,

$$(Vo + \delta Vo) - Vo = Vs\left(\frac{R1+\delta R1}{R1+\delta R1+R2} - \frac{R1}{R1+R2}\right) \qquad (2.7.11)$$

If $\delta R1$ is much smaller than $R1$ the equation 2.7.11 can be written as

$$\delta Vo \approx Vs\left(\frac{\delta R1}{R1+R2}\right) \qquad (2.7.12)$$

We can say that change in resistance $R1$ produces a change in output voltage. Thus we can convert a change in resistance signal into voltage signal.

3. Pulse modulation

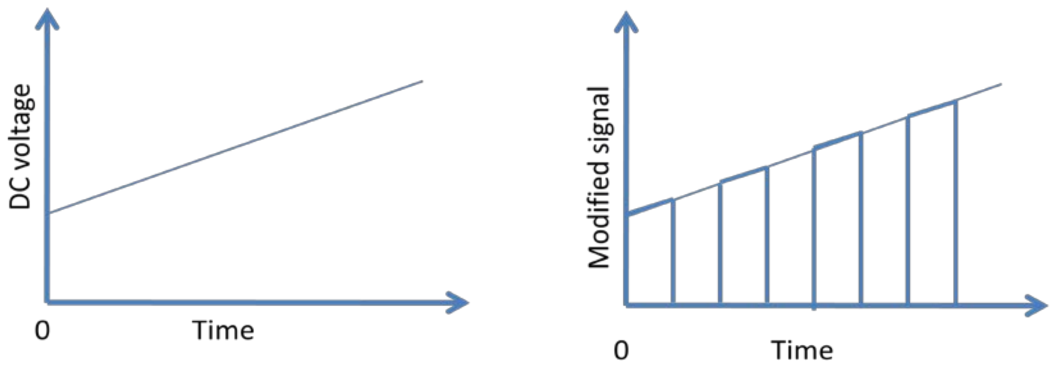


Figure 2.7.4 Pulse amplitude modulation

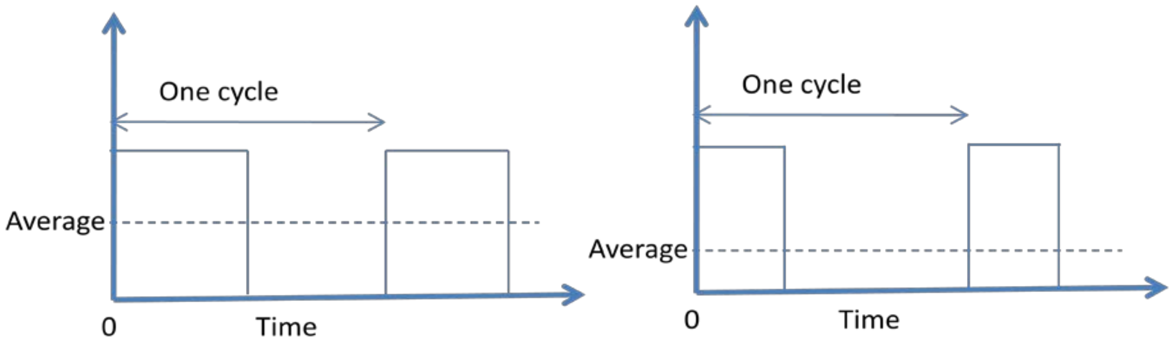


Figure 2.7.5 Pulse width modulation

During amplification of low level DC signals from a sensor by using Op-amp, the output gets drifted due to drift in the gain of Op-amp. This problem is solved by converting the analogue DC signal into a sequence of pulses. This can be achieved by chopping the DC signal in to a chain of pulses as shown in Figure 2.7.4. The heights of pulses are related to the DC level of the input signal. This process is called as Pulse Width Modulation (PWM). It is widely used in control systems as a mean of controlling the average value of the DC voltage. If the width of pulses is changed then the average value of the voltage can be changed as shown in Figure 2.7.5. A term Duty Cycle is used to define the fraction of each cycle for which the voltage is high. Duty cycle of 50% means that for half of the each cycle, the output is high.

Quiz:

1. State the applications of Wheatstone bridge in Mechatronics based Manufacturing Automation. Explain one of them in detail.
2. Why do we need pulse width modulation?
3. How Zener diode is different than ordinary diode?

Module 5: Sensors and signal processing

Lecture 8

Data conversion devices

Data Conversion Devices are very important components of a Machine Control Unit (MCU). MCUs are controlled by various computers or microcontrollers which are accepting signals only in Digital Form i.e. in the form of 0s and 1s, while the signals received from signal conditioning module or sensors are generally in analogue form (continuous). Therefore a system is essentially required to convert analog signals into digital form and vis-à-vis. Analog to Digital Converter is abbreviated as ADC. Figure 2.8.1 shows a typical control system with data conversion devices.

Based on the signals received from sensors, MCU generates actuating signals in the Digital form. Most of the actuators e.g. DC servo motors only accept analogue signals. Therefore the digital signals must be converted into Analog form so that the required actuator can be operated accordingly. For this purpose Digital to Analog Converters are used, which are abbreviated as DACs. In subsequent sections we will be discussing about various types of ADC and DAC devices, their principle of working and circuitry.

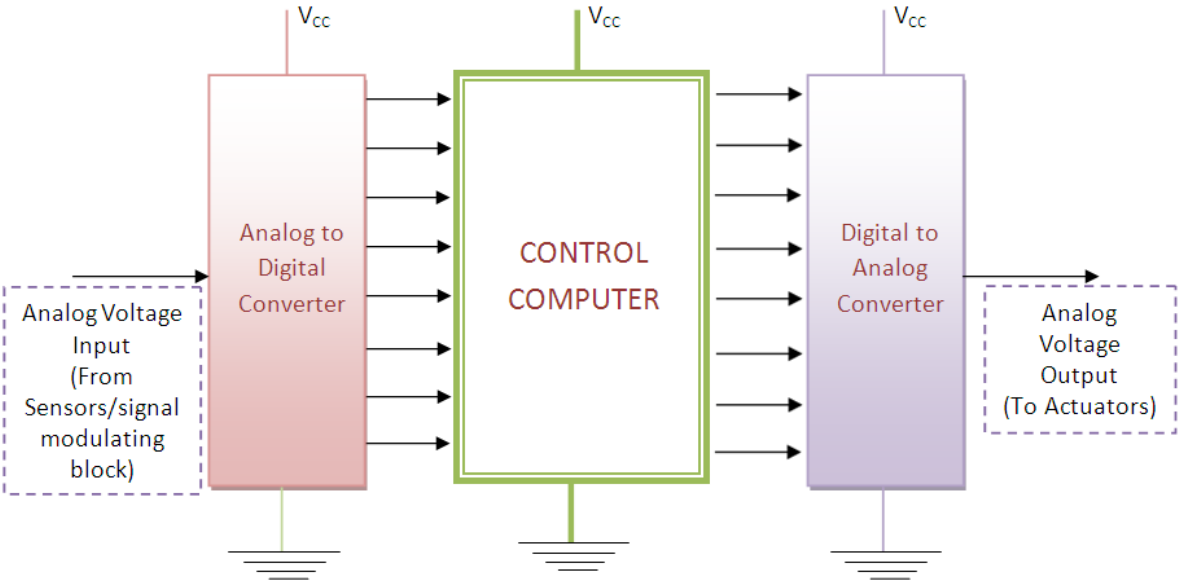


Figure 2.8.1 A control system with ADC and DAC devices

Basic components used in ADCs and DACs

1. Comparators

In general ADCs and DACs comprise of Comparators. Comparator is a combination of diodes and Operational Amplifiers. A comparator is a device which compares the voltage input or current input at its two terminals and gives output in form of digital signal i.e. in form of 0s and 1s indicating which voltage is higher. If V+ and V- be input voltages at two terminals of comparator then output of comparator will be as

$$V+ > V- \rightarrow \text{Output } 1$$

$$V+ < V- \rightarrow \text{Output } 0$$

2. Encoders

Though the output obtained from comparators are in the form of 0s and 1s, but can't be called as binary output. A sequence of 0s and 1s will be converted into binary form by using a circuit called Encoder. A simple encoder converts 2^n input lines into 'n' output lines. These 'n' output lines follow binary algebra.

3. Analog to Digital Converter (ADC)

As discussed in previous section ADCs are used to convert analog signals into Digital Signals. There are various techniques of converting Analog Signals into Digital signals which are enlisted as follows. However we will be discussing only Direct Conversion ADC, detail study of other techniques is out of the scope of the present course.

1. Direct Conversion ADC or Flash ADC
2. Successive Approximation ADC
3. A ramp-compare ADC
4. Wilkinson ADC
5. Integrating ADC
6. Delta-encoded ADC or counter-ramp
7. Pipeline ADC (also called subranging quantizer)
8. Sigma-delta ADC (also known as a delta-sigma ADC)
9. Time-interleaved ADC

3.1 Direct Conversion ADC or Flash ADC

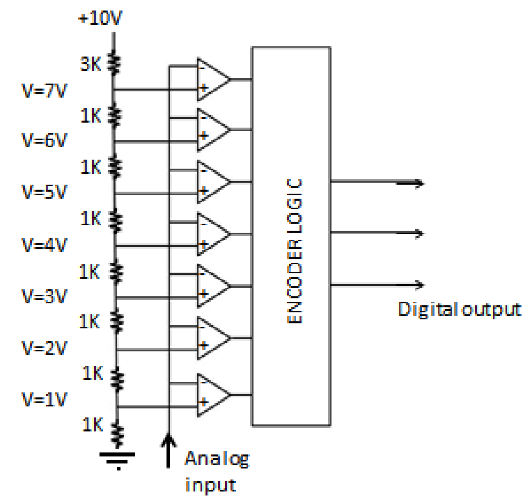


Figure 2.8.2 Circuit of Flash ADC

Figure 2.8.2 shows the circuit of Direct conversion or Flash ADC. To convert a digital signal of N-bits, Flash ADC requires 2^N-1 comparators and 2^N resistors. The circuit provides the reference voltage to all the comparators. Each comparator gives an output of 1 when its analog voltage is higher than reference voltage or otherwise the output is 0. In the above circuit, reference voltages to comparators are provided by means of resistor ladder logic.

The circuit described in figure 2.8.2 acts as 3 Bit ADC device. Let us assume this ADC works between the range of 0-10 Volts. The circuit requires 7 comparators and 8 resisters. Now the voltages across each resistor are divided in such a way that a ladder of 1 volt is built with the help of 1K-Ohm resistances. Therefore the reference voltages across all the comparators are 1-7 volts.

Now let us assume that an input voltage signal of 2.5 V is to be converted into its related digital form. As 2.5V is greater than 1V and 2V, first two comparators will give output as 1, 1. But 2.5V is less than 3,4,5,6,7 V values therefore all other comparators will give 0s. Thus we will have output from comparators as 0000011 (from top). This will be fed to the encoder logic circuit. This circuit will first change the output in single high line format and then converts it into 3 output lines format by using binary algebra. Then this digital output from ADC may be used for manipulation or actuation by the microcontrollers or computers.

4. Digital to Analog Converters

As discussed in previous section DACs are used to convert digital signals into Analog Signals. There are various techniques of converting Digital Signals into Analog signals which are as follows however we will be discussing only few important techniques in detail:

1. Pulse-width modulator
2. Oversampling DACs or interpolating DACs
3. The binary-weighted DAC
4. Switched resistor DAC
5. Switched current source DAC
6. Switched capacitor DAC
7. The R-2R ladder
8. The Successive-Approximation or Cyclic DAC,
9. The thermometer-coded DAC

4.1 Binary Weighted DAC

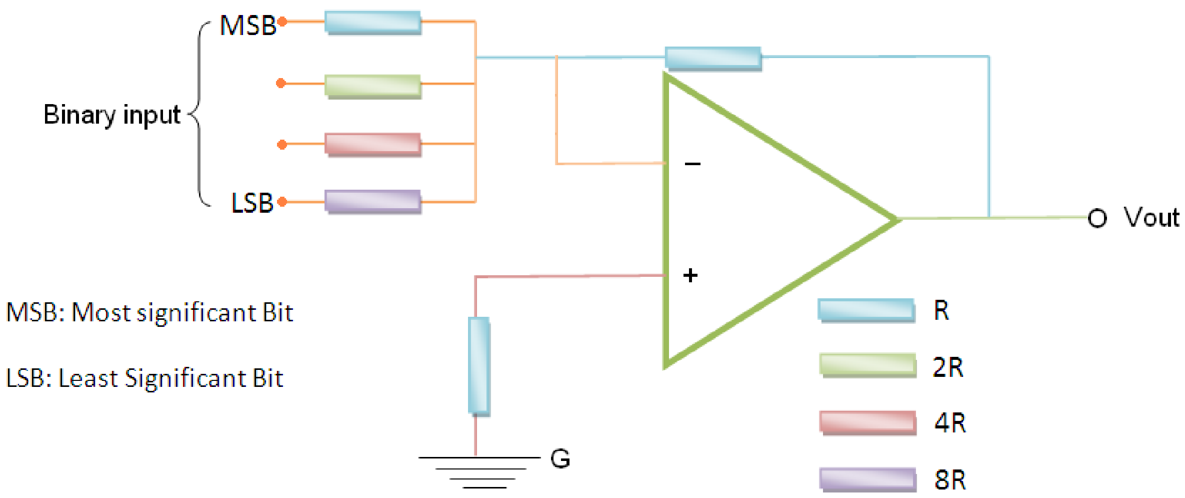


Figure 2.8.3 Circuit of binary weighted DAC

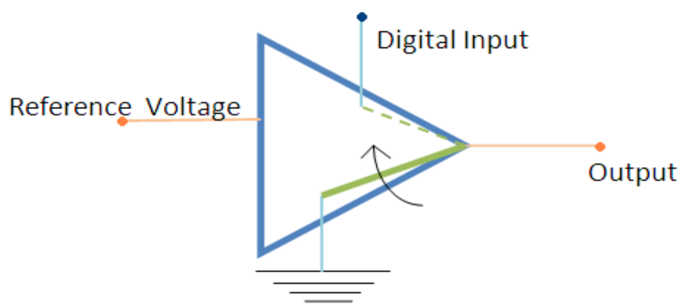


Figure 2.8.4 An op-amp used in DAC

As name indicates, in binary weighted DAC, output voltage can be calculated by expression which works on binary weights. Its circuit can be realized in Figure 2.8.3. From the figure it can be noted that most significant bit of digital input is connected to minimum resistance and vice versa. Digital bits can be connected to resistance through a switch which connects resistance-end to the ground. The digital input is zero when former bit is connected to reference voltage and if it is 1. This can be understood from Figure 2.8.4. DAC output voltage can be calculated from property of operational amplifiers. If V1 be input voltage at MSB (most significant bit), V2 be input voltage at next bit and so on then for four bit DAC we can write,

$$\frac{V1}{R}+\frac{V2}{2R}+\frac{V3}{4R}+\frac{V4}{8R}=\frac{Vout}{R} \qquad (2.8.1)$$

Note: Here V1,V2 V3,V4 will be Vref if digital input is 1 or otherwise it will be zero.

Hence output voltage can be found as:

$$V_{OUT}\ \alpha\ (2^3 * V1 + 2^2 * V2 + 2^1 * V3 + 2^0 * V4) \qquad (2.8.2)$$

However Binary weighted DAC doesn’t work for multiple or higher bit systems as the value of resistance doubles in each case.

Thus simple and low bit digital signals from a transducer can be converted into a related continuous value of voltages (analogue) by using binary weighted DAC. These will further be used for manipulation or actuation.

4.2 R-2R Ladder based DAC

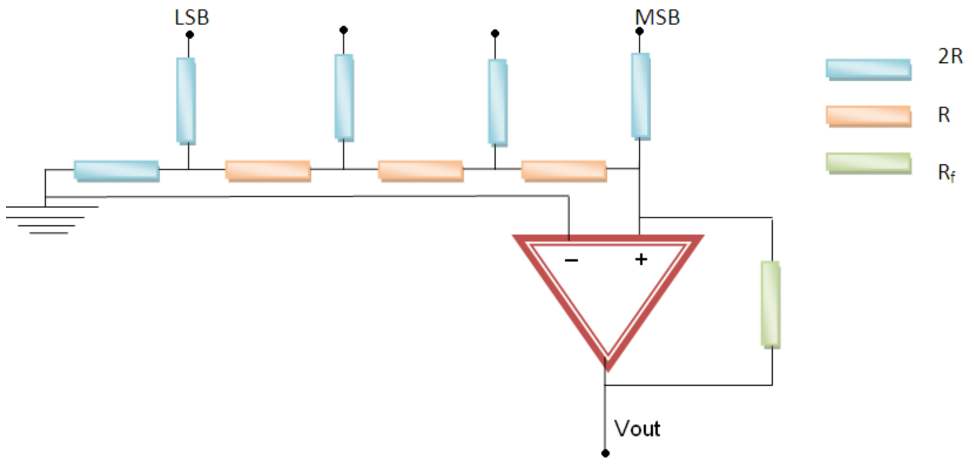


Figure 2.8.5 R-2R Ladder based DAC

In R-2R ladder logic, shortcoming of Binary Logic has been removed by making the value of maximum resistance double however the rest of the circuit remains same. Figure 2.8.5 shows the circuit of R-2R Ladder based DAC. If we apply voltage division rule in above case, then we can calculate that output voltage as,

$$V_{out} = \frac{V_{ref} * R_f}{R} * VAL \qquad (2.8.3)$$

Where VAL can be calculated from the digital signal input as,

$$VAL = \frac{D_0}{2^4} + \frac{D_1}{2^3} + \frac{D_2}{2^2} + \frac{D_3}{2^3} \qquad (2.8.4)$$

In this way output voltage is obtained by converting the digital signals received from microprocessor/ microcontroller. These voltages will further be used to actuate the desired actuator viz. DC/AC motors.

In this module we have studied the principle of operation of various sensors which are commonly used in mechatronics and manufacturing automation. Also the signal conditioning operations and the devices which are used to generate the proper signals for desired automation application have been studied. In the next module we will study the construction and working of microprocessor and the devices which are being used in controlling the various operations of automation using the microprocessors.

Quiz

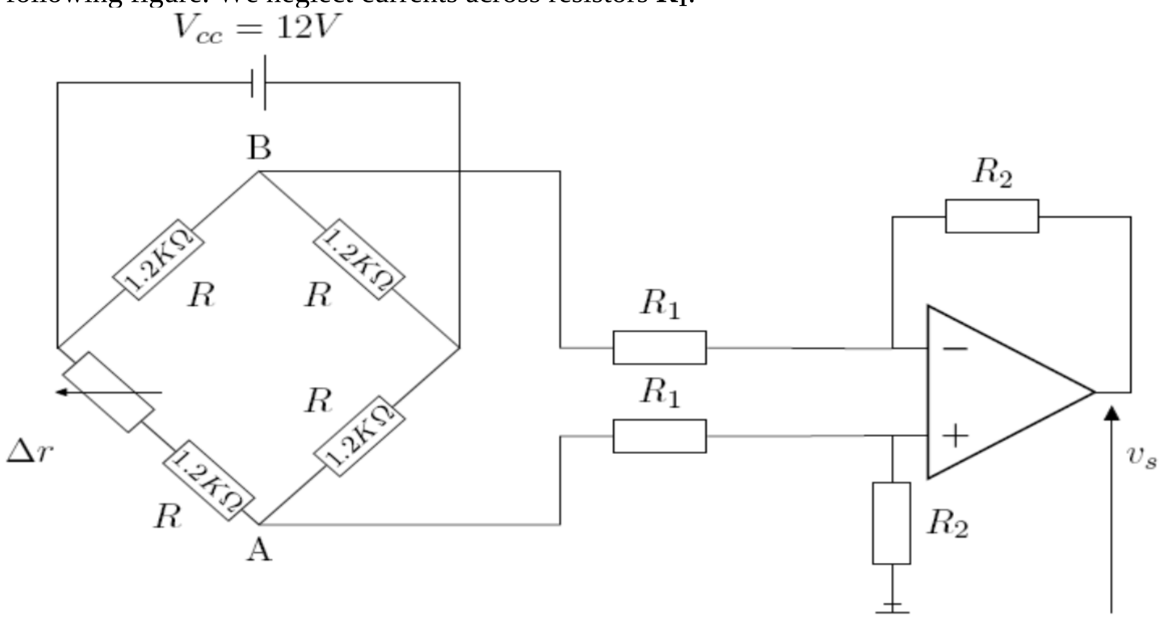
1. Differentiate between Binary weighted DAC and R-2R ladder based DAC.
2. Explain the importance of data conversion devices in mechatronics with suitable example.

Exercise 1:

A platinum resistance thermometer has a resistance $\mathbf{r_0 = 100\Omega}$ at θ = **0°C** and $\mathbf{r_0 = 138\Omega}$ at **θ= 100°C.**

1. Let **Δr** be the temperature variation of the platinum resistance thermometer in the range [0, 100°C]. Explains **Δr = f(θ)**.

2. The platinum resistance thermometer is inserted into a Wheatstone bridge as shown in the following figure. We neglect currents across resistors $\mathbf{R_1}$.



a) Explains $\mathbf{V_{AB} = f(V_A, V_B)}$

b) Explains **Vs** with respect to $\mathbf{V_A, V_B, R_1}$ and $\mathbf{R_2}$. $\mathbf{(R_1 = 11k\Omega, R_2 = 220k\Omega)}$

c) Explains $\mathbf{V_B}$

d) Explains $\mathbf{V_A}$ with resect to $\mathbf{V_{cc}}$ and **Δr**

e) Deduce the expression of $\mathbf{V_s}$ with respect to **Δr**.

If we assume that **Δr << R**, show that the last expression becomes $\mathbf{V_s = a\Delta r}$

f) Compute **a** (V/Ω) if $\mathbf{V_{cc} = 12V}$

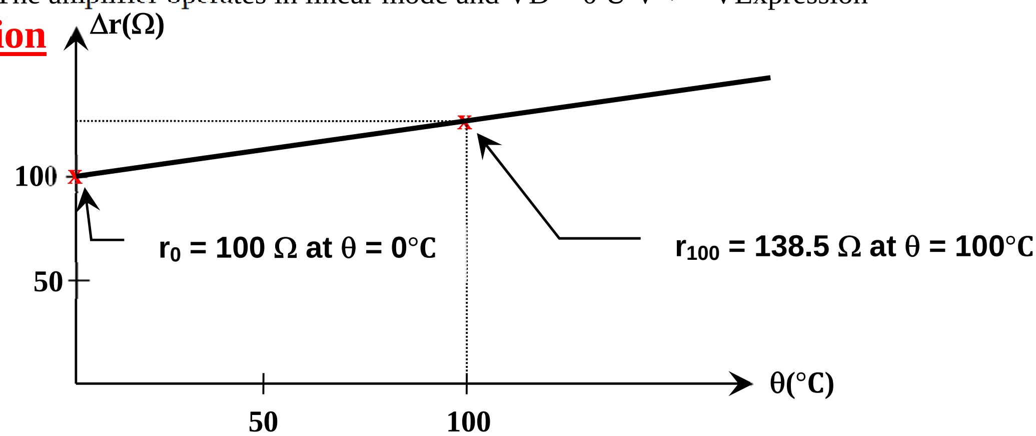
g) Plot the graph of **Vs = a**Δr for 0 <Δr < 40Ω.

Ideal a.o.p operational amplifier simplifying assumptions:

- The input currents i + and i- are zero.
- The amplifier operates in linear mode and VD = 0 Û V + = VExpression

Solution

1.



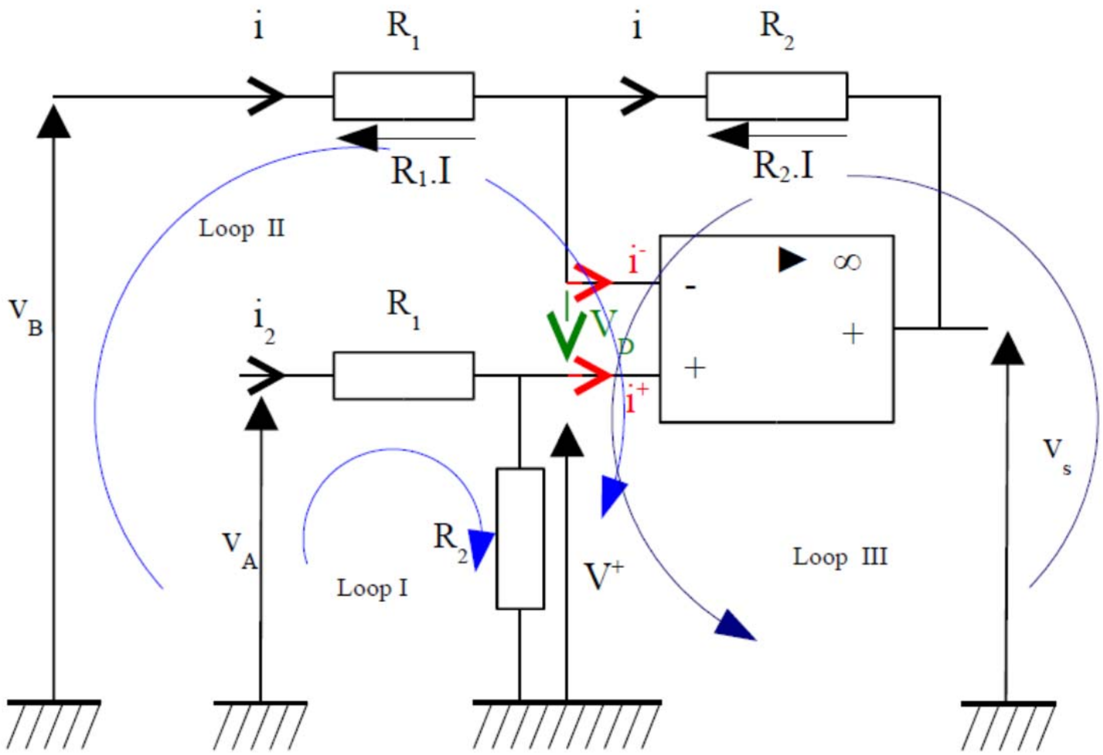
$$\text{The plot of } \Delta r = f(\theta) \text{ is line. The slope of this line is } \frac{138.5 - 100}{100 - 0} = 0.385\ \Omega/°C$$

$\Delta r = 0.385\theta$

2.

a) $V_{AB} = V_A - V_B$

b)



Loop 1:

$$V^+ = \frac{R_2}{R_1 + R_2} V_A \quad \text{Using KVL} \quad : \text{Equation1}$$

Loop 2:

$$\begin{cases} V_B - R_1 i + V_D - V^+ = 0 \\ V_D = 0 \end{cases} \quad \Rightarrow i = \frac{V_B - V^+}{R_1}$$

Loop 3:

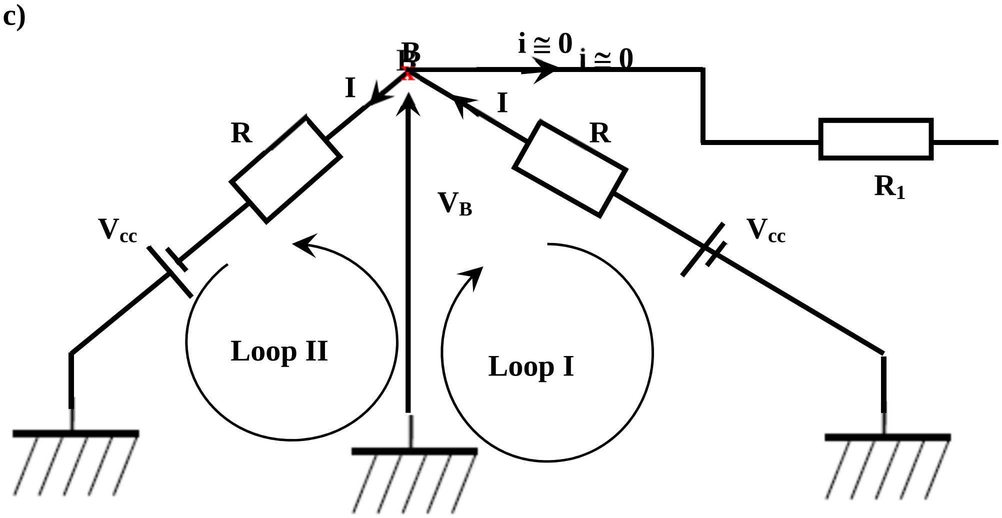
$$\begin{cases} -V_S - R_2 i - V_D + V^+ = 0 \\ V_D = 0 \end{cases} \quad \Rightarrow V^+ = V_S + R_2 i$$

$$\begin{cases} V^+ = V_S + R_2 i \\ i = \dfrac{V_B - V^+}{R_1} \end{cases} \quad \Rightarrow V^+ = \frac{R_2 V_B + R_1 V_S}{R_1 + R_2} : \text{Equation2}$$

$$\text{Equation1} = \text{Equation} \Rightarrow \begin{cases} V^+ = \dfrac{R_2 V_B + R_1 V_S}{R_1 + R_2} \\ V^+ = \dfrac{R_2}{R_1 + R_2} V_A \end{cases} \Rightarrow \frac{R_2}{R_1 + R_2} V_A = \frac{R_2 V_B + R_1 V_S}{R_1 + R_2}$$

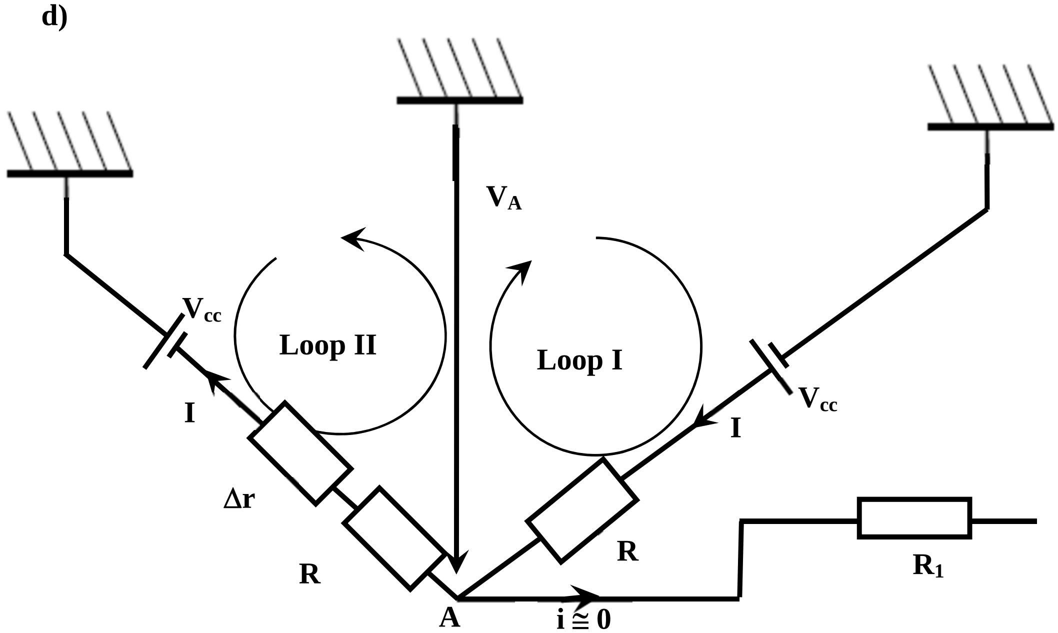
$$\frac{R_2}{R_1 + R_2} V_A = \frac{R_2 V_B + R_1 V_S}{R_1 + R_2} \Rightarrow R_2 V_B + R_1 V_S = R_2 V_A \Rightarrow V_S = \frac{R_2}{R_2}(V_A - V_B)$$

c)



$$\left.\begin{aligned} \text{LoopII}: V_B - RI + V_{CC} = 0 \\ \text{LoopI}: V_B + RI - V_{CC} = 0 \end{aligned}\right\} \Rightarrow 2\, V_B = 0 \Rightarrow V_B = 0$$

d)



$$\text{LoopI}: -V_A + RI + V_{CC} = 0 \Rightarrow I = \frac{V_{CC} - V_A}{R}$$

$$\text{LoopII: } -V_A + (R+\Delta r)I - V_{CC} = 0 \Rightarrow V_A = (R+\Delta r)I - V_{CC}$$

$$\Rightarrow V_A = (R+\Delta r)\frac{V_{CC} - V_A}{R} - V_{CC} \Rightarrow RV_A = (R+\Delta r)(V_{CC} - V_A) - RV_{CC}$$

$$\Rightarrow RV_A + (R+\Delta r)V_A = \Delta r V_{CC} \Rightarrow V_A = \frac{\Delta r V_{CC}}{(2R+\Delta r)}$$

e)

$$V_S = \frac{R_2}{R_2}(V_A - V_B) \Rightarrow V_S = \frac{R_2}{R_2}V_A = \frac{R_2}{R_2}\frac{\Delta r}{(2R+\Delta r)}V_{CC}$$

When $\Delta r <<< R \Rightarrow (2R+\Delta r) \cong 2R$

$$\begin{cases} (2R+\Delta r) \cong 2R \\ V_S = \dfrac{R_2}{R_2}\dfrac{\Delta r}{(2R+\Delta r)}V_{CC} \end{cases} \Rightarrow V_S = \frac{R_2}{R_2}\frac{\Delta r}{(2R)}V_{CC}$$

$$\begin{cases} V_S = \dfrac{R_2}{R_2}\dfrac{V_{CC}}{(2R)}\Delta r \\ V_S = a\,\Delta r \end{cases} \Rightarrow a = \frac{R_2}{R_2}\frac{V_{CC}}{(2R)}$$

f)

$$a = \frac{R_2}{R_2}\frac{V_{CC}}{(2R)} = \frac{220}{11}\frac{12}{(2x1.2x10^3)} = 0.1V/\Omega$$

g)

$V_S = a\,\Delta r \Rightarrow V_S = f(\Delta r)$ is line. The slope of this line is $a = 0.1V/\Omega$