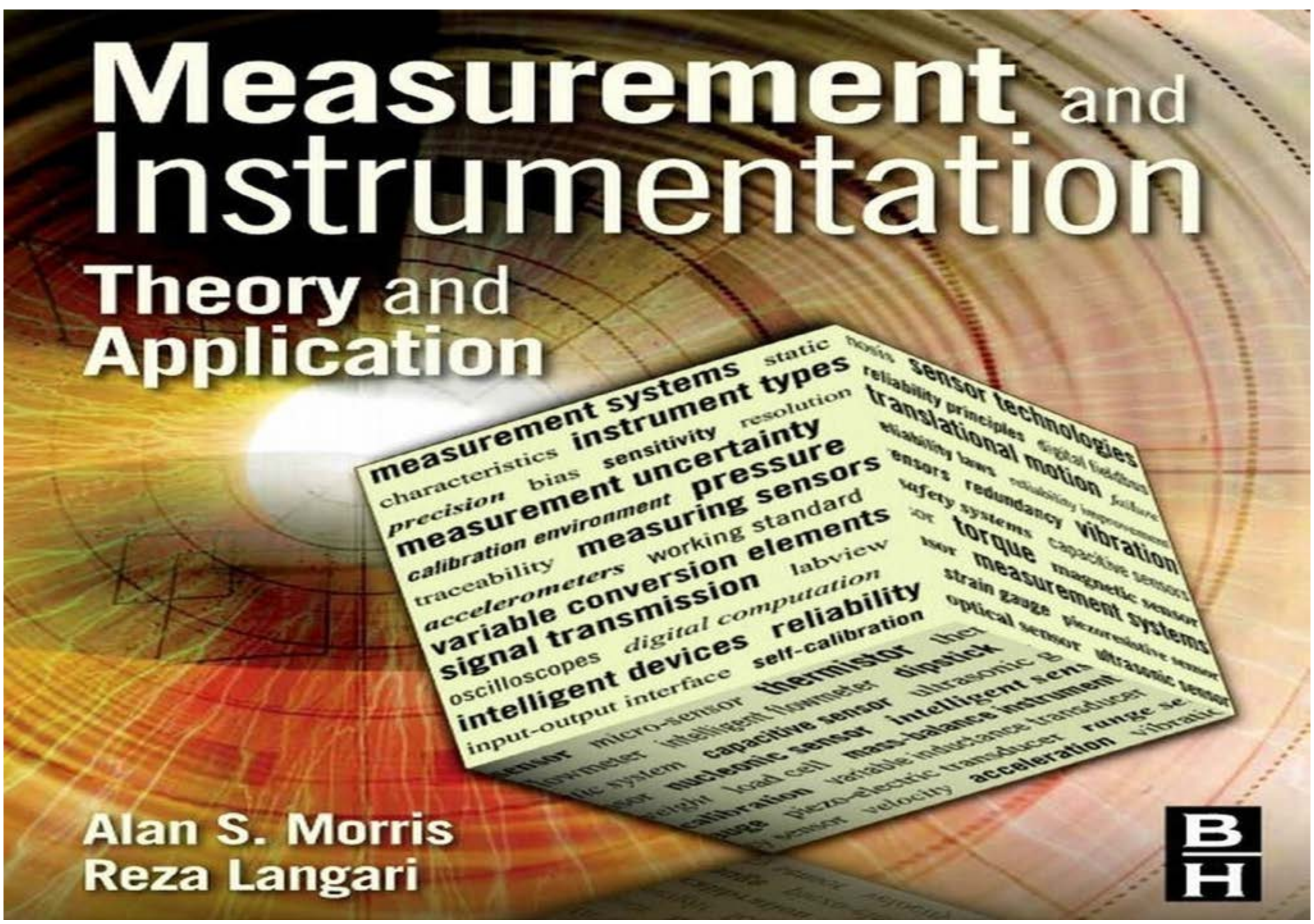
Course: 236CCE

Electronic Measurements

Lectures Notes of Chapter 2

Measurements Errors and Dimensions

This lectures notes are extracted from the text book: " Alan S Morris and Reza Langari, Measurement and Instrumentation: Theory and Application, Second edition, Academic Press, 2015. ISBN-13: 978-0128008843. From CHAPTER 2: Chapter 2 Instrument Types and Performance Characteristics (From page23 to page 94)



Measurements Errors

Input Threshold

It is the smallest stimulus or signal that results in a detectable output.

Zero Shift

It is the drift in the zero indication of an instrument without any change in the measured variable.

Error

It is the difference between the true (actual value) to be measured and the measured value (output) from the measuringsystem. The error value and absolute error value are

$$e = X_{actual} - X_{measured}$$

$$|e| = |X_{actual} - X_{measured}|$$

Error Range

It is the mean value of the error in both sides of the average value of the measured values.

$$E_{up} = X_{max} - X_{ave}$$

$$E_{down} = X_{ave} - X_{min}$$

$$E_{range} = \mp \frac{E_{up} + E_{down}}{2}$$

Error Percentage

It is the mean value of the error in both sides of the average value of the measured values.

$$E = \frac{|e|}{X_{actual}} \times 100\%$$

Example 1

A set of independent voltage measurements taken by four observers was recorded as 117.02V, 117.11V, 117.08V, and 117.03V. Calculate

1. The average voltage value?
2. Error range?

Solution 1

$$E_{ave} = \frac{1}{N}\sum_{n=1}^{N} E_n = \frac{117.02 + 117.11 + 117.08 + 117.03}{4} = 117.06V$$

$$E_{up} = X_{max} - X_{ave} = 117.11 - 117.06 = 0.05V$$

$$E_{down} = X_{ave} - X_{min} = 117.06 - 117.02 = 0.04V$$

$$E_{range} = \mp \frac{E_{up} + E_{down}}{2} = \mp \frac{0.05 + 0.04}{2} = \mp 0.045V$$

Relative Accuracy

$$A_r = \frac{X_{actual} - |e|}{X_{actual}} = 1 - \frac{|e|}{X_{actual}}$$

Accuracy Percentage

$$A = \frac{X_{actual} - |e|}{X_{actual}} \times 100\% = \left(1 - \frac{|e|}{X_{actual}}\right) \times 100\%$$

Example 2

The expected voltage across a resistor is 80V and the measured value is 79V. Calculate

1. The absolute error?

2. The error percentage?
3. The relative accuracy?
4. The accuracy percentage?

Solution 2

$$|e| = |X_{actual} - X_{measured}| = |80 - 79| = 1V$$

$$E = \frac{|e|}{X_{actual}} \times 100\% = \frac{1}{80} \times 100\% = 1.25\%$$

$$A_r = \frac{X_{actual} - |e|}{X_{actual}} = 1 - \frac{1}{80} = 0.9875$$

$$A = \left(1 - \frac{|e|}{X_{actual}}\right) \times 100\% = \left(1 - \frac{1}{80}\right) \times 100\% = 98.75\%$$

Precision of a Given Measured Value

$$P_n = 1 - \left|\frac{X_n - X_{ave}}{X_{ave}}\right|$$

whereX_n is the value of the n^{th} measurement.

Example 3

Calculate the precision of the sixth measurement value?

n	1	2	3	4	5	6	7	8	9	10
X_n	98	101	102	97	101	100	103	98	106	99

Solution 3

$$X_{ave} = \frac{1}{N}\sum_{n=1}^{N} X_n = \frac{98 + 101 + 102 + 97 + 101 + 100 + 103 + 98 + 106 + 99}{10} = 100.5V$$

$$P_6 = 1 - \left|\frac{X_6 - X_{ave}}{X_{ave}}\right| = 1 - \left|\frac{100 - 100.5}{100.5}\right| = 0.995$$

Example 4

Calculate the precision of $X_1 = 98V$ and $X_2 = 98.5V$ if the mean in $X_{ave} = 101V$?

Solution 4

$$P_1 = 1 - \left|\frac{X_1 - X_{ave}}{X_{ave}}\right| = 1 - \left|\frac{98 - 101}{101}\right| = 0.97$$

$$P_2 = 1 - \left|\frac{X_2 - X_{ave}}{X_{ave}}\right| = 1 - \left|\frac{98.5 - 101}{101}\right| = 0.975$$

The second value is more precise.

Measuring Device Resistance

Each measuring device has an internal resistance which is given by

$$R_i = \text{Sensitivity} \times FSO$$

Sources of Errors

1. Faulty design of instrument.
2. Insufficient knowledge of quantity and design conditions.
3. Improper maintenance of the instrument.
4. Sudden change in the parameter to be measured.
5. Unskilled operator.
6. Effects of environmental conditions.

Classification of Static Errors

Static errors can be classified as

1. Systematic errors.

2. Random errors (Noise/Interference).
3. Gross errors.

Random Error in the Measurement System

It occurs by chance for unknown causes.This error can take valueson either side of an average value. Random error is an accumulation of many small effects.

Probable Error

It is the error with a 50% or higher chance of occurrence.A statement of probable error is of little value.

Gross Error (Personal Errors)

This error mainly covers human mistakes that occur due to

- Carelessness of human while reading, recording and calculating results.
- Incorrect adjustments of instruments.
- Improper use of an instrument like poor initial adjustments, improper zero setting, or using leads of high resistance.
- Loading effect like connecting a well calibrated voltmeter across a high resistance circuit.

Complete elimination of gross errors is probably impossible but these errors can be minimized.

Example 5

A voltmeter with a sensitivity of $1000\Omega/V$ reads 100V on its 150V full scale when connected across an unknown resistor in series with a milliammeter. The milliammeter reads 5mA with a negligible resistance. Calculate

1. The measured value of the unknown resistance?
2. The actual value of the unknown resistance?
3. The Error percentage due to the loading effect of the voltmeter?

Solution 5

The total circuit resistance is

$$R_T = \frac{V_T}{I_T} = \frac{100V}{5mA} = 20K\Omega$$

The voltmeter resistance is

$$R_i = \text{Sensitivity} \times FSO = \frac{1000\Omega}{V} \times 150V = 150K\Omega$$

The voltmeter is connected in parallel with the measured resistance. Thus the measured resistance is

$$R_T = \frac{R_i R}{R_i + R} \qquad \rightrightarrows \qquad R = \frac{R_i R_T}{R_i - R_T}$$

$$R = \frac{R_i R_T}{R_i - R_T} = \frac{150K \times 20K}{150K - 20K} = 23.077K\Omega$$

$$E = \frac{|X_{actual} - X_{measured}|}{X_{actual}} \times 100\% = \frac{|23.077K - 20K|}{20K} \times 100\% = 15.38\%$$

Example 6

A voltmeter with a sensitivity of $1000\Omega/V$ reads 40V on its 150V full scale when connected across an unknown resistor in series with a milliammeter. The milliammeter reads 800mA with a negligible resistance. Calculate

1. The measured value of the unknown resistance?
2. The actual value of the unknown resistance?
3. The Error percentage due to the loading effect of the voltmeter?

Solution 6

The total circuit resistance is

$$R_T = \frac{V_T}{I_T} = \frac{40V}{800mA} = 50\Omega$$

The voltmeter resistance is

$$R_i = \text{Sensitivity} \times FSO = \frac{1000\Omega}{V} \times 150V = 150K\Omega$$

The voltmeter is connected in parallel with the measured resistance. Thus the measured resistance is

$$R_T = \frac{R_i R}{R_i + R} \qquad \rightrightarrows \qquad R = \frac{R_i R_T}{R_i - R_T}$$

$$R = \frac{R_i R_T}{R_i - R_T} = \frac{150K \times 50}{150K - 50} = 50.017\Omega$$

$$E = \frac{|X_{actual} - X_{measured}|}{X_{actual}} \times 100\% = \frac{|50.017 - 50|}{50} \times 100\% = 0.034\%$$

The error in Example 5 is high while it is low in Example 6. That means this voltmeter cannot be used for measuring voltage across high resistance element.

Systematic Error

It is a constant uniform deviation of operation in an instrument due to the material used in the instrument like worn parts, or ageing effects.

Types of Systematic Error

1. **Instrumental error:** It is inherent in the instrument due to its mechanical structure. For example, Friction in bearings, Irregular spring tension, or variation in air gap.

2. **Environmental error:**It is related to the external condition affecting the measurement including surrounding area conditions such as change in temperature, humidity, barometer pressure, etc.
3. **Observational error:** This error is introduced by the observer such as parallax error and estimation error.

Limiting Errors (Guarantee Errors)

They are the deviations from the nominal value of a particular quantity. The value of the limiting error is specified by the manufacturer and he guarantee that error in the instrument is no greater than the limit set. If the limiting error is $\pm\delta X$ and the nominal value is X, then the measured value will be

$$\delta X = (1 - A_r)FSO$$

The measured value will be in the following range

$$X_m = X \pm \delta X \in [X - \delta X, X + \delta X]$$

Relative Limiting Error

It is the ratio of the limiting error to the nominal value.

$$\delta_x = \frac{\delta X}{X}$$

The measured value will be in the following range

$$X_m = X \pm \delta X = X \pm \delta_x X = (1 \pm \delta_x)X$$

Example 7

A 600V voltmeter has an accuracy of 98%. Calculate the limiting error and relative limiting error when the voltmeter is used to measure a voltage of 250V?

Solution 7

$$\delta X = (1 - A_r)FSO = (1 - 0.98) \times 600 = 12V$$

$$\delta_x = \frac{\delta X}{X} = \frac{12}{250} = 0.048$$

Therefore the measured voltage value will be

$$X_m = 250 \pm 12 \in [238,262]$$

Example 8

A 150V voltmeter has a guaranteed accuracy of 99% (1% of full scale reading). Calculate the following

1. Limiting error?
2. Relative limiting error when the voltmeter reading is 75V?
3. The range of the measured voltage value?

Solution 8

$$\delta X = (1 - A_r)FSO = (1 - 0.99) \times 150 = 1.5V$$

$$\delta_x = \frac{\delta X}{X} = \frac{1.5}{75} = 0.02$$

Therefore the measured voltage value will be

$$X_m = 75 \pm 1.5 \in [73.5,76.5]$$

Part2 : Propagation of Errors

1) Basic Rules

Suppose two measured quantities x and y have uncertainties, Δx and Δy, determined by procedures described in previous sections: we would report (x ± Δx), and (y ± Δy). From the measured quantities a new quantity, z, is calculated from x and y. What is the uncertainty, Δz, in z? For the purposes of this course we will use a simplified version of the proper statistical treatment. The formulas for a full statistical treatment (using standard deviations) will also be given. The guiding principle in all cases is to consider the most pessimistic situation. Full explanations are covered in statistics courses.
The examples included in this section also show the proper rounding of answers, which is covered in more detail in next Section. The examples use the propagation of errors using average deviations.

(a) Addition and Subtraction: z = x + y or z = x - y

Derivation: We will assume that the uncertainties are arranged so as to make z as far from its true value as possible. (dz=dx+dy or dz=dx-dy)
Average deviations **Δz = |Δx| + |Δy|** in both cases
With more than two numbers added or subtracted we continue to add the uncertainties.

Using simpler average errors		Using standard deviations	
$\Delta z = \ |\Delta x| + |\Delta y| + \cdots.$	**Eq.1a**	$\Delta z = \sqrt{|\Delta x|^2 + |\Delta y|^2 + \cdots}.$	**Eq.1b**

Example: w = (4.5 ± 0.02) cm, x = (2.0 ± 0.2) cm, y = (3.0 ± 0.6) cm. Find z = x + y - w and its uncertainty.

z = x + y - w = 2.0 + 3.0 - 4.5 = 0.5 cm

Δz = Δx + Δy + Δw = 0.2 + 0.6 + 0.02 = 0.82 rounding to 0.8 cm So z = (0.5 ± 0.8) cm	Solution with standard deviations, Eq. 2a, Δz = 0.633cm z = (0.5 ± 0.6) cm

Notice that we round the uncertainty to one significant figure and round the answer to match.

For multiplication by an exact number, multiply the uncertainty by the same exact number.

Example: The radius of a circle is x = (3.0 ± 0.2) cm. Find the circumference and its uncertainty.
C = 2 π x = 18.850 cm
ΔC = 2 π Δx = 1.257 cm (The factors of 2 and π are exact)
C = (18.8 ± 1.3) cm
We round the uncertainty to two figures since it starts with a 1, and round the answer to match.

Example: x = (2.0 ± 0.2) cm, y = (3.0 ± 0.6) cm. Find z = x - 2y and its uncertainty.

Δz = Δx +2 Δy = 0.2 + 2*0. 6=1.4cm So z = (-4.0 ± 1.4) cm	Using Eq 1b, z = (-4.0 ± 0.9) cm.

The 0 after the decimal point in 4.0 is significant and must be written in the answer. The uncertainty in this case starts with a 1 and is kept to two significant figures. (More on rounding in Section 7.)

(b) Multiplication and Division: z = x y or z = x/y

Derivation: We can derive the relation for multiplication easily. Take the largest values for x and y, that is

z + Δz = (x + Δx)(y + Δy) = xy + x Δy + y Δx + Δx Δy

Usually Δx << x and Δy << y so that the last term is much smaller than the other terms and can be neglected. Since z = xy,

Δz = y Δx + x Δy

which we write more compactly by forming the relative error, that is the ratio of Δz/z, namely

$$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \cdots.$$

The same rule holds for multiplication, division, or combinations, namely add all the relative errors to get the relative error in the result.

Using simpler average errors		Using standard deviations	
$\frac{\Delta z}{z} = \frac{\Delta x}{x} + \frac{\Delta y}{y}.$	Eq.2a	$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} + \cdots.$	Eq.2b

Example: w = (4.52 ± 0.02) cm, x = (2.0 ± 0.2) cm. Find z = w x and its uncertainty.

$z = w\,x = (4.52)\,(2.0) = 9.04\mathrm{cm}^2$

$\frac{\Delta z}{9.04\mathrm{cm}^2} = \frac{\Delta x}{x} + \frac{\Delta w}{w} = \frac{0.2cm}{2.0cm} + \frac{0.02}{4.52} = 0.1044$ $\Delta z = 0.1044 * 9.04\mathrm{cm}^2 = 0.944\mathrm{cm}^2$ which we round to $0.9\ \mathrm{cm}^2$, $z = (9.0 \pm 0.9)\mathrm{cm}^2$	Using Eq. 2b we get $\Delta z = 0.944\mathrm{cm}^2$ and $z = (9.0 \pm 0.9)\mathrm{cm}^2$

The uncertainty is rounded to one significant figure and the result is rounded to match. We write 9.0 $\mathbf{cm^2}$ rather than 9 $\mathbf{cm^2}$ since the 0 is significant.

Example: x = (2.0 ± 0.2) cm, y = (3.0 ± 0.6) sec Find z = x/y..

$$z = \frac{x}{y} = \frac{2.0}{3.0} = 0.6667\mathrm{cms}^{-1}$$

$\frac{\Delta z}{0.6667\text{cms}^{-1}} = \frac{\Delta x}{x} + \frac{\Delta w}{w} = \frac{0.2cm}{2.0cm} + \frac{0.6s}{3s} = 0.3$ $\Delta z = 0.3 * 0.6667\text{cms}^{-1} = 0.2\text{cms}^{-1}$ **Also we round** 0.6667cms^{-1} **to** $0.6\ \text{cms}^{-1}$, $z = (0.6 \pm 0.2)\text{cms}^{-1}$	Using Eq. 2b we get $\Delta z = 0.15\text{cms}^{-1}$ **and** $z = (0.67 \pm 0.15)\text{cms}^{-1}$

Note that in this case we round off our answer to have no more decimal places than our uncertainty..

(c) Products of powers: . $z = x^m y^n$

The results in this case are

Using simpler average errors		Using standard deviations	
$\frac{\Delta z}{z} = \lvert m \rvert \frac{\Delta x}{x} + \lvert n \rvert \frac{\Delta y}{y}.$	**Eq.3a**	$\frac{\Delta z}{z} = \sqrt{\left(m\frac{\Delta x}{x}\right)^2 + \left(n\frac{\Delta y}{y}\right)^2 + \cdots}.$	**Eq.3b**

Example: w = (4.52 ± 0.02) cm, A = (2.0 ± 0.2) cm^2 , y = (3.0 ± 0.6) cm. Find $z = \frac{wy^2}{\sqrt{A}}$

$$z = \frac{wy^2}{\sqrt{A}} = \frac{4.52\text{cm}(3\text{cm})^2}{\sqrt{2\text{cm}^2}} = 28.765\text{cm}^2$$

m=1, n=2, l=-1/2

$\frac{\Delta z}{28.765\text{cm}^2} = 1 * \frac{\Delta w}{w} + 2\frac{\Delta y}{y} + 0.5\frac{\Delta A}{A}$ The second relative error, (Δy/y), is multiplied by 2 because the power of y is 2. The third relative error, (ΔA/A), is multiplied by 0.5 since a square root is a power of one half. $\frac{\Delta z}{28.765\text{cm}^2} = \frac{0.02}{4.52} + 2\frac{0.6}{3} + 0.5\frac{0.2}{2} = 0.49$ $\Delta z = 28.765\text{cm}^2 * 0.49 = 14.09\text{cm}^2$ **Also we round** 28.765cm^2 **to** $29\ \text{cm}^2$ **and** 14.09cm^2 **to** $14\ \text{cm}^2$, $z = (29 \pm 14)\text{cm}^2$	Using Eq. 3b we get $\Delta z = 12\text{cm}^2$ **and** $z = (14 \pm 12)\text{cm}^2$

Because the uncertainty begins with a 1, we keep two significant figures and round the answer to match.

(d) Mixtures of multiplication, division, addition, subtraction, and powers.

If z is a function which involves several terms added or subtracted we must apply the above rules carefully. This is best explained by means of an example.

Example: w = (4.52 ± 0.02) cm, x = (2.0 ± 0.2) cm, y = (3.0 ± 0.6) cm. Find z = w x +y^2

$z = wx + y\text{^}2 = 18.0\ cm^2$.

First we compute v = wx as in the example in (b) to get $v = (9.0 \pm 0.9)\ cm^2$. Next we compute $\frac{\Delta y^2}{y^2} = 2\frac{\Delta y}{y} = 2\frac{0.6cm}{3cm} = 0.4 \Rightarrow \Delta y^2 = y^2 * 0.4$ $= (3cm)^2 * 0.4$ $\Delta y^2 = 3.6cm^2$ Finally, we compute $\Delta z = \Delta v + \Delta y^2 = 0.9 + 3.6$ $= 4.5cm^2$ rounding to 4 Hence z $= (18 \pm 4)\ cm^2$	We have v = wx = (9.0 ± 0.9)cm. The calculation of the uncertainty in $\mathbf{y^2}$ is the same as that shown to the left. Then from Eq. 1b $\Delta z = 3.7cm^2$ $z = (18 \pm 4)\ cm^2$

(e) Other Functions: e.g.. z = sin x. The simple approach.

For other functions of our variables such as sin(x) we will not give formulae. However you can estimate the error in z = sin(x) as being the difference between the largest possible value and the average value. and use similar techniques for other functions.
Thus

$$\Delta(\sin x) = \sin(x + \Delta x) - \sin(x)$$

Example: Consider S = x cos (θ) for x = (2.0 ± 0.2) cm, θ = 53 ± 2 °. Find S and its uncertainty.
S = (2.0 cm) cos 53° = 1.204 cm
To get the largest possible value of S we would make x larger, (x + Δx) = 2.2 cm, and θ smaller, (θ - Δθ) = 51°. The largest value of S, namely (S + ΔS), is (S + ΔS) = (2.2 cm) cos 51° = 1.385 cm.
The difference between these numbers is ΔS = 1.385 - 1.204 = 0.181cm which we round to 0.18 cm.
Then S = (1.20 ± 0.18) cm.

(f) Other Functions: Getting formulas using partial derivatives

The general method of getting formulas for propagating errors involves the total differential of a function. Suppose that z = f(w, x, y, ...) where the variables w, x, y, etc. must be **independent variables**!
The total differential is then

$$dz = \left(\frac{\partial f}{\partial w}\right) dw + \left(\frac{\partial f}{\partial x}\right) dx + \left(\frac{\partial f}{\partial y}\right) dy + \cdots \dots \dots \dots \dots \dots$$

We treat the dw = Δw as the error in w, and likewise for the other differentials, dz, dx, dy, etc.
The numerical values of the partial derivatives are evaluated by using the average values of w, x, y, etc. The general results are

Using simpler average errors	
$$\Delta z=\left(\frac{\partial f}{\partial w}\right)\Delta w+\left(\frac{\partial f}{\partial x}\right)\Delta x+\left(\frac{\partial f}{\partial y}\right)\Delta y+\cdots\ldots\ldots\ldots\ldots\ldots$$.	**Eq.4a**
Using standard deviations	
$$\Delta z=\sqrt{\left(\left(\frac{\partial f}{\partial w}\right)\Delta w\right)^2+\left(\left(\frac{\partial f}{\partial x}\right)\Delta x\right)^2+\left(\left(\frac{\partial f}{\partial y}\right)\Delta y\right)^2+\cdots}.$$	**Eq.4b**

Example:
Consider S = x cos (θ) for x = (2.0 ± 0.2) cm, θ = (53 ± 2) °= (0.9250 ± 0.0035) rad. Find S and its uncertainty. Note: **the uncertainty in angle must be in radians!**
S = 2.0 cm cos 53° = 1.204 cm.

$$ds=\left(\frac{\partial S}{\partial x}\right)dx+\left(\frac{\partial S}{\partial \theta}\right)d\theta=\cos(\theta)dx-x\sin(\theta)d\theta$$

We take the absolute value of each term.

$$\Delta s=\cos(\theta)\Delta x+x\sin(\theta)\Delta\theta$$
$$=\cos(53°)*0.2cm+2cm\sin(53°)2°=\cos(53°)*0.2cm+2cm\sin(53°)0.0035°$$
$$=0.126cm$$

For standard deviations approach,

$$\Delta S=\sqrt{\left(\left(\frac{\partial S}{\partial x}\right)\Delta x\right)^2+\left(\left(\frac{\partial S}{\partial \theta}\right)\Delta\theta\right)^2}=\sqrt{(\cos(\theta)\Delta x)^2+(x\sin(\theta)\Delta\theta)^2}=0.120cm$$

Hence S = (1.20 ± 0.13) cm (using average deviation approach) or S = (1.20± 0.12) cm (using standard deviation approach.)

2. Rounding off answers in regular and scientific notation.

In the above examples we were careful to round the answers to an appropriate number of significant figures. The uncertainty should be rounded off to one or two significant figures. If the leading figure in the uncertainty is a 1, we use two significant figures, otherwise we use one significant figure. Then the answer should be rounded to match.

Example: Round off z = 12.0349 cm and Δz = 0.153 cm.
Since Δz begins with a 1, we round off Δz to two significant figures:
Δz = 0.15 cm. Hence, round z to have the same number of decimal places:

z = (12.03 ± 0.15) cm.

When the answer is given in scientific notation, the uncertainty should be given in scientific notation with the **same power of ten**. Thus, if
$z = \mathbf{1.43x10^6}$s and $\Delta z = \mathbf{2x10^4} s$,
we should write our answer as
z = (1.43± 0.02) x s.

$$z = (\mathbf{1.43} \pm \mathbf{0.02})\ \mathbf{x10^6}\text{s}$$

This notation makes the range of values most easily understood. The following is technically correct, but is hard to understand at a glance.
$\mathbf{z} = (\mathbf{1.43x10^6} \pm \mathbf{2x10^4})$ s ***Don't write like this!***

Problem: Express the following results in proper rounded form, x ± Δx.
(i) m = 14.34506 grams, Δm = 0.04251 grams.
(ii) t = 0.02346 sec, Δt = 1.623 x 10-3sec.
(iii) M = 7.35 x 10^{22} kg ΔM = 2.6 x 10^{20} kg.
(iv) m = 9.11 x 10^{-33} kg Δm = 2.2345 x 10^{-33} kg

Answer:

(i) m = 14.34506 grams, Δm =0.04251 grams.	m = (14.35 ± 0.04) g
(ii) t = 0.02346 sec, Δt = 1.623 x 10^{-3} sec.	t = (0.0235 ± 0.0016) s **or** t = (2.35 ± 0.16) x 10^{-3}s **or** (2.35 ± 0.16) ms
(iii) M = 7.35 x 10^{22} kg, ΔM = 2.6 x10^{20} kg.	M = (7.35 ± 0.03) x10^{22} kg
(iv) m = 9.11 x 10^{-33} kg, Δm = 2.2345 x 10^{-33} kg	m = (9 ± 2) x 10^{-33}kg

Ecercise1: Problems on Uncertainties and Error Propagation.

Try the following problems to see if you understand the details of this part . The answers are at the end.

(a) Find the average and the average deviation of the following measurements of a mass.
4.32, 4.35, 4.31, 4.36, 4.37, 4.34 grams.

(b) Express the following results in proper rounded form, x ± Δx.
(i) m = 14.34506 grams, Δm = 0.04251 grams.
(ii) t = 0.02346 sec, Δt = 1.623 x10^{-3} sec.
(iii) M = 7.35 x10^{22} kg ΔM = 2.6 x10^{20} kg.
(iv) m = 9.11 x 10^{-33}kg Δm = 2.2345 x10^{-33} kg

(c) Are the following numbers equal within the expected range of values?
(i) (3.42 ± 0.04) m/s and 3.48 m/s?
(ii) (13.106 ± 0.014) grams and 13.206 grams?

(iii) (2.95 ± 0.03) x m/s and 3.00 x m/s

(d) Calculate z and Δz for each of the following cases.

(i) z = (x - 2.5 y + w) for x = (4.72 ± 0.12) m, y = (4.4 ± 0.2) m, w = (15.63 ± 0.16) m.

(ii) z = (w x/y) for w = (14.42 ± 0.03) m/s^2 , x = (3.61 ± 0.18) m, y = (650 ± 20) m/s.

(iii) z =x^3 for x = (3.55 ± 0.15) m.

(iv) z = v (xy + w) with v = (0.644 ± 0.004) m, x = (3.42 ± 0.06) m, y = (5.00 ± 0.12) m, w = (12.13 ± 0.08) .

(v) z = A sin y for A = (1.602 ± 0.007) m/s, y = (0.774 ± 0.003) rad.

(e) How many significant figures are there in each of the following?

(i) 0.00042 (ii) 0.14700 (ii) 4.2 x (iv) -154.090 x 10-27

(f) I measure a length with a meter stick which has a least count of 1 mm I measure the length 5 times with results in mm of 123, 123, 124, 123, 123 mm. What is the average length and the uncertainty in length?

Answers

(a).

$$\boldsymbol{Average\ value\ = M_{av} = \frac{1}{N}\sum_{i=1}^{N} M_i = \frac{1}{6}(M_1 + M_2 + M_3 + M_4 + M_5 + M_6)}$$

$$\boldsymbol{= \frac{1}{6}(4.32 + 4.35 + 4.31 + 4.36 + 4.37 + 4.34) = 4,34167g = 4,342g}$$

Maximum value of masst $\boldsymbol{M_{max} = 4.37g}$

$\boldsymbol{Range_{up} = M_{max} - M_{av} = 4.37 - 4,342 = 0.028m}$ ·

Minimum value of current $\boldsymbol{M_{min} = 4.31m}$

$\boldsymbol{Range_{down} = -M_{min} + M_{av} = 4.342 - 4.31 = 0.032m}$ ·

$$\boldsymbol{Therefore\ average\ range\ of\ error\ is = \frac{Range_{up} + Range_{down}}{2} = \frac{0.028 + 0.032}{2}}$$

$$\boldsymbol{= \pm 0.03g}$$

Masse=(4.342 ± 0.03) grams

(b).

i) (14.34 ± 0.04) grams

ii) (0.0235 ± 0.0016) sec or (2.35 ± 0.16) x10^{-2} sec

iii) (7.35 ± 0.03) x10^{22} kg

iv) (9.11 ± 0.02) x 10^{-31}kg

(c).

Yes for (i) and (iii), no for (ii)

(d) i)

z= x - 2.5 y + w = 4.72-2.5*4.4+15.63=9.35m which we round to 9. 4m

Δz =Δ (x - 2.5 y + w) = Δx + 2.5Δy + Δw=0.78m which we round to 0. 8m.

z=(9.4 ± 0.8) m

ii)

$$\boldsymbol{z = \frac{wx}{y} = \frac{14.42 * 3.61}{650} = 0.080m/s}$$

$$\boldsymbol{\frac{\Delta z}{z} = \frac{\Delta z}{0.080} = \frac{0.18}{3.61} + \frac{20}{650} + \frac{0.03}{14.42} = 0.083}$$

$$\boldsymbol{\Delta z = 0.1044 * 9.04cm^2 = 0.00661m/s\ \text{which we round to}\ 0.007m/s}$$

$\boldsymbol{\text{which we round to } 0.9\ cm^2}$,

$$\boldsymbol{z = (0.080 \pm 0.007)m/s}$$

iii)

$$z = (x)^3 = (3.55)^3 = 44.738875 \text{ which we round to } 45$$
$$\frac{\Delta z}{z} = 3\frac{\Delta x}{x} = 3 * \frac{0.15}{3.55} = 0.01 \Rightarrow \Delta z = 0.01 * z = 5.7043 \text{ which we round to } 6$$

$$z = (45 \pm 6)$$

iv)

(iv) z = v (xy + w) with v = (0.644 ± 0.004) m, x = (3.42 ± 0.06) m, y = (5.00 ± 0.12) m, w = (12.13 ± 0.08) .

$$z = v\,(xy + w) = 0.644\,(3.42 * 5 + 12.13) = 18.82412 = 18.8$$
$$\frac{\Delta z}{z} = \frac{\Delta v}{v} + \frac{\Delta(xy + w)}{xy + w} = \frac{\Delta v}{v} + \frac{\frac{\Delta x}{x} + \frac{\Delta y}{y} + \Delta w}{xy + w} = \frac{0.004}{0.644} + \frac{\frac{0.06}{3.42} + \frac{0.12}{5} + 0.08}{3.42 * 5 + 12.13} = 0.0104$$

$$\frac{\Delta z}{z} = 0.0104 \Rightarrow \Delta z = 0.0104 * z = 0.195 \text{ which we round to } 0.2$$
$$z = (18.8 \pm 0.2)$$
v)

$$z = A\sin(y) = 1.602 * \sin(0.774) = 1.1198 m/s \text{ which we round to } 1.12 m/s$$
$$\frac{\Delta z}{z} = \frac{\Delta A}{A} + \frac{\Delta \sin(y)}{\sin(y)} = \frac{\Delta A}{A} + \frac{\sin(y + \Delta y) - \sin(y)}{\sin(y)}$$
$$= \frac{0.007}{1.602} + \frac{\sin(0.774 + 0.003) - \sin(0.774)}{\sin(0.774)} = 0.0074342$$

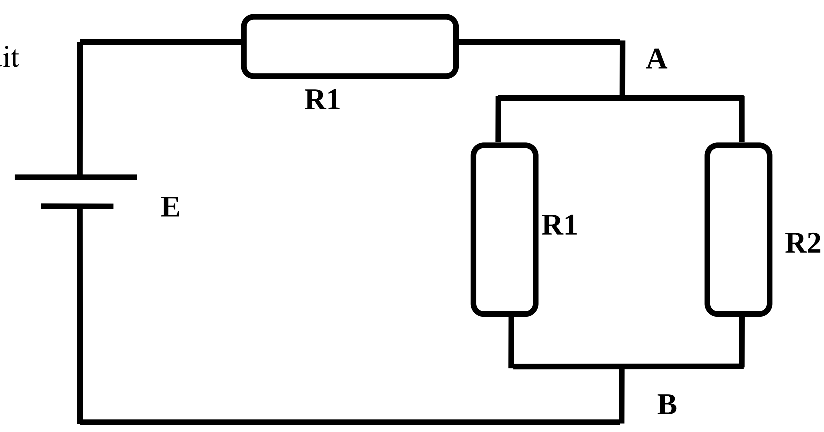
$$z = (1.12 \pm 0.008) m/s$$

(e) i) 2 ii) 5 iii) 2 iv) 6

(f) (123 ± 1) mm (I used the ILE = least count since it is larger than the average deviation.)

Ecercise2:

Consider the following circuit



R1 = 8Ω ± 1 % , R2 = 4 Ω ± 2 % et **E = 30 V ± 3 %**

a. Calculate the resistance **RAB** (between **A** and **B**).

b. Calculate $\frac{\Delta RAB}{RAB}$ and ΔRAB

3- Find **UAB**, $\frac{\Delta UAB}{UAB}$ and ΔUAB

Answer:

a.

$$R_{AB} = \frac{R1}{R2} = \frac{R1 * R2}{R1 + R2} = \frac{8 * 4}{8 + 4} = 2.67\Omega \text{ which we round to } 2.7\Omega$$

b.

$$\frac{\Delta R_{AB}}{R_{AB}} = \frac{\Delta R1}{R1} + \frac{\Delta R1}{R1} + \frac{\Delta(R1 + R2)}{R1 + R2} = \frac{\Delta R1}{R1} + \frac{\Delta R1}{R1} + \frac{\Delta R1 + \Delta R2}{R1 + R2}$$

$$\frac{\Delta R_{AB}}{R_{AB}} = \frac{0.01}{8} + \frac{0.02}{4} + \frac{0.01 + 0.02}{8 + 4} = \frac{\Delta R1}{R1} + \frac{\Delta R1}{R1} + \frac{\Delta R1 + \Delta R2}{R1 + R2} = 0.00875\Omega$$

$$\Delta R_{AB} = R_{AB} * 0.00875 = 12 * 0.00875 = 0.105\Omega \text{ which we round to } 0.1\Omega$$

c.

$$U_{AB} = \frac{R_{AB} * E}{R_{AB} + R1} = \frac{12 * 30}{2.7 + 8} = 18V$$

$$\frac{\Delta U_{AB}}{U_{AB}} = \frac{\Delta RAB}{RAB} + \frac{\Delta E}{E} + \frac{\Delta(R1 + RAB)}{R1 + RAB} = \frac{\Delta RAB}{RAB} + \frac{\Delta E}{E} + \frac{\Delta R1 + \Delta RAB}{R1 + RAB}$$

$$\frac{\Delta U_{AB}}{U_{AB}} = \frac{0.1}{2.7} + \frac{0.03}{30} + \frac{0.01 + 0.1}{8 + 2.7}$$

Part 3: Dimensions
Units, Dimensional Analysis, Problem Solving, and Estimation

International System of Units

The system of units most commonly used throughout science and technology today is the System International (SI). It consists of seven base quantities and their corresponding base units:

Table 1 International System of Units

Base Quantity	Base Unit
Length	meter (m)
Mass	kilogram (kg)
Time	second (s)
Electric Current	ampere (A)
Temperature	kelvin (K)
Amount of Substance	mole (mol)
Luminous Intensity	candela (cd)

- Kilogram: The mass of a cylinder of platinum–iridium kept under given conditions at BIPM, Paris.
- second: A particular fraction of a certain oscillation within a caesium 133 atom.
- degree kelvin: The temperature of the triple point of water, on an absolute scale, divided by 273.16. The degree kelvin is the unit of temperature difference as well as the unit of thermodynamic temperature.
- radian: The angle subtended at the centre of a circle by an arc equal in length to the radius.
- ampere: The electrical current which if maintained in two straight parallel conductors ofinfinite length and negligible cross-section, placed 1 m apart in a vacuum, produces a force between them of 2×10^{-7} N per metre length.
- mol: The amount of substance containing as many elementary units (atoms or molecules) as there are in 12 g of carbon 12.

We shall refer to the *dimension* of the base quantity by the quantity itself, for example
dim length ≡ length ≡ L, dim mass ≡ mass ≡ M, dim time ≡ time ≡ T, dim Current ≡ I..
Mechanics is based on just the first three of these quantities, the MKS or meterkilogram-second system. An alternative metric system to this, still widely used, is the socalled CGS system (centimeter-gram-second).
The SI system is very logical and, in a scientific and industrial context, has a great many advantages over previous systems of units. However, it is usually criticised on two counts. First that the names given to certain derived units, such as the pascal for the unit of pressure, of themselves mean nothing and that it would be better to remain with, for example, the kilogram per square metre. This is erroneous; the definitions of newton, joule, watt and pascal are simple and straightforward if the underlying principles are understood. Derived units which have their own symbols, and which are encountered in this book, are listed in Table 2.

Table 2 Some derived SI units

Name	Symbol	Quantity represented	Basic units
newton	N	Force	$Kg.m.s^{-2}$
joule	J	Energy or work	$N.m$
watt	W	Power	$J.s^{-1}$
pascal	Pa	Pressure	$N.m^{-2}$
hertz	Hz	Frequency	s^{-1}
volt	V	Electrical potential	$W.A^{-1}$

Dimensions of Commonly Encountered Quantities

Many physical quantities are derived from the base quantities by set of algebraic relations defining the physical relation between these quantities. The dimension of the derived quantity is written as a power of the dimensions of the base quantities. For example velocity is a derived quantity and the dimension is given by the relationship.

$$dim\ velocity = [velocity] = (length)/(time) = L\,T^{-1}.$$

where L ≡ length , T ≡ time . Force is also a derived quantity and has dimension

$$dim\ force = [force] = \frac{(mass)(dim\ velocity)}{(\text{time})}$$

where M ≡ mass . We can also express force in terms of mass, length, and time by the relationship.

$$dim\ force = [force] = \frac{(mass)(length)}{(\text{time})^2} = ML\,T^{-2}$$

The derived dimension of kinetic energy is

$$dim\ kinetic\ energy = [kinetic\ energy\,] = (mass)(dim\ velocity)^2,$$

which in terms of mass, length, and time is

$$dim\ kinetic\ energy = [kinetic\ energy\,] = \frac{(mass)(length)^2}{(\text{time})^2} = ML^2\,T^{-2}$$

The derived dimension of work is

$$dim\ work = [work\,] = (dim\ force)(length),$$

which in terms of our fundamental dimensions is

$$dim\ work = [work\,] = \frac{(mass)(length)^2}{(\text{time})^2} = ML^2\,T^{-2}$$

So work and kinetic energy have the same dimensions. Power is defined to be the rate of change in time of work so the dimensions are

$$dim\ power = [power\,] = \frac{dim\ work}{(time)} = \frac{(dim\ force)(length)}{(time)} = \frac{(mass)(length)^2}{(\text{time})^3}$$
$$= ML^2\,T^{-3}$$

Table 3 Dimensions of Some Common Mechanical Quantities

M ≡ mass , L ≡ length , I ≡ current, T ≡ time

Quantity	Dimension	MKS unit
Angle	$dimensionless$	$Dimensionless = radian$
Solid Angle	$dimensionless$	$Dimensionless = sterradian$
Area	L^2	m^2
Volume	L^3	m^3
Frequency	T^{-1}	$s^{-1} = hertz = Hz$
Velocity	$L.T^{-1}$	ms^{-1}
Acceleration	$L.T^{-2}$	ms^{-2}
Angular Velocity	T^{-1}	$rad.s^{-1}$
Angular Acceleration	T^{-2}	$rad.s^{-2}$
Density	$M.L^{-3}$	$Kg.m^{-3}$
Momentum	$M.L.T^{-1}$	$Kg.m.s^{-1}$
Angular Momentum	$M.L^2.T^{-1}$	$Kg.m^2.s^{-1}$
Force	$M.L.T^{-2}$	$Kg.m.s^{-2} = newton = N$
Work, Energy	$M.L^2.T^{-2}$	$Kg.m^2.s^{-2} = joule = J$
Torque	$M.L^2.T^{-2}$	$Kg.m^2.s^{-2}$
Power	$M.L^2.T^{-3}$	$Kg.m^2.s^{-3} = watt = W$
Pressure	$M.L^{-1}.T^{-2}$	$Kg.m^{-1}.s^{-2} = pascal = Pa$

In Table 2 we include the derived dimensions of some common mechanical quantities in terms of mass, length, and time.

Exercise1

All mathematical relationships which are used to describe physical phenomena should be dimensionally consistent. That is, the dimensions (and hence the units) should be the same on each side of the equality. Determine the dimensions of the following Equation.

$$P = \rho g h$$

Answer

Using square brackets to denote dimensions or units, the dimensions of the terms on the right-hand side are as follows:

$$[\rho] = M.L^{-3}$$

$$[g] = L.T^{-2}$$

$$[h] = L$$

Thus the dimensions of equation $P = \rho g h$ are

$$[\rho g h] = (M.L^{-3})(L.T^{-2})(L) = M.L^{-1}.T^{-2}$$

Pressure is a force per unit area, force is given by the product of mass and acceleration and therefore the dimensions of the equation $P = \rho gh$ are

$$[P\,] = \frac{[\boldsymbol{mass}][\boldsymbol{acceleration}]}{[\mathbf{area}]}$$

Or

$$[P\,] = \frac{(M)(L.T^{-2})}{(\mathrm{L}^{-2})}$$

Or

$$[P\,] = M.L^{-1}.T^{-2}$$

Similarly the units must be the same on each side of the equation. This is simply a warning not to mix SI and non-SI units and to be aware of prefixes. The units of the various quantities in equation $P = \rho gh$ are

$$[P\,] = Pa\,; [\rho\,] = kg\,m^{-3};\ [g\,] = m.s^{-2}; [h\,] = m$$

and therefore the right-hand side of $P = \rho gh$ becomes

$$[\rho gh\,] = (\,kg\,m^{-3})(m.s^{-2})(m) = kg\,m^{-1}s^{-2}$$

As a force of **1 N**, acting upon a body of mass 1 kg, produces an acceleration of **1 m s^{-2}**, the units of the equation $P = \rho gh$ are now

$$[\rho gh\,] = kg\,m^{-1}s^{-2} = N.m^{-2}$$

and thus

$$[\rho gh\,] = Pa$$

Exercise2

1. What is the pressure at the base of a column of water 20 m high? The density of water is 1000 kg m^{-3}.
2. What density of liquid is required to give standard atmospheric pressure (101.325 kPa) at the base of a 12 m high column of that liquid?
3. A person of mass 75 kg climbs a vertical distance of 25 m in 30 s. Ignoring inefficiencies and friction,
   (a) how much energy does the person expend?
   (b) how much power must the person develop?
4. Determine the dimensions of the following:
   (a) linear momentum (mass × velocity)
   (b) work
   (c) power
   (d) pressure
   (e) weight
   (f) moment of a force (force × distance)
   (g) angular momentum (linear momentum × distance)

(h) pressure gradient

(i) stress

(j) velocity gradient.

Answer

1.

Given height of water = 20 m, g = 10 m s^{-2}

And density of water = 1000 kg m^{-3}

Pressure = h ρ g

= 20 × 1000 × 10

= 2 × 10^5 N m^{-2}

2.

Given height of liquid = 12 m, g = 10 m s^{-2}

And $\boldsymbol{Pressure = 101.325\ kPa = 101325Pa = 101325N.m^{-2} = 101325kgm^{-1}s^{-2}}$

$\boldsymbol{Pressure = h\ \rho\ g} \Rightarrow$ density of liquid $= \boldsymbol{\rho} = \frac{\boldsymbol{Pressure}}{\boldsymbol{hg}}$

$$\text{density of liquid} = \boldsymbol{\rho} = \frac{\mathbf{101325}}{\mathbf{12x10}} = \mathbf{844.375}\text{kg m}^{-3}$$

3.

Given mass= 75 kg, distance = 25 m

And Time = 30 s

a) $\boldsymbol{energy} = \frac{(\boldsymbol{mass})(\boldsymbol{length})^2}{(\textbf{time})^2} = \frac{\mathbf{75*25^2}}{\mathbf{30^2}}\boldsymbol{Kg.m^2.s^{-2}} = \mathbf{52.083}\boldsymbol{Kg.m^2.s^{-2}} = \mathbf{52.083}\boldsymbol{J}$

b) $\boldsymbol{dim\ power} = [\boldsymbol{power}\ ] = \frac{\boldsymbol{dim\ work}}{(\boldsymbol{time})} = \frac{\boldsymbol{dim\ energy}}{(\boldsymbol{time})} = \frac{(\boldsymbol{mass})(\boldsymbol{length})^2}{(\textbf{time})^3} = \frac{\mathbf{75*25^2}}{\mathbf{30^3}}\boldsymbol{Kg.m^2.s^{-3}} = \mathbf{1.73611}\boldsymbol{Kg.m^2.s^{-3}} = \mathbf{1.73611}\boldsymbol{W}$

4.

a) $\boldsymbol{dim}$ linear momentum $=$ [(mass $\times$ velocity)] $=$ [mass][velocity] $= \boldsymbol{M.L.T^{-1}}$

b) $\boldsymbol{dim}$ work = [work] = $(\boldsymbol{dim\ force})(\boldsymbol{length}) == \frac{(\boldsymbol{mass})(\boldsymbol{length})^2}{(\textbf{time})^2} = \boldsymbol{ML^2\ T^{-2}}$

c) $\boldsymbol{dim}$ power = [power] $= \frac{\text{dim work}}{(\text{time})} = \frac{(\text{dim force})(\text{length})}{(\text{time})} = \frac{(\text{mass})(\text{length})^2}{(\text{time})^3} = \text{ML}^2\ \text{T}^{-3}$

d) $\boldsymbol{dim}$ pressure $= [\boldsymbol{P}\ ] = \frac{[\boldsymbol{mass}][\boldsymbol{acceleration}]}{[\textbf{area}]} = \frac{(\boldsymbol{M})(\boldsymbol{L.T^{-2}})}{(\mathbf{L^{-2}})} = \boldsymbol{M.L^{-1}.T^{-2}}$

e) $\boldsymbol{dim}$ weight $=$ [mass $* \boldsymbol{g}$] $= \boldsymbol{M.L.T^{-2}}$

f)

$\boldsymbol{dim}$ moment of a force = [moment of a force = force $\times$ distance] $= (\boldsymbol{dim\ force})(\boldsymbol{length})$

$$[\text{moment of a force} = \text{force} \times \text{distance}] = \frac{(\boldsymbol{mass})(\boldsymbol{length})^2}{(\textbf{time})^2} = \boldsymbol{ML^2\ T^{-2}}$$

g)

$\boldsymbol{dim}$ angular momentum $=$ ($\boldsymbol{dim}$ linear momentum)($\boldsymbol{length}$) $=$ [mass][velocity][length]

$\boldsymbol{dim}$ angular momentum (linear momentum $\times$ distance) $= \boldsymbol{M.L^2.T^{-1}}$

h) $\boldsymbol{dim}$ **Pressure Gradient** $= \left(\boldsymbol{dim}\ \frac{\text{Pressure}}{\text{Distance}}\right) = \frac{M.L^{-1}.T^{-2}}{L} = \boldsymbol{M.L^{-2}.T^{-2}}$

i)

Mathematically, Stress = Force / Area

Dimensional Formula of Force $= \boldsymbol{M.L.T^{-2}}$

Dimensional Formula of Area $= \mathrm{L}^2 = \boldsymbol{M^0.L^2.T^0}$

Putting these values in above equation we get

Dimensional Formula of Stress $= \boldsymbol{M.L.T^{-2}/M^0.L^2.T^0}$

Dimensional Formula of Stress $= \boldsymbol{M.L^{-1}.T^{-2}}$ SI unit of Stress is Nm^{-2}

j) $\boldsymbol{dim}$ **velocity Gradient** $= \left(\boldsymbol{dim}\ \frac{\text{velocity}}{\text{Distance}}\right) = \frac{L.T^{-1}}{L} = \boldsymbol{T^{-1}}$

Exercise 3

1. Determine the dimensions of the following:

- **a) The electric charge Q and the Surface charge density σ**
- **b) The electric field E**
- **c) electric potential V= E. l**
- **d) the electrical resistance R and the resistivity ρ such that R = ρ L / s (L: length of the wire, s: its section).**
- **e) capacitance C**
- **f) The self-inductance L as induced electromotive force e = - L di / dt; (Homogeneous to V).**

2. Check $\frac{\sigma^2}{4\pi\varepsilon_0}$ **is homogeneous with an electrostatic pressure suffered by a charged conductor (P = F / s)**

3. Verify that RC, L / R and (LC) $^{1/2}$ **are homogeneous at a time.**

Note : q= I.t, V = R.I, Q = C. V, F= Q.E, σ= Q/s

And $[\varepsilon_0] = \boldsymbol{L^{-3}.M^{-1}.T^4.I^2}$ **: the dimensional formula of the permittivity of vacuum**

Answer

a)

$$\mathrm{I} = \frac{dQ}{dt} \Rightarrow \text{Dim Q} = [\mathrm{Q}] = \boldsymbol{I.T}$$

$$\sigma = \frac{Q}{Area} \Rightarrow \text{Dim } \sigma = [\sigma] = \boldsymbol{I.L^{-2}.T}$$

b)

$$\mathrm{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \Rightarrow \text{Dim E} = [\mathrm{E}] = \frac{[Q]}{[4\pi\varepsilon_0][r^2]} = \frac{I.T}{L^{-3}.M^{-1}.T^4.I^2.L^2} = \boldsymbol{L.M.T^{-3}.I^{-1}}$$

c)

Solution 1

$$\textbf{electric potential} = \frac{W}{q_0} \Rightarrow \textbf{Dim electric potential} = [\textbf{electric potential}] = \frac{[W]}{[q_0]}$$

$$[\textbf{electric potential}] = \frac{[W]}{[q_0]} = \frac{M.T^{-2}.L^2}{I.T} = L^2.M.T^{-3}.I^{-1}$$

Solution 2

$$\textbf{electric potential} = E.l \Rightarrow \textbf{Dim electric potential} = [\textbf{electric potential}] = [E].[length]$$

$$[\textbf{electric potential}] = [E].[length] = L.M.T^{-3}.I^{-1}.L = L^2.M.T^{-3}.I^{-1}$$

d)

$$\textbf{resistance R} = \frac{V}{I} \Rightarrow \textbf{Dim R} = [\textbf{R}] = \frac{[V]}{[I]} = \frac{L^2.M.T^{-3}.I^{-1}}{I} = L^2.M.T^{-3}.I^{-2}$$

$$\textbf{R} = \frac{\rho\, L}{s} \Rightarrow \rho = \frac{Rs}{L} \Rightarrow \textbf{Dim } \rho = [\rho] = \frac{[R][s]}{[L]} = \frac{L^2.M.T^{-3}.I^{-2}L^2}{L} = L^3.M.T^{-3}.I^{-2}$$

e)

$$V = \frac{Q}{C} \Rightarrow C = \frac{Q}{V} \Rightarrow \textbf{Dim C} = [C] = \frac{[Q]}{[V]} = \frac{I.T}{L^2.M.T^{-3}.I^{-1}} = L^{-2}.M^{-1}.T^4.I^2$$

e)

$$e = -L\frac{di}{dt} \Rightarrow L = \frac{-eT}{I} \Rightarrow \textbf{Dim L} = [L] = \frac{[e][T]}{[I]} = \frac{[V][T]}{[I]} = \frac{L^2.M.T^{-3}.I^{-1}.T}{I} = L^2.M.T^{-2}.I^{-2}$$

2)

$$dim\frac{\sigma^2}{4\pi\varepsilon_0} = \frac{[\sigma]^2}{[4\pi\varepsilon_0]} = \frac{\left(I.L^{-2}.T\right)^2}{L^{-3}.M^{-1}.T^4.I^2} = L^{-1}.M.T^{-2}$$

$$dim\ P = dim\ \frac{F}{s} = dim\ \frac{qE}{s} = \frac{[q][E]}{[s]} = \frac{I.T.L.M.T^{-3}.I^{-1}}{L^2} = L^{-1}.M.T^{-2}$$

$$dim\frac{\sigma^2}{4\pi\varepsilon_0} = dim\ P \Rightarrow \frac{\sigma^2}{4\pi\varepsilon_0}\ \textbf{is homogeneous with an electrostatic pressure}$$

3)

$$dim\ RC = [RC] = [R][C] = L^2.M.T^{-3}.I^{-2}.\ L^{-2}.M^{-1}.T^4.I^2 = T$$

$$dim\ \frac{L}{R} = \left[\frac{L}{R}\right] = \frac{[L]}{[R]} = \frac{L^2.M.T^{-2}.I^{-2}}{L^2.M.T^{-3}.I^{-2}} = T$$

$$dim\ (LC)^{1/2} = [LC]^{1/2} = ([L][C])^{1/2} = (L^2.M.T^{-2}.I^{-2}.\ L^{-2}.M^{-1}.T^4.I^2)^{1/2} = (T^2)^{1/2} = T$$

Or

$$dim\ (LC)^{1/2} = dim\ \frac{L}{R} = dim\ RC = T \Rightarrow RC,\quad \frac{L}{R}\ \textbf{and}\ (LC)^{1/2}\ \textbf{are homogeneous at a time}$$