Chapter 4
INVENTORY SYSTEM

Introduction: When we speak of inventory, we generally refer to finished goods, raw material, spare purchased parts and supplies. Inventory consists of a stock of items, the size of which is called the inventory level.

Definition: An inventory is the system of managing stock, demand and supply so that there is enough stock to supply according to the demand.

Inventory management involves making decisions concerning ***how much*** inventory to order and ***when***. The basic criterion in making these decisions is to minimize total inventory costs, such as the cost to carry inventory, the cost to order inventory, and the item cost, subject to meeting demand for the items.

Inventory control involves process, procedures, and infrastructure to maintain the inventory at the desired level.

Objectives of Inventory Model

- To minimize the possibility of delays in production through regular supply of raw materials.
- To keep inactive, waste, surplus, scrap and obsolete items at the minimum level.
- To maintain the overall investment in inventory at the lowest level, consistent with operating requirements.
- To exercise economies in ordering, obtaining, and storing of the materials.

Benefits of Inventory Model

- It enables the material to be procured in economic quantities.
- It eliminates delays in production caused by the non-availability of required materials.
- It reduces inventory losses caused by inadequate inspection of incoming materials and losses due to obsolescence, deterioration, waste and theft while in storage.
- It ensures proper execution of policies covering procurement and use of materials.
- It also facilitates timely adjustment with changing conditions in the market.

Basics Concepts of Inventory Systems

1. **Delivery lag (Lead Time)**: It is defined as the time between Purchase Order and Arrival of Item.

Purchase of	Delivery Lag
Car	1 Month
Khabsa	10 mins
Coffee at Bufia	2 mins
Buscuit packet	0 mins

- **Set up cost** : is the common cost of any Purchase.
- **Incremental cost**: is the cost depending on the number of Items Purchased.
- **Purchase cost** = set up cost + incremental Cost
- **Inventory level I(t)** : it is the Quantity of an Item in the Stock.
- **Re Order level (S):** it is the Quantity of an Item in the stock at which the new order for more purchase is given. **Note:** Purchase Order is given when I(t)< S
- **Order quantity:** It is the quantity of an Item is to be Purchased the stock.
- **Holding cost:** it is the Cost of keeping an Item in the stock.

Example: for Keeping (holding) more stock of milk, the holding cost are:
(a) More space in the fridge. b) Electricity. c) Buying Cost.

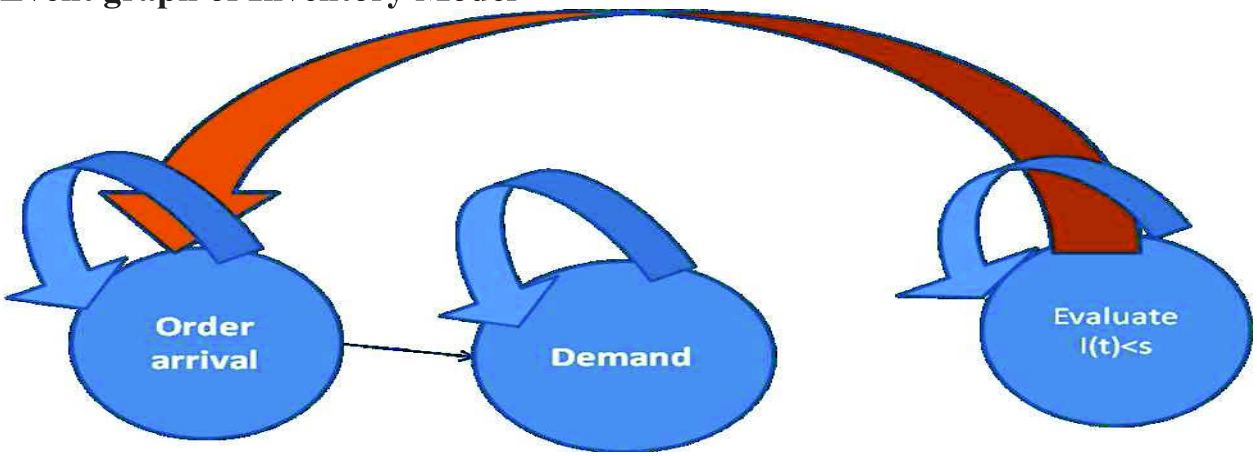
- **Shortage Cost:** It is the loss due to the non-availability of an Item.

Example*:* If there is a shortage of cups in a coffee shop, then the losses are a) No sale of coffee b) No sale of Tea.

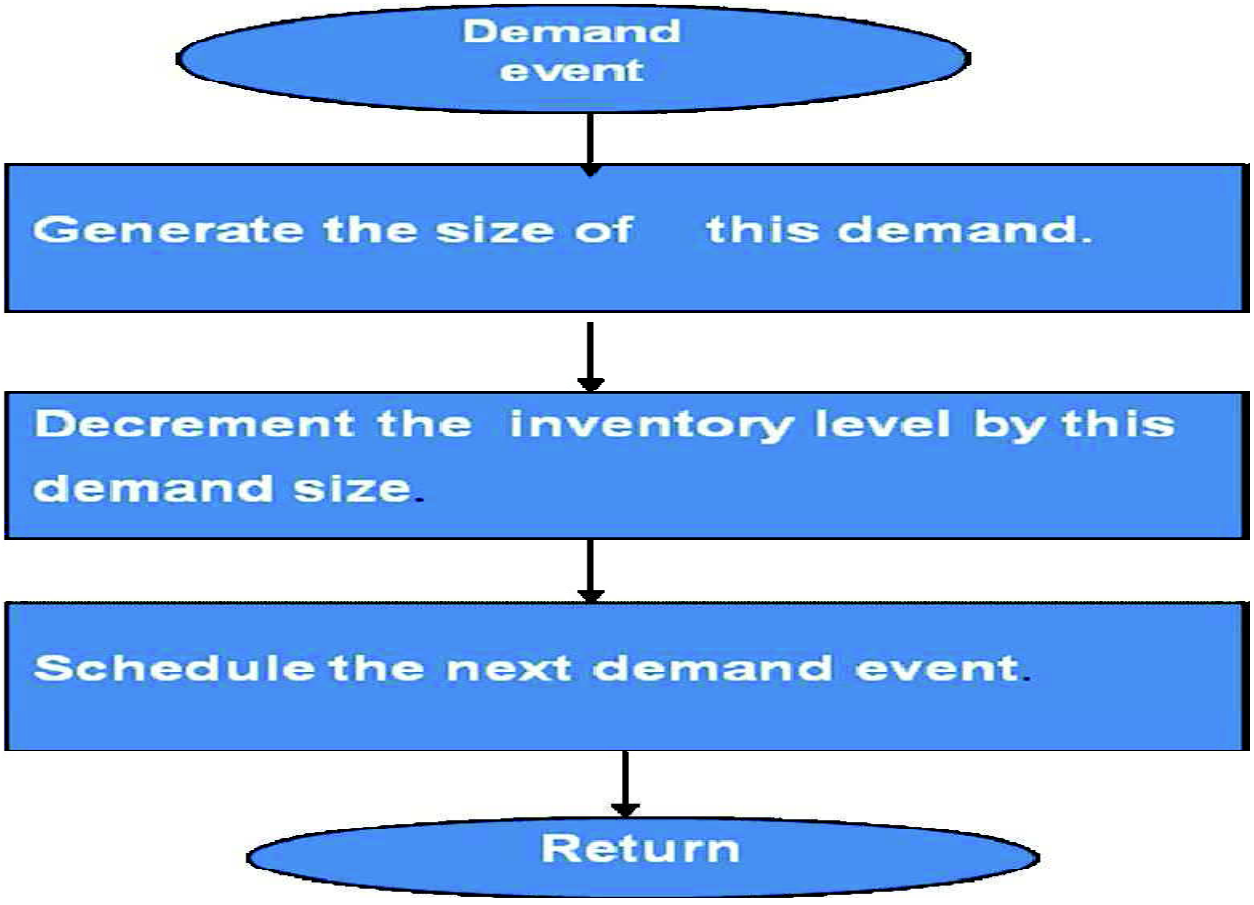
Q1.In a Bufia, the number of Sandwiches at present is 35 and the manager ask for purchase more when the stock is less than 10.
Sol: Inventory Level I(t) = 35
Re order Level S= 10

Q2. Bufia at KKU wants to Purchase 50 sandwiches from Khamis. Taxi charges from KKU to Khamis and return (20 SR). Cost of one sandwich at Khamis is 1SR.
Sol: Set up cost: Taxi charges from KKU to Khamis and return (20 SR).
Incremental Cost: Cost of one sandwich at Khamis is 1SR.
Purchase Cost = set up cost(K) + Incremental Cost(i) x Number of Items(Z)
Purchase cost for 50 sandwiches is = 20 + 1x 50=20+ 50 =70 SR
Purchase cost for 100 sandwiches is = 20 + 1x 100=120 SR

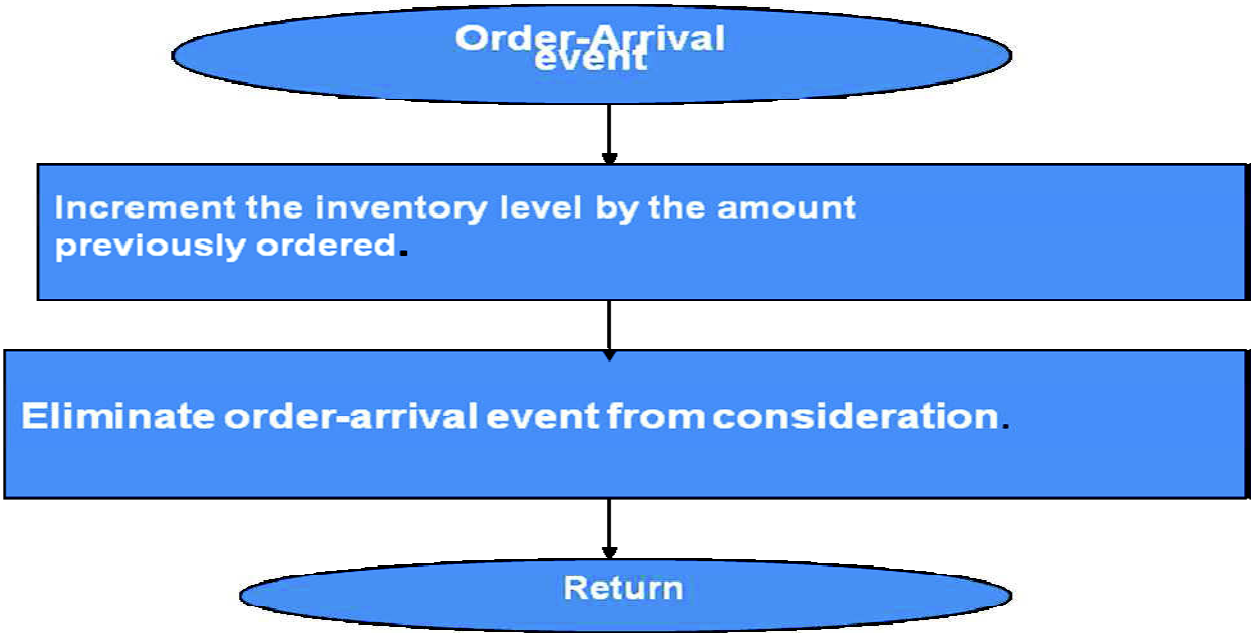
Event graph of Inventory Model



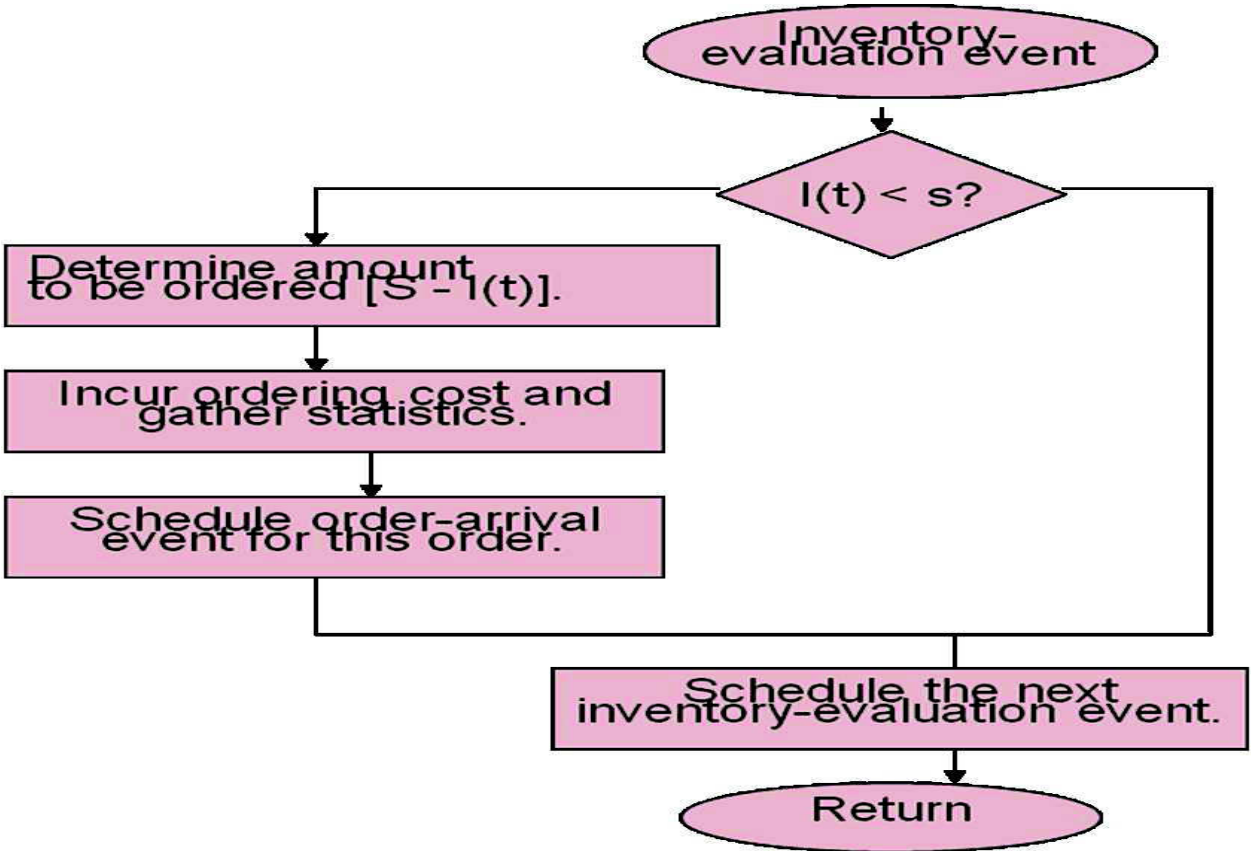
Flow chart for demand routine



Flow chart for order arrival routine



Flow chart for inventory evaluation routine



Deterministic Inventory Models

Notations used in the Models.

Q = Number of units ordered per order.
D = Rate of demand.
N = Number of orders placed per year.
TC = Total inventory cost
Co = Cost of ordering per order
C = Purchase or manufacturing price per unit Ch = Cost
of holding stock per unit per period of time.
Cs = Shortage cost per unit.
R = Reorder point
L = Lead time (weeks or months)
t = The elapsed time between placement of two successive orders6

Model I - Economic Order Quantity with no shortages (Purchase Model)

The main problem while purchasing material is how much to buy at a time. If large quantities are bought, the cost of carrying the inventory would be high. To the contrary, if frequent purchases are made in small quantities, costs relating to ordering will be high. So the problem is of indecision. How this problem can be resolved?

Definition: EOQ is that size of the order for which the cost of maintaining inventories is minimum. Therefore, the quantity to be ordered at a given time should be determined by taking into account two factors, i.e., the acquisition cost and the cost of possessing materials.

Assumptions

1. Demand rate is uniform over time and is known with certainty.
2. The inventory is replenished as soon as the level of the inventory reaches to zero. Thus shortages are not allowed.
3. Lead time is zero.
4. The rate of inventory replenishment is instantaneous.
5. Quantity discounts are not allowed

INVENTORY SYSTEM

Inventory means any stored resource that is used to satisfy a current or a future need; for example, the raw materials and finished goods. Therefore the inventory control is crucial for every company. Reducing on-hand inventory level means reducing costs and thus increases the cash flow. On the other hand frequent stock outs may dissatisfy the customers. Thus a company must make a balance between low and high inventory levels. The major factor to be considered in achieving this balance is the cost minimization.

INVENTORY DECISIONS

There are two fundamental decisions that we have to make when controlling inventory:

1. How much to order or produce?
2. When to order?

ECONOMIC ORDER QUANTITY (EOQ)

This is the basic model in achieving an optimal ordering quantity which minimizes the total inventory costs. However there are some assumptions to be upheld:

(a) Demand is known and constant.
(b) The lead time, the time between the placement of order and the receipt of the order, is known and constant.
(c) The receipt of inventory is instantaneous.
(d) Quantity discounts are not possible.
(e) The only variable costs are the ordering cost and the holding or carrying cost.
(f) Orders are placed so that stock outs or shortages are avoided completely.

Inventory Costs in the EOQ Situation

Let Q be the amount of quantity ordered. Thus an inventory levels increases from 0 to Q units when an order arrives. Since the inventory level changes daily, we use the average inventory level to determine the annual holding or carrying cost.

Therefore,

Maximum inventory level = Q
Average inventory level = $Q/2$

The following variables are important in developing mathematical expressions for the annual ordering and carrying costs.

Q = Ordering quantity
Q^* = EOQ
D = Annual Demand
C_o = Ordering cost per order
Ch = Holding or carrying cost per unit per year

Finding the EOQ

The total costs are minimized when

$$\text{Annual Ordering Cost} = \text{Annual Holding Cost}$$

$$\frac{D}{Q}C_o = \frac{Q}{2}C_h$$

$$Q^2 = \frac{2DC_o}{C_h}$$

$$Q = \sqrt{\frac{2DC_o}{C_h}}$$

Finding the ROP (Re-Order Point)

The second inventory question is 'when to order'. For this, we should know the lead time first. Lead time, L, (or delivery time) means the time between the placing and receipt of an order. This could be few days or few weeks and inventory must meet the demand during these days. Therefore,

ROP = (demand per day) × (lead time) = $d \times L$

This means, an order is placed when the inventory level reaches the ROP, and the new inventory arrives at the same instant the inventory is reaching 0.

Economic Order Quantity (EOQ) Model

Co= Annual Ordering Cost

Ch = Annual Holding Cost

$$\text{EOQ} = Q^* = \sqrt{\frac{2DCo}{Ch}}$$

Annual Ordering Cost = (number of orders per year) × (ordering cost per order)

$$= \frac{\mathbf{D}}{\mathbf{Q}} * C_o$$

Annual Holding Cost = (average inventory) × (carrying cost per unit per order)

$$= \frac{\mathbf{Q}}{\mathbf{2}} * Ch$$

Total cost without material(Annual) = Annual Ordering Cost + Annual Holding Cost

$$= \frac{\mathbf{D}}{\mathbf{Q}} * C_o + \frac{\mathbf{Q}}{\mathbf{2}} * Ch = \sqrt{2\text{DCoCh}}$$

Re-order Point = daily demand * lead time

Problems:

1. A factory requires 3600kg of raw material for producing an item per year. The cost of placing an order is Rs. 36 and the holding cost of the stock is Rs. 2.50 per Kilogram per year. Determine the EOQ.

Solution We are given that

D = 3600 kg per year

C_o = Rs 36 per order

C_h = Rs 2.50 per kg per year

$$\therefore \text{ Now } \quad EOQ = Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

$$= \sqrt{\frac{2 \times 3600 \times 36}{2.5}} = 322 \text{ kg}$$

$$\text{Period of one run } t^* = \frac{Q^*}{D} = \frac{322}{3600}$$

$$= 0.089 \text{ year}$$

$$= 32 \text{ days}$$

$$\text{Number of runs per year } n = \frac{D}{Q^*} = 11.4$$

2. The annual demand for an item is 3200 units. The unit cost is SR 6. The inventory carrying cost is 25% per annum per unit. The cost of one procurement is SR 150. Determine
(i) EOQ
(ii) Number of orders per year
(iii) Time between two consecutive orders
(iv) Total annual cost including material

Solution Given $D = 3200, \quad C = 6, \quad I = 0.25$

Co = SR150
Ch = 25% per annum per unit = 0.25 * 6 = 1.5 = SR1.5

$$\text{(i) EOQ} = Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2\times 3200\times 150}{1.5}}$$

$$= 800 \text{ units}$$

$$\text{(ii) Number of orders } n = \frac{D}{Q^*} = \frac{3200}{800} = 4$$

(iii) Time between two consecutive orders

$$= t^* = \frac{Q^*}{D} = \frac{800}{3200} = \frac{1}{4} = 3 \text{ months}$$

(iv) Total annual cost

= material cost + inventory cost

$$= D \times C + \sqrt{2D\,\text{Co Ch}}$$

$$= 3200 \times 6 + \sqrt{2\times 3200\times 150\times 1.5}$$

$$= 19200 + 1200$$

$$= \text{SR } 20400$$

3. A manufacturing company has to supply 12000 units of a product per year to his customer. Shortages are not permitted and there is no lead time. The inventory holding cost is SR 0.20 per unit per month and the set-up cost per run is SR 350. Determine

(i) The economic size

(ii) The period of one run

(iii) The minimum annual inventory cost [excluding material]

Solution We have D = 12000 units per year
= 1000 units per month

Co = SR350, Ch = 0.20 per unit per month

$$\text{(i) EOQ} = Q^* = \sqrt{\frac{2DC_0}{C_h}} \sqrt{\frac{2\times1000\times350}{0.20}}$$

$$= 1870 \text{ units}$$

$$\text{(ii) Period of one run } t^* = \frac{Q^*}{D} = \frac{1870}{1000}$$

$$= 1.87 \text{ months}$$

$$= 56 \text{ days}$$

(iii) Annual inventory cost

$$= \sqrt{2D\text{Co Ch}} \times 12$$

$$= \sqrt{2\times1000\times350\times(0.2)} \times 12$$

$$= 374 \times 12 = \text{Rs } 4490$$

4. A Car Hardware Spare parts Store in Riyadh sells 2,500 tools in a year and the sales is relatively constant throughout the year. These tools are purchased from a supplier from JAPAN for SR 15.00 each, and the lead time is two days. The holding cost per tool per year is SR 1.50 (or 10% of the unit cost) and the ordering cost per order is SR 18.75. There are 250 working days per year. (i) What is the EOQ? (ii) Given the EOQ, what is the average inventory? (iii) What is the annual holding cost? (iv) In minimizing the cost, how many orders would be made each year? (v) What would be the annual ordering cost. (vi) Given the EOQ, what is the total annual inventory cost (including purchase cost)? (vii) What is the time between orders? (viii) What is the ROP?

Demand D = 2,500 tools per year, Unit Cost C = SR 15.00, Lead time = 2 days
Holding Cost Ch = SR 1.50per tool per year, Ordering Cost Co = SR 18.75 per order, Total number of days in a year = 250

1)

$$EOQ = Q^* = \sqrt{\frac{2DCo}{Ch}}$$

$$= 250 \text{ tools}$$

2) Average inventory = Q/2 = 250/2 = 125 tools

3) Annual Holding Cost = (average inventory)×(carrying cost per unit per order)

$$= \frac{\mathbf{Q}}{\mathbf{2}} * Ch$$

$$= 125 * 1.50 = \text{SR } 187.50$$

4) Number of orders per year $= \frac{\mathbf{D}}{\mathbf{Q}} = \frac{\mathbf{2500}}{250} = 10$

5) Annual Ordering Cost= (number of orders per year)×(ordering cost per order)

$$= \frac{\mathbf{D}}{\mathbf{Q}} * C_o$$

$$= 10 * 18.75 = \text{SR } 187.50$$

6) Total Inventory cost including material = $D*C + \sqrt{2*D*Co*Ch}$ =SR 37875

7) Time between the order = $\frac{\mathbf{Q}}{\mathbf{D}} = \frac{\mathbf{250}}{\mathbf{2500}}$ =0.1 year = 0.1 * 250 days = 25 days

8) Re-order point = Daily demand * Lead time = $\frac{2500}{250} * 2 = 20$

5. **An auto parts supplier sells Hardy-brand batteries to car dealers and auto mechanics. The annual demand is approximately 1,200 batteries. The supplier pays $28 for each battery and estimates that the annual holding cost is 30 percent of the battery's value . It costs approximately $20 to place an order (managerial and clerical costs). The supplier currently orders 100 batteries per month.**

a. Determine the ordering, holding, and total inventory costs for the current order quantity.

b. Determine the economic order quantity (EOQ).

c. How many orders will be placed per year using the EOQ?

d. Determine the ordering, holding, and total inventory costs for the EOQ. How has ordering cost changed? Holding cost? Total inventory cost?

Solution : We are given the following information:

Annual demand: $D = 1200$ batteries per year

item cost: $c = \$28$ per battery

holding cost: $Ch = ic = 0.30*(28) = \$8.40$ per battery per year

order cost: $Co = \$20$ per order

current order quantity: $Q = 100$ batteries per month

a. The current ordering and holding costs are:

Annual Ordering Cost = (number of orders per year) × (ordering cost per order)

$$= \frac{\mathbf{D}}{\mathbf{Q}} * C_o = \frac{\mathbf{1200}}{\mathbf{100}} * 20 = 240$$

Annual Holding Cost = (average inventory) × (carrying cost per unit per order)

$$= \frac{\mathbf{Q}}{\mathbf{2}} * Ch = \frac{\mathbf{100}}{\mathbf{2}} * 8.40 = 420$$

Total cost without material(Annual) = Annual Ordering Cost + Annual Holding Cost

$$= \frac{\mathbf{D}}{\mathbf{Q}} * C_o + \frac{\mathbf{Q}}{\mathbf{2}} * Ch = \sqrt{2DCoCh} = 240 + 420 = 660$$

b. The EOQ = Q* = 75.6 = 76 batteries

c. Number of orders per year = $\frac{D}{Q*}$ = = 15.78 orders

d Annual Ordering Cost = (number of orders per year) × (ordering cost per order)

$$= \frac{\mathbf{D}}{\mathbf{Q*}} * C_o = \frac{\mathbf{1200}}{\mathbf{76}} * 20 = 315.78$$

Annual Holding Cost = (average inventory) × (carrying cost per unit per order)

$$= \frac{\mathbf{Q*}}{\mathbf{2}} * Ch = \frac{\mathbf{76}}{\mathbf{2}} * 8.40 = 319.20$$

Total cost without material(Annual) = Annual Ordering Cost + Annual Holding Cost

$$= \frac{\mathbf{D}}{\mathbf{Q*}} * C_o + \frac{\mathbf{Q*}}{\mathbf{2}} * Ch = \sqrt{2DCoCh} = 634.98$$