Chapter-2

TYPES OF SIMULATION

Introduction: There are various types of simulation. Here we will study mainly four types. These are as follows:

1. Discrete Event System Simulation
2. Continuous Simulation
3. Combined Continuous and Discrete Model
4. Monte Carlo Simulation

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Discrete-Event Simulation: Discrete-event simulation concerns the modeling of a system as it evolves over time by a representation in which the state variables change instantaneously at separate points in time.

Time-Advance Mechanisms: Discrete-event simulation models are dynamic in nature, therefore, we must keep track of the current value of simulated time as the simulation proceeds, and we also need a mechanism to advance simulated time from one value to another.

Simulation clock: Variable that keeps the current value of (simulated) time in the model.
● Must decide on, be consistent about, time units
● Usually no relation between simulated time and (real) time needed to run a model on a computer.

Most common

There are two approaches for time advance:
The first approach is
● ***Next-event time advance*** **(usually used)**

Steps

- ● Initialize simulation clock to 0
- ● Determine times of occurrence of future events – *event list*
- ● Clock advances to next (most imminent) event, which is executed
- ● Event execution may involve updating event list
- ● Continue until stopping rule is satisfied (must be explicitly stated)
- ● Clock "jumps" from one event time to the next, and doesn't "exist" for times between successive events … periods of inactivity are ignored

The second approach is
● ***Fixed-increment time advance*** **(seldom used)**

● Generally introduces some amount of modeling error in terms of when events *should* occur vs. *do* occur

sometimes there will be an error in the modeling when events should happen or do happen

First aproach

Single-server queue -Next-event time advance:

t_i = time of arrival of ith customer ($t_0 = 0$)

$A_i = t_i - t_{i-1}$ = interarrival time between (i-1)st and ith customers (usually assumed to be a random variable from some probability distribution)

S_i = service-time requirement of ith customer (another random variable)

D_i = delay in queue of ith customer

$C_i = t_i + D_i + S_i$ = time ith customer completes service and departs

e_j = time of occurrence of the jth event (of any type), j = 1, 2, 3, …Possible trace of events.

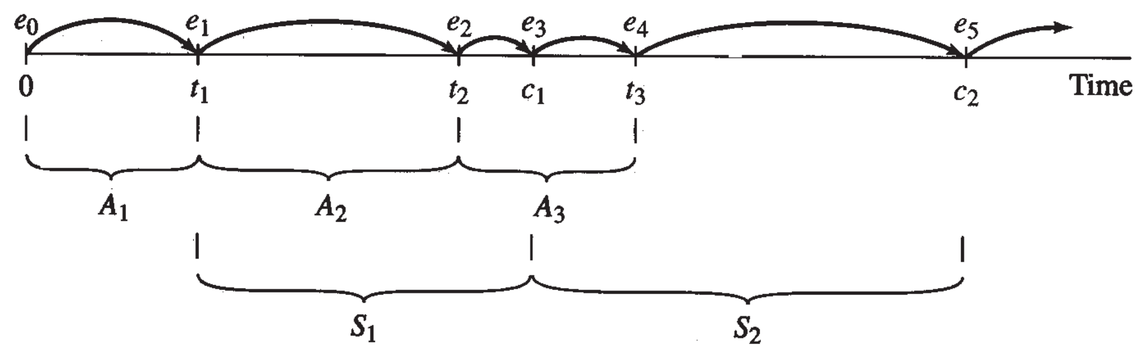

Figure 2.1: Next event time advance

Components and Organization of a Discrete-Event Simulation Model:

- Each simulation model must be customized to target system
- The following components will be found in most discrete-event simulation models using the next-event time-advance approach.

examples of systems using next event time advance

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– *System state* – variables to describe state
– *Simulation clock* – current value of simulated time
– *Event list* – times of future events (as needed)
– *Statistical counters* – to accumulate quantities for output
– *Initialization routine* – initialize model at time 0
– *Timing routine* – determine next event time, type; advance clock
– *Event routines* – carry out logic for each event type
– *Library routines* – utility routines to generate random variates, etc.
– *Report generator* – to summarize, report results at end
– *Main program* – ties routines together, executes them in right order

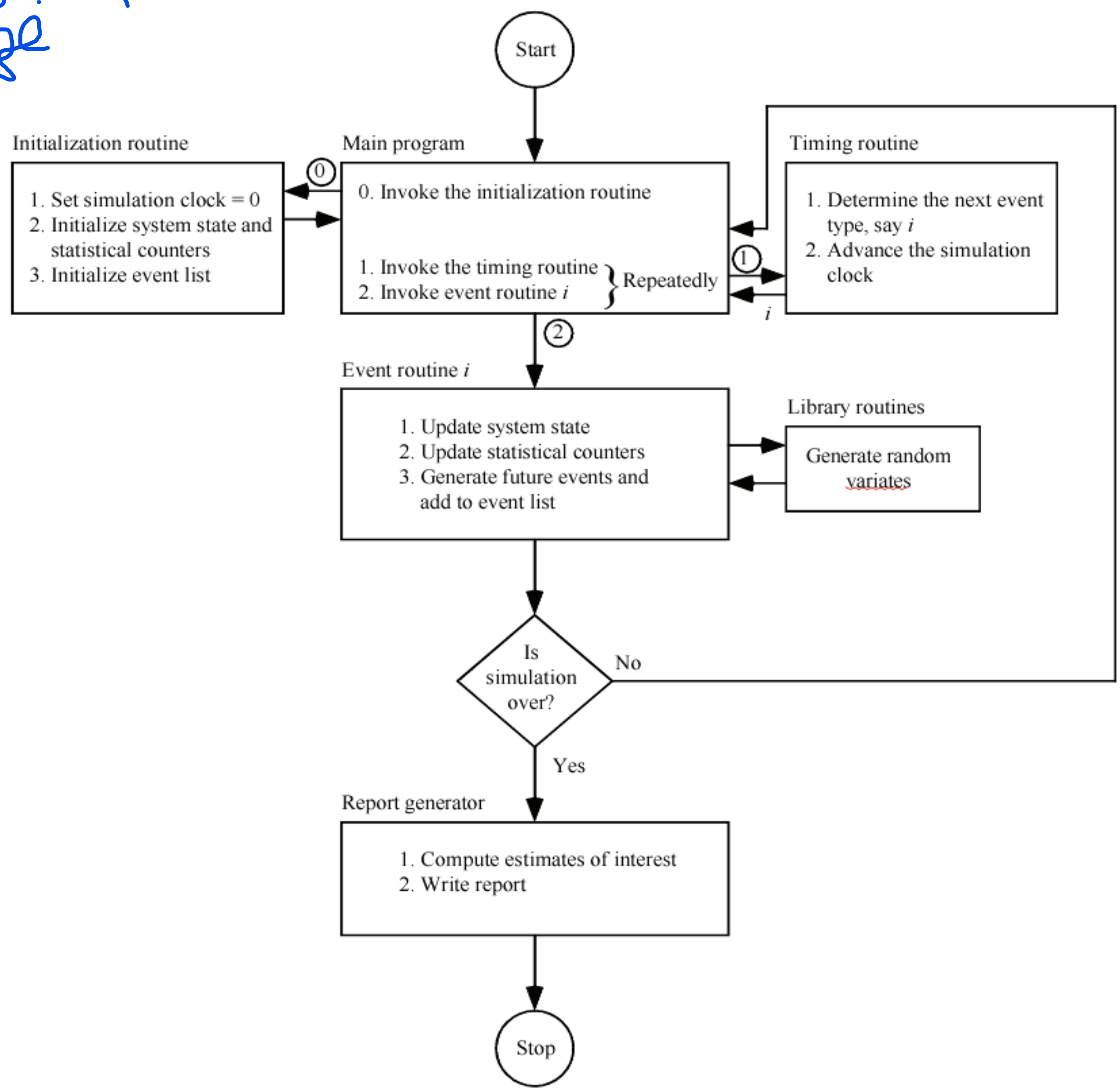

Figure 2.2: Organization of discrete event simulation model

CONTINUOUS SIMULATION

Continuous Simulation refers to a computer model of a physical system that continuously tracks system response over time according to a set of equations typically involving differential equations. These parameters of a system change in a continuous way and thus change the state of the entire system.

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Example: Predator-Prey Model

Predator/prey model :

Predator: is animal which kills other animals. and eat.

Prey: is an animal which is killed by other animals for food. علاقة طردية

Example: Tiger (Predator)-Cat(Prey)

This is a model for Competing Population. As long as the population of the prey is on the rise, the predators population also rises, since they have enough to eat. But very soon the population of the predators becomes too large that the hunting exceeds the recreation of the prey. This leads to a decrease in the prey's population and as a consequence of this it also leads to a decrease of predators population as they do not have enough food to feed.

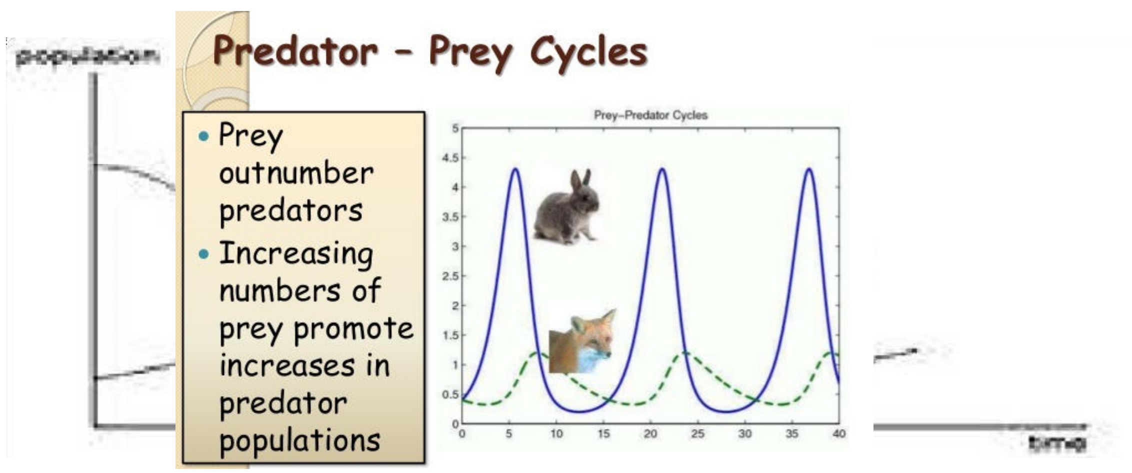

Figure 2.3: Graph of predator-prey model

Notation: Predator-Prey model

- $X(t)$: the population of prey at time=t.
- $Y(t)$: the population of predator at time=t.
- a: rate of change of population of prey due to interaction.
- b: rate of change of population of predators due to interaction.
- r: natural birth and death rate of prey.
- s: natural birth and death rate of predator.

Rate of change of the prey population.

$dx/dt=rx(t)-ax(t)y(t)$

Rate of change of the predator population.

$dy/dt=-sy(t)+bx(t)y(t)$

where a ,b,s,r is a positive constant of proportionality

COMBINED CONTINUOUS AND DISCRETE MODEL

Since some systems are neither completely discrete nor completely continuous, the need may arise to construct a model with the aspects of both discrete-event and continuous simulation, resulting in a combined discrete-continuous simulation.

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Interaction between discrete and Continuous state variables can change both discrete and continuous variable.

1. A discrete event may cause change in the continuous variable.
Ex: switch (ON /OFF) discrete event and cause change in Light(cont. variable)

2. A Continuous event may change in a continuous variable.
Ex: Accelerator (fuel) and speed is continuous variable. Change in Accelerator changes speed.

3. A continuous event may cause change in a discrete variable.
Ex.: In a fridge, when the continuous variable-temperature reaches 18 degrees, the discrete variable switch will change to OFF state.

Example: Oil Refinery Model

Trucks carrying crude oil arrive and form a queue. The oil from the truck is unloaded to the storage tank from the storage tank. Oil is pumped through a pipeline to the refinery. At the refinery, the crude oil is purified and many products are produced.

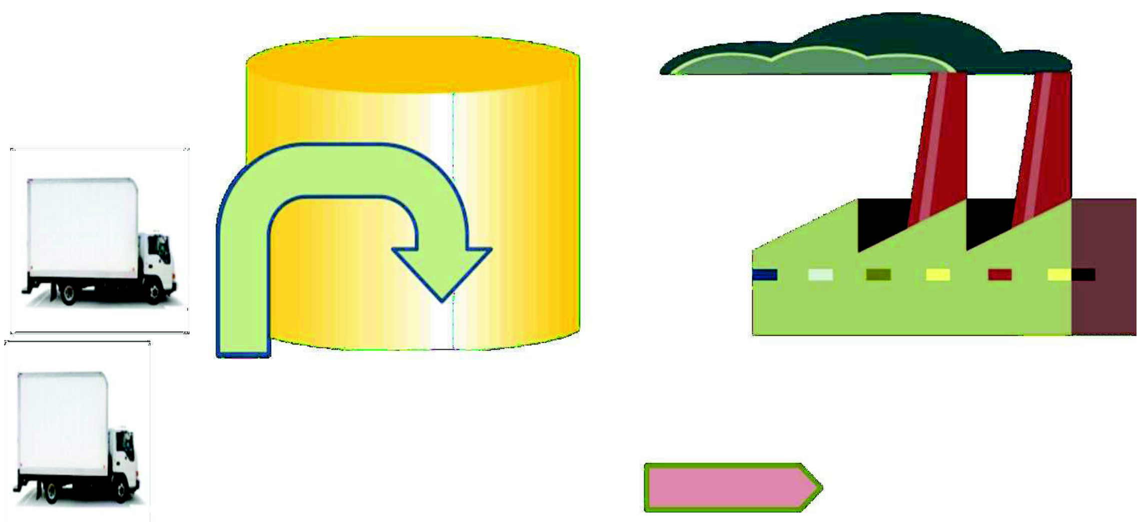

Figure 2.4: Model of oil Refinary

Example

Oil Refinery Model(Assumption)
1. Refinery start at 6.00am and 10.00 pm
2. Unloading of the trucks is FIFO.
3. unloading stops when level of oil in the truck is <= 5%.
4. Unloading stop when level of oil in the storage tank is >= 90%.
5. Unloading resumes when level of oil in the storage tank<=80%
6. Pumping oil to the refinery starts when level of oil in the storage tank >=20%.
7. Pumping oil is stopped when level of oil in the storage tank is <=10 %.
8. Pumping oil is restarts when level of oil in the storage tank is <=15 %.

Events of oil refinery model

1. Arrival of Trucks
2. Unloading of Trucks.
3. Pumping of oil.
4. Working of the Refinery.

State Variable in oil refinery model

1. Level of Oil in the Truck.
2. Level of Oil in the Storage tank.
3. Time (Continuous).
4. Unloading (Discrete: Start/Stop/Stop/ restart).

MONTE CARLO SIMULATION

The Monte Carlo method may be used when the model contains elements that exhibit change in their behaviour. It is used for solving certain stochastic or deterministic problems.

Steps

1. Set up probability distributions for important variables
2. Build a cumulative probability distribution for each variable
3. Establish an interval of random numbers for each variable
4. Generate random numbers
5. Simulate a series of trials

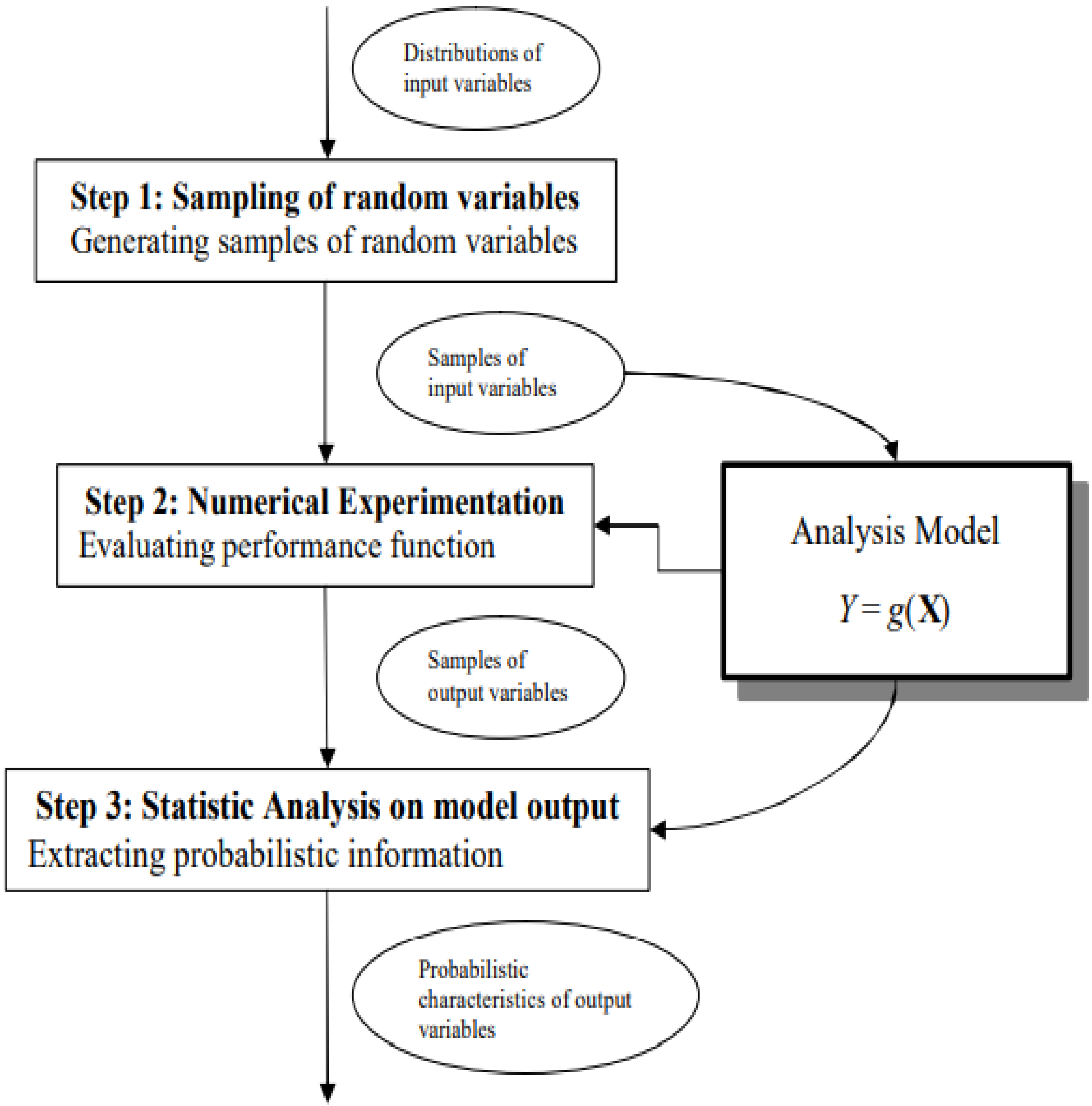

Figure: Flowchart of Monte Carlo Simulation Model

Monte Carlo Simulation – Advantages

- Easy to implement.
- Provides statistical sampling for numerical experiments using the computer.
- Provides approximate solution to mathematical problems.
- Can be used for both stochastic and deterministic problems.

Monte Carlo Simulation – Disadvantages

- Time consuming as there is a need to generate large number of sampling to get the desired output.
- The results of this method are only the approximation of true values, not the exact.
- where fx(x) = l/(b - a) is the probability 'density function of'a U(a, b) random

- *Monte Carlo simulation*
  - No time element (usually)
  - Wide variety of mathematical problems
  - Example: Evaluate a "difficult" integral $I = \int_a^b g(x)\,dx$
    - Let $X \sim U(a, b)$, and let $Y = (b - a)\, g(X)$
    - Then $E(Y) = E[(b-a)g(X)]$

$$
\begin{aligned}
&= (b-a)E[g(X)] \\
&= (b-a)\int_a^b g(x) f_X(x)\,dx \\
&= (b-a)\int_a^b g(x)\frac{1}{b-a}\,dx \\
&= \int_a^b g(x)\,dx \\
&= I
\end{aligned}
$$

    - Algorithm: Generate $X \sim U(a, b)$, let $Y = (b - a)\, g(X)$; repeat; average the Y's … this average will be an unbiased estimator of I