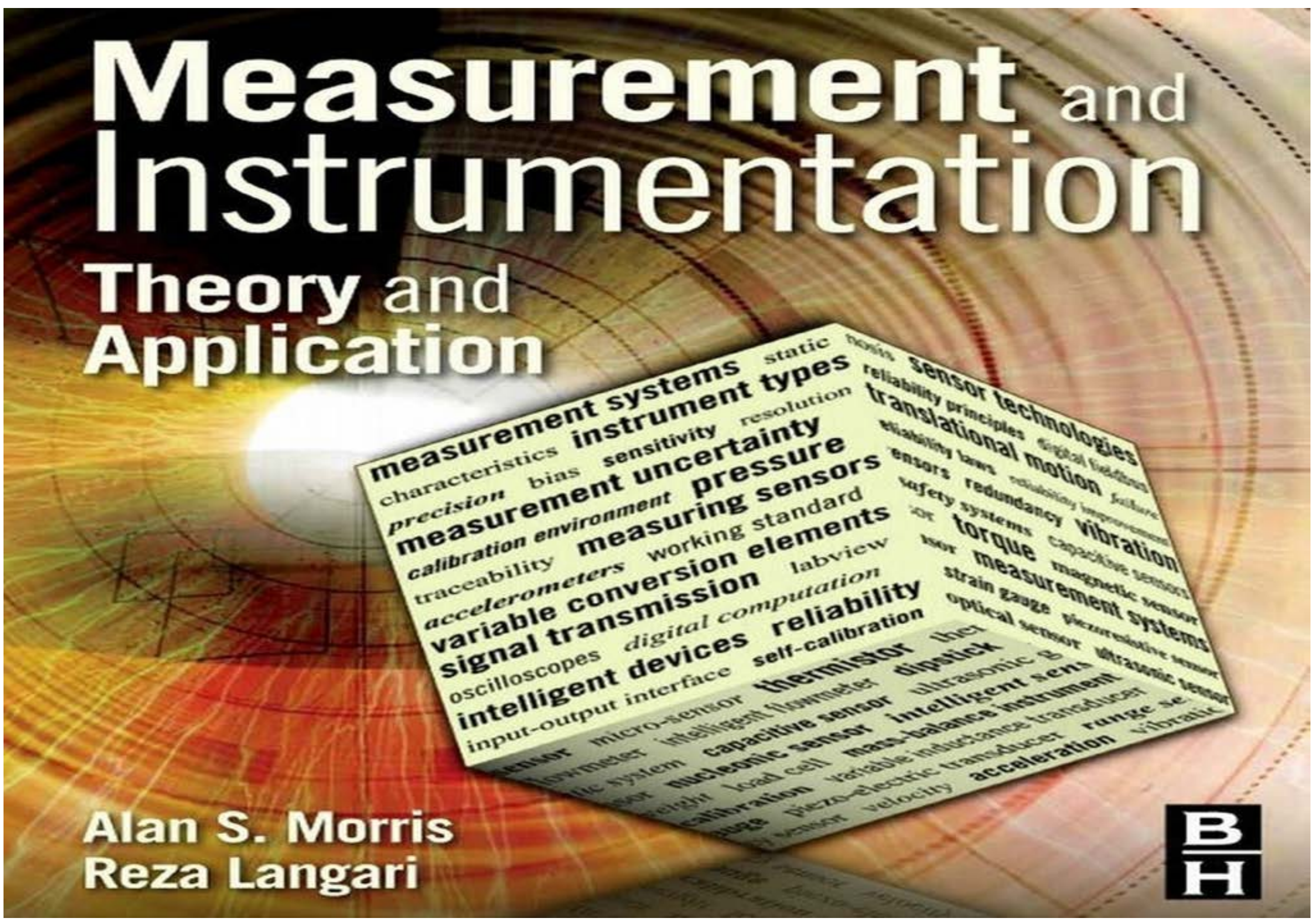
Course: 236CCE

Electronic Measurements

Lectures Notes of Chapter 3

Principals of Measurement Techniques

This lectures notes are extracted from the text book: " **Alan S Morris and Reza Langari, Measurement and Instrumentation: Theory and Application, Second edition, Academic Press, 2015. ISBN-13: 978-0128008843**. From CHAPTER 9: Variable Conversion Elements (From page208 to page 238)



Capacitance Measurement Techniques

There are two methods for measuring capacitance.

1. Approximate method using a variable resistance and two voltmeters.
2. Accurate method using Schering bridge.

Dissipation Factor of a Capacitor (D)

Dissipation factor is a s a measure of loss-rate of energy of a given capacitor. D tells how inefficient the insulating material of a capacitor is.

$$D = \frac{\text{Average Power}}{\text{Reactive Power}}$$

- The dissipated power is in the form of heat that is lost when a capacitor dielectric is exposed to an alternating field of electricity.
- If a capacitor has a low dissipation factor at a particular frequency, this generally means it has a better efficiency at that frequency.
- D is the reciprocal of Q factor, which represents the quality of storage of a capacitor.
- D tells us about the quality of a capacitor, how close the phase angle of the capacitor is to the ideal value of 90^0.
- Typical values of D are between 10^{-4} to 0.1.
- The D factor for parallel RC circuit is:

$$D = \frac{X_C}{R_C} = \frac{1}{\omega C R_C}$$

- The D factor for series RC circuits is:

$$D = \frac{R_C}{X_C} = \omega C R_C$$

Approximate Method for Capacitance Measurement

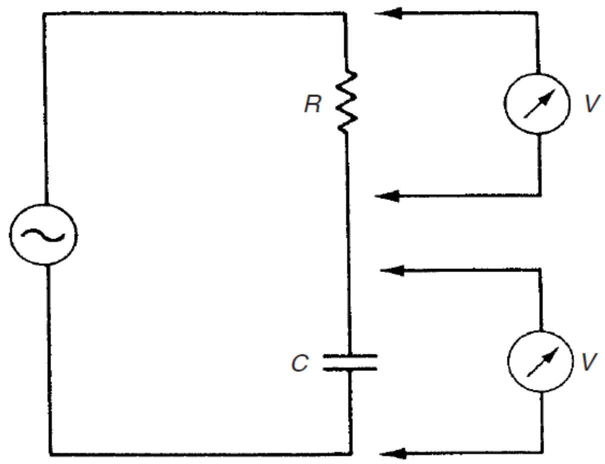
This method connects the unknown capacitance in series with a known resistor, in a circuit excited with a known frequency sinusoidal voltage. The voltage is measured across the resistor, R, and across the capacitor. The value of the capacitance C is:

$$\frac{|X_R|}{|X_C|} = \frac{V_R}{V_C}$$

$$\omega CR = \frac{V_R}{V_C} \qquad \rightarrow \rightarrow \qquad C = \frac{V_R}{2\pi f R V_C}$$

Example 1

The value of the variable resistance of the approximate method for measuring capacitor is $R = 80\Omega$. The voltage across the variable resistance and the capacitor are 20V and 30V. Find the capacitance value if the supply frequency is 60Hz?



Solution 1

$$C = \frac{V_R}{2\pi f R V_C} = \frac{20}{2\pi 60 \times 80 \times 30} = 22.1\mu\text{F}$$

Schering Bridge Circuit

Schering bridge is useful for measuring capacitors with high losses (high Ds) with a good accuracy. It consists of two fixed-value elements, R_3 and C_2 and two variable elements, R_4 and C_3. This bridge measures an unknown capacitance in terms of known capacitances.

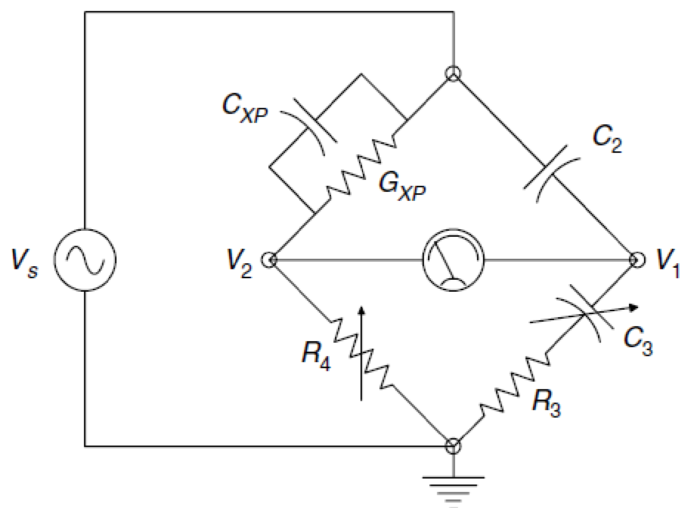
$$\frac{1}{Z_{XP}} = \frac{1}{R_{XP}} + j\omega C_{XP}$$

$$Z_{XP} = \frac{R_{XP}}{1 + j\omega C_{XP} R_{XP}}$$

$$Z_4 = R_4$$

$$Z_3 = R_3 + \frac{1}{j\omega C_3}$$

$$Z_2 = \frac{1}{j\omega C_2}$$



At balance

$$I_1 Z_{XP} = I_2 Z_2 \quad \text{and} \quad I_1 Z_4 = I_2 Z_3$$

$$\frac{I_2}{I_1} = \frac{Z_{XP}}{Z_2} \quad \text{and} \quad \frac{I_2}{I_1} = \frac{Z_4}{Z_3}$$

$$\frac{Z_{XP}}{Z_2} = \frac{Z_4}{Z_3} \quad \Rightarrow \quad Z_{XP} = Z_2 \frac{Z_4}{Z_3}$$

$$|Z_{XP}|\angle\theta_{XP} = |Z_2|\angle\theta_2 \times \frac{|Z_4|\angle\theta_4}{|Z_3|\angle\theta_3}$$

The magnitude balance and phase balance of this circuit are

$$|Z_{XP}| = |Z_2| \times \frac{|Z_4|}{|Z_3|}$$

$$\angle\theta_{XP} = \angle\theta_2 + \angle\theta_4 - \angle\theta_3$$

The real and imaginary parts of the unknown impedance can be calculated as below

$$\frac{R_{XP}}{1 + j\omega C_{XP} R_{XP}} = \frac{1}{j\omega C_2} \frac{R_4}{\left(R_3 + \frac{1}{j\omega C_3}\right)} = \frac{R_4}{j\omega C_2 R_3 + \frac{C_2}{C_3}}$$

$$\frac{1}{R_{XP}} + j\omega C_{XP} = \frac{j\omega C_2 R_3}{R_4} + \frac{C_2}{C_3 R_4}$$

$$R_{XP} = R_4 \frac{C_3}{C_2}$$

$$C_{XP} = C_2 \frac{R_3}{R_4}$$

The dissipation factor is

$$D = \frac{1}{\omega C_{XP} R_{XP}} = \frac{1}{\omega C_3 R_3}$$

Example 2

The fixed-value components of a Schering bridge are $R_3 = 15\Omega$, and $C_2 = 2mF$. At balance $R_4 = 90\Omega$ and $C_3 = 10mF$. The supply frequency of 50 Hz.

1. Calculate the value of the unknown impedance?
2. Calculate D factor?

Solution 2

At balance

$$R_{XP} = R_4 \frac{C_3}{C_2} = 90 \times \frac{10m}{2m} = 450\Omega$$

$$C_{XP} = C_2 \frac{R_3}{R_4} = 2m \times \frac{15}{90} = 0.333mF$$

$$D = \frac{1}{\omega C_{XP} R_{XP}} = \frac{1}{\omega C_3 R_3} = \frac{1000}{2\pi 50 \times 10 \times 15} = 0.0212$$

Inductance Measurement Techniques

There are two methods for measuring inductance.

1. Approximate method using a variable resistance and two voltmeters.
2. Accurate method using Maxwell bridge or Hey bridge.

Quality Factor of a Coil

Quality factor of a coil is a measure of the amount of energy placed in storage by the coil compared to that dissipated. Mathematically, it is the ratio of the coil reactance to the coil resistance.

$$Q = \frac{\text{Reactive Power}}{\text{Average Power}} = \frac{X_L}{R_L} = \frac{\omega L}{R_L}$$

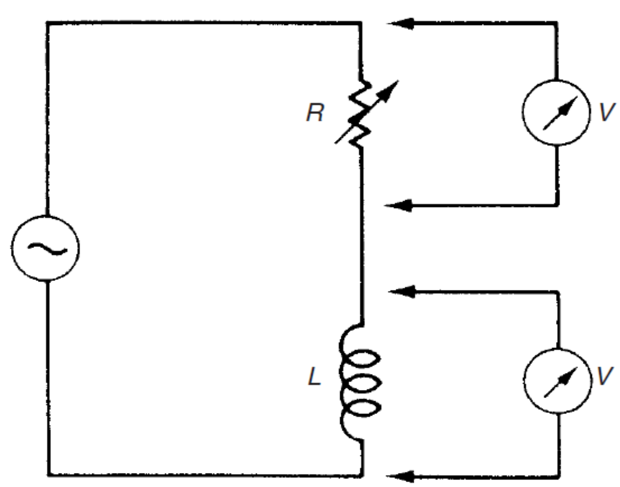
Approximate Method for Inductance Measurement

This method connects the unknown inductance in series with a variable resistance, in a circuit excited with a sinusoidal voltage. The variable resistance, R, is adjusted until the voltage measured across the resistance is equal to that measured across the inductance. At that time, the values of two impedances are then equal, and the value of the inductance L can be calculated from:

$$R = |X_L| = \sqrt{r^2 + \omega^2 L^2}$$

$$R^2 = r^2 + \omega^2 L^2$$

$$L = \frac{\sqrt{(R^2 - r^2)}}{2\pi f}$$



where r is the value of the inductor resistance and f is the excitation frequency.

Example 1

A 2A current is measured through unknown coil when a 20V DC voltage is applied across that coil. A 60Hz AC supply is applied across that coil in series with a variable resistor. The same voltage across the variable resistor is obtained when $R = 100\Omega$. Find the coil inductance?

Solution 1

$$r = \frac{V}{I} = \frac{20V}{2A} = 10\Omega$$

$$L = \frac{\sqrt{(R^2 - r^2)}}{2\pi f} = \frac{\sqrt{(100^2 - 10^2)}}{2\pi 60} = 0.2639H$$

This bridge is commonly used to measure unknown inductances with a good accuracy. It consists of two fixed-value elements, R_3 and C, and two variable resistances, R_1, and R_2. This bridge measures an unknown inductance in terms of a known capacitance.

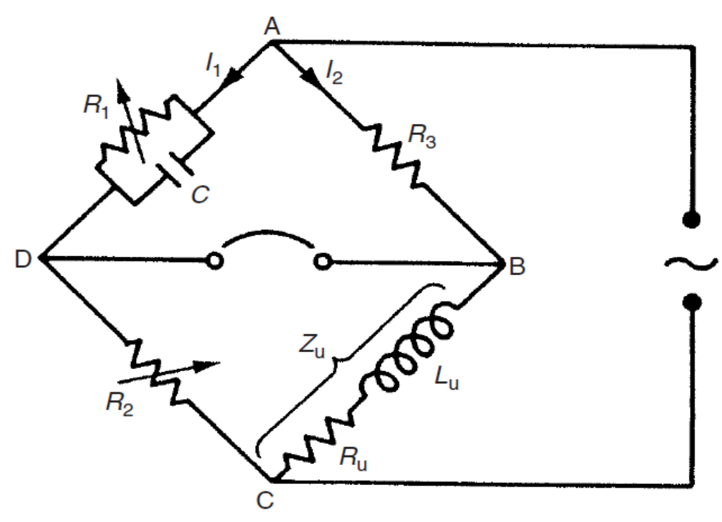
$$\frac{1}{Z_{AD}} = \frac{1}{R_1} + j\omega C$$

$$Z_{AD} = \frac{R_1}{1 + j\omega C R_1}$$

$$Z_{AB} = R_3$$

$$Z_{BC} = R_u + j\omega L_u$$

$$Z_{DC} = R_2$$



At balance

$$I_1 Z_{AD} = I_2 Z_{AB} \quad \text{and} \quad I_1 Z_{DC} = I_2 Z_{BC}$$

$$\frac{I_1}{I_2} = \frac{Z_{AB}}{Z_{AD}} \qquad \text{and} \qquad \frac{I_1}{I_2} = \frac{Z_{BC}}{Z_{DC}}$$

$$\frac{Z_{BC}}{Z_{DC}} = \frac{Z_{AB}}{Z_{AD}} \qquad \rightrightarrows \qquad Z_{BC} = Z_{DC}\frac{Z_{AB}}{Z_{AD}}$$

$$|Z_{BC}|\angle\theta_{BC} = |Z_{DC}|\angle\theta_{DC} \times \frac{|Z_{AB}|\angle\theta_{AB}}{|Z_{AD}|\angle\theta_{AD}}$$

The magnitude balance and phase balance of this circuit are

$$|Z_{BC}| = |Z_{DC}| \times \frac{|Z_{AB}|}{|Z_{AD}|}$$

$$\angle\theta_{BC} = \angle\theta_{DC} + \angle\theta_{AB} - \angle\theta_{AD}$$

The real and imaginary parts of the unknown impedance can be calculated as below

$$R_u + j\omega L_u = \frac{R_2R_3(1 + j\omega CR_1)}{R_1} = \frac{R_2R_3}{R_1} + j\omega CR_2R_3$$

$$R_u = \frac{R_2R_3}{R_1}$$

$$L_u = CR_2R_3$$

The quality factor is

$$Q = \frac{\omega L}{R_L} = \frac{\omega CR_2R_3R_1}{R_2R_3} = \omega CR_1$$

If a constant frequency ω is used:

$$Q \approx R_1$$

Thus, the Maxwell bridge can be used to measure the Q value of a coil directly. This bridge is suitable for medium Q value coils (1-10). It is impractical for high Q value coils since R_1 will be very large.

Example 2

The fixed-value components of a Maxwell bridge are $R_3 = 5\Omega$, and $C = 1mF$. At balance $R_1 = 159\Omega$ and $R_2 = 10\Omega$.

1. Calculate the value of the unknown impedance?
2. Calculate Q factor at a supply frequency of 50 Hz?

Solution 2

At balance

$$R_u = \frac{R_2 R_3}{R_1} = \frac{10 \times 5}{159} = 0.3145\Omega$$

$$L_u = CR_2R_3 = 10^{-3} \times 10 \times 5 = 50mH$$

$$Q = \omega C R_1 = 2\pi \times 50 \times 10^{-3} \times 159 = 49.95$$

Hay Bridge Circuit

This bridge is similar to Maxwell bridge but R_1 is connected in series with C. This bridge is suitable for high Q value coils (>10).

$$Z_{AD} = R_1 + \frac{1}{j\omega C}$$

$$Z_{AD} = \frac{j\omega C R_1 + 1}{j\omega C}$$

$$Z_{AB} = R_3$$

$$Z_{BC} = R_u + j\omega L_u$$

$$Z_{DC} = R_2$$

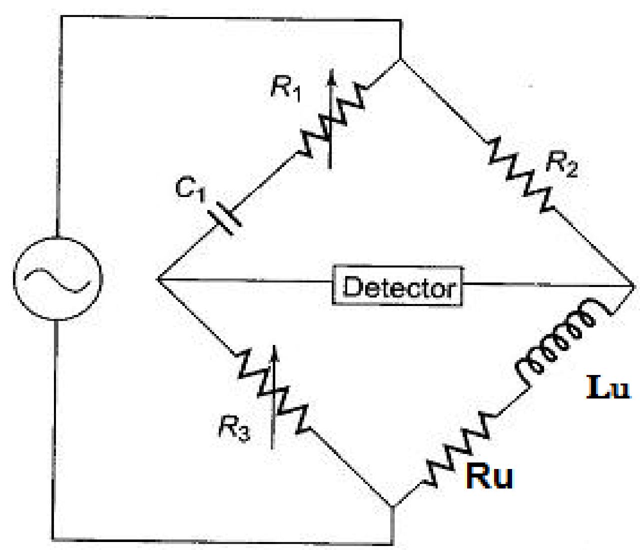


Fig. 11.23 Hay's Bridge

At balance

$$Z_{BC} = Z_{DC}\frac{Z_{AB}}{Z_{AD}} = R_2\frac{j\omega CR_3}{1+j\omega CR_1} = \frac{j\omega CR_2R_3}{1+j\omega CR_1}\frac{1-j\omega CR_1}{1-j\omega CR_1} = \frac{\omega^2C^2R_1R_2R_3 + j\omega CR_2R_3}{1+(\omega CR_1)^2}$$

The real and imaginary parts of the unknown impedance can be calculated as below

$$R_u + j\omega L_u = \frac{\omega^2C^2R_1R_2R_3}{1+(\omega CR_1)^2} + j\frac{\omega CR_2R_3}{1+(\omega CR_1)^2}$$

$$R_u = \frac{\omega^2C^2R_1R_2R_3}{1+(\omega CR_1)^2}$$

$$L_u = \frac{CR_2R_3}{1+(\omega CR_1)^2}$$

The quality factor is

$$Q = \frac{\omega L}{R_L} = \frac{1}{\omega CR_1}$$

Example 3

The fixed-value components of a Hay bridge are $R_3 = 5\Omega$, and $C = 1\mu F$. At balance $R_1 = 15\Omega$ and $R_2 = 10\Omega$. The supply frequency is 50 Hz.

1. Calculate the value of the unknown impedance?
2. Calculate Q factor?

Solution 3

At balance

$$R_u = \frac{\omega^2C^2R_1R_2R_3}{1+(\omega CR_1)^2} = \frac{(2\pi 50)^2\times 10^{-12}\times 15\times 10\times 5}{1+(2\pi 50\times 10^{-6}\times 15)^2} = \frac{7.4022\times 10^{-5}}{1}$$

$$= 7.4022\times 10^{-5}\Omega$$

$$L_u = \frac{CR_2R_3}{1+(\omega CR_1)^2} = \frac{10^{-6}\times 10\times 5}{1} = 50\mu H$$

The quality factor is

$$Q = \frac{\omega L}{R_L} = \frac{1}{\omega CR_1} = \frac{1}{2\pi 50\times 10^{-6}\times 15} = 212.21$$

Example 4

The fixed-value components of a Hay bridge are $R_3 = 650\Omega$, and $C = 4nF$. At balance $R_1 = 240\Omega$ and $R_2 = 1156\Omega$. The supply frequency is 6632 Hz.

1. Calculate the value of the unknown impedance?
2. Calculate Q factor?

Solution 4

At balance

$$R_u = \frac{\omega^2C^2R_1R_2R_3}{1+(\omega CR_1)^2} = \frac{(2\pi 6632)^2\times 4^2\times 10^{-18}\times 240\times 1156\times 650}{1+(2\pi 6632\times 4\times 10^{-9}\times 240)^2} = \frac{5.01}{1} = 5.01\Omega$$

$$L_u = \frac{CR_2R_3}{1+(\omega CR_1)^2} = \frac{4\times 10^{-9}\times 1156\times 650}{1} = 3mH$$

The quality factor is

$$Q = \frac{1}{\omega CR_1} = \frac{1}{2\pi 6632\times 4\times 10^{-6}\times 240} = 25$$

Resistance Measurement Techniques

Resistors are the most fundamental and commonly used of all the electronic components. The value of a resistor depends on the length (l), cross sectional area (A) and resistivity (ρ) of the resistive material it is made from.

$$R = \frac{\rho l}{A}$$

Resistor Tolerance

Resistor tolerance (Δx) indicates how much the measured value of a resistor is different from its theoretical value (nominal value), and it is calculated using percentages.

$$R = x \pm \Delta x$$

where x is the **nominal value** and Δx is the **tolerance**.

Example 1

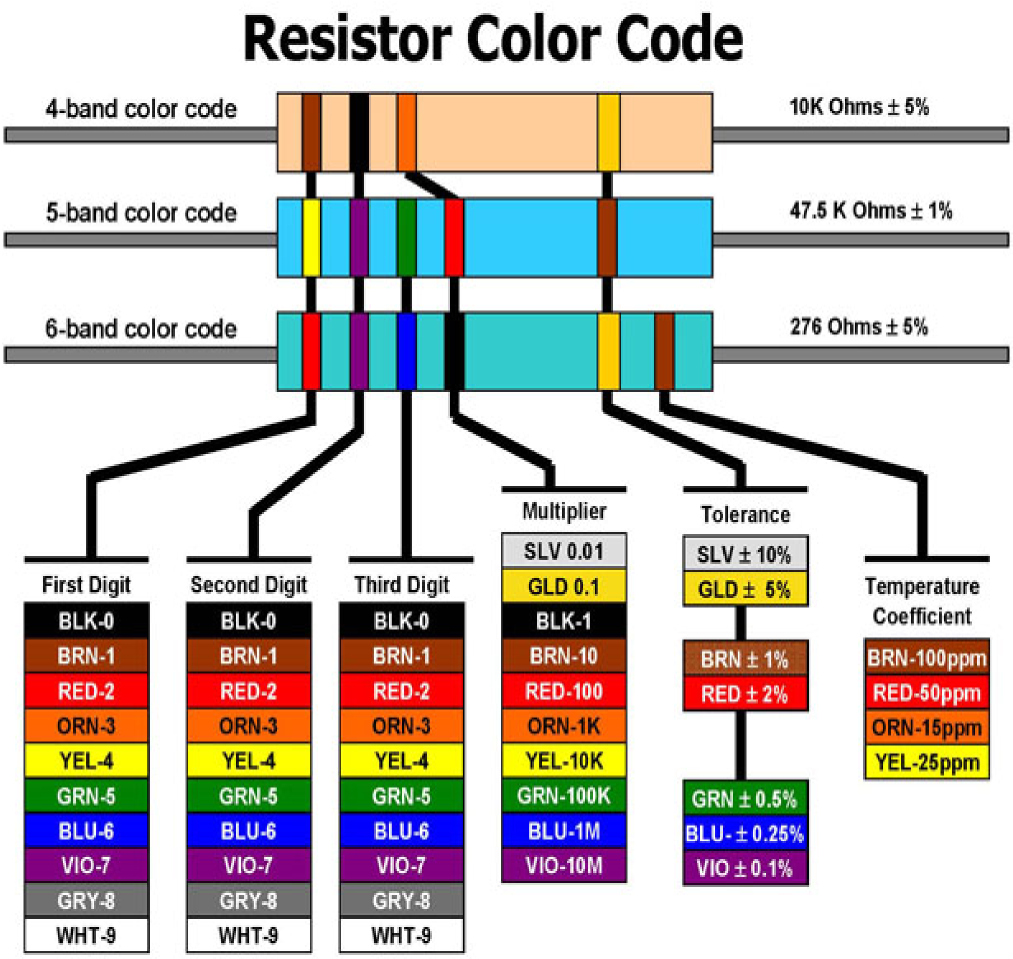
The resistor value range will be

$$R = 1K\Omega \pm 10\% \in [900\Omega, 1100\Omega]$$

Resistor Color Code

The value of a resister can be inferred from the color band across its body. There are many variants of the color band. The tolerance of 3-band color code has a default value of plus or minus 20%. For example, a 1KΩ resistor would have an actual value that measures from 800 to 1200 Ω, since the tolerance would be 200 Ω.

3-band color code	1^{st} Digit	2^{nd} Digit	Multiplier			
4-band color code	1^{st} Digit	2^{nd} Digit	Multiplier	Tolerance		
5-band color code	1^{st} Digit	2^{nd} Digit	3^{rd} Digit	Multiplier	Tolerance	
6-band color code	1^{st} Digit	2^{nd} Digit	3^{rd} Digit	Multiplier	Tolerance	Temperature



Example 2

Find the value of a resistor having the following colors, orange, green, red, red?

Solution 2

$$R = 35 \times 10^2 \pm 2\% = 3500 \pm 2\% \in [3430\Omega, 3570\Omega]$$

Example 3

Find the value of a resistor having the following colors, gray, black, orange, green, silver?

Solution 3

$$R = 803 \times 10^5 \pm 10\% = 80.3M \pm 10\% \in [72.27M\Omega, 88.33M\Omega]$$

Power Rating of a Resistor

It is the maximum power the resistor can dissipate without burning. The size of the resistor decides its power rating.

Temperature Coefficient of a Resistor

It describes the change in the resistance due to a change in temperature. This change is normally quite small over a particular temperature range. The change in value of a resistor with changing temperature is due to mainly a change in the resistivity of the material caused by the activity of the atoms of which the material is made. This change in value is normally quoted in parts per million (ppm).

Example 3

The temperature coefficient 50ppm/°C means that the change in the resistor value due to a temperature change of 1°C will not be more than 50Ω for every 1MΩ of the resistor's value (or 0.05Ω for every 1KΩ of its value).

Resistance Measurement Techniques

- Voltmeter-ammeter method.
- Ohmmeter.
- Substitution.
- Bridge circuits.

Ammeter-Voltmeter Method

The voltmeter-ammeter method is probably the most basic means of measuring resistance. It makes use of Ohm's law and the assumption that the resistance is linear. This method uses a voltmeter to measure the voltage across the resistor and an ammeter to measure the current flowing through that resistor. This method can calculate the resistance with reasonable accuracy. Whilst this method can provide good measurement results, it is not a practical solution to everyday measurement needs. This method is rarely used in practice but mostly used in laboratory. This method requires two meters and calculations.

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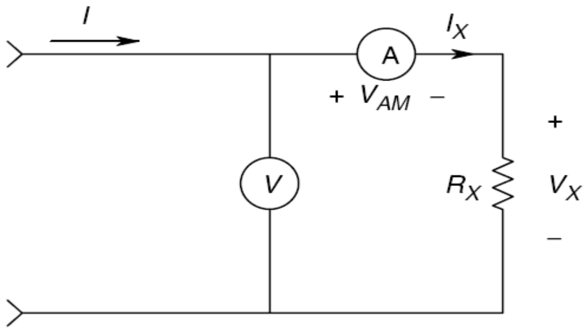
<td>
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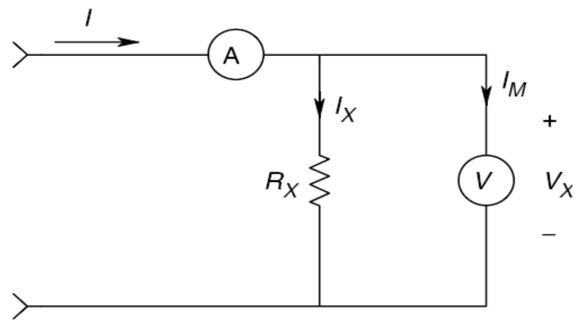
<td>
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<tr>
<td>In this case, the voltmeter measures the voltage drop across the ammeter plus that across the resistor.
$$R = \frac{V}{I_x} = \frac{V_x + V_{AM}}{I_x} = R_x + \frac{V_{AM}}{I_x}$$
$$R_x = R - \frac{V_{AM}}{I_x}$$
If $V_x \gg V_{AM}$ then $R \approx R_x$. Therefore this circuit is suitable to measure large resistances.</td>
<td>In this case, the ammeter measures the current in the voltmeter as well as the resistor.
$$R = \frac{V}{I} = \frac{V}{I_x + I_M} = \frac{R_x}{1 + I_M/I_x}$$
$$R_x = \frac{V}{I_x} = R\left(1 + \frac{I_M}{I_x}\right)$$
If $I_x \gg I_M$ then $R \approx R_x$. Therefore this circuit is suitable to measure small resistances.</td>
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</table>

Example 4

An ammeter-voltmeter method is used to measure the resistance of a given resistor. The voltmeter reading is $20V$, the ammeter reading is $2A$, and the voltage across the ammeter is $1V$.

1. Find R, and R_x?
2. Find the voltage across the resistor?
3. Calculate the error percentage of this measurement?

Solution 4

$$R = \frac{V}{I_x} = \frac{20V}{2A} = 10\Omega$$

$$R_x = R - \frac{V_{AM}}{I_x} = 10\Omega - \frac{1V}{2A} = 9.5\Omega$$

$$V_x = I_x R_x = 2A \times 9.5\Omega = 19\text{V}$$

$$|e| = |X_{actual} - X_{measured}| = |9.5 - 10| = 0.5$$

$$E = \frac{|e|}{X_{actual}} \times 100\% = \frac{0.5}{9.5} \times 100\% = 5.263\%$$

The error percentage is high because the resistance is low.

Example 5

An ammeter-voltmeter method is used to measure the resistance of a given resistor. The voltmeter reading is $20V$, the ammeter reading is $2A$, and the current through the voltmeter is $200mA$.

1. Find R, and R_x?
2. Find the current flowing through the resistor?
3. Calculate the error percentage of this measurement?

Solution 5

$$R = \frac{V}{I} = \frac{20V}{2A} = 10\Omega$$

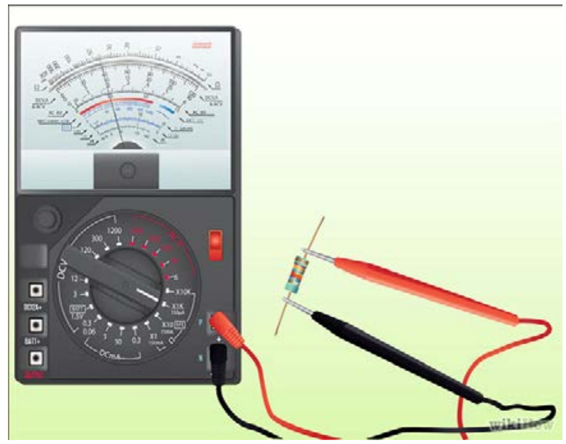
$$I = I_x + I_M \quad \rightrightarrows \quad I_x = I - I_M = 2000m - 200m = 1800mA = 1.8A$$

$$R_x = \frac{V}{I_x} = R\left(1 + \frac{I_M}{I_x}\right) = \frac{20V}{1.8A} = 11.11\Omega$$

$$|e| = |X_{actual} - X_{measured}| = |11.11 - 10| = 1.11$$

$$E = \frac{|e|}{X_{actual}} \times 100\% = \frac{1.11}{11.11} \times 100\% = 0.999\%$$

Ohmmeter Method



Ohmmeter is an instrument for measuring electrical resistance, which is expressed in ohms. Micro-ohmmeters make low resistance measurements while mega-ohmmeters (Megger) measure large values of resistance. Ohmmeter uses only one meter by keeping one parameter constant. In the simplest ohmmeters, the resistance to be measured may be connected to the instrument in **parallel** or in **series**.

Series Ohmmeter Method

In this case, the current will decrease as resistance rises. The ammeter reading is

$$I_M = \frac{V_B}{(R_M + R_S) + R_x}$$

The full-scale deflection is obtained if the output is short-circuited.

$$I_{FSO} = \frac{V_B}{R_M + R_S}$$

The zero-deflection occurs when the output is open-circuited $(R_x = \infty)$. The half-deflection resistance is

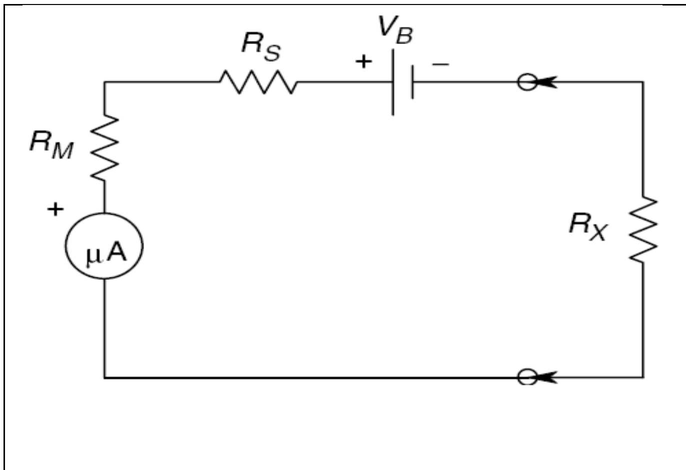
$$R_H = R_M + R_S$$

If $R_x = R_H$, then $I_M = \frac{I_{FSO}}{2}$ so the name. The ohmmeter's sensitivity is

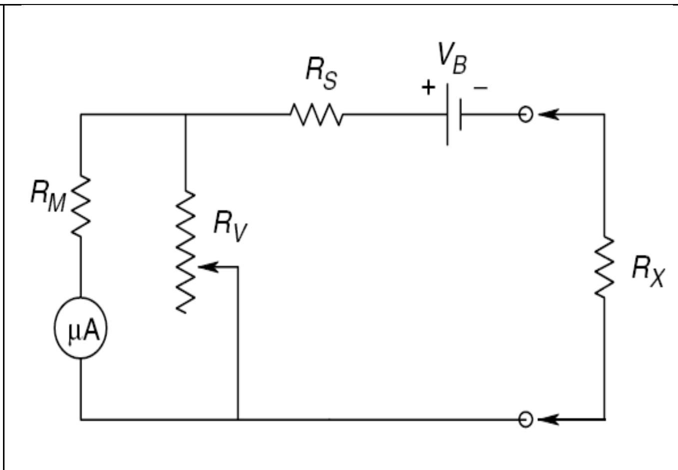
$$S = \frac{R_H}{(R_H + R_X)^2}$$

The resistance value is

$$R_x = \frac{V_B - I_M R_H}{I_M}$$



A simple, series ohmmeter circuit.



A practical series ohmmeter circuit.

Since the voltage, V_B, of the series ohmmeter's battery drops as the battery is used and ages, a variable resistor, R_V, is added in parallel with the microammeter to compensate for the drop in V_B. R_V is adjusted with the meter leads shorted ($R_x = 0$), so $I_M = I_{FSO}$. Thus R_V is used to set the ohmmeter zero in compensation for changes in the value of V_B.

Example 6

A series-ohmmeter is used to measure the resistance of a given resistor. The ammeter reading is $0.8A$, the ammeter resistance is 0.5Ω, the series resistance is 1Ω, and the ohmmeter battery is $3V$.

1. Find the full-scale deflection?
2. Find the half-deflection resistance of the ohmmeter?
3. Determine the resistance of the resistor?
4. Calculate the ohmmeter sensitivity?

Solution 6

$$I_{FSO} = \frac{V_B}{R_M + R_S} = \frac{3V}{1\Omega + 0.5\Omega} = 2A$$

$$R_H = R_M + R_S = 1\Omega + 0.5\Omega = 1.5\Omega$$

$$R_x = \frac{V_B - I_M R_H}{I_M} = \frac{3 - 0.8 \times 1.5}{0.8} = 2.25\Omega$$

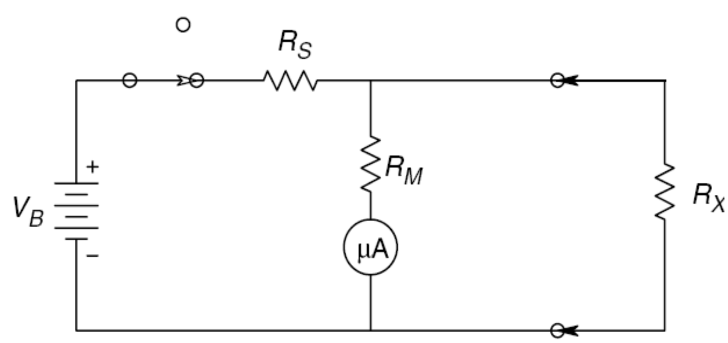
$$S = \frac{R_H}{(R_H + R_X)^2} = \frac{1.5}{(1.5 + 2.25)^2} = 0.1066 = 10.66\%$$

Shunt Ohmmeter Method

In this case, the instrument will draw more current as resistance increases. Shunt ohmmeters are less frequently encountered than are series ohmmeters. Shunt ohmmeters are used to measure low resistances. The battery must supply a substantial current, often in the ampere range. Consequently, a robust battery, such as a lead acid motorcycle type battery, is used. The half deflection resistance, R_H, (the Thevenin resistance) is

$$R_H = \frac{R_M R_S}{R_M + R_S}$$

$$R_{TH} = R_H$$



The full-scale deflection occurs when $R_x = \infty$ and is simply

$$I_{FSO} = \frac{V_B}{R_M + R_S}$$

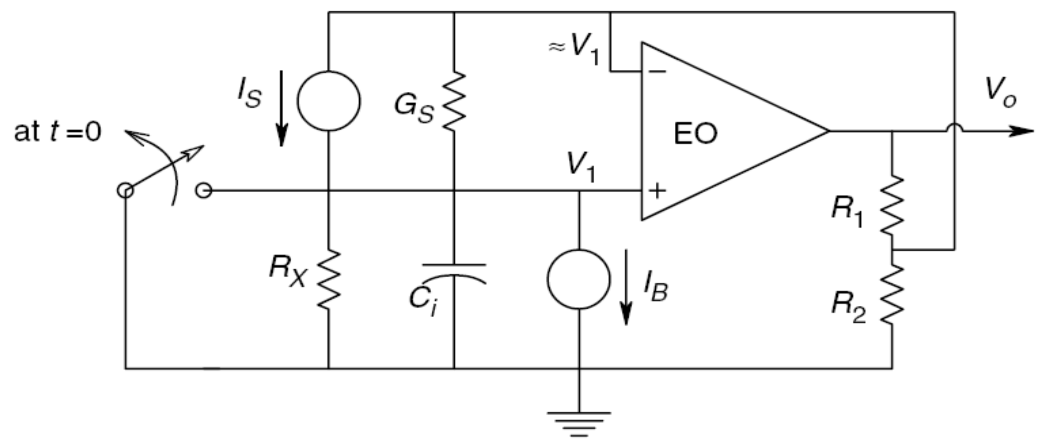
The zero-deflection occurs when the output is short-circuited, $R_x = 0$. The meter current is given by

$$I_M = \frac{\alpha V_B}{R_M + \alpha R_S} \qquad \text{and} \qquad \alpha = \frac{R_x}{R_x + R_S}$$

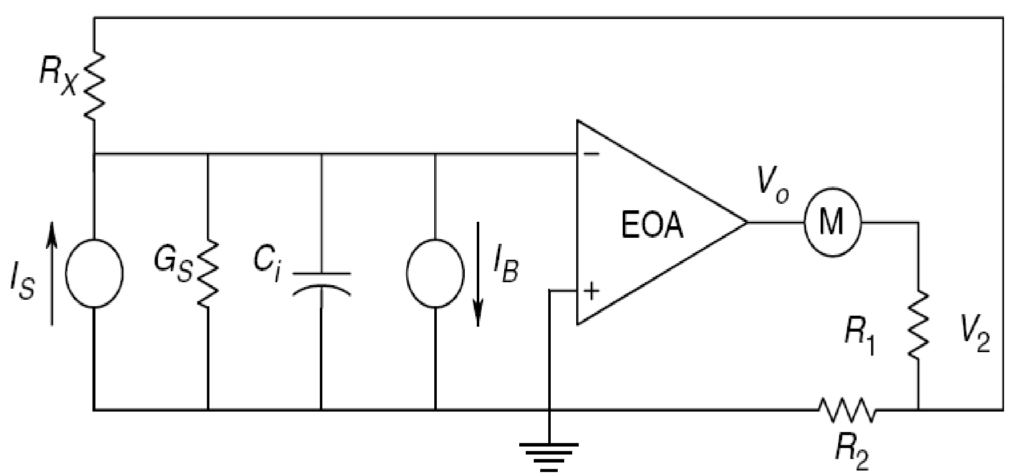
Electronic Ohmmeter Circuits

Electronic ohmmeters can measure resistances ranging $0.1\mu\Omega - 10^{18}\Omega$. They can be

- Normal Mode ohmmeter.
- Fast Mode electronic ohmmeter.



Normal Mode ohmmeter configuration.

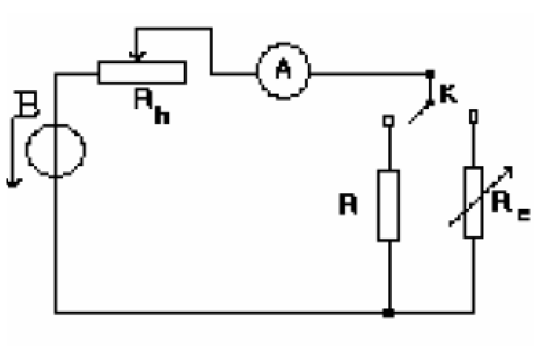


Fast Mode electronic ohmmeter configuration

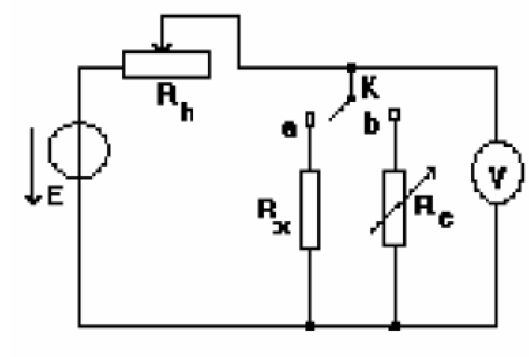
Resistance Measurement Techniques 2

Resistance-Substitution Method

This method avoids the measurement error caused by voltmeter-ammeter method. In this method, the unknown resistance in a circuit is temporarily replaced by a variable resistance. The variable resistance is adjusted until the measured circuit voltage or current are the same as existed with the unknown resistance in place. The variable resistance at this point is equal in value to the unknown resistance.



Substitution method using Ammeter



Substitution method using Voltmeter

Steps for Resistance-Substitution Method

1. Put the unknown resistance R_x in the circuit.
2. Close the switch K to the unknown resistance and read the ammeter reading (voltmeter reading).
3. Close the switch K to the standard variable resistance.
4. Adjust R_s until the ammeter reading (voltmeter reading) becomes as same as the previous value ammeter reading.
5. Read the value of the standard variable resistance which will be the value of the unknown resistance.

Null-type and Deflection-type Instruments

- **Deflection type of instrument:** The value of the quantity being measured is displayed in terms of the amount of movement of a pointer. In terms of usage, the deflection-type instrument is convenient as it is simpler to read the position of a pointer against a scale. The deflection-type instrument is the one that would normally be used in the workplace.
- **Null-type instrument:** Here measurement is made in terms of the value of another element which should be adjusted to reach a null position or point. Null-type instruments are more accurate than deflection types. For calibration duties, the null-type instrument is preferable because of its superior accuracy. The extra effort required to use such an instrument is perfectly acceptable in this case because of the infrequent nature of calibration operations.

Bridge Circuit

Bridge circuit is a null-output-type method, operates on the principle of comparison. That is a known (standard) value is adjusted until it is equal to the unknown value. Bridge circuits are particularly useful in converting resistance changes into voltage signals that can be input directly into automatic control systems. Bridge circuits have very good accuracy. Bridge circuits enable the detection of very small changes in the quantities under consideration about a nominal value.

DC Bridge Circuit

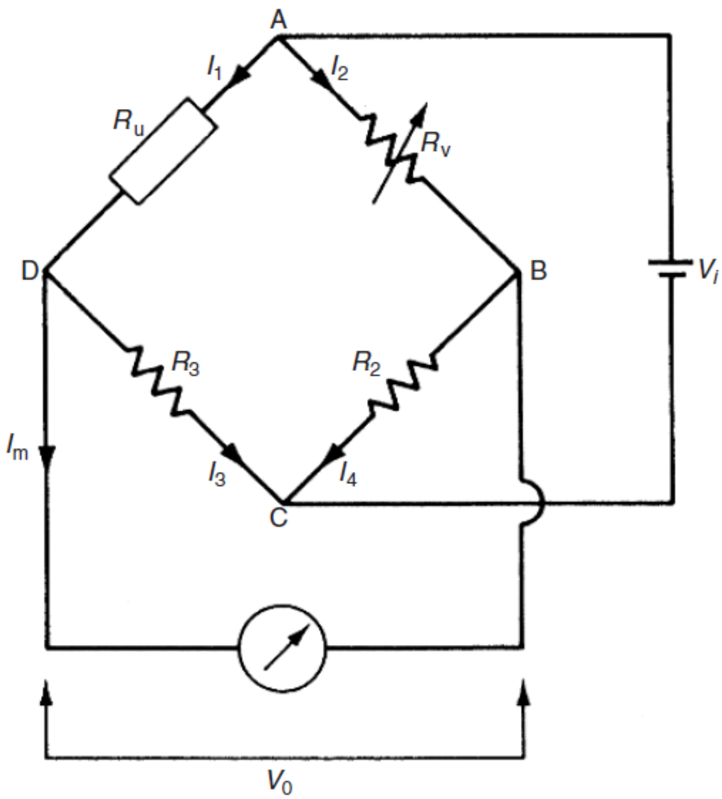
DC bridge circuits are used for measuring resistances. The most popular DC bridge circuit is the Wheatstone bridge which is the most commonly used method of measuring medium value resistance values from 1Ω to $10M\Omega$. Wheatstone bridges are traditionally used to make precise, accurate resistance measurements or to measure some physical quantity, such as temperature, light intensity or strain, which causes a known change in resistance.

AC Bridge Circuit

AC bridge circuits are used for measuring inductance (Maxwell Bridge, and Hay Bridge), capacitance (Schering Bridge), and frequency (Wien Bridge).

Wheatstone Bridge Circuit

The basic Wheatstone bridge circuit consists of four arms. The four arms of the bridge consist of the unknown resistance R_u, two resistors R_2 and R_3 and a variable resistor R_v (usually a decade resistance box). A DC voltage V_i is applied across the points A and C and the resistance R_v is varied until the voltage measured across points B and D is zero. This null point is usually measured with a high sensitivity DC null sensing device like galvanometer or other sensitive current meter.



Wheatstone Bridge Analysis

The current flowing in each arm are I_1, I_2, I_3, I_4. Normally, if a high impedance voltage-measuring instrument is used, the current I_m drawn by the measuring instrument will be very small and can be approximated to zero. If this assumption is made, then $I_m = 0$.

$$I_1 = I_3 \qquad \text{and} \qquad I_2 = I_4$$

The current through the galvanometer depends on the potential difference between points B and D. the bridge is said to be balanced when the potential difference across the galvanometer is zero.

$$V_{AD} = V_{AB} \qquad \text{and} \qquad V_{CB} = V_{CD}$$

$$I_1 R_u = I_2 R_v \qquad \text{and} \qquad I_4 R_2 = I_3 R_3$$

$$\frac{I_2}{I_1} = \frac{R_u}{R_v} \qquad \text{and} \qquad \frac{I_4}{I_3} = \frac{R_3}{R_2}$$

$$\frac{R_u}{R_v} = \frac{R_3}{R_2} \qquad \rightrightarrows \qquad R_u = R_v \frac{R_3}{R_2}$$

Example 1

A Wheatstone bridge has a $R_3 = 200\Omega$, and $R_2 = 300\Omega$. At balance, R_v is adjusted to 600Ω. Find the value of the unknown resistor?

Solution 1

At balance

$$R_u = R_v \frac{R_3}{R_2} = 600 \times \frac{200}{300} = 400\Omega$$

Example 2

A Wheatstone bridge has a ratio arm of 1/100 (R_3/R_2). At first balance, R_v is adjusted to 1000.3Ω. The value of R_u is then changed by temperature. The new value of R_v to achieve the balance condition again is $1002.1\ \Omega$. Find the change of R_u due to the temperature change?

Solution 2

At the first balance

$$R_{u1} = R_v \frac{R_3}{R_2} = 1000.3 \times \frac{1}{100} = 10.003\Omega$$

At the second balance

$$R_{u2} = R_v \frac{R_3}{R_2} = 1002.1 \times \frac{1}{100} = 10.021\Omega$$

The change of R_u due to the temperature change is

$$\Delta R = R_{u2} - R_{u1} = 10.021 - 10.003 = 0.018\Omega$$

Example 3

The decade resistance box inaccuracy is $\mp 0.2\%$. The arms resistances $R_3 = R_2 = 500\Omega \mp 0.1\%$. At balance, R_v is adjusted to 520.4Ω. Determine the error percentage of this measurement?

Solution 3

$$R_2 \mp \Delta R_2 = R_3 \mp \Delta R_3 = 500\Omega \mp 0.1\% = 500\Omega \mp \frac{0.1}{100} \times 500 \in [499.5\Omega, 500.5\Omega]$$

$$R_v \mp \Delta R_v = 520.4\Omega \mp 0.2\% = 520.4\Omega \mp \frac{0.2}{100} \times 520.4\Omega \in [519.3959\Omega, 521.4408\Omega]$$

$R_2 \mp \Delta R_2$	$R_3 \mp \Delta R_3$	$R_v \mp \Delta R_v$	$R_u \mp \Delta R_u$
$R_2 + \Delta R_2$	$R_3 + \Delta R_3$	$R_v + \Delta R_v$	521.4408Ω
$R_2 + \Delta R_2$	$R_3 + \Delta R_3$	$R_v - \Delta R_v$	519.3959Ω
$R_2 + \Delta R_2$	$R_3 - \Delta R_3$	$R_v + \Delta R_v$	520.44277Ω

$R_2 + \Delta R_2$	$R_3 - \Delta R_3$	$R_v - \Delta R_v$	518.35816Ω
$R_2 - \Delta R_2$	$R_3 + \Delta R_3$	$R_v + \Delta R_v$	522.4847Ω
$R_2 - \Delta R_2$	$R_3 + \Delta R_3$	$R_v - \Delta R_v$	520.43573Ω
$R_2 - \Delta R_2$	$R_3 - \Delta R_3$	$R_v + \Delta R_v$	521.4408Ω
$R_2 - \Delta R_2$	$R_3 - \Delta R_3$	$R_v - \Delta R_v$	519.3959Ω

$$R_u \mp \Delta R_u \in [518.35816\Omega, 522.4847\Omega]$$

$$R_u = \frac{518.35816\Omega + 522.4847\Omega}{2} = 520.42143\Omega$$

$$E = \frac{|X_{actual} - X_{measured}|}{X_{actual}} \times 100\% = \frac{|520.42143 - 518.35816|}{520.42143} \times 100\% = 0.4\%$$

$$E = \frac{|X_{actual} - X_{measured}|}{X_{actual}} \times 100\% = \frac{|520.42143 - 522.4847|}{520.42143} \times 100\% = 0.4\%$$

The error percentage is $\mp 0.4\%$.

Whetstone Bridge Analysis with Tolerance

If the tolerances of the bridge resistors are taken into account then the unknown resistor is

$$R_u \mp \Delta R_u = (R_v \mp \Delta R_v)\frac{R_3 \mp \Delta R_3}{R_2 \mp \Delta R_2}$$

Using the first order approximation we can write $R_u \mp \Delta R_u$ as below

$$R_u \mp \Delta R_u = R_v \frac{R_3}{R_2} \mp (\Delta R_2 + \Delta R_3 + \Delta R_v)$$

Example 4

Repeat example 3?

Solution 4

$$R_u \mp \Delta R_u = R_v \frac{R_3}{R_2} \mp (\Delta R_2 + \Delta R_3 + \Delta R_v)$$

$$R_u \mp \Delta R_u = 520.4 \frac{500}{500} \mp (0.1 + 0.1 + 0.2)\% = 520.4 \mp 0.4\%$$

$$R_u \mp \Delta R_u \in [518.32\Omega, 522.48\Omega]$$

Frequency Measuring Techniques

There are four methods for measuring unknown frequency.

1. **Digital counter-timer:** It is the most common instrument for measuring frequency.
2. **Phase-locked loop.**
3. **Oscilloscope:** It is commonly used for obtaining approximate measurements of frequency, especially in circuit test and fault-diagnosis applications.
4. **Wien bridge:** It is used for frequencies within the audio frequency range.

Digital Counter-Timer

A digital counter-timer is the most accurate and flexible instrument available for measuring frequency. It can measure all frequencies between DC and several gigahertz.

- The essential component within a counter-timer instrument is an oscillator that provides a very accurately known and stable reference frequency, which is typically either 100KHz or 1MHz.
- The oscillator output is transformed by a pulse-shaper circuit into a train of pulses and applied to an electronic gate. These successive pulses alternately open and close the gate.
- The input signal of unknown frequency is similarly transformed into a train of pulses and applied to the gate.
- If the reference frequency, f_1, is smaller than the unknown frequency, f_2, then the number of unknown signal pulses that pass through the gate during the open period of the reference signal is a measure of the frequency of the unknown signal.
- If the reference frequency is greater than the unknown signal frequency, then the number of pulses at the reference frequency that pass through the gate during the open period of the unknown signal is then a measure of the frequency of the unknown signal.

The accuracy of measurement obviously depends upon how far the unknown frequency is above or below the reference frequency.

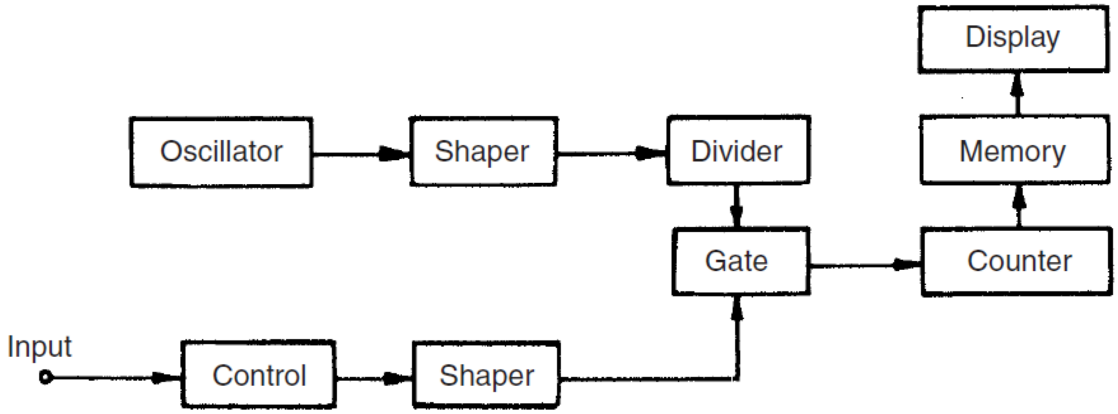


Fig. 7.16 Digital counter-timer system.

$$f \propto N \qquad \rightarrow \qquad \frac{f_2}{f_1} = \frac{N_2}{N_1}$$

$$f_2 = \frac{N_2}{N_1} f_1$$

If $f_1 < f_2$, for the known frequency $N_1 = 1$, then

$$f_2 = N_2 f_1$$

If $f_1 > f_2$, for the unknown frequency $N_2 = 1$, then

$$f_2 = \frac{f_1}{N_1}$$

Example 1

A digital counter-timer of reference frequency $2MHz$. The number of unknown signal pulses are 25 for one reference signal pulse. Find the unknown frequency?

Solution 1

$$f_2 = N_2 f_1 = 25 \times 2M = 50MHz$$

Example 2

A digital counter-timer of reference frequency $2MHz$. The number of reference signal pulses are 25 for one unknown signal pulse. Find the unknown frequency?

Solution 2

$$f_2 = \frac{f_1}{N_1} = \frac{2M}{25} = 80KHz$$

Phase-Locked Loop (PLL)

A phase-locked loop is a circuit consisting of a phase-sensitive detector, a voltage controlled oscillator (VCO), and amplifiers, connected in a closed-loop system. In a VCO, the oscillation frequency is proportional to the applied voltage.

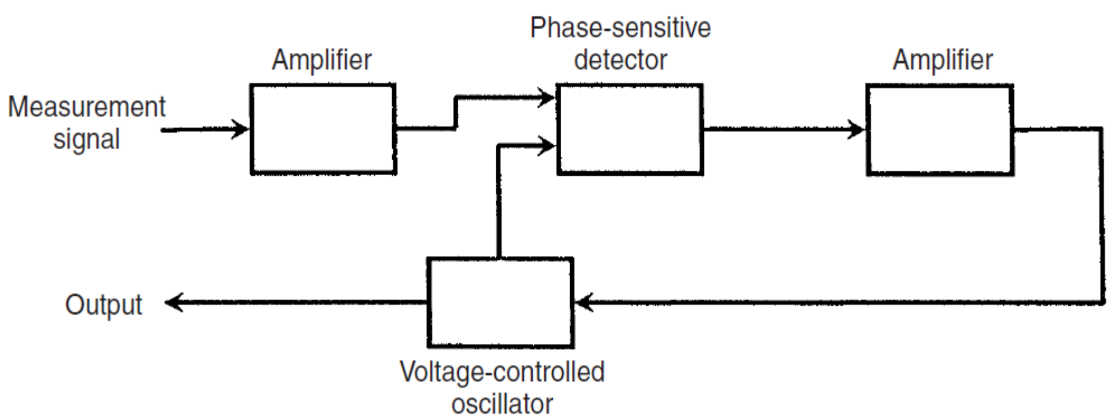


Fig. 7.17 Phase-locked loop.

Phase-locked Loop Operation

- The phase-sensitive detector compares the phase of the amplified input signal with the phase of the VCO output.
- Any phase difference generates an error signal, which is amplified and fed back to the VCO.

- This adjusts the frequency of the VCO until the error signal goes to zero, and thus the VCO becomes locked to the frequency of the input signal.
- The DC output from the VCO is then proportional to the input signal frequency.

$$f \propto V \qquad \rightarrow \qquad \frac{f_1}{f_2} = \frac{V_1}{V_2}$$

$$f_2 = \frac{V_2}{V_1} f_1$$

Example 3

A known frequency of 42KHz is applied to a PLL which looked at input voltage 2V. Find the frequency of unknown signal if the PLL locked at input voltage 5V?

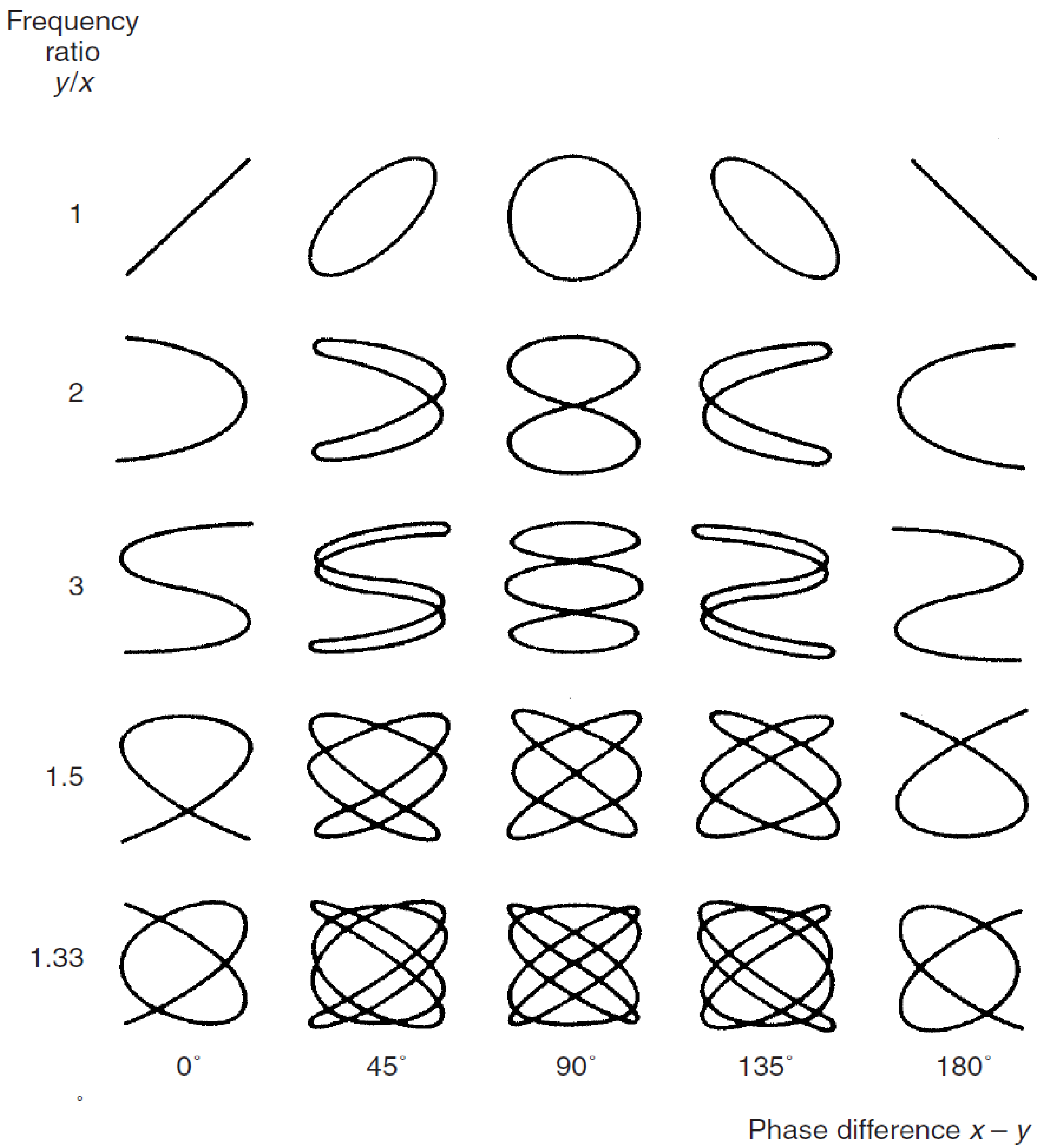
Solution 3

$$f_2 = \frac{V_2}{V_1} f_1 = \frac{5V}{2V} 42K = 105KHz$$

Oscilloscope for Measuring Unknown Frequency

An oscilloscope can measure frequency by generating *Lisajous patterns*. These patterns are produced by applying a known reference-frequency sine wave to the Y input (vertical deflection plates) of the oscilloscope and the unknown frequency sinusoidal signal to the X input (horizontal deflection plates). A pattern is produced on the screen according to the frequency ratio between the two signals, and if the numerator and denominator in the ratio of the two signals both represent an integral number of cycles, the pattern is stationary. Frequency measurement proceeds by adjusting the reference frequency until a steady pattern is obtained on the screen and then calculating the unknown frequency according to the frequency ratio that the pattern obtained represents.

Examples of these patterns are given below, which also shows that phase difference between the waveforms has an effect on the shape.



18 Lisajous patterns.

Wien bridge

It can be used to measure frequencies in the audio range. An alternative use of the instrument is as a source of audio frequency signals of accurately known frequency. It consists of four fixed-value elements, R_1, R_2, C_3, and C_4 and two variable elements, R_4 and R_3. A simple set of headphones is often used to detect the null-output balance condition. At balance, the unknown frequency is calculated according to:

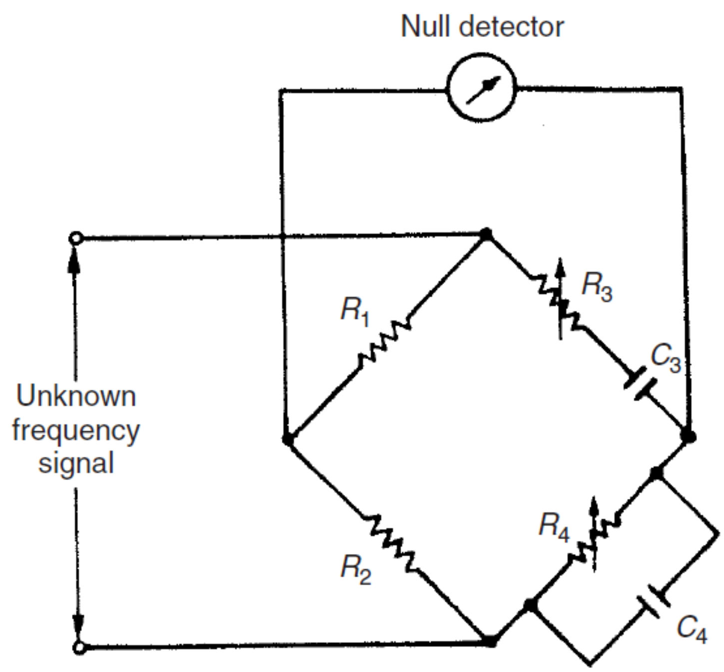
$$Z_1 = R_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3 + \frac{1}{j\omega C_3}$$

$$\frac{1}{Z_4} = \frac{1}{R_4} + j\omega C_4$$

$$Z_4 = \frac{R_4}{1 + j\omega C_4 R_4}$$



At balance

$$I_1 Z_1 = I_3 Z_3 \quad \text{and} \quad I_4 Z_4 = I_2 Z_2$$

$$\frac{I_3}{I_1} = \frac{Z_1}{Z_3} \quad \text{and} \quad \frac{I_4}{I_2} = \frac{Z_2}{Z_4}$$

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4} \quad \rightrightarrows \quad Z_1 = Z_3 \frac{Z_2}{Z_4}$$

$$|Z_1|\angle\theta_1 = |Z_3|\angle\theta_3 \times \frac{|Z_2|\angle\theta_2}{|Z_4|\angle\theta_4}$$

The magnitude balance and phase balance of this circuit are

$$|Z_1| = |Z_3| \times \frac{|Z_2|}{|Z_4|} \qquad \text{and} \qquad \angle\theta_1 = \angle\theta_3 + \angle\theta_2 - \angle\theta_4$$

The real and imaginary parts can be calculated as below

$$Z_1 = Z_3 \frac{Z_2}{Z_4}$$

$$R_1 = \frac{R_2(1 + j\omega C_3 R_3)}{j\omega C_3} \frac{(1 + j\omega C_4 R_4)}{R_4} = \frac{-jR_2(1 + j\omega C_3 R_3)}{\omega C_3} \frac{(1 + j\omega C_4 R_4)}{R_4}$$

$$R_1 = \frac{R_2(\omega C_3 R_3 - j)}{\omega C_3} \frac{(1 + j\omega C_4 R_4)}{R_4} = \frac{R_2\big(\omega C_3 R_3 + \omega C_4 R_4 + j(\omega^2 C_3 R_3 C_4 R_4 - 1)\big)}{\omega C_3 R_4}$$

$$R_1 = \frac{R_2(\omega C_3 R_3 + \omega C_4 R_4)}{\omega C_3 R_4} = R_2 \left(\frac{R_3}{R_4} + \frac{C_4}{C_3}\right)$$

$$0 = R_2 \left(\frac{\omega^2 C_3 R_3 C_4 R_4 - 1}{\omega C_3 R_4}\right)$$

$$\omega^2 C_3 R_3 C_4 R_4 = 1$$

$$\omega = \frac{1}{\sqrt{C_3 R_3 C_4 R_4}} \qquad \text{and} \qquad f = \frac{1}{2\pi\sqrt{C_3 R_3 C_4 R_4}}$$

Example 4

The fixed-value components of a Wien bridge are $R_1 = 100\Omega$, $R_2 = 80\Omega$, $C_3 = 2nF$ and $C_4 = 3nF$. At balance $R_3 = 50\Omega$ and $R_4 = 120\Omega$. Find the supply unknown frequency?

Solution 4

At balance

$$f = \frac{1}{2\pi\sqrt{C_3 R_3 C_4 R_4}} = \frac{1}{2\pi\sqrt{2 \times 10^{-9} \times 50 \times 3 \times 10^{-9} \times 120}} = 838.8KHz$$

Phase Measuring Techniques

It is often necessary to measure the phase difference between two periodic signals of the same frequency. In power distribution systems as example, the power factor angle between the AC supply voltage and the load current is of great economic and practical importance to the power generator and the end user.

$$v_1(t) = v_1(t+T) = v_1(t+kT)$$

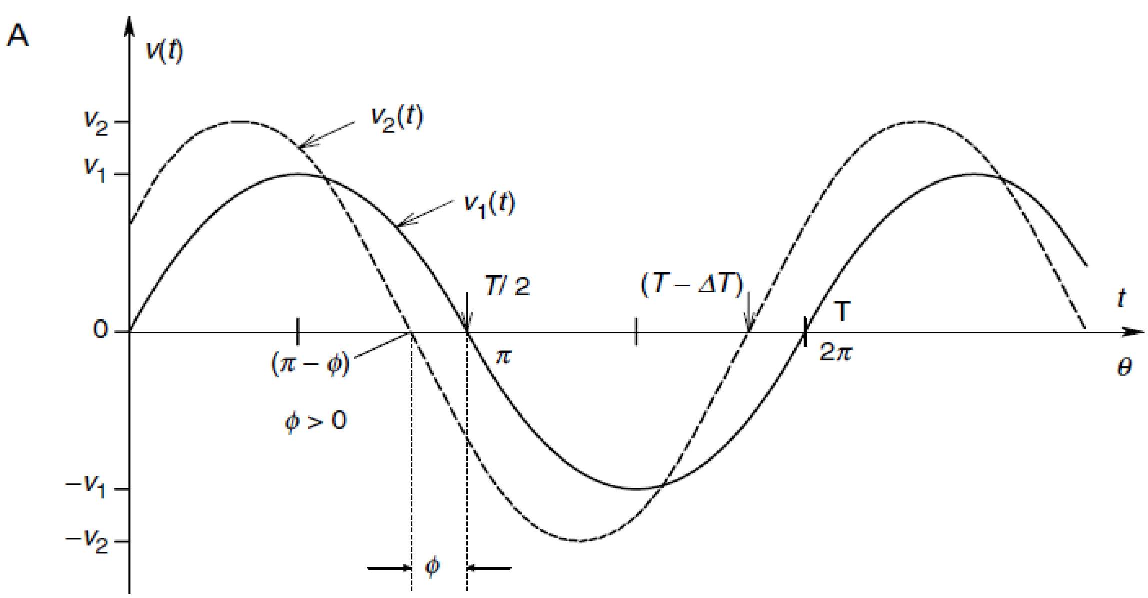
$$v_2(t) = v_2(t+T+\Delta T) = v_2(t+kT+\Delta T)$$

where ΔT is the time shift between $v_1(t)$ and $v_2(t)$, T is the period of $v_1(t)$ and $v_2(t)$, and k is an integer. For the following figure, $v_2(t)$ is said to lead $v_1(t)$ by a phase difference of φ where:

$$\varphi = \frac{\Delta T}{T} 2\pi \quad \text{rad} \qquad\qquad \varphi = \frac{\Delta T}{T} 360^0 \quad \text{degree}$$

$$v_1(t) = V_1 \sin(\omega t)$$

$$v_2(t) = V_2 \sin(\omega t + \varphi)$$



There are many methods for measuring phase but we will discuss only two here.

1. **Digital Counter-timer**: It is the most accurate instrument for measuring the phase difference between two signals.
2. **Lisajous Pattern Method**: It is less accurate than electronic counter-timer method.

Digital Counter-Timer

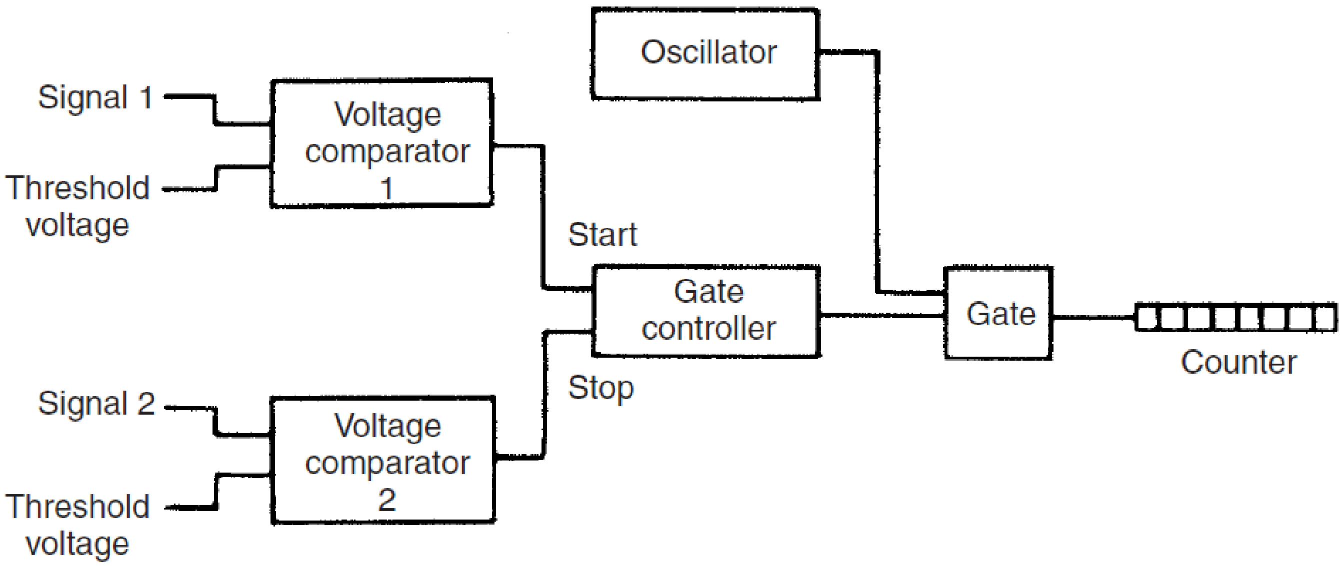
It is the most accurate instrument for measuring phase. In principle, the phase difference between two sinusoidal signals can be determined by measuring the time that elapses between the two signals crossing the time axis. However, in practice, this is inaccurate because the zero crossings are susceptible to noise contamination. The normal solution to this problem is to amplify/attenuate the two signals so that they have the same amplitude and then measure the time that elapses between the two signals crossing some non-zero threshold value.

- The basis of this method of phase measurement is a digital counter-timer with a precision quartz crystal oscillator that provides a very accurately known and stable reference frequency.
- The crossing points of the two signals through the reference threshold voltage level are applied to a gate that starts and then stops pulses from the oscillator into a digital counter.
- The time interval is measured by counting clock pulses generated from a precision quartz crystal oscillator between successive, positive-going, zero crossings of $v_1(t)$ and $v_2(t)$.
- Hence phase difference between the two input signals is then measured in terms of the counter display.

$$\varphi \propto N$$

$$\frac{\varphi}{180^0} = \frac{N_{\Delta T}}{N_T} \qquad \rightarrow \qquad \varphi = \frac{N_{\Delta T}}{N_T} 180^0$$

where $N_{\Delta T}$ is the number of oscillator pulses for the time difference between the two signal and N_T is the number of oscillator pulses for the positive signal duration.



. **7.20** Phase measurement with digital counter-timer.

Example 1

A digital counter-timer of reference frequency $20MHz$ is used for measuring the phase shift between two equal frequency signals. The number of oscillator pulses for the positive signal duration is 25 while it is 5 for the time shift between the two signals. Find the phase shift?

Solution 1

$$\varphi = \frac{N_{\Delta T}}{N_T} 180^0 = \frac{5}{25} 180^0 = 36^0$$

Lisajous Pattern Method

It is the simplest and easiest direct measurement of phase shift between two signals.

- This method uses a dual-beam oscilloscope to show the Lisajous' pattern formed when $v_1(t)$ is inputted to the x (horizontal deflection) axis and $v_2(t)$ is inputted to the y (vertical deflection) axis.
- In general, we see an ellipse on the CRT screen.
- The ellipse must be centered at the origin (center) of the CRT screen.

At $t = 0$,

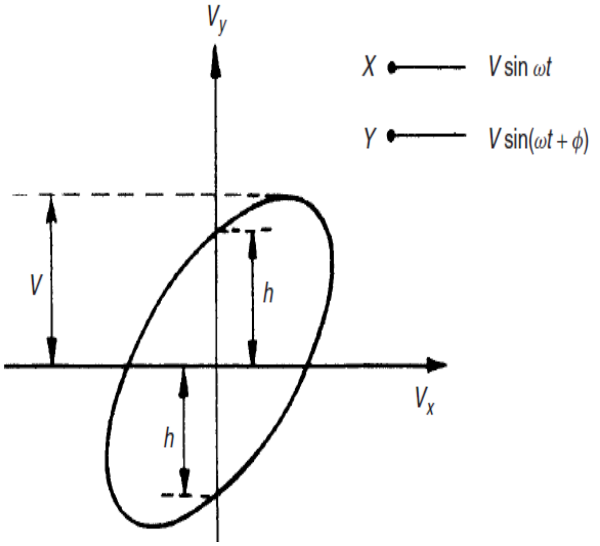
$$v_1(t) = V_1 \sin(\omega t) = V_1 \sin(0) = 0$$

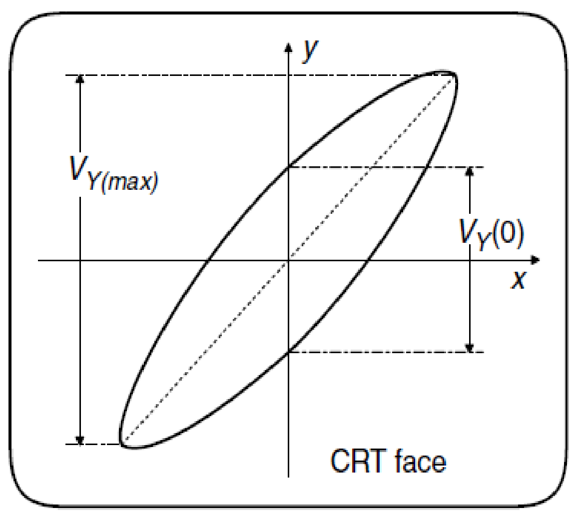
$$v_2(t) = V_2 \sin(\omega t + \varphi) = V_2 \sin(\varphi)$$

From the Lisajous pattern, $V_X = 0$ and $V_Y = \mp h$

$$\sin(\varphi) = \mp \frac{h}{V_2} = \mp \frac{V_Y(0)}{V_Y(\max)}$$

$$\varphi = \sin^{-1}\left(\mp \frac{h}{V_2}\right) = \sin^{-1}\left(\mp \frac{V_Y(0)}{V_Y(\max)}\right)$$





There are four possible values for φ but the ambiguity about which quadrant φ is in can usually be solved by observing the two signals plotted against time on a dual-beam oscilloscope.

When the Lisajous pattern is a straight line, $\varphi = 0^0$ and when it is a circle, $\varphi = 90^0$. Accuracy of the Lissajous figure method of phase measurement is poor near $\varphi = \mp 90^0$. At phase angles near integer multiples of $\varphi = 180^0$, accuracy is better and is limited by the thickness of the oscilloscope trace, as also one's ability to estimate the distances to its intersections with the vertical axis of the CRT.

Example 2

A Lisajous pattern method is used for measuring the phase difference between two signals of 10KHz. Find the phase shift between the two signals if the ellipse zero and peal to peak distances are 3 and 5.5?

Solution 2

$$\varphi = \sin^{-1}\left(\frac{V_Y(0)}{V_Y(\max)}\right) = \sin^{-1}\left(\frac{3}{5.5}\right) = 33^0$$