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Project 2 - Team 4

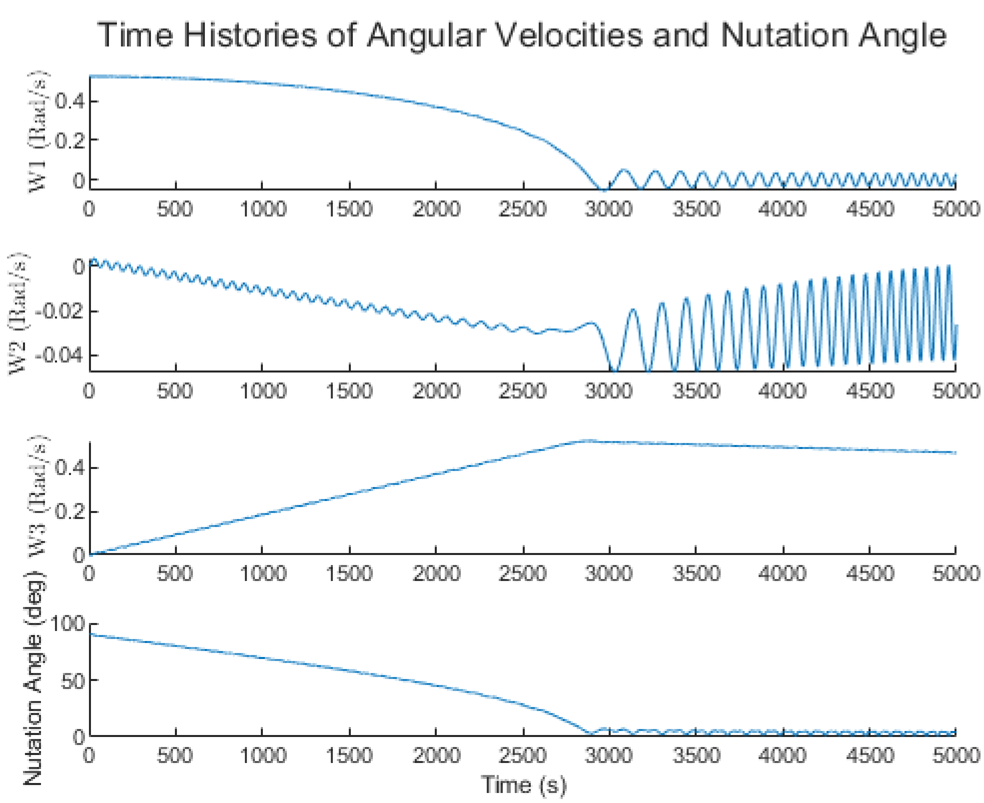
```
clear; clc; close all;
```

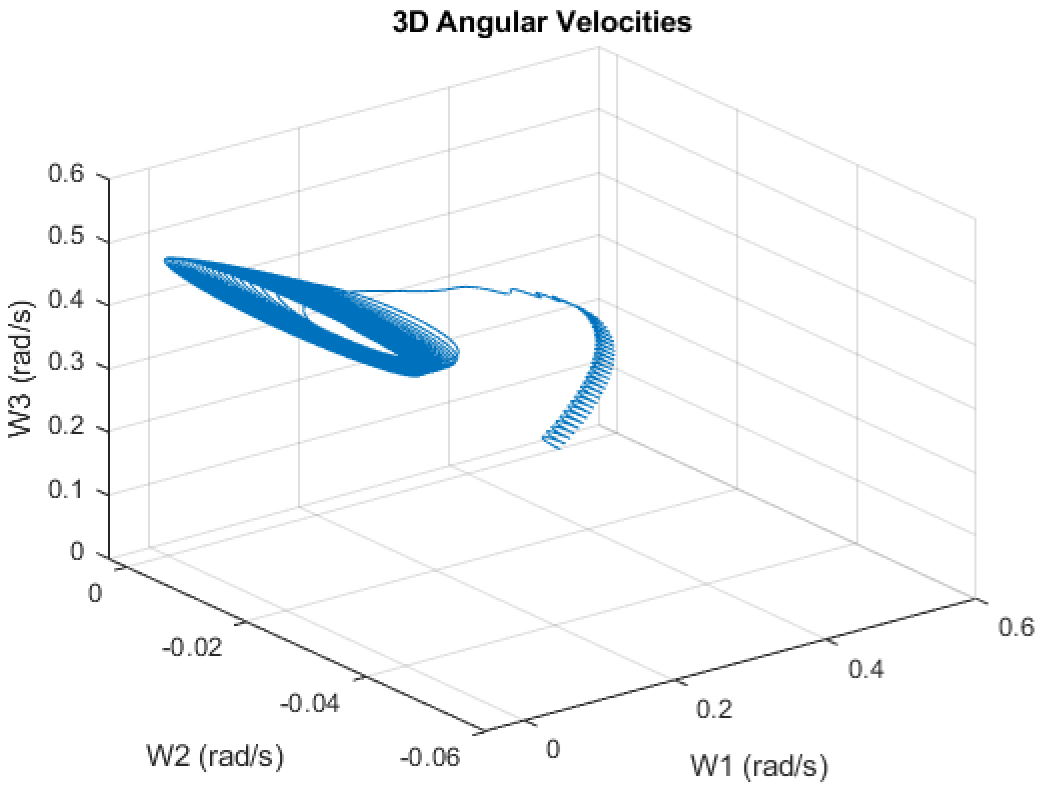
Question 3.1

```
Iw = 0.1; %kg*m^2
initial = [5;0;0;0;0;0]./60.*2.*pi; % initial tumbling angular velocity
(rad/s)
opts = odeset('MaxStep',2.0); % Sets max time step for ode45
ts = 5000; % wheel spin-up time (sec)
hw = 55; % nominal angular momentum (kg*m^2/s)
Iww = [0,0,0;0,0,0;0,0,Iw]; % inertia matrix for wheels (kg*m^2)
J = [500,0,0;0,400,-7;0,-7,440]; % inertia matrix for full satellite system
(kg*m^2)

% Calculate the satellite behavior  during the wheel spin up
[t,y] = ode45(@(t,w) angular_rates(t,w,J,hw,Iw,ts),[0,5000],initial,opts);

% Save data in a vector
y2 = y;

% Calculate the nutation angle at each time step
nutation_angle = nutation(J,Iww,y,[0,0,1]);

% Plot angular velocities of the satellite and the nutation angle
figure(1);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of Angular Velocities and Nutation Angle')
nexttile(tcl)
hold on;
plot(t,y(:,1));
ylabel('W1  (Rad/s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,y(:,2));
ylabel('W2  (Rad/s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,y(:,3));
```

```
ylabel('W3 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,nutation_angle);
ylabel('Nutation Angle (deg)')
xlabel('Time (s)')

% Plot 3D angular velocities
figure(2)
plot3(y(:,1),y(:,2),y(:,3));
title('3D Angular Velocities')
xlabel('W1 (rad/s)'); ylabel('W2 (rad/s)'); zlabel('W3 (rad/s)')
grid on;

% Check initial and final angular momentum to ensure conservation
wi = [5;0;0]./60.*2.*pi;
wf = [y(end,1); y(end,2); y(end,3)];
wwf = [y(end,4); y(end,5); y(end,6)];
H_i = norm(J*wi);
H_f = norm(J*wf+ Iww*wwf);
if abs(H_i-H_f) < 1e-3
    disp('3.1 passes angular momentum conservation check')
else
    disp('3.1 fails angular momentum conservation check')
end
```

```
3.1 passes angular momentum conservation check
```



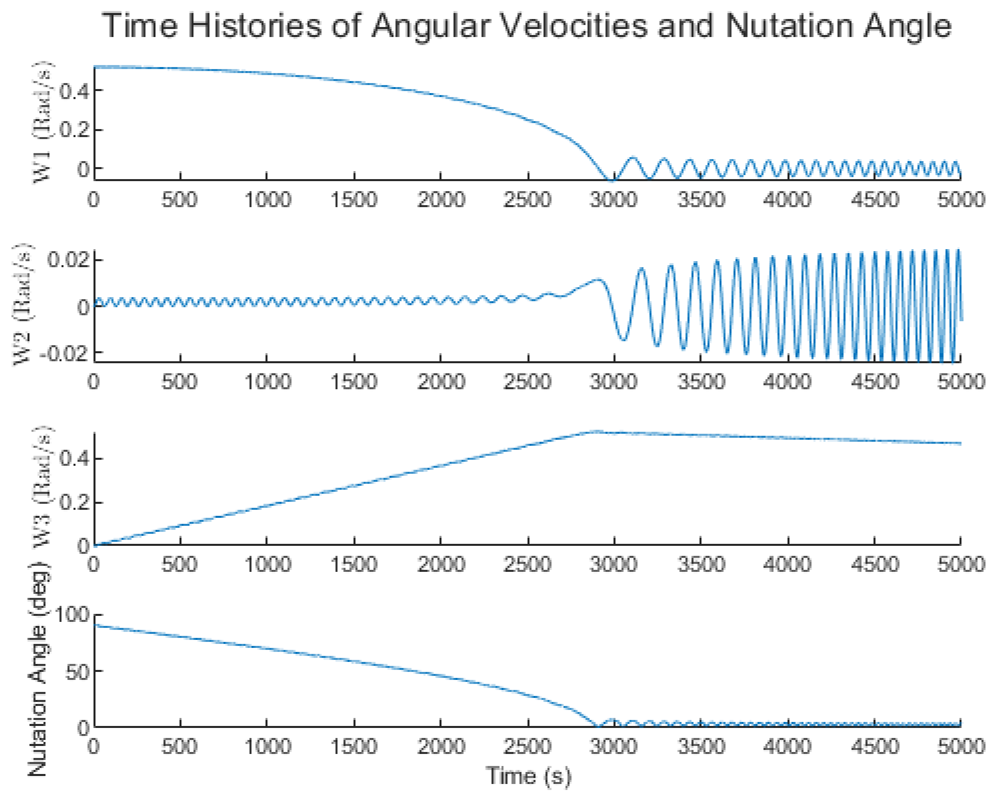


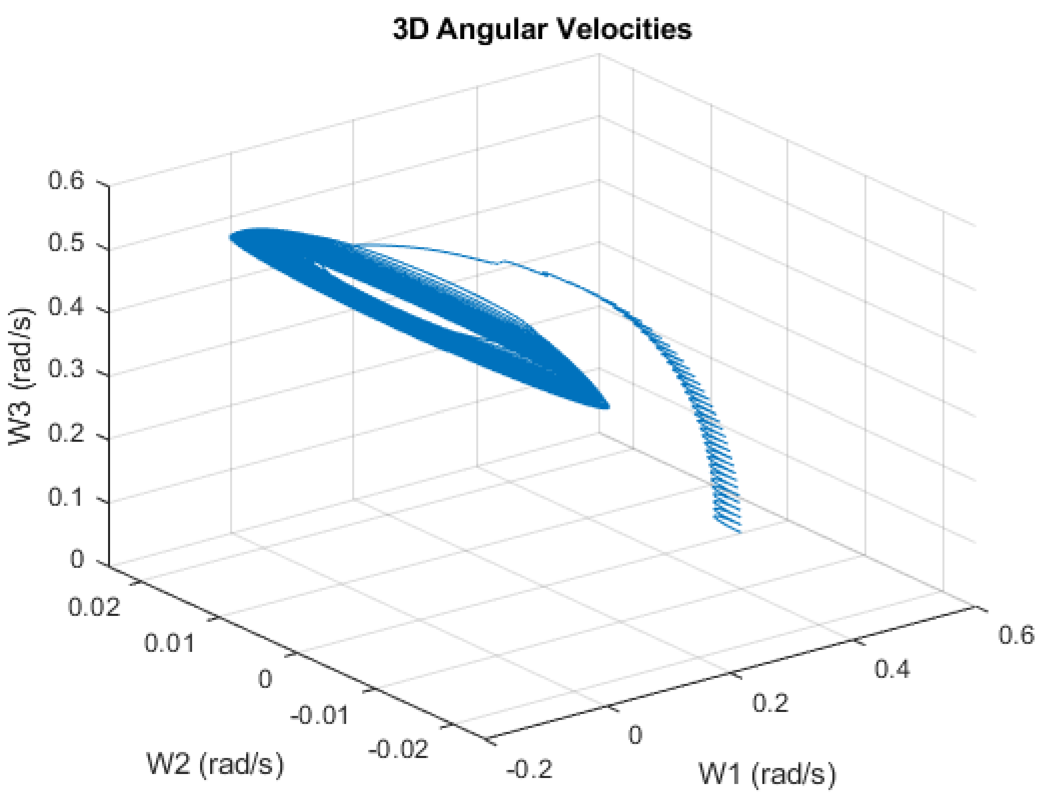
Question 3.2

```
Iw = 0.1; %kg*m^2
initial = [5;0;0;0;0;0]./60.*2.*pi; % initial tumbling angular velocity
(rad/s)
opts = odeset('MaxStep',2.0); % Sets max time step for ode45
ts = 5000; % wheel spin-up time (sec)
hw = 55; % nominal angular momentum (kg*m^2/s)
Iww = [0,0,0;0,0,0;0,0,Iw]; % inertia matrix for wheels (kg*m^2)
J = [500,0,0;0,400,0;0,0,440]; % inertia matrix for full satellite system
(kg*m^2)

% Calculate the satellite behavior  during the wheel spin up
[t,y] = ode45(@(t,w) angular_rates(t,w,J,hw,Iw,ts),[0,5000],initial,opts);

% Calculate the nutation angle at each time step
nutation_angle = nutation(J,Iww,y,[0,0,1]);

% Plot angular velocities of the satellite and the nutation angle
figure(3);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of Angular Velocities and Nutation Angle')
nexttile(tcl)
hold on;
plot(t,y(:,1));
```

```
ylabel('W1 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,y(:,2));
ylabel('W2 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,y(:,3));
ylabel('W3 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,nutation_angle);
ylabel('Nutation Angle (deg)')
xlabel('Time (s)')

% Plot 3D angular velocities
figure(4)
plot3(y(:,1),y(:,2),y(:,3));
title('3D Angular Velocities')
xlabel('W1 (rad/s)'); ylabel('W2 (rad/s)'); zlabel('W3 (rad/s)')
grid on;

% Check initial and final angular momentum to ensure conservation
wi = [5;0;0]./60.*2.*pi;
wf = [y(end,1); y(end,2); y(end,3)];
wwf = [y(end,4); y(end,5); y(end,6)];
H_i = norm(J*wi);
H_f = norm(J*wf+ Iww*wwf);

if abs(H_i-H_f) < 1e-3
    disp('3.2 passes angular momentum conservation check')
else
    disp('3.2 fails angular momentum conservation check')
end
```

3.2 passes angular momentum conservation check


Time Histories of Angular Velocities and Nutation Angle
W1 (Rad/s)
W2 (Rad/s)
W3 (Rad/s)
Nutation Angle (deg)
Time (s)


3D Angular Velocities
W3 (rad/s)
W2 (rad/s)
W1 (rad/s)

Question 4

```
% Initialize values for calculation
Bv1 = [-0.3;-0.1;0.9]; % star tracker observation vector
Bv2 = [0.8;-0.5;0.2]; % sun sensor observation vector
Nv1 = [0;0;1]; % star tracker reference vector
Nv2 = [1;0;0]; % sun sensor reference vector

% Calculate attitude DCM using TRIAD
C_BN_triad = TRIAD(Bv1,Bv2,Nv1,Nv2);

disp('DCM from TRIAD: ')
disp(C_BN_triad)

% Normalize vectors for QUEST use
Bv1 = Bv1./norm(Bv1);
Bv2 = Bv2./norm(Bv2);
Nv1 = Nv1./norm(Nv1);
Nv2 = Nv2./norm(Nv2);
Bv = [Bv1,Bv2];
Nv = [Nv1,Nv2];
w = [2,1]; % weight array, with star tracker twice as accurate

% Calculate attitude DCM using QUEST
C_BN_quest = QUEST(Bv,Nv,w);

disp('DCM from QUEST: ')
disp(C_BN_quest)

% Calculate the principle rotation angle between the two DCMs
angle_diff = PR(C_BN_quest*C_BN_triad');

disp('Principle rotation angle between DCMs: ')
disp(round(angle_diff,4))
```

```
DCM from TRIAD:
    0.8262    0.4674   -0.3145
   -0.5196    0.8479   -0.1048
    0.2177    0.2500    0.9435

DCM from QUEST:
    0.8273    0.4674   -0.3115
   -0.5193    0.8479   -0.1067
    0.2142    0.2500    0.9442

Principle rotation angle between DCMs:
    0.2076
```

Question 5

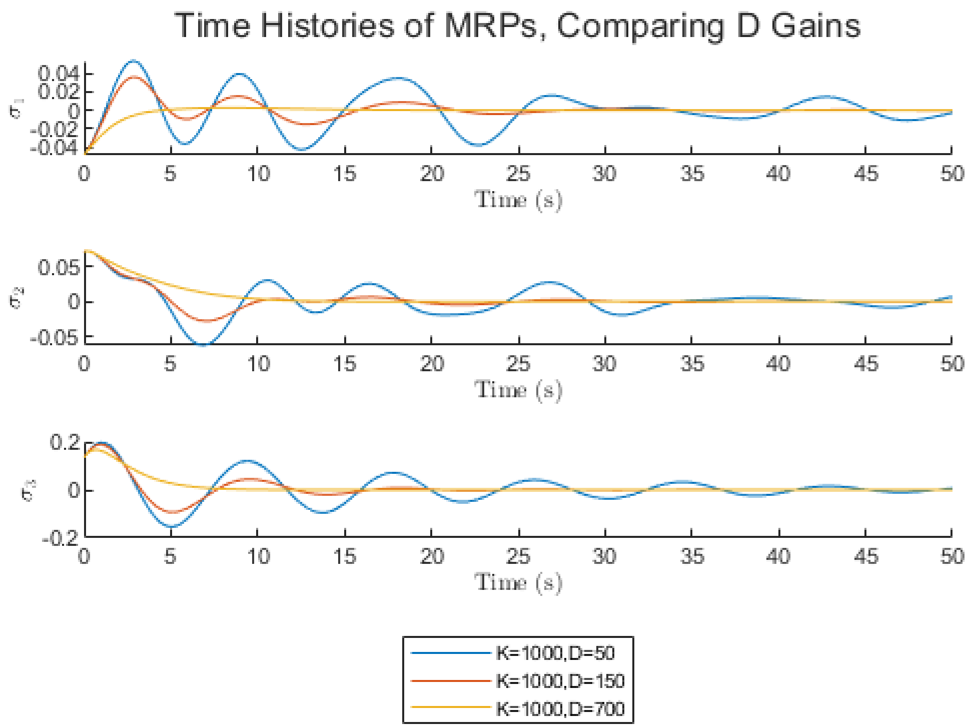
```
Iw = 0.1; %kg*m^2
% Calculate current attitude in MRPs
```

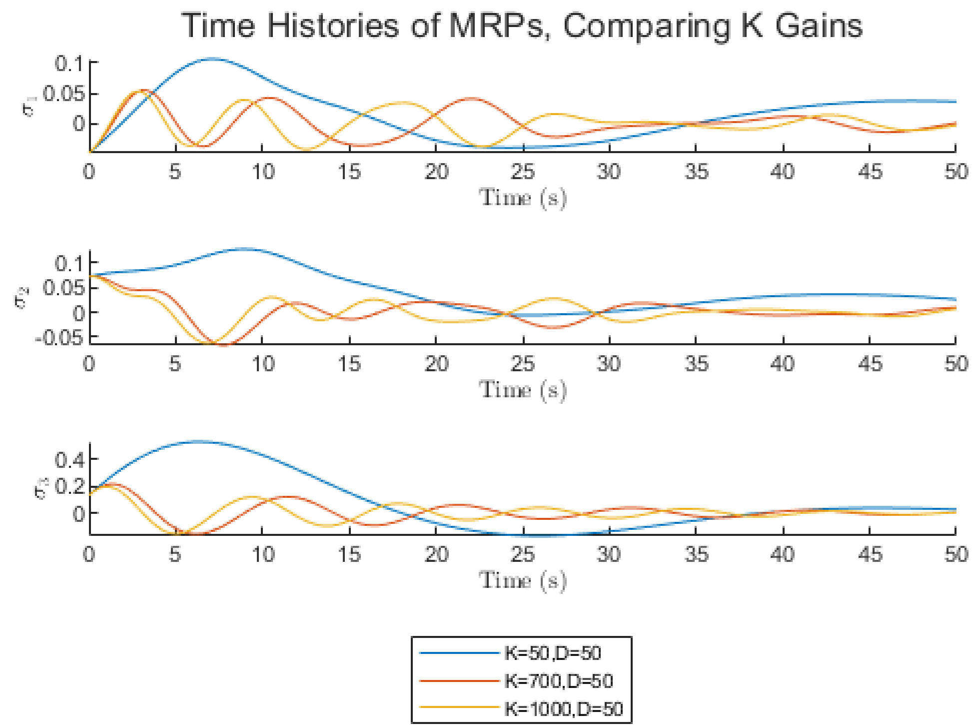
```
mrp = DCM2MRPs(C_BN_quest);
% Set initial conditions for integration
initial = [mrp;y2(end,:)'];
opts = odeset('MaxStep',2); % Sets max time step for ode45
J = [500,0,0;0,400,-7;0,-7,440]; % inertia matrix for full satellite system
(kg*m^2)

states = cell(1,3);
times = cell(1,3);
K = [1000,1000,1000,50,700,1000];
d = [50,150,700,50,50,50];
% Integrate the system utilizing different gains
for i = 1:6
    D = diag([d(i),d(i),d(i)]);
    mrp = [0;0;0];
    [t,s] = ode45(@(t,w) full_dyn(w,J,Iw,D,K(i),mrp),[0,50],initial,opts);
    times{i} = t;
    states{i} = s;
end

% Plot the comparison of D gains (MRP history)
figure(5);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of MRPs, Comparing D Gains')
nexttile(tcl)
hold on;
plot(times{1},states{1}(:,1));
plot(times{2},states{2}(:,1));
plot(times{3},states{3}(:,1));
ylabel('$\sigma _1$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{1},states{1}(:,2));
plot(times{2},states{2}(:,2));
plot(times{3},states{3}(:,2));
ylabel('$\sigma _2$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{1},states{1}(:,3));
plot(times{2},states{2}(:,3));
plot(times{3},states{3}(:,3));
ylabel('$\sigma _3$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
Lgnd = legend('K=1000,D=50', 'K=1000,D=150', 'K=1000,D=700');
Lgnd.Layout.Tile = 4;

% Plot the comparison of K gains (MRP history)
figure(50);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of MRPs, Comparing K Gains')
nexttile(tcl)
hold on;
```

```
plot(times{4},states{4}(:,1));
plot(times{5},states{5}(:,1));
plot(times{6},states{6}(:,1));
ylabel('$\sigma _1$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{4},states{4}(:,2));
plot(times{5},states{5}(:,2));
plot(times{6},states{6}(:,2));
ylabel('$\sigma _2$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{4},states{4}(:,3));
plot(times{5},states{5}(:,3));
plot(times{6},states{6}(:,3));
ylabel('$\sigma _3$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
Lgnd = legend('K=50,D=50', 'K=700,D=50', 'K=1000,D=50');
Lgnd.Layout.Tile = 4;

% Plot the angular velocity history for the chosen gains
figure(6);
tcl = tiledlayout(3,1);
title(tcl,'Time Histories of Angular Velocities')
nexttile(tcl)
hold on;
plot(times{3},states{3}(:,4));
ylabel('W1 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{3},states{3}(:,5));
ylabel('W2 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{3},states{3}(:,6));
ylabel('W3 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")

% Plot the angular velocity of the RWs for the chosen gains
figure(16);
tcl = tiledlayout(3,1);
title(tcl,'Time Histories of Reaction Wheel Speed')
nexttile(tcl)
hold on;
plot(times{3},states{3}(:,7).*60./2./pi);
ylabel('$W_w1$ (RPM)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{3},states{3}(:,8).*60./2./pi);
```

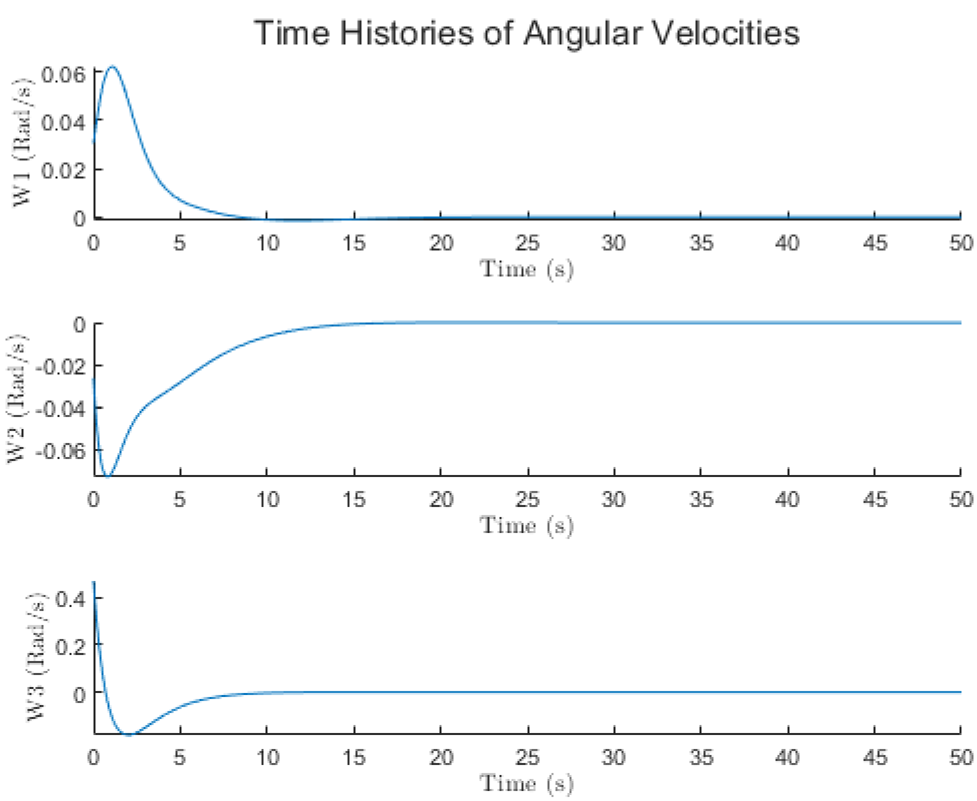
```
ylabel('$W_w2$ (RPM)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{3},states{3}(:,9).*60./2./pi);
ylabel('$W_w3$ (RPM)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")

% Calculate the norm errors for the both the attitude and angular
% velocities, and the torque at each step
goal = [0;0;0];
s_e = [];
w_e = [];
u = []; K = 1000; D = diag([700,700,700]);
J = [500,0,0;0,400,-7;0,-7,440];
V = [];
for i = 1:length(states{3}(:,1))
    sig = states{3}(i,1:3)';
    sig_e = ((1-norm(sig)^2)*goal - (1-norm(goal)^2)*sig +
cross(2*goal,sig))...
            /(1+norm(sig)^2*norm(goal)^2 - 2*dot(sig,goal));
    s_e(i) = norm(sig_e);
    w_e(i) = norm(states{3}(i,4:6));
    w = states{3}(i,4:6)';
    u(i,:) = (-K*sig_e + D*w + skew(w)*J*w)';
    V(i) = lyap(sig,w,J,K);
end

% Plot the angular velocity history for the chosen gains
figure(15);
tcl = tiledlayout(2,1);
title(tcl,'Time Histories of Orientation Errors')
nexttile(tcl)
semilogy(times{3},s_e); hold on;
yline(10^-5);
ylabel('MRP Error Norm')
xlabel('Time (s)')
nexttile(tcl)
semilogy(times{3},w_e); hold on;
yline(10^-5);
ylabel('Angular Velocity Error Norm')
xlabel('Time (s)')

% Plot the angular velocity of the RWs for the chosen gains
figure(17);
tcl = tiledlayout(3,1);
title(tcl,'Time Histories of Reaction Wheel Torque')
nexttile(tcl)
hold on;
plot(times{3},u(:,1));
ylabel('U1 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
```

```
plot(times{3},u(:,2));
ylabel('U2 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(times{3},u(:,3));
ylabel('U3 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")

%Plot the Lyapunov function over time
figure(30)
plot(times{3},V);
xlabel('Time (s)')
ylabel('V(t)'); title('Lyapunov Function')
```

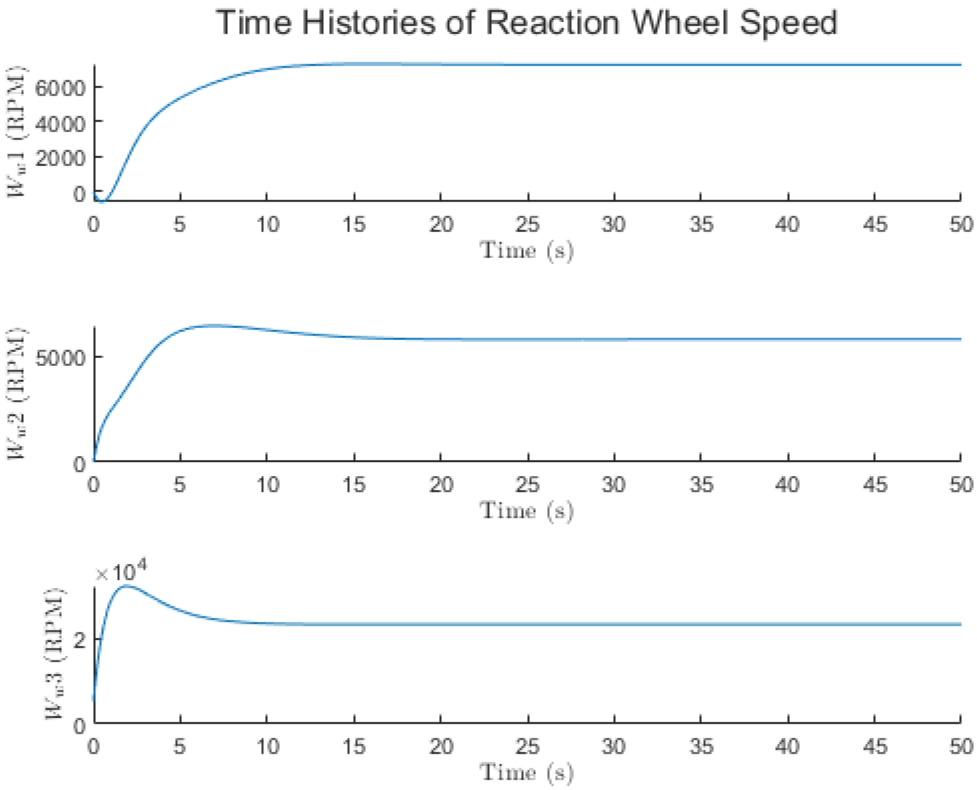


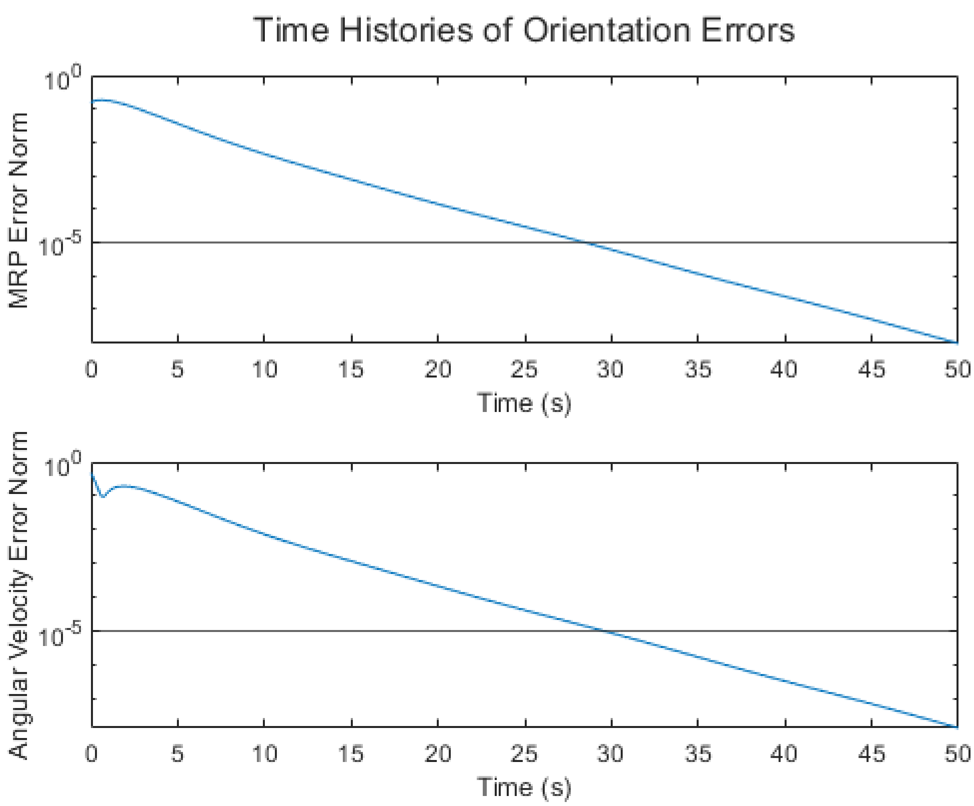


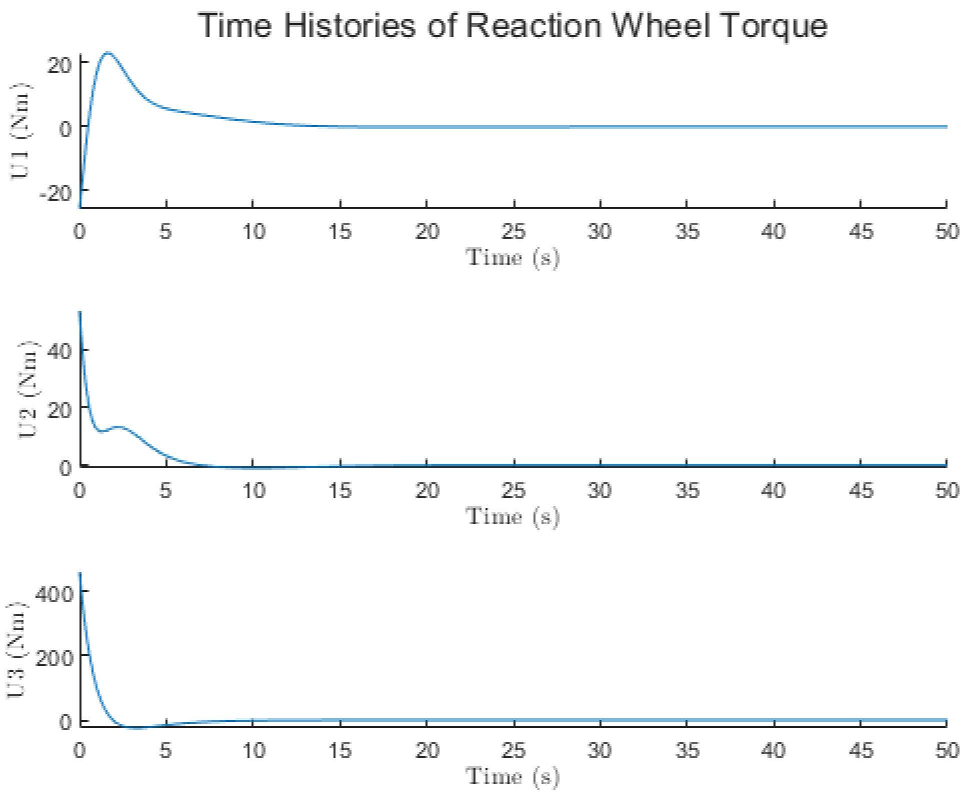
Time Histories of MRPs, Comparing K Gains
σ_1
σ_2
σ_3
Time (s)
K=50,D=50
K=700,D=50
K=1000,D=50

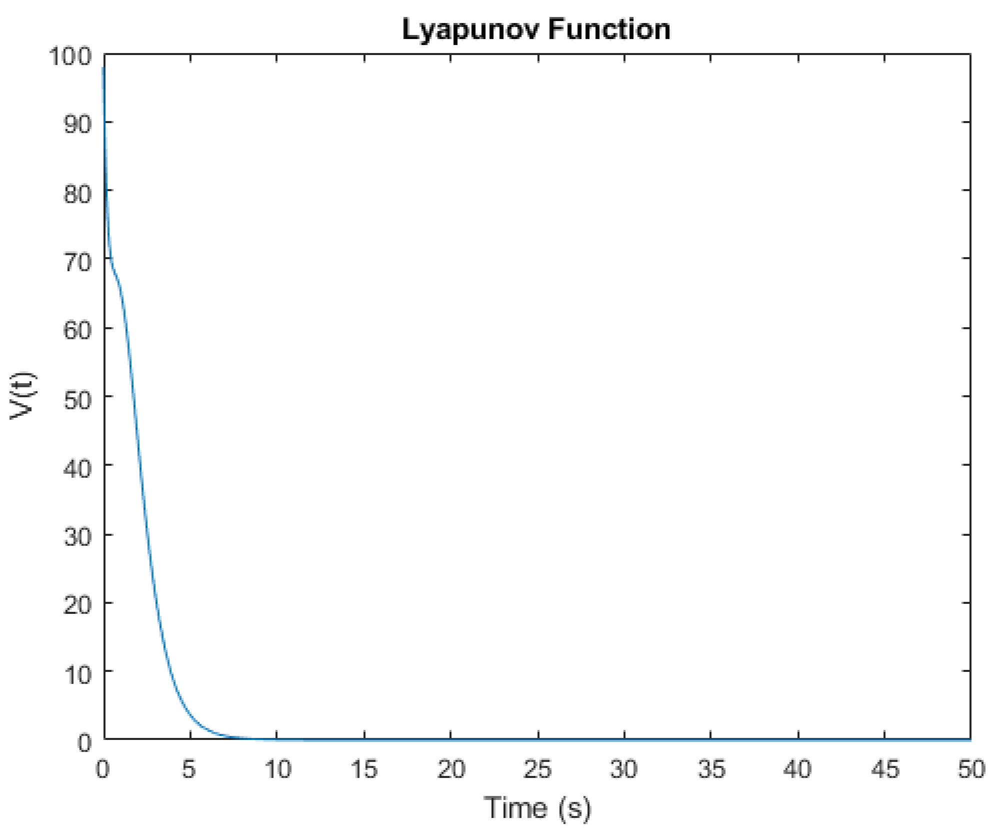


Time Histories of Angular Velocities
W1 (Rad/s)
W2 (Rad/s)
W3 (Rad/s)
Time (s)


Time Histories of Reaction Wheel Speed
W_w1 (RPM)
W_w2 (RPM)
W_w3 (RPM)
Time (s)


Time Histories of Orientation Errors
MRP Error Norm
Angular Velocity Error Norm
Time (s)


Time Histories of Reaction Wheel Torque
U1 (Nm)
U2 (Nm)
U3 (Nm)
Time (s)


Lyapunov Function
V(t)
Time (s)

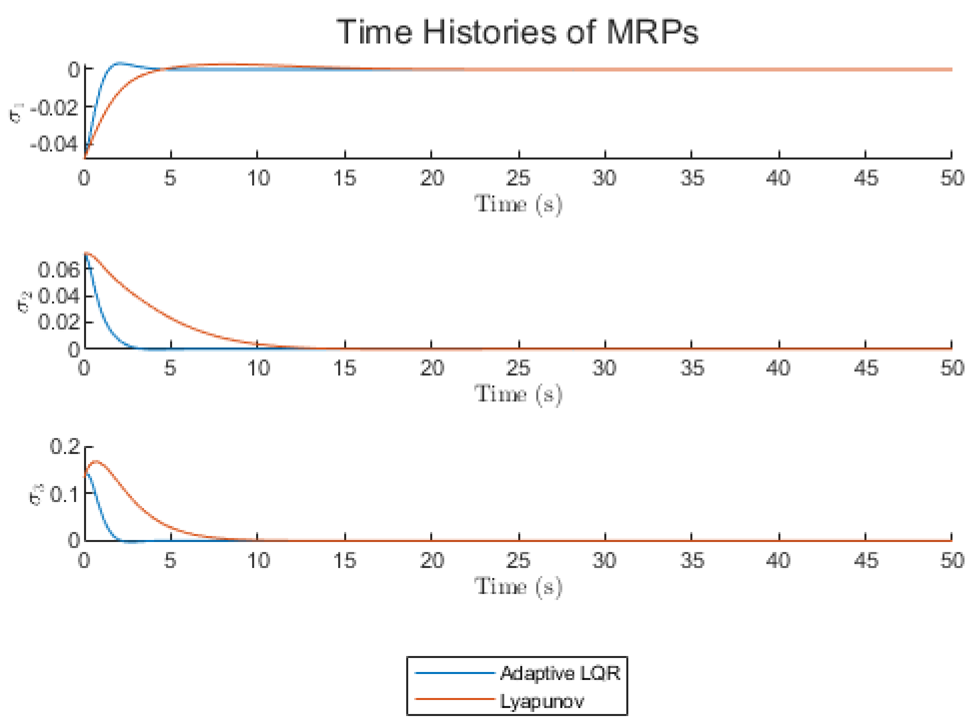
Question 5.3

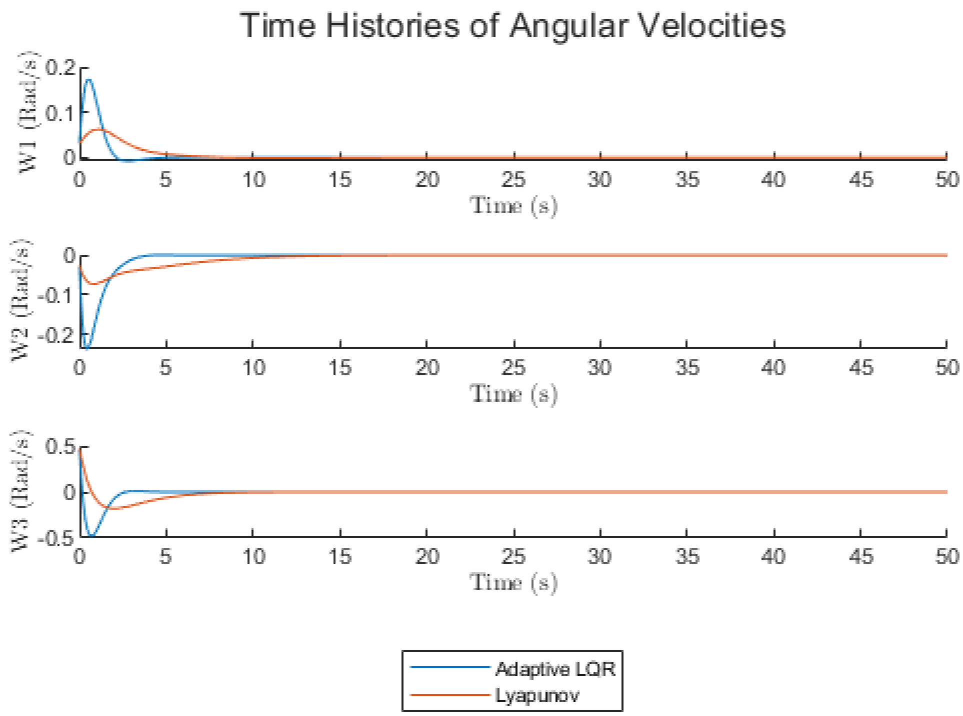
```
Iw = 0.1; %kg*m^2
mrp = DCM2MRPs(C_BN_quest);
initial = [mrp;y2(end,:)'];
opts = odeset('MaxStep',2);
J = [500,0,0;0,400,-7;0,-7,440];
mrp = [0;0;0];
[t,s] = ode45(@(t,w) lin_dyn(w,J,Iw,mrp),[0,50],initial,opts);

figure(7);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of MRPs')
nexttile(tcl)
hold on;
plot(t,s(:,1));
plot(times{3},states{3}(:,1));
ylabel('$\sigma _1$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,s(:,2));
plot(times{3},states{3}(:,2));
ylabel('$\sigma _2$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,s(:,3));
plot(times{3},states{3}(:,3));
ylabel('$\sigma _3$',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
Lgnd = legend('Adaptive LQR', 'Lyapunov');
Lgnd.Layout.Tile = 4;


figure(8);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of Angular Velocities')
nexttile(tcl)
hold on;
plot(t,s(:,4));
plot(times{3},states{3}(:,4));
ylabel('W1 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,s(:,5));
plot(times{3},states{3}(:,5));
ylabel('W2 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,s(:,6));
```

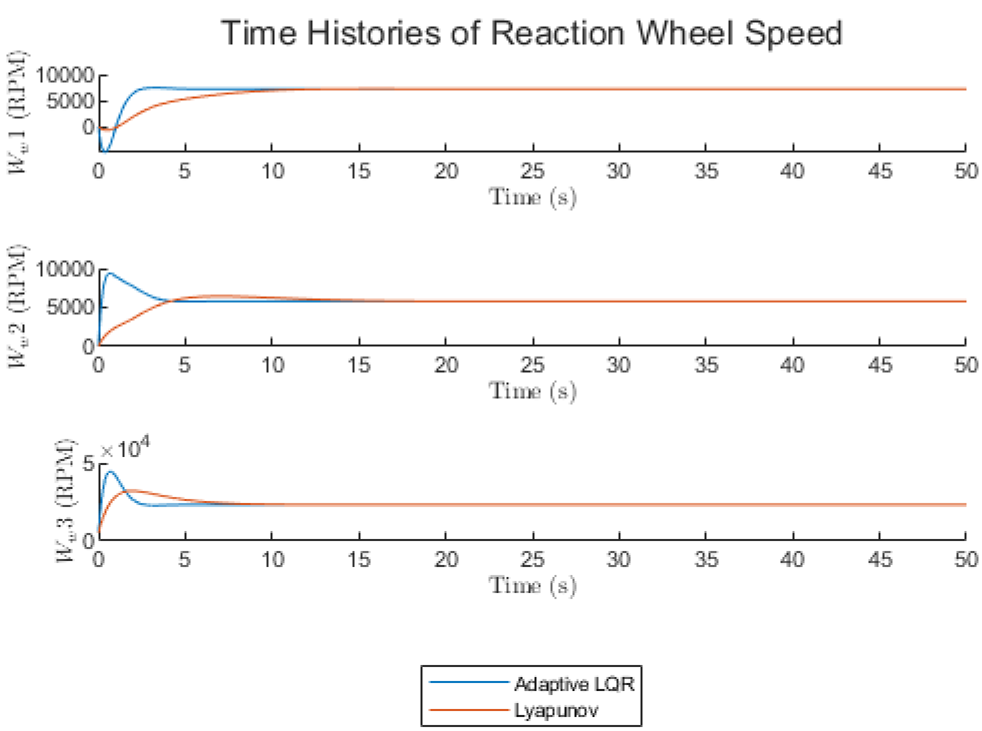
```
plot(times{3},states{3}(:,6));
ylabel('W3 (Rad/s)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
Lgnd = legend('Adaptive LQR', 'Lyapunov');
Lgnd.Layout.Tile = 4;

% Plot the angular velocity of the RWs for the chosen gains
figure(20);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of Reaction Wheel Speed')
nexttile(tcl)
hold on;
plot(t,s(:,7).*60./2./pi);
plot(times{3},states{3}(:,7).*60./2./pi);
ylabel('$W_w1$ (RPM)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,s(:,8).*60./2./pi);
plot(times{3},states{3}(:,8).*60./2./pi);
ylabel('$W_w2$ (RPM)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,s(:,9).*60./2./pi);
plot(times{3},states{3}(:,9).*60./2./pi);
ylabel('$W_w3$ (RPM)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
Lgnd = legend('Adaptive LQR', 'Lyapunov');
Lgnd.Layout.Tile = 4;

% Calculate the norm errors for the both the attitude and angular
% velocities, and the torque at each step
goal = [0;0;0];
s_e2 = [];
w_e2 = [];
u2 = []; K = 1000; D = diag([700,700,700]);
J = [500,0,0;0,400,-7;0,-7,440];
V = [];
for i = 1:length(s(:,1))
    sig = s(i,1:3)';
    sig_e = ((1-norm(sig)^2)*goal - (1-norm(goal)^2)*sig +
cross(2*goal,sig))...
            /(1+norm(sig)^2*norm(goal)^2 - 2*dot(sig,goal));
    s_e2(i) = norm(sig_e);
    w_e2(i) = norm(s(i,4:6));
    w = s(i,4:6)';
    [~,temp_u] = lin_dyn([sig;w;s(i,7:9)'],J,Iw,[0;0;0]);
    u2(i,:) = temp_u';
    V(i) = lyap(sig,w,J,K);
end

% Plot the angular velocity history for the chosen gains
figure(21);
```

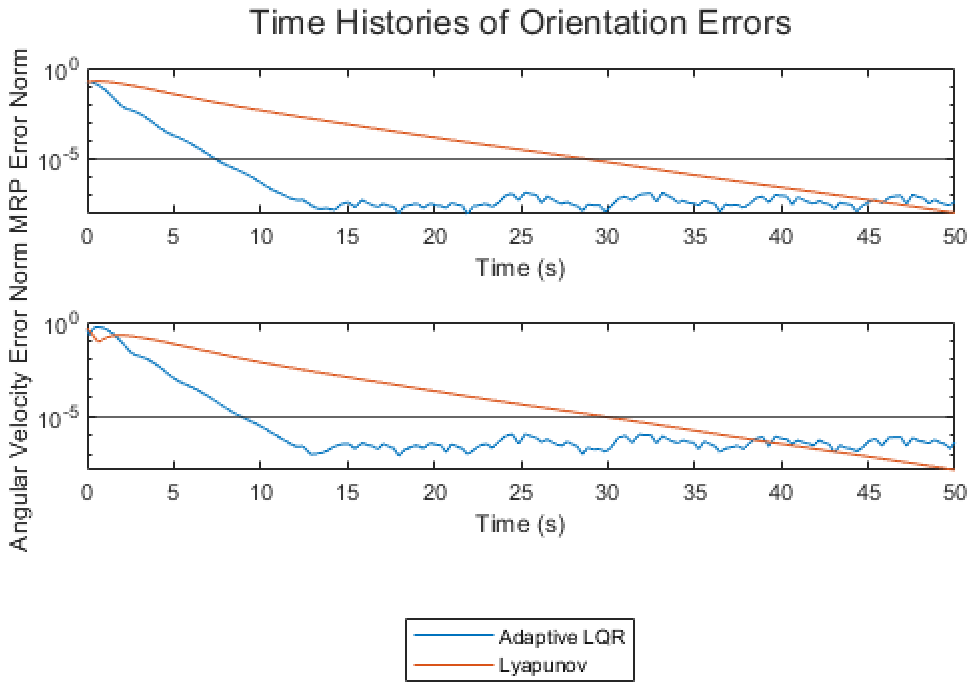
```
tcl = tiledlayout(3,1);
title(tcl,'Time Histories of Orientation Errors')
nexttile(tcl)
semilogy(t,s_e2); hold on;
semilogy(times{3},s_e);
yline(10^-5);
ylabel('MRP Error Norm')
xlabel('Time (s)')
nexttile(tcl)
semilogy(t,w_e2); hold on;
semilogy(times{3},w_e);
yline(10^-5);
ylabel('Angular Velocity Error Norm')
xlabel('Time (s)')
Lgnd = legend('Adaptive LQR', 'Lyapunov');
Lgnd.Layout.Tile = 3;

% Plot the angular velocity of the RWs for the chosen gains
figure(22);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of Reaction Wheel Torque')
nexttile(tcl)
plot(t,u2(:,1)); hold on;
plot(times{3},u(:,1));
ylabel('U1 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,u2(:,2)); hold on;
plot(times{3},u(:,2));
ylabel('U2 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
nexttile(tcl)
hold on;
plot(t,u2(:,3)); hold on;
plot(times{3},u(:,3));
ylabel('U3 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
Lgnd = legend('Adaptive LQR', 'Lyapunov');
Lgnd.Layout.Tile = 4;

% Plot the angular velocity of the RWs for the chosen gains
figure(23);
tcl = tiledlayout(4,1);
title(tcl,'Time Histories of Reaction Wheel Torque')
nexttile(tcl)
plot(t,u2(:,1)); hold on;
plot(times{3},u(:,1));
ylabel('U1 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
xlim([0,5])
nexttile(tcl)
hold on;
plot(t,u2(:,2)); hold on;
```

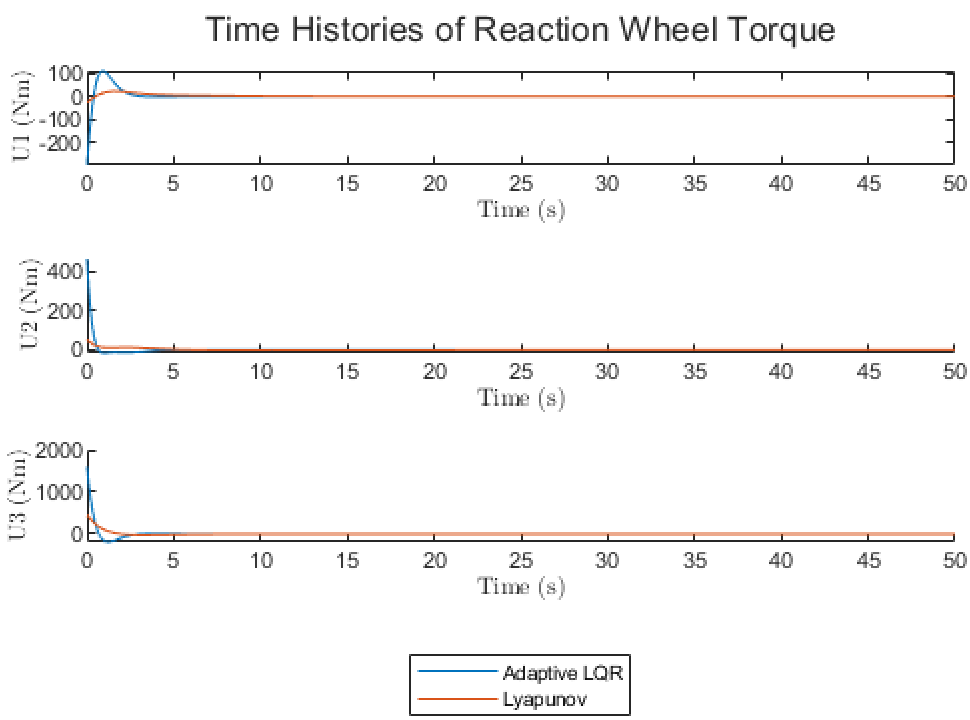
```
plot(times{3},u(:,2));
ylabel('U2 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
xlim([0,5])
nexttile(tcl)
hold on;
plot(t,u2(:,3)); hold on;
plot(times{3},u(:,3));
ylabel('U3 (Nm)',"Interpreter","latex","FontWeight","Bold")
xlabel('Time (s)',"Interpreter","latex","FontWeight","Bold")
xlim([0,5])
Lgnd = legend('Adaptive LQR', 'Lyapunov');
Lgnd.Layout.Tile = 4;

%Plot the Lyapunov function over time
figure(31)
plot(t,V);
xlabel('Time (s)')
ylabel('V(t)'); title('Lyapunov Function')
```

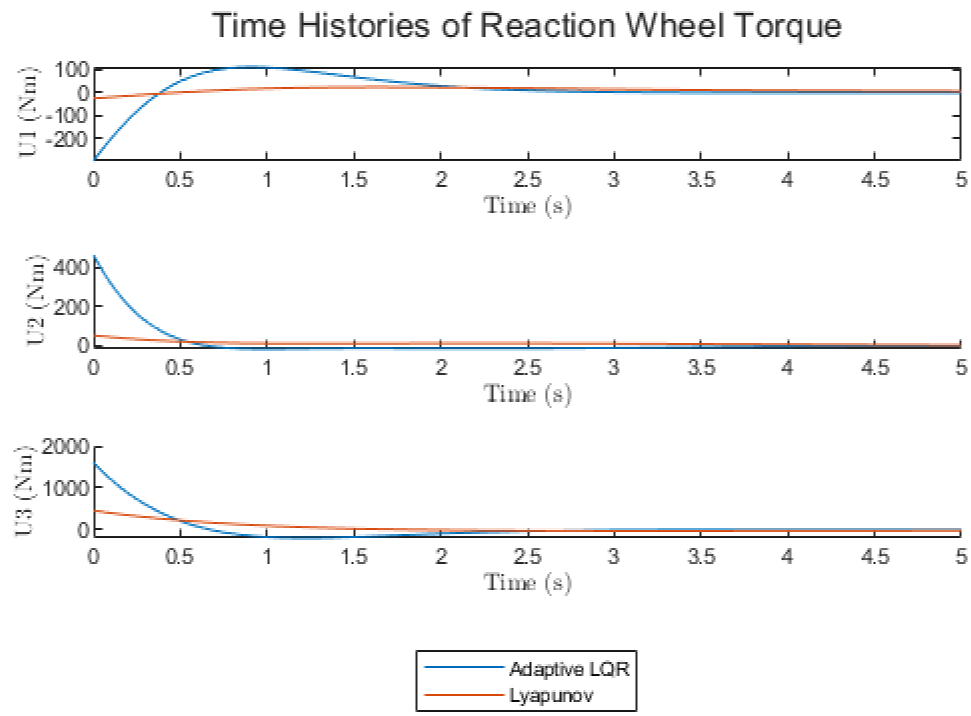


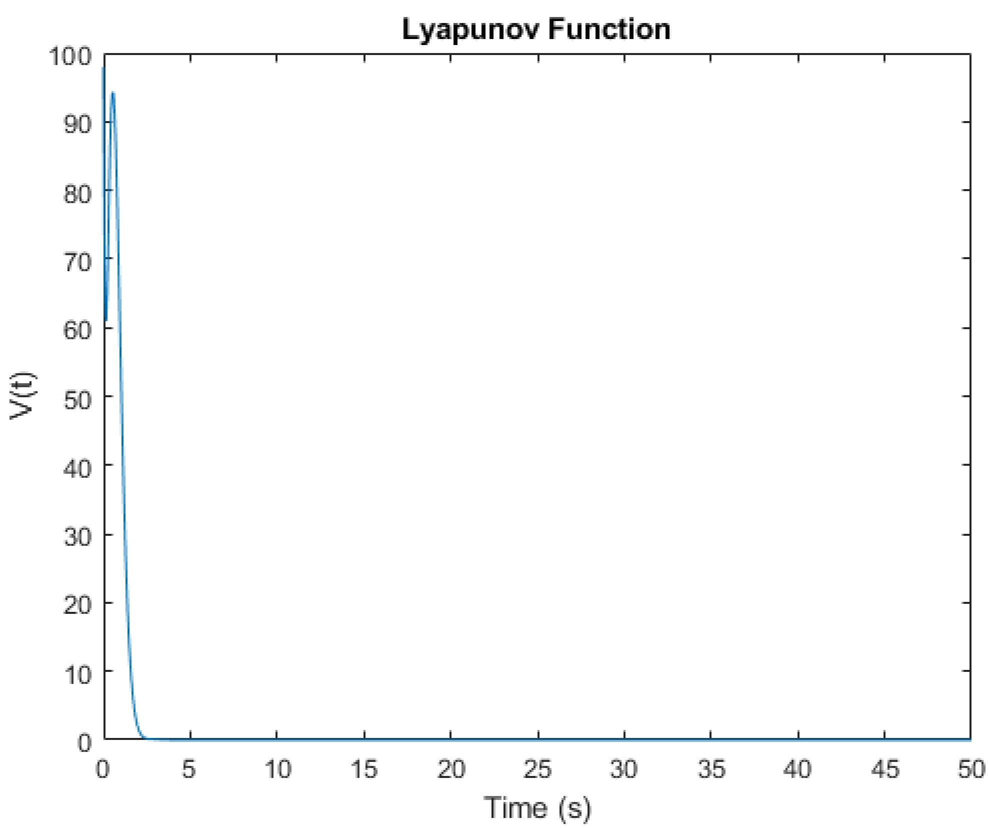

Time Histories of Angular Velocities
W1 (Rad/s)
W2 (Rad/s)
W3 (Rad/s)
Time (s)
Adaptive LQR
Lyapunov


Time Histories of Reaction Wheel Speed
W_w1 (RPM)
W_w2 (RPM)
W_w3 (RPM)
Time (s)
Adaptive LQR
Lyapunov


Time Histories of Orientation Errors
MRP Error Norm
Angular Velocity Error Norm
Time (s)
Adaptive LQR
Lyapunov


Time Histories of Reaction Wheel Torque
U1 (Nm)
U2 (Nm)
U3 (Nm)
Time (s)
Adaptive LQR
Lyapunov


Time Histories of Reaction Wheel Torque
U1 (Nm)
U2 (Nm)
U3 (Nm)
Time (s)
Adaptive LQR
Lyapunov


Lyapunov Function
V(t)
Time (s)

Functions

```
% Full dynamics including Lyapunov control law
function [prime] = full_dyn(x,J,Iw,D,K,goal)
    sig = x(1:3);
    sig2 = sig'*sig;
    % Calculate and switch to shadow MRP if necessary
    if sig2 > 1
        sig = -sig./sig2;
    end
    sig_e = ((1-norm(sig)^2)*goal - (1-norm(goal)^2)*sig +
cross(2*goal,sig))...
            /(1+norm(sig)^2*norm(goal)^2 - 2*dot(sig,goal));

    w = x(4:6);
    ww = x(7:9);

    % Inertia matrix for RWs
    Iww = [Iw,0,0;0,Iw,0;0,0,Iw];
    % Torque calculation using predefined gains
    u = -K*sig_e + D*w + skew(w)*J*w;
    % Angular acceleration of RWs
    ww_dot = inv(Iww)*(skew(ww)*Iww*ww+u);
    % Angular acceleration of the spacecraft
    wdot = J\(-skew(w)*J*w-skew(w)*Iww*ww-Iww*ww_dot);
    % Attitude acceleration of the spacecraft
    sig_dot = 0.25*((1-sig2)*eye(3) + 2*skew(sig) + 2*sig*sig')*w;
    % Propogate solution
    prime = [sig_dot;wdot;ww_dot];
end

% Full dynamics using linear control law
function [prime,u] = lin_dyn(x,J,Iw,goal)
    % Calculate gains using current state
    K = gain(x(1:6));
    sig = x(1:3);
    sig2 = sig'*sig;
    % Calculate and switch to shadow MRP if necessary
    if sig2 > 1
        sig = -sig./sig2;
    end
    sig_e = ((1-norm(sig)^2)*goal - (1-norm(goal)^2)*sig +
cross(2*goal,sig))...
            /(1+norm(sig)^2*norm(goal)^2 - 2*dot(sig,goal));

    w = x(4:6);
    ww = x(7:9);

    % Inertia matrix for RWs
    Iww = [Iw,0,0;0,Iw,0;0,0,Iw];
    % Torque calculation using LQR gains
    u = -K*x(1:6);
    % Angular acceleration of RWs
```

```
    ww_dot = inv(Iww)*(skew(ww)*Iww*ww+u);
    % Angular acceleration of the spacecraft
    wdot = J\(-skew(w)*J*w-skew(w)*Iww*ww-Iww*ww_dot);
    % Attitude acceleration of the spacecraft
    sig_dot = 0.25*((1-sig2)*eye(3) + 2*skew(sig) + 2*sig*sig')*w;
    % Propogate solution
    prime = [sig_dot;wdot;ww_dot];

    % Function to assemble skew matrix from a vector
    function M = skew(s)
        M = [0,-s(3),s(2); s(3),0,-s(1); -s(2),s(1),0];
    end
    % Function to calculate gains using LQR
    function K = gain(x)
        sig_ = x(1:3);
        % Calculate and switch to shadow MRP if necessary
        sig2_ = sig_'*sig_;
        if sig2_ > 1
            sig_ = -sig_./sig2_;
        end

        w_ = x(4:6);
        Iw_ = 0.1;
        J_ = [500,400,440];
        % Calculate linear coefficients for state variables
        sd = (1-sig2_)*eye(3) + 2*skew(sig_) + 2*sig_*sig_';
        w_var = diag(J_)\(-skew(w_)*diag(J_));
        A = [0,0,0,0.25*sd(1,1),0.25*sd(1,2),0.25*sd(1,3);
             0,0,0,0.25*sd(2,1),0.25*sd(2,2),0.25*sd(2,3);
             0,0,0,0.25*sd(3,1),0.25*sd(3,2),0.25*sd(3,3);
             0,0,0,w_var(1,1),w_var(1,2),w_var(1,3);
             0,0,0,w_var(2,1),w_var(2,2),w_var(2,3);
             0,0,0,w_var(3,1),w_var(3,2),w_var(3,3)];

        % Calculate linear coefficients for input variables
        B = zeros(6,3);
        B(4,1) = -J_(1)*Iw_*inv(Iw_);B(5,2) = -J_(2)*Iw_*inv(Iw_);B(6,3) =
-J_(3)*Iw_*inv(Iw_);

        % Weights for each state variable and input
        Q = diag([500,500,500,20,20,20]);
        R = diag([0.00001,0.00001,0.00001]);

        % Calculate gain matrix using LQR
        [K,~,~] = lqr(A,B,Q,R);

    end
end

% Define the differential kinematic equation for 3-2-1 Euler angles
function [rotation_rates] = angular_rates(t,w,J,hw,Iw,ts)

    hw_dot = hw/ts;
    ww_dot = hw_dot/Iw;
```

```
    ww_dot = [0;0;ww_dot];

    Iww = [Iw,0,0;0,Iw,0;0,0,Iw];

    wdot = J\(-skew(w(1:3))*J*w(1:3)-skew(w(1:3))*Iww*w(4:6)-Iww*ww_dot);

    rotation_rates = [wdot;ww_dot];

end

% Function to assemble skew matrix from a vector
function M = skew(s)
    M = [0,-s(3),s(2); s(3),0,-s(1); -s(2),s(1),0];
end

% Calculate the nutation angle
function angle = nutation(J,Iw,w,control)
    for i = 1:length(w(:,1))
        H(i,:) = (J*w(i,1:3)' + Iw*w(i,4:6)')';
        angle(i) = acosd(dot(H(i,:),control)/(norm(H(i,:))*norm(control)));
    end
end

% Calculate attitude DCM using TRIAD
function C_BN = TRIAD(Bv1,Bv2,Nv1,Nv2)
    % Normalize all vectors
    Bv1 = Bv1./norm(Bv1);
    Bv2 = Bv2./norm(Bv2);
    Nv1 = Nv1./norm(Nv1);
    Nv2 = Nv2./norm(Nv2);
    % Calculate vectors in intermediate frame
    Bt1 = Bv1;
    Bt2 = cross(Bv1,Bv2)/norm(cross(Bv1,Bv2));
    Bt3 = cross(Bt1,Bt2);
    Nt1 = Nv1;
    Nt2 = cross(Nv1,Nv2)/norm(cross(Nv1,Nv2));
    Nt3 = cross(Nt1,Nt2);
    % Compute DCMs for both B and N frames
    C_BT = [Bt1,Bt2,Bt3];
    C_NT = [Nt1,Nt2,Nt3];
    % Compute attitude DCM
    C_BN = C_BT*C_NT';
end

% Calculate attitude DCM using QUEST
function C_BN = QUEST(Bv,Nv,w)
    % Calculate intermediate values
    B = zeros(3,3);
    for i = 1:length(w)
        B = B + w(i)*Bv(:,i)*Nv(:,i)';
    end
    S = B + B';
    sig = trace(B);
    Z = [B(2,3)-B(3,2);B(3,1)-B(1,3);B(1,2)-B(2,1)];
```

```
    % Assemble K matrix
    K = [sig,Z';Z,S-sig*eye(3,3)];
    % Calculate max eigenvalue using Newton Raphson
    eig_val = NR(sum(w),1e-10);

    % Calculate Rodrigues parameters (eq. 3.238)
    q = inv((eig_val+sig)*eye(3,3)-S)*Z;

    % Assemble DCM from Rodrigues parameters
    C_BN = (1/(1+q'*q))*[1+q(1)^2-q(2)^2-q(3)^2, 2*(q(1)*q(2)+q(3)),
2*(q(1)*q(3)-q(2))
                          2*(q(1)*q(2)-q(3)), 1-q(1)^2+q(2)^2-q(3)^2,
2*(q(2)*q(3)+q(1))
                          2*(q(1)*q(3)+q(2)), 2*(q(2)*q(3)-q(1)), 1-q(1)^2-
q(2)^2+q(3)^2];

    % Eq 3.236 for Newton Raphson root solving
    function y = f(s)
        y = det(K-s*eye(4,4));
    end
    % 1st derivative of Eq 3.236
    function dy = fp(s)
        dy = -det(K-s*eye(4,4))*trace(inv(K-s*eye(4,4)));
    end
    % Root solving using Newton Raphson for the eigenvalue
    function eig_val = NR(initial,tol)
        e = initial;
        err = f(e);
        while err > tol
            e = e - f(e)/fp(e);
            err = f(e);
        end
        eig_val = e;
    end
end

% Find principle rotation angle of a DCM
function angle = PR(C)
    angle = acosd(0.5*(trace(C)-1));
end

% Convert DCM to MRP
function mrp = DCM2MRPs(DCM)
    c = sqrt(trace(DCM)+1);
    mrp = 1/(c*(c+2)) *[DCM(2,3)-DCM(3,2);DCM(3,1)-DCM(1,3);DCM(1,2)-
DCM(2,1)];
end

% Calculate Lyapunov function value
function V = lyap(sig,w,J,K)
    V = 0.5*w'*J*w + 2*K*log(1+sig'*sig);
end
```

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