# A simple diversity guided firefly algorithm



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## Abstract

**Purpose** – The purpose of this paper is to present a modified firefly algorithm (FA) considering the population diversity to avoid local optimum and improve the algorithm's precision.

**Design/methodology/approach** – When the population diversity is below the given threshold value, the fireflies' positions update according to the modified equation which can dynamically adjust the fireflies' exploring and exploiting ability.

**Findings** – A novel metaheuristic algorithm called FA has emerged. It is inspired by the flashing behavior of fireflies. In basic FA, randomly generated solutions will be considered as fireflies, and brightness is associated with the objective function to be optimized. However, during the optimization process, the fireflies become more and more similar and gather into the neighborhood of the best firefly in the population, which may make the algorithm prematurely converged around the local solution.

**Research limitations/implications** – Due to different dimensions and different ranges, the population diversity is different undoubtedly. And how to determine the diversity threshold value is still required to be further researched.

**Originality/value** – This paper presents a modified FA which uses a diversity threshold value to guide the algorithm to alternate between exploring and exploiting behavior. Experiments on 17 benchmark functions show that the proposed algorithm can improve the performance of the basic FA.

**Keywords** Optimization, Firefly algorithm, Metaheuristic, Population diversity

**Paper type** Research paper

## 1. Introduction

The firefly algorithm (FA) is a new population-based optimization strategy. It was introduced by Xin-She Yang (Huan and Yang, 2013; Yang, 2010b), which is nature-inspired by behavior of the flashing characteristics of fireflies. It has been successfully applied to many areas such as: function optimization (Yang, 2013), products online (Banati and Bajaj, 2012), economic dispatch (Chandrasekaran and Simon, 2012; Sulaiman *et al.*, 2012), image compression (Horng, 2012), wireless sensor networks (Xu and Liu, 2013), and so on. However, some shortcomings of this novel algorithm still need to be solved. One of these shortcomings is that the FA trends to converge

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prematurely or stagnate during the optimization process and causes to lower precision (Ventresca and Tizhoosh, 2008).

Many researchers have made tremendous efforts to improve the effectiveness, efficiency and precision of FA in solving optimization problems. Dos Santos Coelho *et al.* (2011) in their paper proposed a combination of FA with chaotic maps in order to improve the convergence of the basic FA. Use of the chaos sequences was shown to be especially effective by easier escape from the local optima. As the first hybridization of the FA, Yang (Yang and De, 2010) formulated a new metaheuristic search method, called Eagle Strategy (ES), which combines the Lévy flight search with the FA. In the paper of Hassanzadeh *et al.* (Hassanzadeh and Meybodi, 2012), cellular learning automata were hybridized with the FA. In this metaheuristic, the cellular learning automata were responsible for making diverse solutions in the FA, while FA improved these solutions in the sense of local search. Farahani (Farahani *et al.*, 2012) proposed three classes of algorithms for improving the performance of the basic FA. In the first class, learning automata were used for adapting the absorption and randomization parameters in the FA. The second class hybridized the genetic algorithm with FA in order to balance the exploration and exploitation properties of this proposed metaheuristic by time. The last class used random walk based on a Gaussian distribution in order to move the fireflies over the search space. In 2013, a self-adaptive step FA (Yu *et al.*, 2013) was proposed, the core idea is to set the step of each firefly according to each firefly's historical information and current situation.

Population premature convergence around a local optimum is a common problem for population-based algorithms. When premature convergence occurs, the FA's search ability of exploration is reduced and it will have a low possibility to explore new search areas. In this paper a simple diversity guided FA is proposed. This work is different from existing ones. The proposed algorithm measures distribution of fireflies' current positions and can reflect fireflies' dynamics. At the beginning of the algorithm, diversity is highest when all fireflies have been randomly generated. As the algorithm progresses, diversity decreases due to sample generating bias or selection pressure and therefore the difficulty of escaping a local optima (Ventresca and Tizhoosh, 2008; Cheng, 2013) is increased. This paper presents a modified FA which uses a diversity threshold value to guide the algorithm alternate between exploring and exploiting behavior. Experiments on 17 benchmark functions show that the proposed algorithm can improve the performance of the algorithm.

The rest of the paper is organized as follows. Section 2 describes the basic FA and population diversity. Section 3 explains the proposed method to improve the performance of the basic FA. Section 4 is dedicated to the experiments and obtained results. The last section concludes the paper and puts forward the scope of our future works.

## 2. Background

This section will describe the basic FA. Also, a brief description of the population diversity will be provided.

### *2.1 The bionic principle of FA*

In the basic FA, there are two important issues: the brightness and attractiveness. Brightness shows the advantages or disadvantages of the fireflies' position and determines the moving direction, while attractiveness determines the moving distance.

Through the brightness and attractiveness are constantly updated, the algorithm can achieve the objective optimization. FA is based upon three idealized rules as following (Yang, 2009):

(1) All fireflies are unsexing so that one firefly is attracted to other fireflies regardless of their sexes. The brightness of a firefly is determined by the scenario of the objective function, the better position has the higher brightness.
(2) Attractiveness is proportional to their brightness, thus for any two fireflies, the less brighter one will be attracted to the brighter one and the attractiveness decreases as their distance increases.
(3) If there is no brighter one than a particular firefly, it will move randomly.

*2.2 The description and analysis of FA*

As mentioned above, there are two important issues in the FA. We can always suppose that the attractiveness of a firefly is determined by its brightness which is equal to the encoded objective function. As the distance from the source increases, the variations of brightness and attractiveness should be monotonically decreasing functions. In most applications, the attractiveness can be approximated using the following form:

$$I(r) = I_0 e^{-\gamma r^2} \tag{1}$$

where $\gamma$ is the light absorption coefficient which can be taken as a constant. Since a firefly's attractiveness is proportional to the brightness, we can now define the attractiveness $\beta$ of a firefly as follows:

$$\beta(r) = \beta_0 e^{-\gamma^2} \tag{2}$$

where $\beta_0$ is the attractiveness at $r=0$. The movement of a firefly $i$ is attracted to another more attractive firefly $j$ is determined by:

$$x_{i+1} = x_i + \beta\left(x_j - x_i\right) + \alpha(rand - 0.5) \tag{3}$$

where $x_i$, $x_j$ are the space coordinate of fireflies $i$ and $j$. The second term is associated with the attractiveness. It is worth pointing out that Yang (2010a) has recently replaced this term with Levy distribution and has shown that it can further improve the FA. That is, the step size is a random number drawn from:

$$L(s) = As^{-(1+\lambda)}, \quad A = \lambda\Gamma(\lambda) sin\left(\frac{\lambda\pi}{2}\right)\frac{1}{\pi} \tag{4}$$

where $\Gamma(\lambda)$ is a $\gamma$ function and $\lambda$ is the exponent of the distribution.

The third term is randomization with the parameter $\alpha$ which is step size factor. The rand is a random number generator uniformly distributed in [0, 1]. The most important influence factor of an optimization algorithm's performance is its ability of "exploration" or "exploitation." Exploration means the ability of a search algorithm to explore different areas of the search space so as to obtain high probability to discover good solutions. Exploitation means the ability to focus on the search around a promising area so as to refine a candidate solution. A good optimization algorithm should optimally balance this two conflicted objectives (Cheng, 2013). In this paper we considered the population diversity factor and modify the Equation (3) to improve the FA's search ability.

The basic steps of the FA are summarized as the pseudo code shown in Pseudo code 1:

Pseudo code 1: Pseudo code of the basic FA

**begin**
  *Objective function f(X),* $X = (x_1, \ldots, x_d)^T$
  *Generate initial population of fireflies* $X_i (i = 1, \ldots, n)$
  *Brightness* $I_i$ *at* $X_i$ *is determined by* $f(X_i)$
  *Define light absorption coefficient* $\gamma$
  **while** $(t < MaxGeneration)$
  **for** $i = 1{:}n$ *all n fireflies*
    **for** $j = 1{:}n$ *all n fireflies*
      **if** $(I_j > I_i)$
        *Move firefly i towards j in d-dimension*
      **end if**
      *Attractiveness varies with distance via* $\exp[-\gamma r^2]$
      *Evaluate new solutions and update brightness*
    **end for** *j*
  **end for** *i*
  *Rank the fireflies and find the current best*
  **end while**
  *Post process results and visualization*
**end**

### *2.3 Population diversity*

Population diversity of FA measures the distribution of fireflies, and the diversity's changing rate is a way to monitor the degree of convergence or divergence of FA search process. Therefore population diversity of FA is helpful for measuring and dynamically adjusting its ability of exploration or exploitation accordingly:

*Definition 1.* The d-dimensional component of the population diversity is defined as (Vesterstrøm and Riget, 2002):

$$Diversity(t) = \frac{1}{NL} \sum_{i=1}^{N} \sqrt{\sum_{j=1}^{D} \left( x_{ij}(t) - \overline{x_j(t)} \right)^2} \tag{5}$$

where $N$ is the population size, $L$ is the length of longest the diagonal in the search space, $D$ is the dimensionality of the problem, $x_{ij}(t)$ is the $j$th value of the $i$th firefly and $\overline{x_j(t)}$ is the average of the $j$-dimension over all fireflies. That is:

$$\overline{x_j(t)} = \frac{\sum_{i=1}^{N} x_{ij}(t)}{N} \tag{6}$$

This definition shows that the population diversity is dependent on population size, the dimensionality of the problem and the search range in each dimension.

## 3. Proposed approach

Diversity loss is a serious problem for FA during the search process. Our target is not only to observe the diversity, but to control the diversity, so that the state of the exploration or exploitation could be dynamically adjusted. Since adding random noise may increases population diversity (Cheng, 2013), we use current position in addition to the current average position and add noise in FA to observe its impacts and modify the Equation (3) as follows: where $\overline{x(t)}$ is the average of $t$th iteration location for all fireflies:

$$x_{i+1}(t) = x_i(t) + \beta\left(x_j(t) - x_i(t)\right) + \alpha Rand()\left(x_i(t) - \overline{x(t)}\right) \tag{7}$$

According to the increase or decrease in diversity measure, we define two phases of the search process. In first phase the fireflies will then attract each other as basic FA. When the diversity drops below diversity threshold value, the algorithm switches to another phase we called noisy phase. In this phase the fireflies update the position by Equation (7). Where Rand() is the random function to generate uniformly distributed random number in the range [0, 1] and is different for each dimension and each firefly. When the diversity of population drops below the predefined threshold value, it switches over to this noisy phase, with the hope of increasing the diversity of the population and therefore helping the population to escape the possible local regions. When the position diversity is measured and adjusted, it can be beneficial to dynamically adjust the algorithm's ability of exploration or exploitation.

Contrasted to the basic FA, the proposed FA which updates the position by Equation (7) measures distribution of fireflies' current positions and can reflect fireflies' dynamics. Position diversity gives the current position distribution information of fireflies, whether the fireflies are going to diverge or converge could be reflected from this measurement. Thus useful search information can be obtained. From Equation (5) we can find that different problems, different dimensions and different ranges, the population diversity are different undoubtedly, how to determine the diversity threshold value is still required to be further researched.

In the proposed FA, we try to update the position according to the diversity threshold value. The flowchart of diversity guided FA is shown in Figure 1.

## 4. Experiments and results

In order to make a fair comparison of the basic FA and the proposed FA, we used a test suite of 17 standard benchmark functions and the same settings.

### *4.1 Benchmark functions*

The test suite is composed of a diverse set of problems of different dimensions including unimodal and multimodal functions. The benchmark functions have been listed in Table I.

### *4.2 Settings for the experiments*

The proposed FA and basic FA were tested on above 17 standard benchmark functions. All the programs were run in Matlab 2010b with 2 GB of RAM under Windows XP. To eliminate stochastic unconformity, in each case study, it adopted 100 independent runs for each of the algorithm. As suggested by Yang (2009), the number of fireflies was 30, the dimension was 20 and the

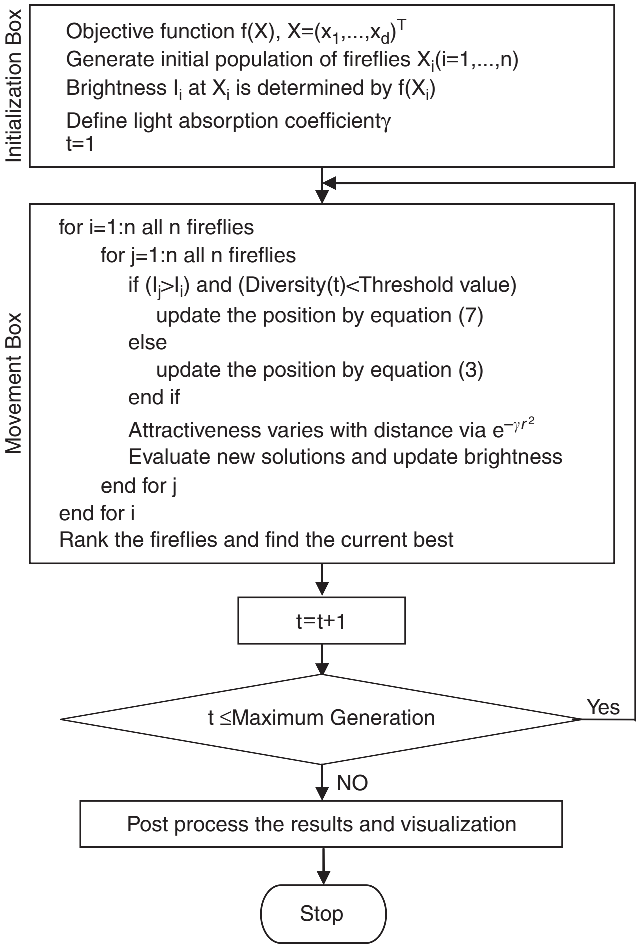


**Figure 1.**
Flowchart of diversity guided firefly algorithm

maximum iteration number was 1,000. Other parameters are set as follows: step factor $\alpha = 0.2$, the light absorption coefficient $\gamma = 1.0$ and the attractiveness $\beta_0 = 1.0$.

*4.3 Experimental results*

*4.3.1 Performance evaluation.* Table II shows the comparison results of best value, worst value and standard deviation on 17 benchmark functions. For simplicity, functions starting from $f_1$ to $f_{15}$ are of Dimensions 2 and $f_{16}$ to $f_{17}$ are 20 dimensions. For $f_1$, $f_3$, $f_8$-$f_{10}$ and $f_{16}$, the proposed FA gave a better result than the basic FA. On $f_2$, $f_4$-$f_7$, $f_{11}$-$f_{13}$, $f_{15}$ and $f_{17}$, the proposed FA and basic FA obtained the similar best value. But the worst value or standard deviation was better than the basic FA got. Only for $f_{14}$ the basic FA outperforms than the proposed FA. On the whole, the proposed FA is much better than the basic FA. Some significant improvements are achieved from this method.

Population diversity measures distribution of fireflies' current positions, therefore, can reflect fireflies' dynamics. Population diversity gives the current position

**Table I.**
Benchmark functions

| Function | Name | Formulation | Range |
|---|---|---|---|
| $f_1$ | Rastrigin's function | $\min f(x) = x_1^2 + x_2^2 - cos(18x_1) - cos(18x_2)$ | $[-1, 1]$ |
| $f_2$ | Six-hump camel back | $\min f(x) = \left(4 - 2.1x_1^2 + \frac{x_1^4}{3}\right)x_1^2 + x_1x_2 + \left(-4 + 4x_2^2\right)x_2^2$ | $[-1, 1]$ |
| $f_3$ | Becher and Lago | $\min f(x) = (\lvert x_1\rvert - 5)^2 + (\lvert x_2\rvert + 5)^2$ | $[-10, 10]$ |
| $f_4$ | Branins's function | $\min f(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$ | $[-5, 5]$ |
| $f_5$ | Modified Rosenbrock | $\min f(x) = 100(x_2 - x_1)^2 + \left((6.4(x_2 - 0.5)^2 - x_1 - 0.6\right)^2$ | $[-5, 5]$ |
| $f_6$ | De Jong's function 2 | $\min f(x) = 100\left(x_2 - x_1^2\right)^2 + (x_1 - 1)^2$ | $[-2.048, 2.048]$ |
| $f_7$ | Schwefel' function | $\min f(x) = (x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$ | $[-10, 10]$ |
| $f_8$ | Goldstein-Price | $\min f(x) = ((1 + (x_1 + x_2 + 1))^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)^2(30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2))$ | $[-2, 2]$ |
| $f_9$ | Schaffer's F6 | $\min f(x) = 0.5 + \frac{\left(\sin\sqrt{x_1^2 + x_2^2}\right)^2 - 0.5}{1 + 0.001\left(x_1^2 + x_2^2\right)}$ | $[-10, 10]$ |
| $f_{10}$ | Rastrigin's function 6 | $\max f(x) = -\left(20 + \left(x_1^2 - 10cos(2\pi x_1)\right)\right) + \left(x_2^2 - 10cos(2\pi x_2)\right)$ | $[-2.048, 2.048]$ |
| $f_{11}$ | Easom 2D | $\max f(x) = \cos(x_1)\cos(x_2)e^{-\left((x_1-\pi)^2 - (x_2-\pi)^2\right)}$ | $[-20, 20]$ |
| $f_{12}$ | Styblinski | $\max f(x) = 280 - \frac{x_1^4 - 16x_1^2 + 5x_1}{2} + \frac{x_2^4 - 16x_2^2 + x_2^2}{2}$ | $[-5, 5]$ |
| $f_{13}$ | Mishra | $\max f(x) = \ln\left\{\left[\left(\sin(\cos x_1 + \cos x_2)^2\right)^2 - \left(\cos(\sin x_1 + \sin x_2)^2\right)^2 + x_1\right]^2\right\} - 0.1\left((x_1 - 1)^2 + (x_2 - 1)^2\right)$ | $[-10, 10]$ |
| $f_{14}$ | Shubert | $\max f(x) = -\left[\sum_{i=1}^{5} i\cos(i+1)x_1 + i\right]\left[\sum_{i=1}^{5} i\cos(i+1)x_2 + i\right]$ | $[-10, 10]$ |
| $f_{15}$ | Himmelblau | $\max f(x) = 660 - \left(x_1^2 - 11\right)^2 - \left(x1 + x_2^2 - 7\right)^2$ | $[-6, 6]$ |
| $f_{16}$ | Ellipsoidal | $\min f(x) = \sum_{i=1}^{n}(x_i - 1)^2$ | $[-2, 2]$ |
| $f_{17}$ | Xin-She Yang's function | $\min f(x) = \sum_{i=1}^{n}\lvert x_i\rvert e^{-\sum_{i=1}^{n} x_i^2}$ | $[-10, 10]$ |

**Table II.**
Results of objective functions in 100 runs

| Function | Best value | | Worst value | | Mean value | | SD | |
|---|---|---|---|---|---|---|---|---|
| | Basic FA | Proposed FA | Basic FA | Proposed FA | Basic FA | Proposed FA | Basic FA | Proposed FA |
| $f_1$ | −1.999999 | *−2.000000* | −1.999330 | −1.878901 | −1.999831 | −1.997578 | 0.000161 | 0.016954 |
| $f_2$ | −1.031628 | *−1.031628* | −0.215442 | *−0.215453* | −1.015298 | *−1.015303* | 0.114264 | 0.114264 |
| $f_3$ | 25.000021 | *25.000014* | 25.002015 | *25.001790* | 25.000584 | *25.000555* | 0.000397 | *0.000386* |
| $f_4$ | 0.397887 | *0.397887* | 0.397901 | *0.397897* | 0.397891 | *0.397889* | 0.000003 | *0.000002* |
| $f_5$ | 0.000000 | *0.000000* | 0.000325 | 0.001987 | 0.000054 | *0.000020* | 0.000055 | 0.000198 |
| $f_6$ | 0.000000 | *0.000000* | 0.000058 | 0.289610 | 0.000010 | 0.004164 | 0.000011 | 0.030551 |
| $f_7$ | 0.000000 | *0.000000* | 0.000019 | *0.000015* | 0.000003 | 0.000003 | 0.000004 | *0.000003* |
| $f_8$ | 3.000001 | *3.000000* | 30.000070 | 30.072638 | 3.270318 | 5.970828 | 2.686441 | 8.450057 |
| $f_9$ | 0.000002 | *0.000001* | 0.033118 | *0.019424* | 0.009203 | *0.007424* | 0.004873 | *0.004363* |
| $f_{10}$ | −0.000002 | *−0.000587* | −1.990340 | *−2.087121* | −0.950829 | −0.925023 | 0.651393 | *0.528977* |
| $f_{11}$ | 1.000000 | *1.000000* | 0.000081 | 0.249257 | 0.774594 | 0.968769 | 0.414540 | *0.110610* |
| $f_{12}$ | 358.332331 | *358.332331* | 348.816361 | *344.195602* | 358.237122 | 357.938302 | 0.946822 | 2.249208 |
| $f_{13}$ | 2.205095 | *2.205095* | 1.658718 | 1.658723 | 2.005913 | 2.036703 | 0.218351 | *0.209574* |
| $f_{14}$ | 186.730823 | 186.730897 | 186.697968 | 186.704466 | 186.726027 | 186.72742 | 0.005369 | *0.004043* |
| $f_{15}$ | 660.000000 | *660.000000* | 659.999896 | *659.999810* | 659.999977 | *659.999976* | 0.000022 | 0.000027 |
| $f_{16}$ | 0.382123 | *0.000025* | 0.777203 | 0.0554770 | 0.568804 | *0.015246* | 0.083603 | *0.012238* |
| $f_{17}$ | 0.000000 | *0.000000* | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

distribution information of fireflies, whether the fireflies are going to diverge or converge could be reflected from this measurement. From diversity measurements, useful search information can be obtained. The result of this is an algorithm that alternates between phases of exploiting and exploring. Position diversity has strong vibration which indicates fireflies are spread into a large space and thus avoids local optimum. This strategy could improve the algorithm's exploration ability and the precision.

*4.3.2 Statistical analysis of trials based on the success ratio*. There are many standards in the literature for evaluating the performance of the algorithms (Akbari and Ziarati, 2011; Taillard, 2003). When two algorithms are compared for a given set of test suite, we can use the success ratio to show whether a solution has better quality than the solution produced by another method for the same problem instance. Therefore, the success ratio is an important measure in optimization problems and it determines the success probability of an algorithm. Here the success ratio is defined as $SR = N_{successful} / N_{all}$, where $N_{successful}$ is the number of trials which found the solution is successful and $N_{all}$ is the number of all trials. In our experiments, $N_{all} = 100$. Figure 2 gives the success ratio of the basic FA and the proposed FA for all the test functions. The results show the significant improvements obtained by the proposed FA. Only for $f_{12}$, the basic FA is slightly better than the proposed FA. So the proposed FA surpasses the basic FA.

*4.3.3 Non-parametric test for analysing the algorithms*. In order to further evaluate the performance of the algorithms, a non-parametric statistical tool, Wilcoxon's test is also conducted. Table III shows the results of applying Wilcoxon's test for functions $f_1$-$f_{17}$. The level of significance considered is $\alpha = 0.05$.

In functions $f_1$, $f_2$, $f_4$-$f_6$, $f_8$-$f_{11}$, $f_{14}$, $f_{16}$, Wilcoxon's test obtain $p$-values smaller than the level of significance $\alpha = 0.05$. In functions $f_3$, $f_7$, $f_{12}$, $f_{13}$, $f_{15}$, Wilcoxon's test obtain $p$-values (0.613, 0.142, 0.936, 0.419, 0.990) greater than the level of significance. For $f_{17}$, the $p$-value is 1.000 because two algorithms obtain the same results, we can see in Table II. According to the suggestion given in literature (García *et al.*, 2009), the smaller the $p$-value, the stronger the evidence against the null hypothesis, and the results in Table III are therefore showing a significant difference in the performance of the proposed FA and the basic FA for most of the functions. For the functions with large $p$-values the test is showing no statistical significant difference in the performance of the algorithms, which agrees with the results displayed for these same functions in Figure 2. Overall it is apparent from the results of the Wilcoxon's tests that the proposed FA has significantly better performance than the basic FA.

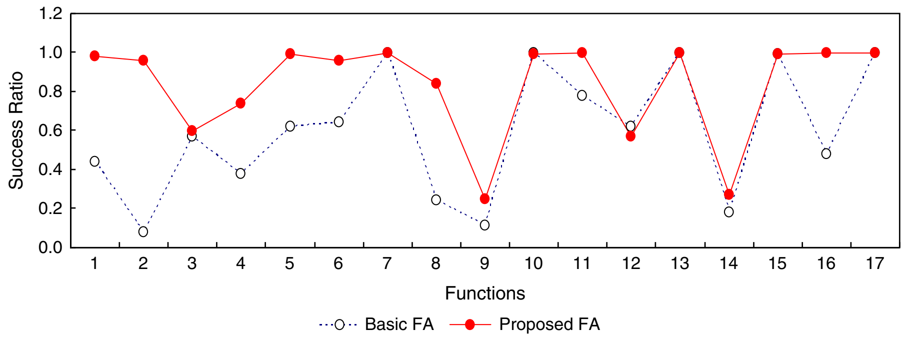


**Figure 2.**
Success ratio from all the test functions

**Table III.**
Wilcoxon's test considering functions $f_1$-$f_{17}$

| | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ |
|---|---|---|---|---|---|---|---|---|---|
| Wilcoxon W | 5,250.000 | 6,112.000 | 9,843.000 | 7,666.000 | 5,199.500 | 5,738.000 | 9,457.000 | 6,486.000 | 9,331.000 |
| Z | −12.486 | −10.543 | −0.506 | −5.919 | −12.670 | −11.315 | −1.468 | −9.025 | −2.345 |
| $p$-value (2-tailed) | 0.000 | 0.000 | 0.613 | 0.000 | 0.000 | 0.000 | 0.142 | 0.000 | 0.019 |
| | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{16}$ | $f_{17}$ | |
| Wilcoxon W | 8,289.500 | 9,089.000 | 10,017.000 | 9,720.000 | 9,123.500 | 10,045.000 | 5,050.000 | 10,050.000 | |
| Z | −4.302 | −2.355 | −0.081 | −0.808 | −2.264 | −0.012 | −12.217 | 0.000 | |
| $p$-value (2-tailed) | 0.000 | 0.019 | 0.936 | 0.419 | 0.024 | 0.990 | 0.000 | 1.000 | |

We also compare the population diversity between the basic FA and our proposed FA in order to provide insight into the diversity of each algorithm. Limited by space, Figures 3-6 measure the difference of the basic FA and the proposed FA with diversity control for $f_1$, $f_3$, $f_4$, and $f_7$.

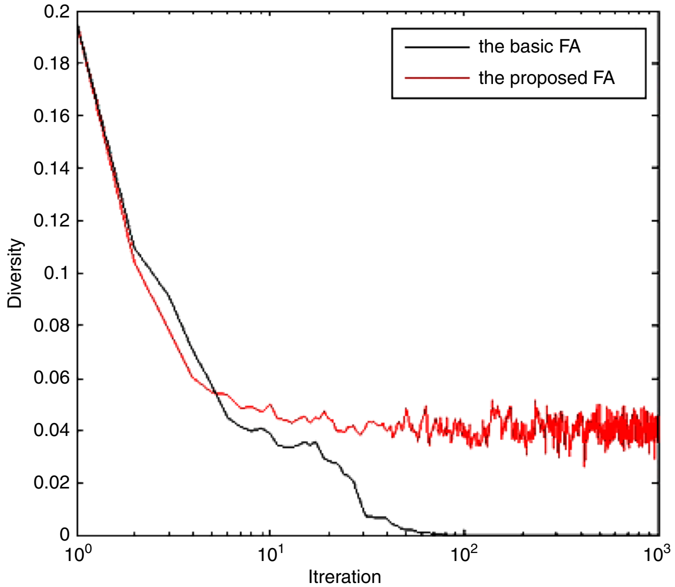


**Figure 3.**
Diversity comparison of $f_1$ between the basic FA and proposed FA

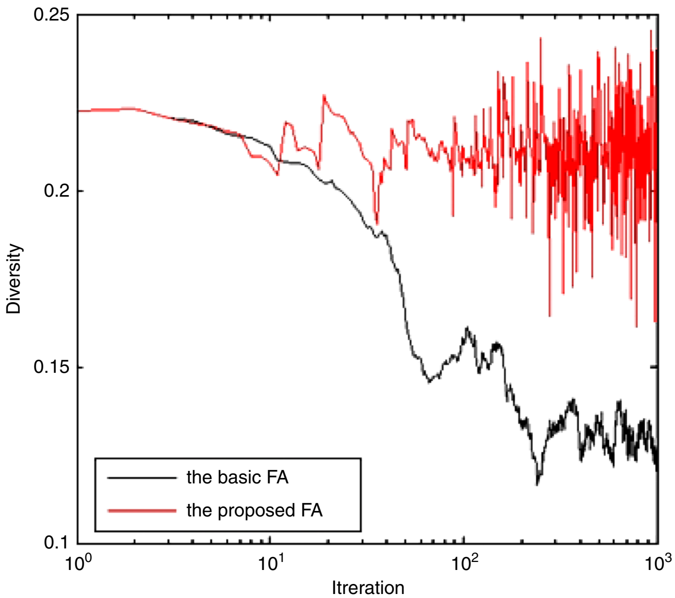


**Figure 4.**
Diversity comparison of $f_3$ between the basic FA and proposed FA

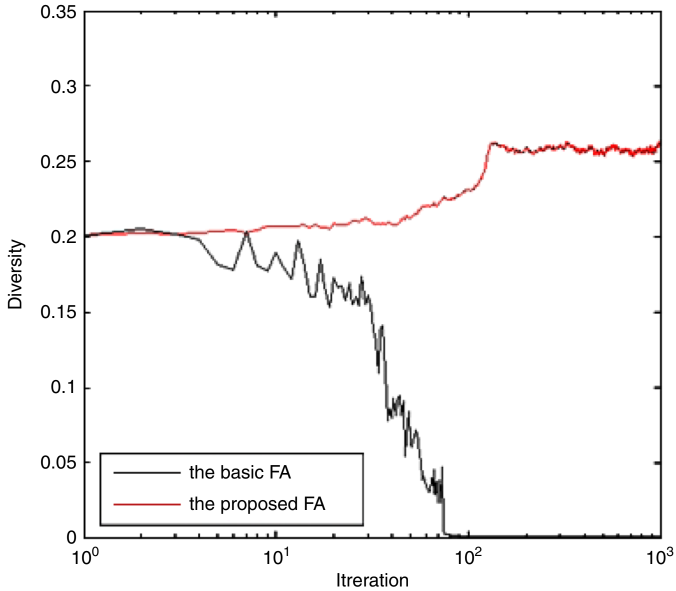


**Figure 5.**
Diversity comparison of $f_4$ between the basic FA and proposed FA

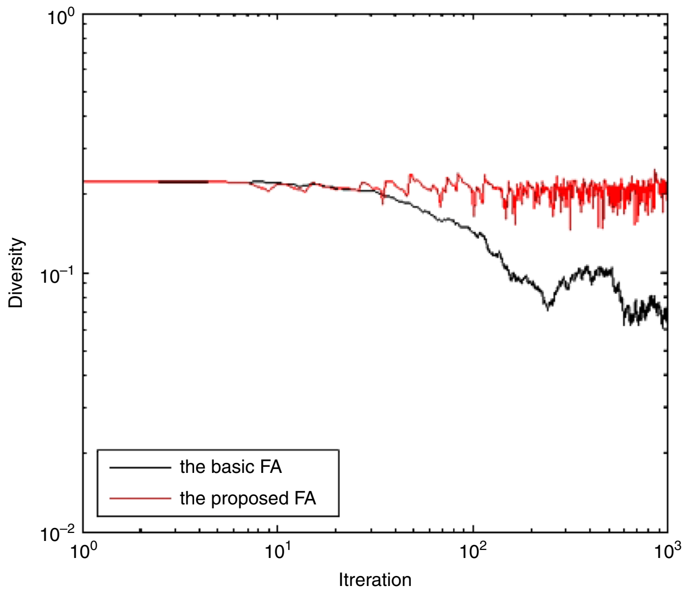


**Figure 6.**
Diversity comparison of $f_7$ between the basic FA and proposed FA

From the Figures 3-6, we can find that the proposed FA shows enormous peaks right. It is apparent that the maintained diversity results in a slower convergence rate as it explores more of the search space. This is a great need of high diversity throughout the search process and it can improve the

FA's search ability. So the proposed method can improve the precision of the algorithm.

## 5. Conclusion

Based on the population diversity, a novel position update equation to modify the diversity during FA search process is presented. Experiments on 17 standard benchmark functions and non-parametric test for analysing the algorithms show that our method can improve accuracy of the basic FA. Certainly, as for different problems during different states of FA searching, the population diversity are different, how to determine the diversity threshold value needs to be further study. Also, how to apply the method in practice such as vehicle routing problem (VRP), and job-shop scheduling problem (JSP) is our future work.

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