1.Introduction :-

1. LoRaWAN is a popular LPWAN technology for IoT sensor and actuator networks due to its energy-efficient long-range communications and resilience to noise.

2. The LoRaWAN MAC protocol has been simplified to keep end devices cheap and easy to implement, but this has resulted in open issues, particularly related to interference management.

3. LoRaWAN operates in the ISM band, which means multiple independent networks using the same frequency range may be in close proximity, leading to interference problems.

4. Efficient power allocation and channel access schemes can help mitigate interference in LoRaWAN networks, but their design requires an accurate model of network operation and the interference environment.

5. The current LoRaWAN MAC protocol lacks sophisticated interference management mechanisms, leading to suboptimal network performance in high-interference scenarios.

6. Ongoing research and development efforts are focused on enhancing the LoRaWAN MAC protocol to improve interference management.

7. Techniques being explored include adaptive data rate selection, interference detection and avoidance, dynamic channel allocation, cooperative communication, and the use of machine learning and AI.

8. Accurate interference modeling is essential for designing effective interference management techniques and evaluating different MAC protocols and optimization algorithms.

9. The goal is to optimize LoRaWAN network performance, reliability, and scalability in the presence of interference, enabling successful IoT deployments in various environments.

Also ,

1. While extensive research has been conducted on LoRa/LoRaWAN, most studies have focused on the physical layer (PHY) aspects, with limited attention given to the medium access control (MAC) layer.

2. Existing research on the LoRaWAN MAC layer has been scarce, with analytical models receiving little attention.

3. Some research has evaluated LoRaWAN network performance through simulations to understand its limitations and behavior in different scenarios.

4. The few existing analytical models for LoRaWAN mostly consider the unacknowledged transmission mode and assume Poisson distribution for total network traffic.

5. These analytical models often employ the M/M/1 queuing model, but they have limitations and do not fully capture the complexities of real-world LoRaWAN networks.

6. Further research is needed to develop more accurate analytical models that account for the specifics of the LoRaWAN MAC protocol and its interaction with the PHY layer.

Here the authors address the need for a more advanced mathematical model to accurately describe the network operation of LoRaWAN, considering acknowledgements (ACKs), retransmissions, propagation losses, and the capture effect.

The previous approach based on ALOHA modeling was found to provide incorrect results for basic LoRaWAN operation with ACKs and retransmissions. Therefore, the authors extend the approach introduced in a previous study to incorporate propagation losses and the capture effect.

The proposed model aims to optimize, design, and evaluate the performance of LoRaWAN MAC layer solutions, particularly related to channel access, power control, and spreading factor control.

The paper is organized as follows:
- Section II explains LoRaWAN channel access and its important features for modeling.
- Section III describes the scenario that is being modeled.
- Section IV presents the mathematical model of LoRaWAN channel access, incorporating the capture effect.
- Section V provides numerical results to demonstrate the accuracy of the proposed model.
- Finally, the conclusion is given in Section VI.

The main objective of the paper is to introduce an advanced mathematical model that accurately represents the network behavior of LoRaWAN, considering various factors such as acknowledgements, retransmissions, propagation losses, and the capture effect. The model can be used to optimize and evaluate different aspects of the LoRaWAN MAC layer, contributing to the improvement of its performance and design.

II. LORAWAN CHANNEL ACCESS DESCRIPTION

1. A typical LoRaWAN network consists of a server, gateways (GWs), and end devices called motes.

2. GWs act as relays between the server and motes, connected via IP network and LoRa links.

3. Class A devices are commonly used in LoRaWAN networks for sporadic uplink data transmission.

4. LoRaWAN networks operate on multiple wireless channels simultaneously, with three main channels and one downlink channel being common in Europe.
5. When a mote wants to transmit a data frame, it randomly selects one of the main channels.
6. The GW receives the data frame and sends two acknowledgements (ACKs) to the mote.

7. The first ACK (ACK1) is sent $T1$ time after frame reception in the same channel.

8. The second ACK (ACK2) is sent after a timeout $T2 = T1 + 1$ second in the downlink channel.

9. If the mote does not receive any ACK, it initiates a retransmission.

10. The recommended retransmission time is randomly chosen from the interval $[1, 1 + W]$ seconds, where $W = 2$.

11. These points summarize the main components, channel operation, data transmission process, and ACKs in a LoRaWAN network with class A devices.

Also ,

1. LoRaWAN uses Chirp Spread Spectrum modulation at the PHY layer.

2. Different spreading factors can be used simultaneously on the same channel, allowing signals to be distinguished and received.

3. The data rate in LoRaWAN is determined by the channel width, coding rate, and spreading factor.

4. Lower data rates provide better reliability over longer distances.
5. Data frames in LoRaWAN are transmitted at a rate determined by the gateway (GW), but the rate allocation algorithm is not specified in the standard.
6. The first ACK is sent at a rate lower than the data rate used for frame transmission, with a configurable offset (which can be set to zero).

7. The second ACK is always sent at a fixed data rate, typically the lowest available rate by default.

8. These points summarize the key aspects of LoRaWAN's PHY layer, including Chirp Spread Spectrum modulation, data rate determination, frame transmission rate, and characteristics of the first and second ACKs.

III. PROBLEM STATEMENT

In a LoRaWAN network with a GW and N motes, operating in F main channels and one downlink channel, the following conditions apply:

- Motes use data rates ranging from 0 to R, which are set by the GW.
- Motes generate frames according to a Poisson process with a total intensity of λ, representing the network load.
- All motes transmit frames with a 51-byte Frame Payload, the maximum size allowed at the lowest data rate.
- Frames are transmitted in the acknowledged mode, and ACKs do not carry any frame payload.
- Motes do not have a queue, meaning that if multiple messages are generated, only the most recent one is transmitted.
- Motes have a retry limit RL and drop frames after RL retransmission attempts.

- A frame is considered successful if, during its duration, its power exceeds the combined power of noise and interfering frames transmitted in the same channel and at the same rate by at least CR dB. CR is the co-channel rejection parameter specified in LoRa chip datasheets.

Given this scenario, the objective is to determine the maximal network load (λ) at which the network can provide reliable communications. The problem involves finding the packet error rate (PER) as a function of the network load λ. The PER represents the probability of a frame transmission attempt being unsuccessful.

In summary, the goal is to establish the maximum network load at which the LoRaWAN network can maintain reliable communications by calculating the packet error rate (PER) as a function of the network load λ.

IV. MATHEMATICAL MODEL

1. To solve the problem, a mathematical model of the transmission process is developed.
2. The first and subsequent transmission attempts are considered separately.
3. For the first transmission attempts, an approach used for evaluating ALOHA networks is extended to account for ACKs.
4. The model takes into account the possibility of multiple frames being transmitted simultaneously, with successful reception based on sufficient power.
5. The model considers the Poisson process for the first transmission attempts, allowing calculation of the packet error rate (PER) in these cases.
6. However, this approach is not applicable to retransmissions, as they do not form a Poisson process.
7. To address retransmissions, a different method is proposed to account for their non-Poisson nature.

8. The proposed mathematical model considers both first transmission attempts and retransmissions, providing a comprehensive analysis of the transmission process.

9. By using this model, it becomes possible to analyze the network's performance and determine the maximum network load at which reliable communications can be achieved.

10. These points summarize the approach taken to develop the mathematical model and its implications for analyzing the transmission process in a LoRaWAN network.

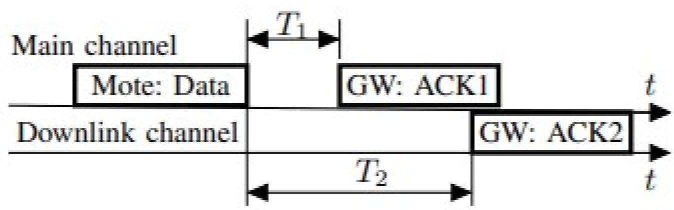

Fig. 1: LoRaWAN channel access

Section IV-C we study a specific case, when the motes are distributed uniformly in a circle around the GW and the path-loss is described by the Okumura-Hata model.

A. The First Transmission Attempt

Let pi be the probability of a mote using data rate i. Its first transmission attempt is successful with probability:

$$P_i^{S,1} = P_i^{Data} P_i^{Ack},$$

The factors affecting the success of a data frame transmission, taking into account frame intersections and ACK collisions are :-

1. Where $P\ Data\ i$ is the probability that the data frame is transmitted without collision at data rate i .

2. P_i^{Ack} is the probability that at least one ACK is received by the mote, provided that the data frame is successful.

3. Since packets transmitted in different channels and at different rates do not collide.

4. We need to consider separately each combination of channel and data rate.

1. The load at data rate i and one of F channels is given by $r_i = \lambda p_i/F$, where λ is the network load and p_i is the probability of a mote using data rate i.

2. The probability of a successful data frame transmission consists of several factors.

3. A data frame is considered successful if it does not intersect with another frame or an ACK from a previous frame.

4. The probability of no frame intersecting with the considered frame in the interval $[-T_i^{Data}, T_i^{Data}]$ **is** e**^(-2**$r_i T_i^{Data}$**)**, assuming a Poisson process for frame generation.

5. If an ACK is being transmitted, collision with an ACK occurs only if the ACK is generated in the interval $[-T_i^{Ack}, 0]$.

6. The rate of ACK generation is $P_i^{Data} r_i$, and the probability of avoiding collision with an ACK is

$$e^{-r_i P_i^{Data} T_i^{Ack}}.$$

1. Secondly, the data frame is also successful, if it intersects with another frames, but the interfering signal is weaker.
2. The probability of k motes transmitting their frames in time intervals that intersect the frame equals

$$e^{-2r_i T_i^{Data}} \left(2r_i T_i^{Data}\right)^k / k!.$$

3. If we sum over all possible k values, we obtain the equation for P_i^{Data} :

$$P_i^{Data} = e^{-(2T_i^{Data} + P_i^{Data} T_i^{Ack}) r_i} + \\ + \sum_{k=1}^{N-1} \frac{\left(2 r_i T_i^{Data}\right)^k}{k!} e^{-2 r_i T_i^{Data}} \mathbb{W}_{i,k}^{GW},$$

where $\mathbb{W}_{i,k}^{GW}$ is the probability of total interfering signal from k motes (in dBm) having less power than the considered mote's signal plus co-channel rejection CR.

In this case, the signal power is measured at the GW.

The probability that at least one ACK arrives is calculated according to the inclusion-exclusion principle:

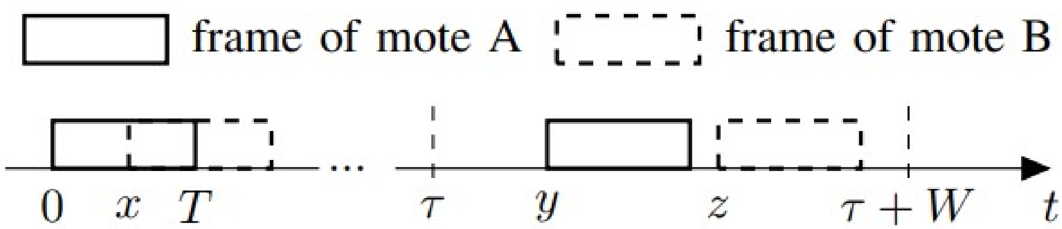

Fig. 2: Retransmission

$P_i^{Ack} = P_i^{Ack1} + P_i^{Ack2} - P_i^{Ack1} P_i^{Ack2}$, where P_i^{Ack1} and P_i^{Ack2} are the probabilities that the first and the second ACK, respectively, is transmitted successfully, provided that data was transmitted at rate i.

The first ACK is delivered if no data frame intersects it or if the interfering signal is weaker:

$$P_i^{Ack1} = e^{-\left(\min\left(T_1, T_i^{Data}\right) + T_i^{Ack}\right) r_i} + \\ + \sum_{k=1}^{N-1} \frac{\left(r_i T_i^{Ack}\right)^k}{k!} e^{-r_i T_i^{Ack}} \mathbb{W}_{i,k}^{Mote}, \qquad (3)$$

1. In the first summand, the duration of the data frame, T *Data* i, is compared with $T1$ (the time at which the first ACK is sent).

2. If the data frame exceeds $T1$, it breaks the acknowledged frame, but this event is already accounted for in P *Data* i. Hence, the minimum of T *Data* i and $T1$ is considered.

3. The second summand accounts for the scenario where k motes start their transmission after the beginning of the ACK. However, their total power is still less than the power of the GW's signal.

4. The probability of this event occurring is denoted as W*Mote* i,k.

5. The signal power is measured at the recipient mote of the ACK, so the distributions of signal power in equations (2) and (3) are different.

6. These points highlight the considerations made for the duration of data frames and the power of interfering signals during ACK transmission.

The second ACK is transmitted without collision if no data frame is successful in any other channel or at any other data rate, such that its second ACK would begin before the considered one and intersect it:

$$P_i^{Ack2} = e^{-T_0^{Ack} \lambda \left(1 - \frac{p_i P_i^{Data}}{F}\right) \sum_{j=0}^{R} p_j P_j^{Data}},$$

where in the exponent we multiply the interval during which the ACK should not start by the total intensity of successful frame process from all channels and all data rates except the one used by the considered mote.