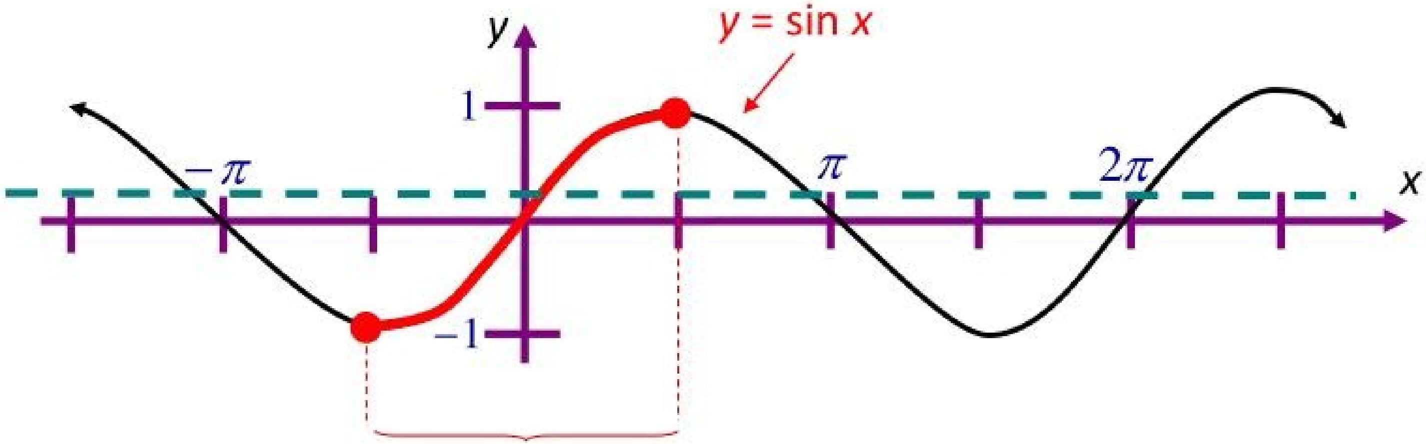
INVERSE TRIGONOMETRIC FUNCTIONS CLASS XII

Inverse Sine Function

Recall that for a function to have an inverse, it must be a one-to-one function and pass the Horizontal Line Test.

$f(x) = \sin x$ does not pass the Horizontal Line Test and must be restricted to find its inverse.



Sin *x* has an inverse function on this interval.

The **inverse sine function** is defined by

$$y = \arcsin x \qquad \text{if and only if} \qquad \sin y = x.$$

Angle whose sine is x

The domain of $y = \arcsin x$ is $[-1, 1]$.

The range of $y = \arcsin x$ is $[-\pi/2\ ,\ \pi/2]$.

Example:

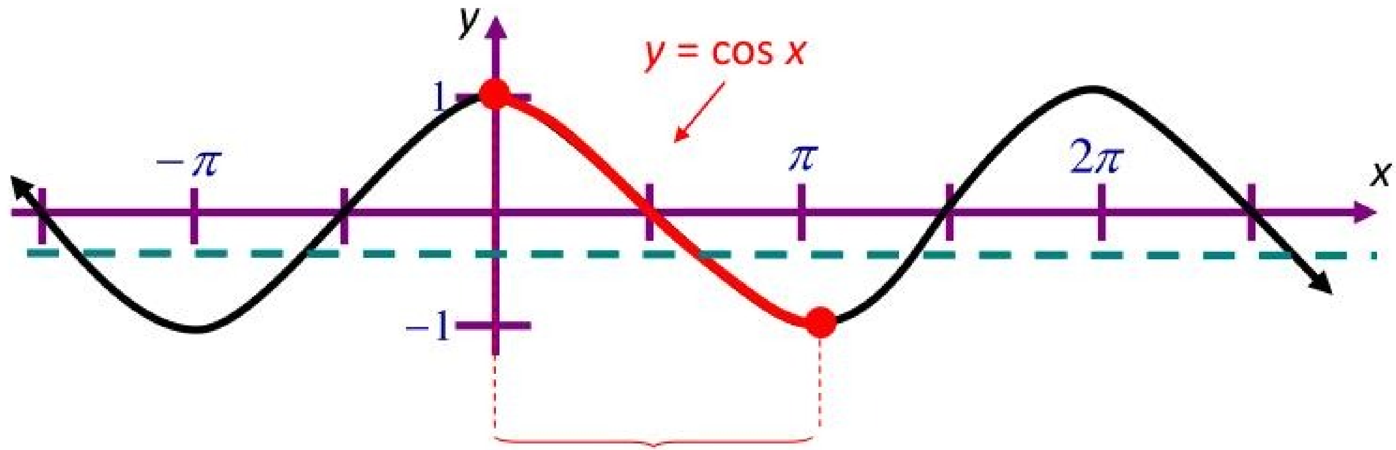
a. $\arcsin \dfrac{1}{2} = \dfrac{\pi}{6}$ $\dfrac{\pi}{6}$ is the angle whose sine is $\dfrac{1}{2}$.

b. $\sin^{-1} \dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$ $\sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

This is another way to write $\arcsin x$.

Inverse Cosine Function

$f(x) = \cos x$ must be restricted to find its inverse.



Cos x has an inverse function on this interval.

The **inverse cosine function** is defined by

$$y = \arccos x \qquad \text{if and only if} \qquad \cos y = x.$$

Angle whose cosine is x

The domain of $y = \arccos x$ is $[-1, 1]$.

The range of $y = \arccos x$ is $[0, \pi]$.

Example:

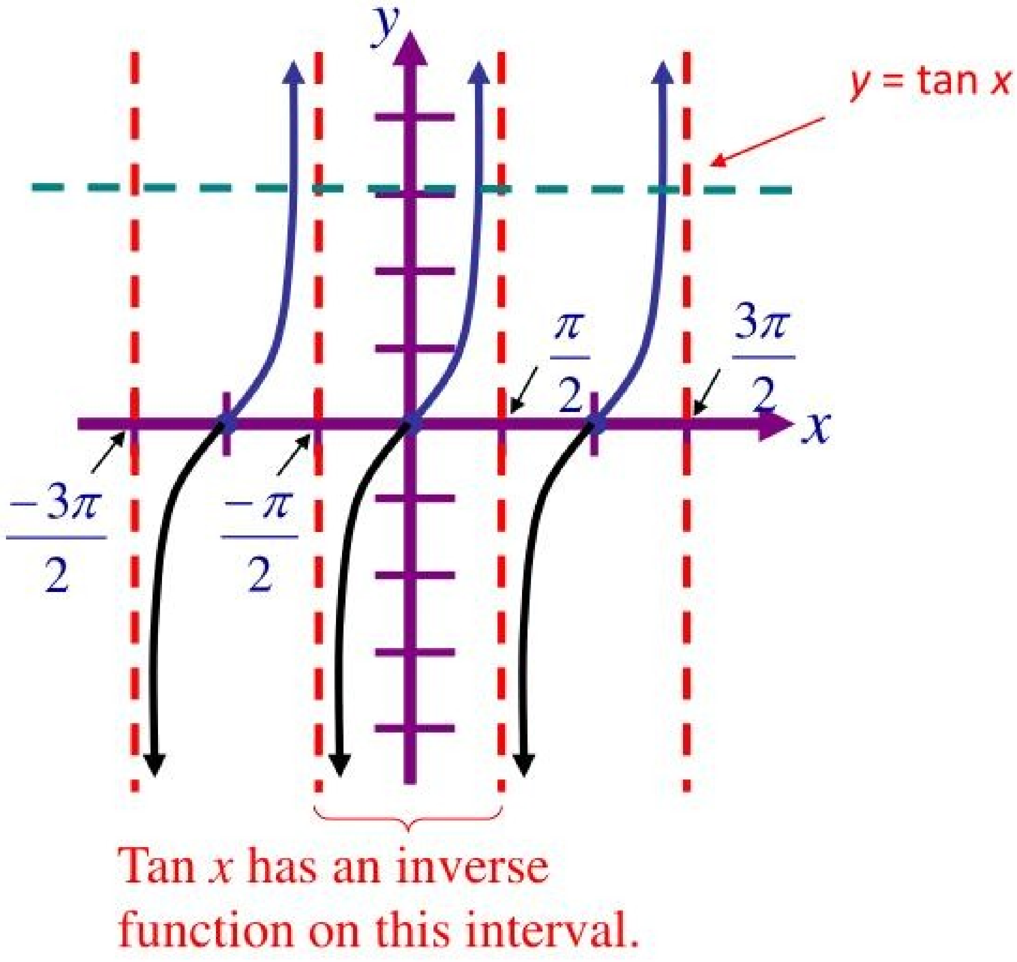
a.) $\arccos \frac{1}{2} = \frac{\pi}{3}$ $\Big\}$ $\frac{\pi}{3}$ is the angle whose cosine is $\frac{1}{2}$.

b.) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ $\Big\}$ $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

This is another way to write $\arccos x$.

Inverse Tangent Function

$f(x) = \tan x$ must be restricted to find its inverse.



The **inverse tangent function** is defined by

$y = \arctan x \qquad$ if and only if $\qquad \tan y = x.$

Angle whose tangent is x

The domain of $y = \arctan x$ is $(-\infty, \infty)$.

The range of y = arctan x is [−π/2 , π/2].

Example:

a.) $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$ $\Big\}$ $\frac{\pi}{6}$ is the angle whose tangent is $\frac{\sqrt{3}}{3}$.

b.) $\tan^{-1}\sqrt{3} = \frac{\pi}{3}$ $\Big\}$ $\tan \frac{\pi}{3} = \sqrt{3}$

This is another way to write arctan x.

Composition of Functions:

$$f(f^{-1}(x)) = x \qquad \text{and} \qquad (f^{-1}(f(x)) = x.$$

Inverse Properties:

If $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$, then

$$\sin(\arcsin x) = x \;\; \text{and} \;\; \arcsin(\sin y) = y.$$

If $-1 \le x \le 1$ and $0 \le y \le \pi$, then

$$\cos(\arccos x) = x \;\; \text{and} \;\; \arccos(\cos y) = y.$$

If x is a real number and $-\pi/2 < y < \pi/2$, then

$$\tan(\arctan x) = x \;\; \text{and} \;\; \arctan(\tan y) = y.$$

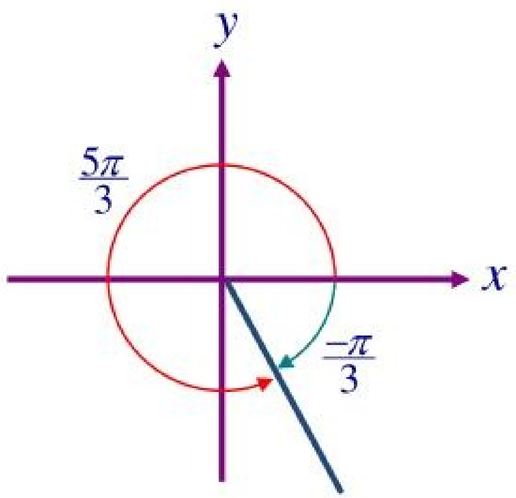
Example: $\tan(\arctan 4) = 4$

Example:

a. $\sin^{-1}(\sin(-\pi/2)) = -\pi/2$

b. $\sin^{-1}\left[\sin\left(\frac{5\pi}{3}\right)\right]$

$\frac{5\pi}{3}$ does not lie in the range of the arcsine function, $-\pi/2 \le y \le \pi/2$.



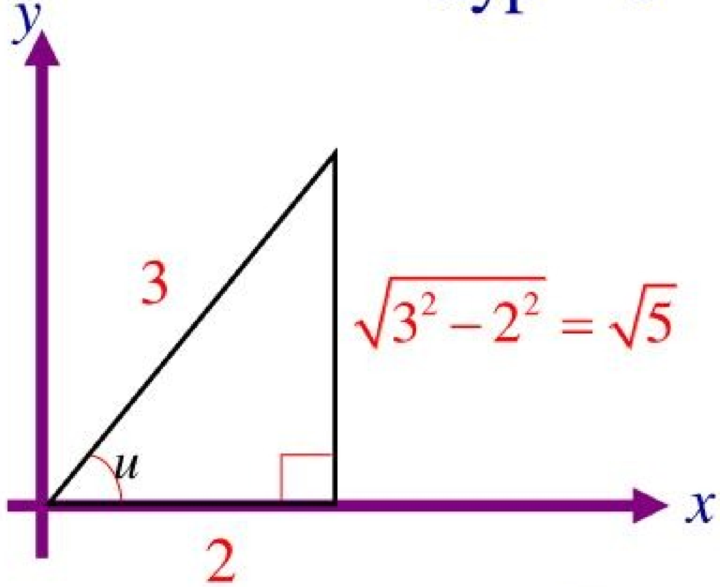
However, it is coterminal with $\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$ which does lie in the range of the arcsine function.

$$\sin^{-1}\left[\sin\left(\frac{5\pi}{3}\right)\right] = \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}$$

Example:

Find the exact value of $\tan\left(\arccos\frac{2}{3}\right)$.

Let $u = \arccos\frac{2}{3}$, then $\cos u = \frac{\text{adj}}{\text{hyp}} = \frac{2}{3}$.



$$\tan\left(\arccos\frac{2}{3}\right) = \tan u = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{5}}{2}$$