$$T(n) = T(i) + T(n-i) + cn$$ $\Rightarrow$ Prob. of any value $= \frac{1}{n}$

$$E[T(i)] = \frac{1}{n} \sum_{j=0}^{n-1} T(j)$$

Since $T(i)$ & $T(n-1)$ are equally likely & take same value

$$E[T(n-1)] = \frac{1}{n} \sum_{j=0}^{n-1} T(j)$$

$$T(n) = \frac{2}{n} \sum_{j=0}^{n-1} T(j) + cn$$

$$n\,T(n) = \frac{2}{n} \sum_{j=0}^{n-1} T(j) + cn^2$$

$$(n-1)\,T(n-1) = 2 \sum_{j=0}^{n-2} T(j) + c(n-1)^2$$

$$nT(n) - (n-1)\,T(n-1) = 2T(n-1) + cn^2 - c(n-1)^2$$

$$nT(n) - 2T(n-1) + 2cn - c$$

$$nT(n) = T(n-1)[2+n-1] + 2cn - c$$

$$nT(n) = T(n-1)[n+1] + 2cn$$

$$\frac{T(n)}{n+1} = \frac{T(n-1)}{n+1} + \frac{2c}{n+1}$$

$n = n-1$

$$\frac{T(n-1)}{n} = \frac{T(n-1)}{n} + \frac{2c}{n+1}$$

$n = n-2$

$$\frac{T(n-2)}{n-1} = \frac{T(n-3)}{n-2} + \frac{2c}{n-1}$$

$$\frac{T(n)}{n+1} = \frac{T(n-2)}{n-1} + \frac{2c}{n} + \frac{2c}{n+1}$$

$$\frac{T(n)}{n+1} = \frac{T(n-2)}{n-2} + \frac{2c}{n-1} + \frac{2c}{n} + \frac{2c}{n+1}$$

$$\frac{T(n)}{n+1} = \frac{T(1)}{2} + 2c\left[\frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1} + \dots\right]$$

$$= \frac{T(1)}{2} + 2c\left[\log n + C_1\right]$$

$$= 2c \log n + C_1$$

$$T(n) = 2c \log n\,[n+1] + C_1\,[n+1]$$

$$T(n) = 2c\, n\log n + 2c\log n + C_1 n + C_1$$

$$T(n) = \underline{n \log n}$$