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CSC 520 FALL 2023

[ASSIGNMENT 3]

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Q1. <Solution>

1) • All lions are animals.

$$\forall x [\text{lion}(x) \rightarrow \text{animal}(x)]$$

• The head of a lion is the head of an animal.

$$\forall h \forall x [\text{head}(h, x) \wedge \text{lion}(x) \rightarrow (\exists y [\text{head}(h, y) \wedge \text{animal}(y)])]$$

2) Logic Statements to CNF.

GIVEN STATEMENTS:

$$\bullet \forall x [\text{lion}(x) \rightarrow \text{animal}(x)]$$

$$\bullet \forall h \forall x [\text{head}(h, x) \wedge \text{lion}(x) \rightarrow$$

$$(\exists y [\text{head}(h, y) \wedge \text{animal}(y)])]$$

STEP 1: Remove implication

$$i) \forall x [\neg \text{lion}(x) \vee \text{animal}(x)]$$

$$ii) \forall h \forall x [\neg (\text{head}(h, x) \wedge \text{lion}(x)) \vee (\exists y [\text{head}(h, y) \wedge \text{animal}(y)])]$$



STEP 2: Apply quantifier distribution
& push negation inside

- i) $\forall x [\neg \text{lion}(x) \vee \text{animal}(x)]$
- ii) $\forall h \forall x [(\neg \text{head}(h, x) \vee \neg \text{lion}(x)) \vee \exists y [\text{head}(h, y) \wedge \text{animal}(y)]]$

STEP 3: SKOLEMIZATION

Adding function $f(h, x)$ to define y

- i) $\forall x [\neg \text{lion}(x) \vee \text{animal}(x)]$
- ii) $\forall h \forall x [(\neg \text{head}(h, x) \vee \neg \text{lion}(x)) \vee \exists y [\text{head}(h, \text{f}(h, x)) \wedge \text{animal}(\text{f}(h, x))]]$

STEP 4: Distribute \vee over \wedge :

- ii) $\forall h \forall x [(\neg \text{head}(h, x) \vee \neg \text{lion}(x) \vee \text{head}(h, \text{f}(h, x))) \wedge (\neg \text{head}(h, x) \vee \neg \text{lion}(x) \vee \text{animal}(\text{f}(h, x)))]$

STEP 5: Apply the Distributive law
to expand \wedge :



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ii) $\forall h \forall x [((\neg \text{head}(h, x) \vee \neg \text{lion}(x) \vee \text{head}(h, f(H, x))) \wedge (\neg \text{head}(h, x) \vee \neg \text{lion}(x) \vee \text{animal}(f(H, x))))]$.

↑ This statement is now in CNF form.

3) FOPL Resolution

Given Statements

i) $\forall x [\neg \text{lion}(x) \vee \text{animal}(x)]$

ii) $\forall h \forall x [((\neg \text{head}(h, x) \vee \neg \text{lion}(x) \vee \text{head}(h, f(H, x))) \wedge (\neg \text{head}(h, x) \vee \neg \text{lion}(x) \vee \text{animal}(f(H, x))))]$.

STEP 1: Resolve clause 2 with itself.

Use substitution of H with

$f(H, x) : \neg \text{head}(f(H, x), x) \vee \neg \text{lion}(x) \vee \text{animal}(f(H, x))$

$[((\neg \text{head}(f(H, x), x) \vee \neg \text{lion}(x) \vee \text{head}(f(H, x), f(f(H, x), x))) \wedge (\neg \text{head}(f(H, x), x) \vee \neg \text{lion}(x) \vee \text{animal}(f(f(H, x), x))))]$



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STEP 2: Substitute 'x' for 'a' for a fresh variable.

Clause 1: $[\neg \text{lion}(a) \vee \text{animal}(a)]$

Clause 2: $[(\neg \text{head}(f(H, a), a) \vee \neg \text{lion}(a) \vee \text{head}(f(H, x), f(f(H, x), a))) \wedge (\neg \text{head}(f(H, a), a) \vee \neg \text{lion}(a) \vee \text{animal}(f(f(H, a), a)))]$.

STEP 3: Resolve the subformulas in ②:

Subformula 1:

$[\neg \text{head}(f(H, a), a) \vee \neg \text{lion}(a) \vee \text{head}(f(H, x), f(f(H, x), a))]$

Subformula 2:

$[\neg \text{head}(f(H, a), a) \vee \neg \text{lion}(a) \vee \text{animal}(f(f(H, a), a))]$

STEP 4: Resolution

$[(\neg \text{head}(f(H, a), a) \vee \neg \text{lion}(a) \vee \text{head}(f(H, x), f(f(H, x), a))) \vee (\neg \text{head}(f(H, a), a) \vee \neg \text{lion}(a) \vee \text{animal}(f(f(H, a), a)))]$



Now,

$$[(\neg \text{lion}(a) \vee \text{head}(f(H, x), f(f(H, x), a))) \\ \vee (\neg \text{lion}(a) \vee \text{animal}(f(f(H, a), a)))]$$

STEP 5: Distribute ' \vee ' operator:

$$[\neg \text{lion}(a) \vee \text{head}(f(H, x), f(f(H, x), a)) \\ \vee \neg \text{lion}(a) \vee \text{animal}(f(f(H, a), a))]$$

Then:

$$[\text{head}(f(H, x), f(f(H, x), a)) \vee \\ \text{animal}(f(f(H, a), a))]$$

Hence the result.

The resulting clause does not have any conflicting literal.

Therefore, second sentence follows the first using FOPL



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$$T(1) \Rightarrow \text{lion}(x) \wedge \neg \text{animal}(x)$$

If we consider resolving $\text{lion}(x)$ with

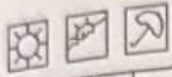
$$T(2) \text{ lion}(x) \quad \neg \text{lion}(x) \vee \neg \text{head}(h(x)) \vee \text{animal}(y)$$

$[\phi] \rightarrow$ query is entailed

$$\neg \text{animal}(x) \neg \text{lion}(x) \vee \neg \text{head}(h(x)) \vee \text{animal}(y)$$

$$\text{unify}(x, y) = [\phi]$$

So, query is satisfied.



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Q3.

Solution: PROBLEM - Predicting chances of getting into an university

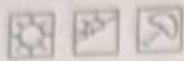
Criteria - to determine chances.

- standardized test scores.
- GPA
- recommendation letters
- other activities

Why?

- Universities can use to screen students (candidates).
- Students are aided to make informed decision.
- Eligibility of scholarships can be determined.





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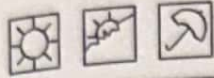
Knowledge Base:

- 1) $(SAT(x) > 1300) \wedge (GPA(x) > 3.5) \rightarrow Admit(x)$
- 2) $Recommendation(x) \wedge Extracur(x) \rightarrow Admit(x)$
- 3) $(GPA(x) < 2.0) \vee Misconduct(x) \rightarrow Reject(x)$
- 4) $SAT(x) > 1500 \rightarrow Fix(x)$
- 5) $\forall x Fix(x) \rightarrow Admit(x)$
- 6) $(SAT(x) < 1000) \wedge (GPA(x) < 2.0) \rightarrow FixReject(x)$
- 7) $\forall x FixReject(x) \rightarrow Reject(x)$
- 8) $\neg admit(x) \vee \neg reject(x) \rightarrow waitlist(x)$

This knowledge base can be used to provide informed decisions.

Sample Query:

$(SAT(x) = 1400) \& (GPA(x) = 3.0) \& (Recommendation(x)) \& \neg Misconduct(x)$



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* BACKWARD CHAINING RESOLUTION STEPS:

1. Resolve Rule 1 with Query:

$$(SAT(x) > 1300) \wedge (GPA(x) > 3.5)$$

$$\rightarrow \text{Admit}(x)$$

The result: Admit.

The student is admitted on basis of rule 1 (and rule 3).

HENCE THE RESULT