2201-MTL106: ASSIGNMENT-3

Q1. An airline knows that 5 percent of the people making reservation on a certain flight will not show up. Consequently, their policy is to sell 55 tickets for a flight that can hold only 53 passengers. Assume that passengers come to airport are independent with each other. What is the probability that there will be a seat available for every passenger who shows up?

Q2. The probability of hitting an aircraft is 0.005 for each shot. Assume that the number of hits when n shots are fired is a random variable having a binomial distribution. How many shots should be fired so that the probability of hitting with two or more shots is above 0.95?

Q3. For any $X \sim \mathcal{B}(n, p)$, show that

$$\mathbb{E}\Big[\frac{1}{1+X}\Big] = \frac{1}{(n+1)p}\big(1-(1-p)^{n+1}\big).$$

Q4. A communications system consists of n components, each of which will, independently, function with probability p. The total system will be able to operate effectively if at least one-half of its components function. For what values of p is a 5-component system more likely to operate effectively than a 3-component system?

Q5. For any $X \sim \mathcal{P}(\lambda)$, prove that

a) $\mathbb{E}[X^n] = \lambda\mathbb{E}[(1+X)^{n-1}], \quad n = 1, 2, 3, \ldots$

b) $\mathbb{P}(X \text{ is even}) = \frac{1}{2}(1+e^{-2\alpha})$ and $\mathbb{P}(X \text{ is odd}) = \frac{1}{2}(1-e^{-2\alpha})$.

Q6. Suppose that the average number of accidents occurring weekly on a particular highway equals 5. Calculate the probability that there is at least two accident this week.

Q7. A manufacturer produces light-bulbs that are packed into boxes of 100. If quality control studies indicate that 0.5% of the light-bulbs produced are defective, what percentage of the boxes will contain: (a) no defective? (b) two or more defectives?

Q8. A large lot of tires contains 5% defectives. 6 tires are to be chosen for a car.

i) Find the probability that you find 3 defective tires before 6 good ones.

ii) Find the mean and variance of the number of defective tires you find before finding 6 good tires.

Q9. For any $X \sim \mathcal{B}_N(r, p)$ and $Y \sim \mathcal{B}(n, p)$, show that $\mathbb{P}(X > n) = \mathbb{P}(Y < r)$.

Q10. A stick of length 1 is split at a point U that is uniformly distributed over $(0, 1)$. Determine the expected length and variance of the piece that contains the point $\frac{1}{3}$.

Q11. A point X is chosen at random in the interval $[-1, 3]$. Find the pdf of $Y = X^2$.

Q12. Suppose X and Y are random variables, where X is the speed and $Y = \frac{180}{X}$ is the duration. If X is uniformly distributed between $[30, 60]$, what is the pdf of Y ?

Q13. Consider the marks of MTL106 examination. Suppose that marks are distributed normally with mean 76 and standard deviation 15. It is observed that 15% of the students obtained A as grade and 10% of the students fail in the course. (a) Find the minimum mark to obtain A as a grade. (b) Find the minimum mark to pass the course.

Q14. A manufacturer produces bolts that are specified to be between 1.19 and 1.2 inches in diameter. If its production process results in a bolt's diameter being normally distributed with mean 1.20 inches and standard deviation .005, what percentage of bolts will not meet specifications?

Q15. Let $f(x)$ denote the probability density function of a normal random variable with mean μ and variance σ^2. Show that $\mu - \sigma$ and $\mu + \sigma$ are points of inflection of this function. That is, show that $f''(x) = 0$ when $x = \mu - \sigma$ or $x = \mu + \sigma$.

Q16. Find the density function of $Y := e^X$ for $X \sim \mathcal{N}(\mu, \sigma^2)$.

Q17. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1$

i) What is the probability that a repair time exceeds 2 hours?
ii) What is the conditional probability that a repair takes at least 3 hours, given that its duration exceeds 2 hours?

Q18. The number of years a radio functions is exponentially distributed with parameter $\lambda = 18$. If Jones buys a used radio, what is the probability that it will be working after an additional 8 years?

Q19. The density function of a continuous random variable T, the time (in operating hours) to failure of a compressor, is given by

$$f(t) = \begin{cases} \frac{0.001}{(0.001t+1)^2}, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

i) Find the failure rate function.
ii) What is its reliability for a 100 hours operating life.
iii) Find the design life if a reliability of 0.95 is desired.
iv) Find the mean time of failure (MTTF).

Q20. Compute the failure rate function of X when $X \sim \mathcal{U}(0, 2)$. Find the failure rate function of $3X$.

Q21. Suppose that the time taken to choose a car in Honda showroom is gamma distributed with mean 2 hours and variance 2 hours^2.

i) What is the probability that it takes at most 4.5 hours?
ii) What is the 90th percentile waiting time.

Q22. Show that if X is a standard Cauchy distributed random variable, then $\frac{1}{X}$ is also a standard Cauchy random variable.

Q23. The probability density function of a beta distributed random variable is given by

$$f(x) = kx^{-\frac{1}{2}}(1-x)^{\frac{1}{2}}, \quad 0 < x < 1.$$

Find its mean and variance.

Q24. Examine whether uniform distribution (continuous case) is a particular case of beta distribution or not.