Answer

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Question: In a locality 'A', the probability of a convective storm event is 0.7 with a density function,

$$f_{X_1}(x_1) = e^{-x_1}, \quad x_1 > 0 \tag{1}$$

The probability of tropical cyclone-induced storm in the same location is given by the density function,

$$f_{X_2}(x_2) = 2e^{-2x_2}, \quad x_2 > 0 \tag{2}$$

The probability of occuring more than 1 unit of storm event is

Solution:

Laplace Transform

Let X be a random variable such that

$$X = X_1 + X_2 \tag{3}$$

Given,

$$p_{X_1}(x) = 0.7e^{-x} \tag{4}$$

$$p_{X_2}(x) = 0.3 \times 2e^{-2x} \tag{5}$$

$$= 0.6e^{-2x} \tag{6}$$

Now,

$$M_X(s) = M_{X_1}(s) \times M_{X_2}(s) \tag{7}$$

$$M_{X_1}(s) = \int_{-\infty}^{\infty} p_{X_1}(x) \times e^{-sx} \tag{8}$$

$$= 0.7 \int_{0}^{\infty} e^{-x} \times e^{-sx} \tag{9}$$

$$= \frac{-0.7}{s+1} \tag{10}$$

$$M_{X_2}(s) = \int_{-\infty}^{\infty} p_{X_2}(x) \times e^{-sx} \tag{11}$$

$$= 0.6 \int_{0}^{\infty} e^{-2x} \times e^{-sx} \tag{12}$$

$$= \frac{-0.6}{s+2} \tag{13}$$

Using (10) and (13) in (7)

$$M_X(s) = \frac{-0.7}{s+1} \times \frac{-0.6}{s+2} \tag{14}$$

$$= \frac{0.42}{(s+1)(s+2)} \tag{15}$$

$$p_X(x) = L^{-1}[M_X(s)] \tag{16}$$

$$= L^{-1}\left[\frac{0.42}{(s+1)(s+2)}\right] \tag{17}$$

$$= 0.42L^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right] \tag{18}$$

$$= 0.42\left(e^{-x} - e^{-2x}\right) \tag{19}$$

Now

$$\Pr(X > 1) = \int_{1}^{\infty} p_X(x)\,dx \tag{20}$$

$$= \int_{1}^{\infty} 0.42\left(e^{-x} - e^{-2x}\right)dx \tag{21}$$

$$= 0.42\left(-e^{-x} + \frac{1}{2}e^{-2x}\right)\Bigg|_{1}^{\infty} \tag{22}$$

$$= 0.42\left(e^{-1} - \frac{1}{2}e^{-2}\right) \tag{23}$$

$$= 0.126 \tag{24}$$