

Assignment

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Question:- The random variable X can take only the values 0, 1, 2. Given that $\Pr(X = 0) = \Pr(X = 1) = p$ and that $E(X^2) = E(X)$, find the value of p .

Solution: Given that X is a random variable such that

$$X = (0, 1, 2) \quad (1)$$

$$\Pr(X = 0) = \Pr(X = 1) = p \quad (2)$$

Then,

$$\Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) = 1 \quad (3)$$

$$\implies p + p + \Pr(X = 2) = 1 \quad (4)$$

$$\implies \Pr(X = 2) = 1 - 2p \quad (5)$$

Expectation is defined as:

$$E(X) = \sum_{k=0}^2 k \Pr(X = k) \quad (6)$$

$$= 0 \Pr(X = 0) + 1 \Pr(X = 1) + 2 \Pr(X = 2) \quad (7)$$

$$= 2 - 3p \quad (8)$$

And

$$E(X^2) = \sum_{k=0}^2 k^2 \Pr(X = k) \quad (9)$$

$$= 0 \Pr(X = 0) + 1 \Pr(X = 1) + 4 \Pr(X = 2) \quad (10)$$

$$= 4 - 7p \quad (11)$$

Given,

$$E(X) = E(X^2) \quad (12)$$

using (8) and (11)

$$\implies 2 - 3p = 4 - 7p \quad (13)$$

$$\implies p = \frac{1}{2} \quad (14)$$