Answer

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Question: Let X be a random sample of size 1 from a population with cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - (1-x)^{\theta} & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1, \end{cases} \tag{1}$$

where $\theta > 0$ is an unknown parameter. To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$, consider using the critical region $(x \in \mathbb{R} : x < 0.5)$. If α and β denote the level and power of the test, respectively, then $\alpha + \beta$ (rounded off to two decimal places) equals

Solution: Given that,

$$H_0 : \theta = \theta_0 = 1 \tag{2}$$
$$H_1 : \theta = \theta_1 = 2 \tag{3}$$

So, the likelihood function is:

$$L = \prod_{i=1}^{1} F(x) \tag{4}$$

Level of test:

$$\alpha = \Pr\left(\text{reject } H_0 | \, H_0 \text{ is true}\right) \tag{5}$$
$$= \Pr\left(x < 0.5 | \, \theta_0\right) \tag{6}$$
$$= F(0.5) \tag{7}$$
$$= 1 - (1 - 0.5) \tag{8}$$
$$= \frac{1}{2} \tag{9}$$

Power of test:

$$\beta = \Pr\left(\text{reject } H_0 | \, H_1 \text{ is true}\right) \tag{10}$$
$$= \Pr\left(x < 0.5 | \, \theta_1\right) \tag{11}$$
$$= F(0.5) \tag{12}$$
$$= 1 - (1 - 0.5)^2 \tag{13}$$
$$= \frac{3}{4} \tag{14}$$

Now,

$$\alpha + \beta = \frac{1}{2} + \frac{3}{4} \tag{15}$$
$$= 1.25 \tag{16}$$