Solution of Q11.16.3.7

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Question:- A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.5 for each tail that turns up.
From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.
Solution: According to the Question:

Variable	Description	Value
n	Number of tosses	4
A	Amount gained/lost	A
p	Profit when it is heads	Re 1
q	Loss when it is tails	Rs 1.5
X	Number of heads in n tosses	X
Y	Number of tails in n tosses	Y

$$X + Y = n \tag{1}$$

The amount of money the person will have after n tosses is:

$$A = (X \times 1) - (Y \times 1.5) \tag{2}$$
$$A = (X \times 1) - ((n - X) \times 1.5) \tag{3}$$
$$A = (2.5X) - (1.5n) \tag{4}$$

For the given question the value of n = 4

$$A = (2.5X) - 6 \tag{5}$$

The probability of getting a profit/loss is:

$$p_X(k) = \binom{4}{k}(0.5)^k(0.5)^{4-k} = \binom{4}{k}(0.5)^4 \tag{6}$$

Let $F_X(k)$ denote the cumulative distribution function of X:

$$F_X(k) = p(X \le k) = \sum_{i=0}^{k} \binom{4}{i}\left(\frac{1}{2}\right)^4 \tag{7}$$

Let $F_A(k)$ denote the cumulative distribution function of A:

$$F_A(k) = p(A \le k) \tag{8}$$
$$= p(2.5X - 6 \le k) \tag{9}$$
$$= p\left(X \le \frac{k+6}{2.5}\right) \tag{10}$$
$$= F_X\left(\frac{k+6}{2.5}\right) \tag{11}$$
$$\tag{12}$$

By (7)

$$= \sum_{i=0}^{\frac{k+6}{2.5}} \binom{4}{i}\left(\frac{1}{2}\right)^4 \tag{13}$$

$$p_A(k) = \begin{cases} \binom{4}{\frac{k+6}{2.5}}\left(\frac{1}{2}\right)^4, & \frac{k+6}{2.5} \in I \text{ and } 0 \le \frac{k+6}{2.5} \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Now, for 4 tosses as given in the question the different amount of money and its probability = $\left\{4, \frac{1}{16}\right\}, \left\{1.5, \frac{1}{4}\right\}, \left\{-1, \frac{3}{8}\right\}, \left\{-3.5, \frac{1}{4}\right\}, \left\{-6, \frac{1}{16}\right\}$