Assignment

Dhruv Parashar-EE22BTECH11019

Question:- Two cards are drawn successively without replacement from a well shuffled deck of cards. Find the mean and standard variation of random variable X where X is the number of aces.

Solution: Let X be a random variable such that

RV	Value	Description
	0	Drawing no ace
	1	Drawing only 1 ace
X	2	Drawing both aces

TABLE I
RANDOM VARIABLE DECLARATION

$$p_X(0) = \frac{48}{52} \times \frac{47}{51} \tag{1}$$

$$= \frac{188}{221} \tag{2}$$

$$p_X(1) = \frac{4}{52} \times \frac{48}{51} + \frac{48}{52} \times \frac{4}{51} \tag{3}$$

$$= \frac{32}{221} \tag{4}$$

$$p_X(2) = \frac{4}{52} \times \frac{3}{51} \tag{5}$$

$$= \frac{1}{221} \tag{6}$$

Now,

$$E(X) = \sum_{k=0}^{2} k p_X(k) \tag{7}$$

$$= 0 p_X(0) + 1 p_X(1) + 2 p_X(2) \tag{8}$$

$$= \frac{34}{221} \tag{9}$$

$$= \frac{2}{13} \tag{10}$$

And

$$E\left(X^2\right) = \sum_{k=0}^{2} k^2 p_X(k) \tag{11}$$

$$= 0 p_X(0) + 1 p_X(1) + 4 p_X(2) \tag{12}$$

$$= \frac{36}{221} \tag{13}$$

Now,

$$Var(X) = E\left(X^2\right) - (E(X))^2 \tag{14}$$

Using (10) and (13)

$$Var(X) = \frac{36}{221} - \left(\frac{2}{13}\right)^2 \tag{15}$$

$$= \frac{400}{2873} \tag{16}$$

$$\implies \sqrt{Var(X)} = \sqrt{\left(\frac{400}{2873}\right)} \tag{17}$$

$$\approx 0.373 \tag{18}$$