Answer

Dhruv Parashar - EE22BTECH11019*

Question: Let X be a random sample of size 1 from a population with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 1 - (1-x)^{\theta} & \text{if } 0 \le x < 1 \\ 1 & \text{if } x \ge 1, \end{cases} \tag{1}$$

where $\theta > 0$ is an unknown parameter. To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$, consider using the critical region $(x \in \mathbb{R} : x < 0.5)$. If α and β denote the level and power of the test, respectively, then $\alpha + \beta$ (rounded off to two decimal places) equals
Solution: Given that,

$$H_0 : \theta = \theta_0 = 1 \tag{2}$$
$$H_1 : \theta = \theta_1 = 2 \tag{3}$$

PDF can be defined as:

$$\Pr(X = x) = \frac{d}{dx} F_X(x) \tag{4}$$
$$= \begin{cases} \theta(1-x)^{\theta-1} & \text{if } 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Level of test:

$$\alpha = \Pr(\text{reject } H_0 |\, H_0 \text{ is true}) \tag{6}$$
$$= \Pr(x < 0.5 |\, \theta_0) \tag{7}$$
$$= F_X(0.5) \tag{8}$$
$$= 1 - (1 - 0.5) \tag{9}$$
$$= \frac{1}{2} \tag{10}$$

Power of test:

$$\beta = \Pr(\text{reject } H_0 |\, H_1 \text{ is true}) \tag{11}$$
$$= \Pr(x < 0.5 |\, \theta_1) \tag{12}$$
$$= F_X(0.5) \tag{13}$$
$$= 1 - (1 - 0.5)^2 \tag{14}$$
$$= \frac{3}{4} \tag{15}$$

Now,

$$\alpha + \beta = \frac{1}{2} + \frac{3}{4} \tag{16}$$
$$= 1.25 \tag{17}$$