

Answer

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Question: Let X be a random sample of size 1 from a population with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - (1 - x)^\theta & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1, \end{cases} \quad (1)$$

where $\theta > 0$ is an unknown parameter. To test $H_0 : \theta = 1$ against $H_1 : \theta = 2$, consider using the critical region $(x \in \mathbb{R} : x < 0.5)$. If α and β denote the level and power of the test, respectively, then $\alpha + \beta$ (rounded off to two decimal places) equals

Solution: Given that,

$$H_0 : \theta = \theta_0 = 1 \quad (2)$$

$$H_1 : \theta = \theta_1 = 2 \quad (3)$$

PDF can be defined as:

$$p_X(x) = \frac{d}{dx} F_X(x) \quad (4)$$

$$= \begin{cases} \theta(1 - x)^{\theta-1} & \text{if } 0 \leq x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Level of test:

$$\alpha = \Pr(\text{reject } H_0 | H_0 \text{ is true}) \quad (6)$$

$$= \Pr(x < 0.5 | \theta_0) \quad (7)$$

$$= F_X(0.5) \quad (8)$$

$$= 1 - (1 - 0.5) \quad (9)$$

$$= \frac{1}{2} \quad (10)$$

Power of test:

$$\beta = \Pr(\text{reject } H_0 | H_1 \text{ is true}) \quad (11)$$

$$= \Pr(x < 0.5 | \theta_1) \quad (12)$$

$$= F_X(0.5) \quad (13)$$

$$= 1 - (1 - 0.5)^2 \quad (14)$$

$$= \frac{3}{4} \quad (15)$$

Now,

$$\alpha + \beta = \frac{1}{2} + \frac{3}{4} \quad (16)$$

$$= 1.25 \quad (17)$$