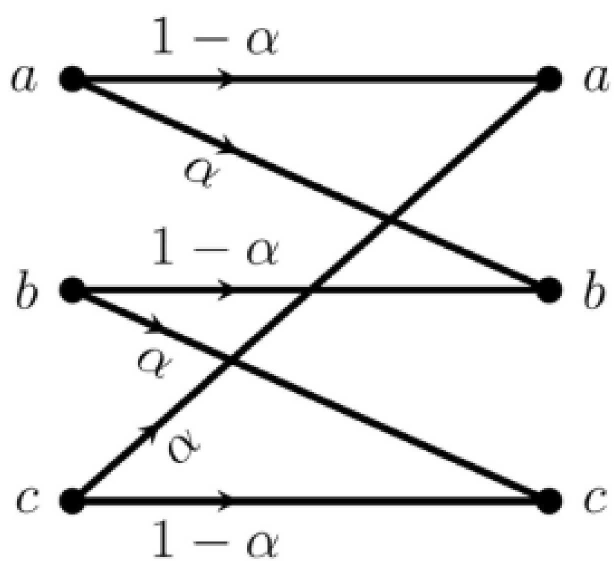
Assignment

Dhruv Parashar-EE22BTECH11019

Question:- The transition diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked. The parameter α lies in the interval $[0.25, 1]$. The value of α for which the capacity of this channel is maximized, is
Solution:



Variable	Description	Value
X	Input variable	$X = \{0, 1, 2\}$
Y	Output variable	$Y = \{0, 1, 2\}$
C	Channel Capacity	C
I	Mutual Information	I
$H(X)$	Entropy	$H(X) = -\sum p_X(k) \log p_X(k)$

$$C = \max_{p(X,Y)} I(X,Y) \tag{1}$$

$$I(X,Y) = H(Y) - H(Y|X) \tag{2}$$

$$= \log_2 3 + \alpha \log_2 \alpha + (1-\alpha)\log_2(1-\alpha) \tag{3}$$

$$\implies \frac{d}{d\alpha} I(X,Y) = (\log_2 \alpha - \log_2(1-\alpha)) \tag{4}$$

For maxima or minima $\frac{d}{d\alpha} I(X,Y) = 0$

$$\log_2 \alpha - \log_2(1-\alpha) = 0 \tag{5}$$

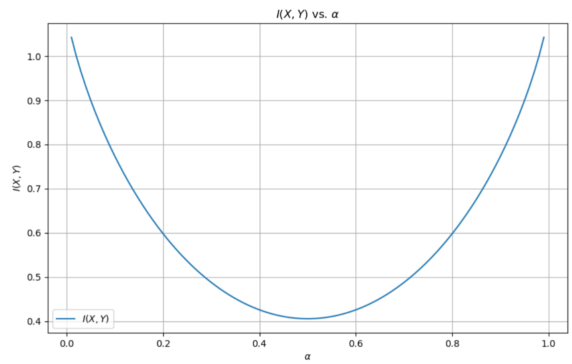
$$\alpha = \frac{1}{2} \tag{6}$$

$$\frac{d^2}{d\alpha^2} I(X,Y) = \frac{1}{\log 2}\left(\frac{1}{\alpha} + \frac{1}{1-\alpha}\right) \tag{7}$$

$$\frac{d^2}{d\alpha^2} I(X,Y)\Big|_{\alpha=\frac{1}{2}} = \frac{4}{\log 2} \tag{8}$$

$$\geq 0 \tag{9}$$

At $\alpha = \frac{1}{2}$, $I(X,Y)$ is minimum



As $I(X,Y)$ is maximum at $\alpha = 0, 1$
But $\alpha \in [0.25, 1]$,

$$\alpha = 1 \tag{10}$$