Solution of 11.16.3.14

Dhruv Parashar-EE22BTECH11019

Question:- If the letters of the word ASSASSINATION are arranged at random. Find the Probability that

(a) Four $S's$ come consecutively in the word
(b) Two $I's$ and two $N's$ come together
(c) All $A's$ are not coming together
(d) No two $A's$ are coming together

Solution: Number of letters in word 'ASSASSINATION' = 13
Letter's are $3A's, 4S's, 2I's, 2N's, 1T's$ and $1O's$

Variable	Description	Value
S	Total ways to arrange letters	S
E_1	$4S's$ together	E_1
E_2	$2I's$ and $2N's$ together	E_2
E_3	All $A's$ together	E_3
E_4	No $2A's$ together	E_4

Total ways to arrange the letters:

$$n(S) = \frac{13!}{3!4!2!2!} \tag{1}$$

(a) Grouping $4S's$ together
Now number of letters is 10 i.e., $(SSSS), N, N, I, I, A, A, A, T, O$

$$n(E_1) = \frac{10!}{3!2!2!} \tag{2}$$

$$P(E_1) = \frac{n(E_1)}{n(S)} \tag{3}$$

$$= \frac{\frac{10!}{3!2!2!}}{\frac{13!}{3!4!2!2!}} \tag{4}$$

$$= \frac{2}{143} \tag{5}$$

(b) $2I's$ and $2N's$ come together
Number of letters is 10 i.e., $(IINN), A, A, A, S, S, S, S, T, O$

$$n(E_2) = \frac{10!}{3!4!} \times \frac{4!}{2!2!} \tag{6}$$

$$P(E_2) = \frac{n(E_2)}{n(S)} \tag{7}$$

$$= \frac{\frac{10!4!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}} \tag{8}$$

$$= \frac{2}{143} \tag{9}$$

(c) Grouping $3A's$ together
Number of letters is 11 i.e., $(AAA), S, S, S, S, I, I, N, N, T, O$

$$n(E_3) = \frac{11!}{4!2!2!} \tag{10}$$

$$P(E_3) = \frac{n(E_3)}{n(S)} \tag{11}$$

$$= \frac{\frac{11!}{4!2!2!}}{\frac{13!}{3!4!2!2!}} \tag{12}$$

$$= \frac{1}{26} \tag{13}$$

$$P(E_3') = 1 - \frac{1}{26} \tag{14}$$

$$= \frac{25}{26} \tag{15}$$

(d) No $2A's$ together
Arranging alphabets except $A's$ i.e., $S, S, S, S, I, I, N, N, T, O$

$$\text{Number of ways} = \frac{10!}{4!2!2!} \tag{16}$$

11 vacant places are present between these alphabets
Three gaps for $3A's$ can be selected in ${}^{11}C_3$

ways

$$n(E_4) = {}^{11}C_3 \times \frac{10!}{4!2!2!} \tag{17}$$

$$P(E_4) = \frac{n(E_4)}{n(S)} \tag{18}$$

$$= \frac{\frac{11!10!}{3!8!4!2!2!}}{\frac{13!}{3!4!2!2!}} \tag{19}$$

$$= \frac{15}{26} \tag{20}$$