

# Answer

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Question: In a locality 'A', the probability of a convective storm event is 0.7 with a density function,

$$f_{X_1}(x_1) = e^{-x_1}, \quad x_1 > 0 \quad (1)$$

The probability of tropical cyclone-induced storm in the same location is given by the density function,

$$f_{X_2}(x_2) = 2e^{-2x_2}, \quad x_2 > 0 \quad (2)$$

The probability of occurring more than 1 unit of storm event is

**Solution:**

**Laplace Transform**

Let  $X$  be a random variable such that

$$X = X_1 + X_2 \quad (3)$$

Given,

$$f_{X_1}(x) = e^{-x} \quad (4)$$

$$f_{X_2}(x) = 2e^{-2x} \quad (5)$$

Now,

$$M_X(s) = M_{X_1}(s) \times M_{X_2}(s) \quad (6)$$

$$M_{X_1}(s) = \int_{-\infty}^{\infty} f_{X_1}(x) \times e^{-sx} \quad (7)$$

$$= \int_0^{\infty} e^{-x} \times e^{-sx} \quad (8)$$

$$= \frac{-1}{s+1} \quad (9)$$

$$M_{X_2}(s) = \int_{-\infty}^{\infty} f_{X_2}(x) \times e^{-sx} \quad (10)$$

$$= \int_0^{\infty} e^{-2x} \times e^{-sx} \quad (11)$$

$$= \frac{-1}{s+2} \quad (12)$$

Using (9) and (12) in (6)

$$M_X(s) = \frac{-1}{s+1} \times \frac{-1}{s+2} \quad (13)$$

$$= \frac{1}{(s+1)(s+2)} \quad (14)$$

$$p_X(x) = L^{-1}[M_X(s)] \quad (15)$$

$$= L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] \quad (16)$$

$$= L^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right] \quad (17)$$

Region of Convergence(ROC) of laplace transform:

$$Re(s) > -1 \cap Re(s) > -2 \quad (18)$$

$$\implies Re(s) > -1 \quad (19)$$

So, now

$$p_X(x) = (e^{-x} - e^{-2x})u(x) \quad (20)$$

Where,  $u(x)$  is unit step function.

CDF of X:

$$F_X(x) = \int_x^{\infty} p_X(x) dx \quad (21)$$

$$= \int_x^{\infty} (e^{-x} - e^{-2x})u(x) dx \quad (22)$$

As  $x > 0$ ,

$$F_X(x) = \left(-e^{-x} + \frac{1}{2}e^{-2x}\right)\Big|_x^{\infty} \quad (23)$$

$$= \left(e^{-x} - \frac{1}{2}e^{-2x}\right) \quad (24)$$

Now,

$$\Pr(X > 1) = F_X(1) \quad (25)$$

$$= \left(e^{-1} - \frac{1}{2}e^{-2}\right) \quad (26)$$

$$= 0.30 \quad (27)$$

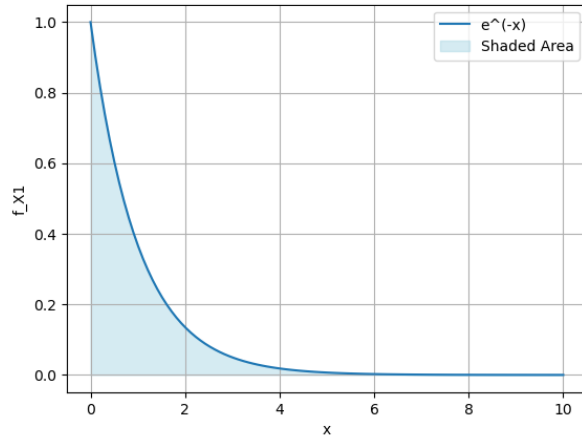


Fig. 0. Density Function of  $X1$

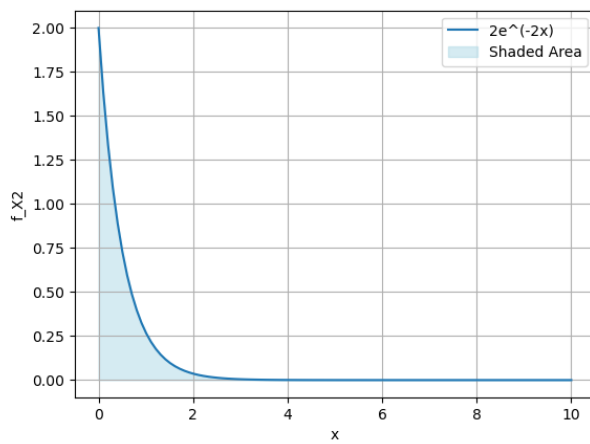


Fig. 0. Density Function of  $X2$