Solution of 11.16.3.14

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Question:- If the letters of the word ASSASSI-NATION are arranged at random. Find the Probability that

(a) Four $S's$ come consecutively in the word
(b) Two $I's$ and two $N's$ come together
(c) All $A's$ are not coming together
(d) No two $A's$ are coming together

Solution: Number of letters in word 'ASSASSINATION' = 13
Letter's are $3A's, 4S's, 2I's, 2N's, 1T's$ and $1O's$

$$\text{Total ways of arranging letters} = \frac{13!}{3!4!2!2!} \quad (1)$$

(a) Let X be a random variable such that

$$X = \begin{cases} 1, & \text{If All } S's \text{ are together as a unit} \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

Then,

$$p_X(1) = \frac{\frac{10!}{3!2!2!}}{\frac{13!}{3!4!2!2!}} \quad (3)$$

$$= \frac{2}{143} \quad (4)$$

(b) Let Y be a random variable such that

$$Y = \begin{cases} 1, & 2I's \text{ and } 2N's \text{ together} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

Then,

$$p_Y(1) = \frac{\frac{10!4!}{3!4!2!2!}}{\frac{13!}{3!4!2!2!}} \quad (6)$$

$$= \frac{2}{143} \quad (7)$$

(c) Let Z be a random variable suct that

$$Z = \begin{cases} 0, & \text{All } A's \text{ together} \\ 1, & \text{Only } 2A's \text{ together} \\ 2, & \text{No } 2A's \text{ are together} \end{cases} \quad (8)$$

Then,

$$p_Z(0) = \frac{\frac{11!}{4!2!2!}}{\frac{13!}{3!4!2!2!}} \quad (9)$$

$$= \frac{1}{26} \quad (10)$$

$$\implies p\prime_Z(0) = 1 - \frac{1}{26} \quad (11)$$

$$= \frac{25}{26} \quad (12)$$

(d)

$$p_Z(2) = \frac{\frac{11!10!}{3!8!4!2!2!}}{\frac{13!}{3!4!2!2!}} \quad (13)$$

$$= \frac{15}{26} \quad (14)$$