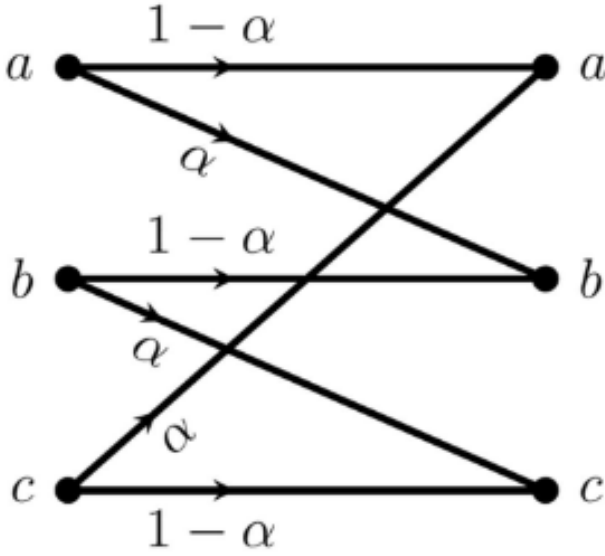# Assignment

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Question:- The transition diagram of a discrete memoryless channel with three input symbols and three output symbols is shown in the figure. The transition probabilities are as marked. The parameter α lies in the interval $[0.25, 1]$. The value of α for which the capacity of this channel is maximized, is

Solution:

Now,

$$\sum_{x=0}^{2} p_X(x) = 1 \tag{7}$$

$$\sum_{y=0}^{2} p_Y(y) = 1 \tag{8}$$

$$H(Y) = -\sum_{y=0}^{2} p_Y(y) \log_2 p_Y(y) \tag{9}$$

$$H(Y|X) = -\sum_{x=0}^{2}\sum_{y=0}^{2} p_X(x)\, p_{Y|X}(y|x) \log_2 \left(p_{Y|X}(y|x)\right) \tag{10}$$

$$= -\sum_{x=0}^{2} p_X(x) \sum_{y=0}^{2} p_Y(y|x) \log_2\left(p_Y(y|x)\right) \tag{11}$$

$$= -\sum_{y=0}^{2} p_Y(y|x) \log_2\left(p_Y(y|x)\right) \tag{12}$$

Using (9) and (12) in (6)

$$I(X,Y) = -\sum_{y=0}^{2} p_Y(y) \log_2 p_Y(y) + \sum_{y=0}^{2} p_Y(y|x) \log_2\left(p_Y(y|x)\right) \tag{13}$$

Variable	Description	Value
$p_X(x)$	Input probability	$p_X(0), p_X(1), p_X(2)$
$p_Y(y)$	Output probability	$p_Y(0), p_Y(1), p_Y(2)$
C	Channel Capacity	C
I	Mutual Information	I
H	Entropy	H

$$C = \max_{p(X,Y)} I(X,Y) \tag{1}$$

$$I(X,Y) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)\,p(y)} \tag{2}$$

$$= \sum_{x,y} p(x,y) \log_2 \frac{p(y|x)}{p(y)} \tag{3}$$

$$= -\sum_{x,y} p(x,y) \log_2 p(y) + \sum_{x,y} p(x,y) \log_2 p(y|x) \tag{4}$$

$$= -\sum_{y} p(y) \log_2 p(y) - \left(-\sum_{x,y} p(x,y) \log_2 p(y|x)\right) \tag{5}$$

$$= H(Y) - H(Y|X) \tag{6}$$