Answer

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Question: In a locality 'A', the probability of a convective storm event is 0.7 with a density function,

$$f_{X_1}(x_1) = e^{-x_1}, \quad x_1 > 0 \tag{1}$$

The probability of tropical cyclone-induced storm in the same location is given by the density function,

$$f_{X_2}(x_2) = 2e^{-2x_2}, \quad x_2 > 0 \tag{2}$$

The probability of occuring more than 1 unit of storm event is

Solution:

Laplace Transform

Let X be a random variable such that

$$X = X_1 + X_2 \tag{3}$$

Given,

$$f_{X_1}(x) = e^{-x}u(x) \tag{4}$$
$$f_{X_2}(x) = 2e^{-2x}u(x) \tag{5}$$

Where, $u(x)$ is unit step function.

Now,

$$M_X(s) = M_{X_1}(s) \cdot M_{X_2}(s) \tag{6}$$
$$M_{X_1}(s) = \int_{-\infty}^{\infty} f_{X_1}(x) \cdot e^{-sx} dx \tag{7}$$
$$= \int_{0}^{\infty} e^{-x} \cdot e^{-sx} u(x)\, dx \tag{8}$$
$$= \frac{1}{s+1} \tag{9}$$

Region of Convergence(ROC) of $M_{X_1}(s)$:

$$Re(s) > -1 \tag{10}$$

Now,

$$M_{X_2}(s) = \int_{-\infty}^{\infty} f_{X_2}(x) \cdot e^{-sx} dx \tag{11}$$
$$= \int_{0}^{\infty} 2e^{-2x} \cdot e^{-sx} u(x)\, dx \tag{12}$$
$$= \frac{2}{s+2} \tag{13}$$

ROC of $M_{X_2}(s)$:

$$Re(s) > -2 \tag{14}$$

Using (9) and (13) in (6)

$$M_X(s) = \frac{1}{s+1} \times \frac{2}{s+2} \tag{15}$$
$$= \frac{2}{(s+1)(s+2)} \tag{16}$$

$$p_X(x) = L^{-1}\left[M_X(s)\right] \tag{17}$$
$$= L^{-1}\left[\frac{2}{(s+1)(s+2)}\right] \tag{18}$$
$$= 2L^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right] \tag{19}$$

ROC of laplace transform:

$$Re(s) > -1 \cap Re(s) > -2 \tag{20}$$
$$\implies Re(s) > -1 \tag{21}$$

So, now

$$p_X(x) = 2\left(e^{-x} - e^{-2x}\right)u(x) \tag{22}$$

CDF of X:

$$F_X(x) = \int_{-\infty}^{x} p_X(x)\, dx \tag{23}$$
$$= 2\int_{-\infty}^{x} \left(e^{-x} - e^{-2x}\right)u(x)\, dx \tag{24}$$

But $p_X(x)$ is integrable for $x > 0$

$$F_X(x) = 2\int_{0}^{x} \left(e^{-x} - e^{-2x}\right) dx \tag{25}$$
$$= 2\left(-e^{-x} + \frac{1}{2}e^{-2x}\right)\Bigg|_0^x \tag{26}$$
$$= 2\left(-e^{-x} + \frac{1}{2}e^{-2x} + \frac{1}{2}\right) \tag{27}$$
$$= -2e^{-x} + e^{-2x} + 1 \tag{28}$$

Now,

$$\Pr(X > 1) = F_X(1) \tag{29}$$
$$= -2e^{-1} + e^{-2} + 1 \tag{30}$$
$$= 0.39 \tag{31}$$

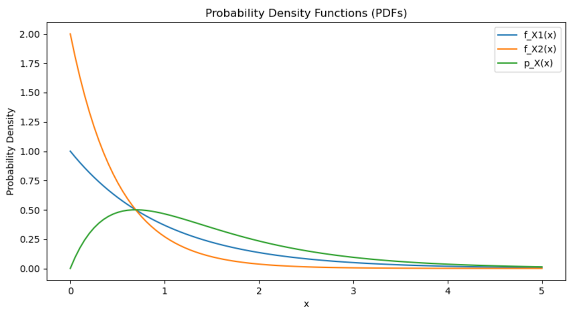

Fig. 0. Probability Density Functions

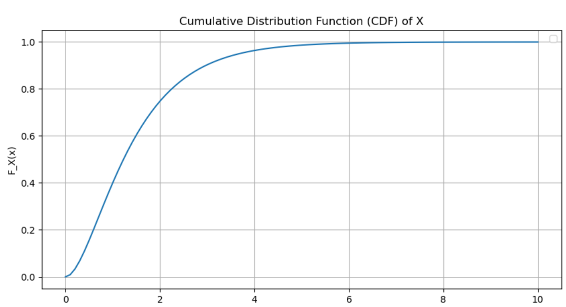

Fig. 0. CDF of X