Assignment

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Question:- A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

(a) $\left(\frac{9}{10}\right)^5$

(b) $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(c) $\frac{1}{2}\left(\frac{9}{10}\right)^5$

(d) $\frac{1}{2}\left(\frac{9}{10}\right)^4 + \left(\frac{9}{10}\right)^5$

Solution: Let X be a random variable such that

Variable	Description	Value
X	Defective pens drawn in sample of 5	$\{0, 1, 2, 3, 4, 5\}$
p	Probability of drawing defective pens	$\frac{1}{10}$

$$p_X(k) = {}^5C_k \left(\frac{1}{10}\right)^k \left(\frac{9}{10}\right)^{5-k} \tag{1}$$

$$p_X(k \le 1) = p_X(0) + p_X(1) \tag{2}$$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 \tag{3}$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4 \tag{4}$$