Answer

Dhruv Parashar - EE22BTECH11019*

Question: In a locality 'A', the probability of a convective storm event is 0.7 with a density function,

$$f_{X_1}(x_1) = e^{-x_1}, \quad x_1 > 0 \tag{1}$$

The probability of tropical cyclone-induced storm in the same location is given by the density function,

$$f_{X_2}(x_2) = 2e^{-2x_2}, \quad x_2 > 0 \tag{2}$$

The probability of occuring more than 1 unit of storm event is

Solution:

Laplace Transform

Let X be a random variable such that

$$X = X_1 + X_2 \tag{3}$$

Given,

$$f_{X_1}(x) = e^{-x} \tag{4}$$
$$f_{X_2}(x) = 2e^{-2x} \tag{5}$$

Now,

$$M_X(s) = M_{X_1}(s) \times M_{X_2}(s) \tag{6}$$
$$M_{X_1}(s) = \int_{-\infty}^{\infty} f_{X_1}(x) \times e^{-sx} \tag{7}$$
$$= \int_{0}^{\infty} e^{-x} \times e^{-sx} \tag{8}$$
$$= \frac{-1}{s+1} \tag{9}$$

Region of Convergence(ROC) of $M_{X_1}(s)$:

$$Re(s) > -1 \tag{10}$$

Now,

$$M_{X_2}(s) = \int_{-\infty}^{\infty} f_{X_2}(x) \times e^{-sx} \tag{11}$$
$$= \int_{0}^{\infty} e^{-2x} \times e^{-sx} \tag{12}$$
$$= \frac{-1}{s+2} \tag{13}$$

ROC of $M_{X_2}(s)$:

$$Re(s) > -2 \tag{14}$$

Using (9) and (13) in (6)

$$M_X(s) = \frac{-1}{s+1} \times \frac{-1}{s+2} \tag{15}$$
$$= \frac{1}{(s+1)(s+2)} \tag{16}$$

$$p_X(x) = L^{-1}\left[M_X(s)\right] \tag{17}$$
$$= L^{-1}\left[\frac{1}{(s+1)(s+2)}\right] \tag{18}$$
$$= L^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right] \tag{19}$$

ROC of laplace transform:

$$Re(s) > -1 \cap Re(s) > -2 \tag{20}$$
$$\implies Re(s) > -1 \tag{21}$$

So, now

$$p_X(x) = \left(e^{-x} - e^{-2x}\right) u(x) \tag{22}$$

Where, $u(x)$ is unit step function.

CDF of X:

$$F_X(x) = \int_{x}^{\infty} p_X(x)\, dx \tag{23}$$
$$= \int_{x}^{\infty} \left(e^{-x} - e^{-2x}\right) u(x)\, dx \tag{24}$$

As $x > 0$,

$$F_X(x) = \left(-e^{-x} + \frac{1}{2}e^{-2x}\right)\Big|_{x}^{\infty} \tag{25}$$
$$= \left(e^{-x} - \frac{1}{2}e^{-2x}\right) \tag{26}$$

Now,

$$\Pr(X > 1) = F_X(1) \tag{27}$$
$$= \left(e^{-1} - \frac{1}{2}e^{-2}\right) \tag{28}$$
$$= 0.30 \tag{29}$$

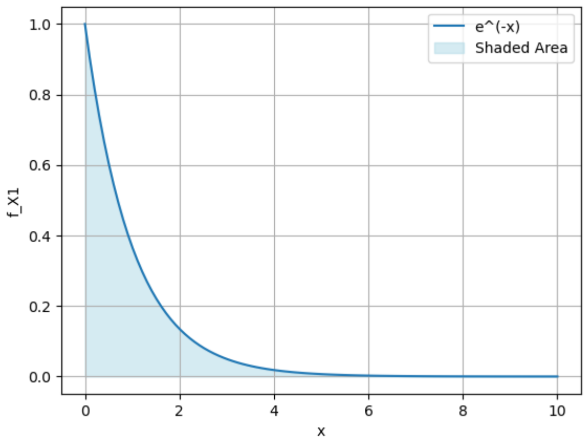

Fig. 0. Density Function of X1

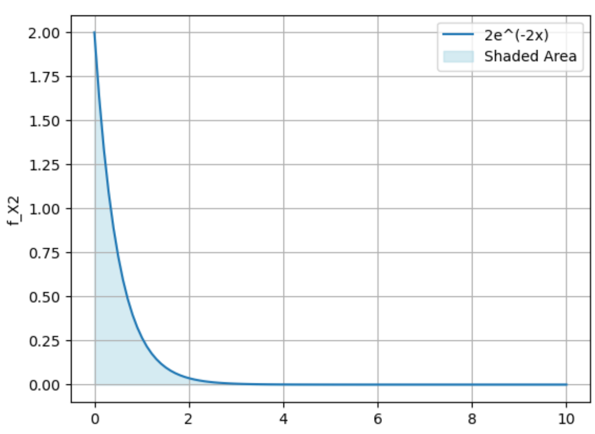

Fig. 0. Density Function of X2