

Assignment

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Question:- A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

- (a) $\left(\frac{9}{10}\right)^5$
 (b) $\frac{1}{2} \left(\frac{9}{10}\right)^4$
 (c) $\frac{1}{2} \left(\frac{9}{10}\right)^5$
 (d) $\frac{1}{2} \left(\frac{9}{10}\right)^4 + \left(\frac{9}{10}\right)^5$

Solution: Let X be a random variable such that

RV	Value	Description
X	0	No defective pen drawn
	1	Only 1 defective pen drawn
	2	Otherwise

TABLE I

RANDOM VARIABLE DECLARATION

$$p_X(k) = {}^5C_k \left(\frac{1}{10}\right)^k \left(\frac{9}{10}\right)^{5-k} \quad (1)$$

$$p_X(k \leq 1) = p_X(0) + p_X(1) \quad (2)$$

$$= {}^5C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^5 + {}^5C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^4 \quad (3)$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2} \left(\frac{9}{10}\right)^4 \quad (4)$$