

Solution of 1.2.2

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Question:- We are given the three vertices of a triangle (A, B, C) and the midpoints $\mathbf{D}, \mathbf{E}, \mathbf{F}$ of the sides AB, BC, AC respectively. We have to find the equations of the sides AD, BE, CF .

Solution:

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad (2)$$

$$\mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (3)$$

The mid points $\mathbf{D}, \mathbf{E}, \mathbf{F}$ of sides AB, BC, AC are :-

$$\mathbf{D} = \frac{1}{2} \begin{pmatrix} -7 \\ 1 \end{pmatrix} \quad (4)$$

$$\mathbf{E} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (5)$$

$$\mathbf{F} = \frac{1}{2} \begin{pmatrix} -3 \\ 5 \end{pmatrix} \quad (6)$$

Now, the slope vector of line $FC(\mathbf{m})$ is :-

$$\mathbf{m} = \mathbf{F} - \mathbf{C} \quad (7)$$

$$\Rightarrow \mathbf{m} = \frac{1}{2} \begin{pmatrix} 3 \\ 15 \end{pmatrix} \quad (8)$$

Now, we have to find \mathbf{n} ,

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \quad (9)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 15 \end{pmatrix} \quad (10)$$

$$= \frac{1}{2} \begin{pmatrix} 15 \\ -3 \end{pmatrix} \quad (11)$$

Normal form of line CF is :

$$\mathbf{n}^T (\mathbf{x} - \mathbf{C}) = 0 \quad (12)$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{C} \quad (13)$$

$$\frac{1}{2} (15 \ -3) \mathbf{x} = \frac{1}{2} (15 \ -3) \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (14)$$

$$(15 \ -3) \mathbf{x} = (15 \ -3) \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (15)$$

$$(15 \ -3) \mathbf{x} = -30 \quad (16)$$

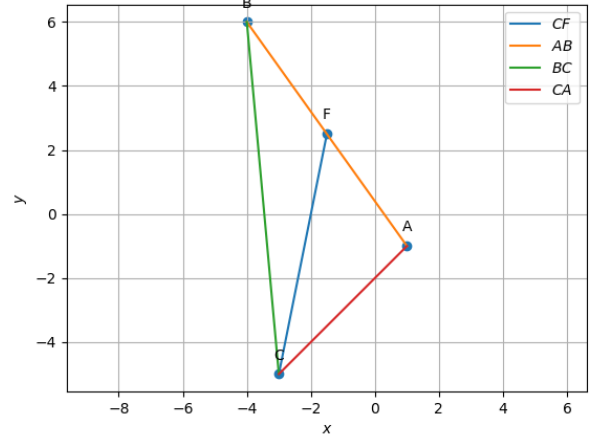


Fig. 0. Line CF