

Answer

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Question: In a locality 'A', the probability of a convective storm event is 0.7 with a density function,

$$f_{X_1}(x_1) = e^{-x_1}, \quad x_1 > 0 \quad (1)$$

The probability of tropical cyclone-induced storm in the same location is given by the density function,

$$f_{X_2}(x_2) = 2e^{-2x_2}, \quad x_2 > 0 \quad (2) \quad \text{Now}$$

The probability of occurring more than 1 unit of storm event is

Solution:

Laplace Transform

Let X be a random variable such that

$$X = X_1 + X_2 \quad (3)$$

Given,

$$p_{X_1}(x) = 0.7e^{-x} \quad (4)$$

$$\begin{aligned} p_{X_2}(x) &= 0.3 \times 2e^{-2x} \\ &= 0.6e^{-2x} \end{aligned} \quad (5) \quad (6)$$

Now,

$$M_X(s) = M_{X_1}(s) \times M_{X_2}(s) \quad (7)$$

$$M_{X_1}(s) = \int_{-\infty}^{\infty} p_{X_1}(x) \times e^{-sx} \quad (8)$$

$$= 0.7 \int_0^{\infty} e^{-x} \times e^{-sx} \quad (9)$$

$$= \frac{-0.7}{s+1} \quad (10)$$

$$M_{X_2}(s) = \int_{-\infty}^{\infty} p_{X_2}(x) \times e^{-sx} \quad (11)$$

$$= 0.6 \int_0^{\infty} e^{-2x} \times e^{-sx} \quad (12)$$

$$= \frac{-0.6}{s+2} \quad (13)$$

Using (11) and (13) in (8)

$$M_X(x) = \frac{-0.7}{s+1} \times \frac{-0.6}{s+2} \quad (14)$$

$$= \frac{0.42}{(s+1)(s+2)} \quad (15)$$

$$p_X(x) = L^{-1}[M_X(x)] \quad (16)$$

$$= L^{-1}\left[\frac{0.42}{(s+1)(s+2)}\right] \quad (17)$$

$$= 0.42L^{-1}\left[\frac{1}{s+1} - \frac{1}{s+2}\right] \quad (18)$$

$$= 0.42(e^{-x} - e^{-2x}) \quad (19)$$

$$\Pr(X > 1) = \int_1^{\infty} p_X(x) dx \quad (20)$$

$$= \int_1^{\infty} 0.42(e^{-x} - e^{-2x}) dx \quad (21)$$

$$= 0.42\left(-e^{-x} + \frac{1}{2}e^{-2x}\right)\Bigg|_1^{\infty} \quad (22)$$

$$= 0.42\left(e^{-1} - \frac{1}{2}e^{-2}\right) \quad (23)$$

$$= 0.126 \quad (24)$$