

Assignment

Dhruv Parashar-EE22BTECH11019

Question:- Let A and B be two events such that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A + B) = \frac{3}{4}$. Then $P(A|B) \cdot P(A'|B)$ is equal to

(a) $\frac{2}{5}$

(b) $\frac{3}{8}$

(c) $\frac{3}{20}$

(d) $\frac{6}{25}$

Solution: Given

$$P(A) = \frac{3}{8} \quad (1)$$

$$P(B) = \frac{5}{8} \quad (2)$$

$$P(A + B) = \frac{3}{4} \quad (3)$$

As we know

$$P(A + B) = P(A) + P(B) - P(AB) \quad (4)$$

$$\implies P(AB) = P(A) + P(B) - P(A + B) \quad (5)$$

$$\implies P(AB) = \frac{1}{4} \quad (6)$$

Now,

$$P(A|B) = \frac{P(AB)}{P(B)} \quad (7)$$

$$P(A'|B) = \frac{P(A'B)}{P(B)} \quad (8)$$

$$= \frac{P(B) - P(AB)}{P(B)} \quad (9)$$

From (7) and (9)

$$P(A|B) \cdot P(A'|B) = \frac{P(AB)}{P(B)} \times \frac{P(B) - P(AB)}{P(B)} \quad (10)$$

$$= \frac{\left(\frac{1}{4}\right)}{\frac{5}{8}} \times \frac{\left(\frac{5}{8} - \frac{1}{4}\right)}{\frac{5}{8}} \quad (11)$$

$$= \frac{6}{25} \quad (12)$$