Module - 3

Mathematical Induction

* It is a technique or method to prove mathematical statement or formula or theorem which is thought to be true for each and every natural number 'n' by generalizing this in the form of the principle which we would be use to prove any mathematical statement is principle of mathematical induction.

Principle of Mathematical Induction

Any statement $P(n)$ which is for 'n' natural number can be proved using the principle of mathematical induction by following steps :

1) Verify if statements is true for ~~in~~trivial cases $(n=1)$ i.e., check if $P(1)$ is true.

2) Assume that the statement is true for $n=k$ for some $k \geq 1$ i.e., $P(k)$ is true.

3) If the truth of $P(k)$ implies the truth of $P(k+1)$ then the statement $P(n)$ is true for all $n \geq 1$

Problems

1) P.T sum of the first 'n' integers for all integers $n \geq 1$. $1+2+3+ \cdots +n = \frac{n(n+1)}{2}$

Sol: Let

$$P(n) = 1+2+3+ \cdots +n = \frac{n(n+1)}{2}$$

put $n=1$

$P(1) \Rightarrow 1 = \frac{1(1+1)}{2} \Rightarrow 1 = 1$

$\therefore P(1)$ is true.

ii) Assume the result is true for $n = k$

$P(k) = 1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}.$

$\therefore P(k)$ is true.

iii) We must S.T $P(k+1)$ is true

$n = k+1, \quad 1 + 2 + 3 + \cdots + k+1 = \frac{k+1\,(k+2)}{2}.$

By using the step ②

$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}$

Both side adding $k+1$.

$$k + (k+1) = \frac{k\cancel{+1}(k+1)}{2} + \frac{(k+1)}{1}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{k^2 + k + 2k + 2}{2}$$

$$= \frac{k^2 + 3k + 2}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$LHS \Rightarrow \frac{(k+1)(k+1+1)}{2} \Rightarrow \frac{(k+1)(k+2)}{2}$$

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

$LHS = RHS$

$\therefore P(k+1)$ is true.

By using principle of mathematical induction //

2) S.T $1+3+5+ \cdots +(2n-1) = n^2$

Sol:- Let

$$p(n) = 1+3+5+ \cdots +(2n-1) = n^2$$

Put $n=1$

$$(2(n)-1) = n^2$$

$$(2(1)-1) = (1)^2$$

$$1 = 1$$

LHS = RHS

$\therefore$ p(1) is true.

Assume the result is true for $n=k$

$$p(k) = 1+3+5+ \cdots +(2k-1) = k^2$$

$\therefore$ P(k) is true.

we must S.T P(k+1) is true.

put $n=k+1$.

$$1+3+5+ \cdots + (2(k+1)-1) = (k+1)^2$$

$$1+3+5+ \cdots + (2k+1) = (k+1)^2$$

By using step-2

$$1+3+5+ \cdots +(2k+1) = (k+1)^2$$

Both sides add $(2k-1+k^2)$

$$1+3+5+ \cdots + k^2 + (2k+1)(2k-1) = 1 = k+1)^2$$

$$k^2+2k+2-1 = (k+1)^2$$

$$k^2+2k+1 = (k+1)^2$$

$$(k+1)^2 = (k+1)^2$$

LHS = RHS

$\therefore$ P(k+1) is true

3) $1^2+2^2+3^2+ --- + n^2 = \frac{n(n+1)(2n+1)}{6}$

Sol:- Let

$$p(n) = 1^2+2^2+3^2+ --- + n^2 = \frac{n(n+1)(2n+1)}{6}$$

~~P(n)~~ put $n=1$, $(1)^2 = \frac{1(1+1)(2(1)+1)}{6}$

$$(1)^2 = \frac{1(2)(3)}{6}$$

$$1 = 1$$

$$LHS = RHS$$

$\therefore$ $p(1)$ is true.

Assume $n = k$.

$$1^2+2^2+3^2+ --- + n^2 = \frac{n(n+1)(2n+1)}{6}$$

put $n = k+1$

$$1^2+2^2+3^2+ --- + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$(k+1)^2 = \frac{k+1(k+2)(2k+3)}{6}$$

by using step-2

$$1^2+2^2+3^2+ --- + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Both sides adding $(k+1)^2$

$$k^2+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} + \frac{(k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1)(\cancel{k+1}) + 6(k+1))}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)[2k^2+k+6k+6]}{6}$$

$= (k+1)(2k^2+7k+6)$

$= k+1\,(2k^2+4k+3k+6)$

$= k+1\,(2k(k+2)+3(k+2))$

$= (k+1)\,((k+2)(2k+3))$

2) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

sol:- Let

$P(n) = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Let $n=1$

$\frac{1}{n(n+1)} = \frac{n}{n+1} \Rightarrow \frac{1}{1(1+1)} = \frac{1}{1+1}$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$

$P(1)$ is true

2) Assume the result $n=k$.

$P(k) = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

$P(k)$ is true.

put $n = k+1$

$\frac{1}{n(n+1)} = \frac{n}{n+1} \Rightarrow \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$

By using step-2,

$\frac{1}{k(k+1)} = \frac{k}{k+1}$

Add $\frac{1}{(k+1)(k+2)}$ we get

$\frac{1}{(k+1)(k+2)} + \frac{1}{k(k+1)} = \frac{1}{(k+1)(k+2)} + \frac{k}{k+1}$

$$= \frac{1 + k(k+2)}{(k+1)(k+2)}$$

$$= \frac{1 + k^2 + 2k}{(k+1)(k+2)}$$

$$= \frac{(k+1)^{\cancel{2}}}{(\cancel{k+1})(k+2)}$$

$$= \frac{k+1}{k+2}$$

p(k+1) is true by mathematical Induction

Q) $n(n+1) = \frac{n(n+1)(n+2)}{3}$

Sol:- ① put $n=1$

$$1(1+1) = \frac{1(1+1)(1+2)}{3} \Rightarrow 2 = \frac{6}{3}$$

$$\Rightarrow 2 = 2$$

LHS = RHS

P(1) is true

② Assume $n=k$.

put $n=k$.

$$n(n+1) = \frac{n(n+1)(n+2)}{3} \Rightarrow k(k+1) = \frac{k(k+1)(k+2)}{3}$$

$\therefore$ P(k) is true.

③ put $n = k+1$

$$n(n+1) = \frac{n(n+1)(n+2)}{3} \Rightarrow (k+1)(k+2) = \frac{k+1(k+2)(k+3)}{3}$$

by step ②

$$k(k+1) = \frac{k(k+1)(k+2)}{3}$$

Add $(k+1)(k+2)$ on both sides

$$k(k+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + \frac{(k+1)(k+2)}{1}$$

Q)

Sol

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k+2)(k+3)}{3} \quad // \text{LHS}$$

$\therefore$ LHS = RHS

$\therefore P(k+1)$ is true.

Q) $1.2 + 3.4 + 5.6 + \cdots + (2n-1)2n = \frac{n(2n+1)(4n-1)}{3}$

sol:- ① Put $n = 1$

$$(2n-1)2n = \frac{n(n+1)(4n-1)}{3}$$

$$(2(1)-1)(2(1)) = \frac{1(1+1)(4(1)-1)}{3} \Rightarrow 2 = \frac{6}{3}$$

$$\Rightarrow 2 = 2$$

LHS = RHS

$\therefore P(1)$ is true

② Assume $n = k$

$$(2n-1)2n = \frac{n(n+1)(4n-1)}{3}$$

$$(2k-1)2k = \frac{k(k+1)(4k-1)}{3}$$

$\therefore P(k)$ is true

③ put $n = k+1$.

$$1.2 + 3.4 + 5.6 + \cdots + (2(k+1)-1)(2(k+1)) = \frac{k+1(k+1+1)(4(k+1)-1)}{3}$$

$$(2k+1)(2k+2) = \frac{k+1(k+2)(4k+3)}{3}$$

By using step-2.

$$(2k-1)(2k) = \frac{k(k+1)(4k-1)}{3}$$

Add $(2k+1)(2k+2)$ on both sides, we get

$$(2k+1)(2k+2) + (2k-1)(2k)\cancel{+2} = \frac{k(k+1)(4k-1)}{3} + \frac{(2k+1)(2k+2)}{1}$$

$$= \frac{k(k+1)(4k-1) + 3[(2k+1)(2k+2)]}{3}$$

$$= \frac{k(k+1)(4k-1) + 3[(2k+1)(2(k+1))]}{3}$$

$$= \frac{(k+1)[k(4k-1) + 3(2k+1)(2)]}{3}$$

$$= \frac{k+1[4k^2 - k + 3(4k+2)]}{3}$$

$$= \frac{k+1(4k^2 - k + 12k + 6)}{3}$$

$$= \frac{k+1(4k^2 + 11k + 6)}{3}$$

$$= \frac{k+1(4k^2 + 8k + 3k + 6)}{3}$$

$$= \frac{k+1(4k(k+2) + 3(k+2))}{3}$$

$$= \frac{(k+1)((k+2)(4k+3))}{3}$$

$$= \frac{(k+1)(k+2)(4k+3)}{3} //$$

$\therefore$ LHS = RHS

P(k+1) is true.

$\therefore$ By Mathematical Induction given problem is true.

Basics of Counting :- Two principles of counting from a foundation for most counting techniques, the first involves addition and the second one involves multiplication.

* There are two decompositions rules
  1) Sum Rule Principle
  2) Product Rule Principle

1) **Sum Rule Principle :-** It states that if some event A can occur in "m-ways" and second event B can occurs in "n-ways" and suppose both events can not occur simultaneously. The A or B occur in "m+n" ways.

⇒ In General, the above principle can be extended to three or more events. That is suppose on event A_1 can occur in n_1 ways. a second event A_2 can occur in n_2 ways and following A_2 ; a that event A_3 can occur in n_3 ways and so on

⇒ If no two events can occur at the same time, then one of the event can occur in.

$$n_1 + n_2 + n_3 + \text{------} \text{ ways}$$

⇒ If A & B are two events with $A \cap B = \phi$ then

$$|A+B| = |A| + |B|$$

⇒ In general $|A+B+C \text{--}| = |A| + |B| + |c| + \text{----}$

2) **Product Rule Principle :-** It states that if there is an event - A which can occur in 'm' ways and independent of this event, there is a second event - B which can occur in 'n' ways. then combinations of A & B can occur in 'm x n' ways.

⇒ The above principle can be generalized to any number of events.

$$n_1, n_2, n_3 \text{ ---- Ways}$$

* If A and B are any two finite sets then

$|A \times B| = |A| \cdot |B|$

Cartesian product $A \times B = \{(a,b),\ a \in A,\ b \in B\}$

* In general, when $A_1, A_2 \dots A_n$ are n finite sets then $|A_1 \times A_2 \times \dots A_n| = |A_1|\,|A_2| \dots |A_n|$

<u>examples</u>

1) How many ways can be draw a club (or) a diamod from pack of cards.

<u>Sol:-</u> ~~Th~~ They are 13 clubs

They are 13 diamonds

the no.of ways a club (or) a diamond may be drawn.

$13 + 13 = 26$

2) How many ways can be drawn an Ace or a king from an ordinary deck of playing cards.

<u>Sol:</u> There are 4 Ace & 4 kings

the no.of ways An ace & a king may be drawn.

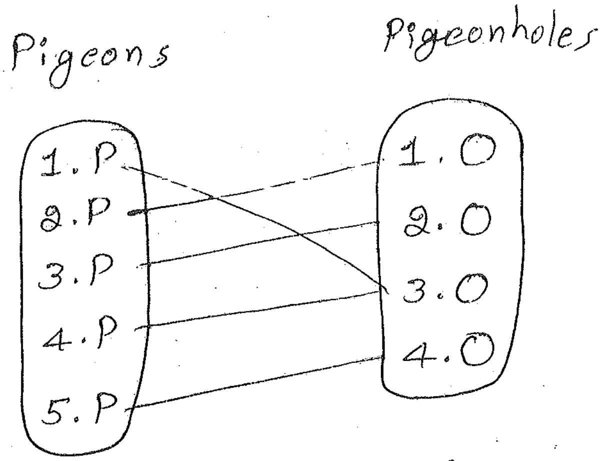
$4 + 4 = 8$ ways.

3) How many different car Licence plates can be made if each plate contains a sequence of 3 upper case english letters and followed by 3- digits.

<u>Sol:-</u> $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17,576,000$

<u>Pigeonhole Principle</u>

→ The pigeonhole principle states that if n-pegions fly into m-pigenholes and $n > m$ then atleat one pegionhole must contain two or more pigeons, this principle is illustrated in the above figure for $n=5$ & $m=4$ the below figure shows that the correspondense from. pigeons to pigeonholes. It is sometimes called Birichlet Box.



Pigeonhole Principle and Proof

Sta :- If n-pigeons are assigned to m-pigeonholes and $m < n$ then some pigeonholes contains atleast 2-pigeons.

Proof :- Let $h_1, h_2, --- h_m$ denotes the m-pigeonholes and $P_1, P_2, P_3, --- P_m, P_{n+1}, --- P_n$ denotes the n-pigeons. (where $m < n$) we consider the assignment of the n-pigeons to m-pigeonholes as follows $n > m$.

Assign. pigeon P_1 to the pigeonhole h_1,

pigeon P_2 to the pigeonhole h_2,

⋮

pigeon P_m to the pigeonhole h_m.

This assign. as many pigeons possible to individual. pigeonholes.

$\because m < n$ there are P_i, Pigeons [which is n-m pigeon] that've not yet been assigned atleast one pigeonhole will be assigned second pigeon.

* A function from one finite set to a smaller finite set can not be one-to-one there must be a least 2-elements in the domain that has the same image. that've the same image in the co-domain, has an arrow diagram for a function from a finite set to a smaller finite set must have atleast 2 arrows from the domain that point to the same elements of the co-domain. In the below figure P1 & P4 both points to a pigeonhole-3.

* In formula pigeonhole principle can be written as
  - → If a function $f: A \to B$ maps a finite set-A with $|A| = k+1$ objects to finite set B with $|B| = k$ boxes then f is not one-to-one.

<u>Generilize PhP</u>

If $n \geq 0$ objects are placed in $k > 1$ boxes then atleast one box contains atleast 'k' related objects.

Permutations & Combinations

<u>Permutations</u> :- It is an arrangement of objects in which the order does matter. The no. of permutations of n-objects chosen 'r' at a time denoted by "np_r" where $0 < r \leq n$.

* A permutations of n-objects taken 'r' at a is also called r-arrangements this can also be represented as $P(n, r)$.

* The no. of permutations of n-objects taken 'r' at a time is given by $np_r = \frac{n!}{(n-r)!}$

1) $7! - 6!$

Sol:- $7! = 5040 \quad 6! = 720$

$$7! - 6! = 5040 - 720$$

$$= 4320$$

2) Find the value of i) ${}^{12}P_4$ ii) ${}^{7}P_3$ iii) ${}^{6}P_6$

Sol:- i) $\frac{12!}{(12-4)!} = \frac{12\times11\times10\times9\times\cancel{8\times7\times6\times5\times4\times3\times2\times1}}{\cancel{1\times2\times3\times4\times5\times6\times7\times8}}$

$$= 11{,}880$$

ii) $\frac{7!}{(7-3)!} = \frac{7\times6\times5\times\cancel{4\times3\times2\times1}}{\cancel{1\times2\times3\times4}} = 210$

iii) $\frac{6!}{(6-6)!} = 6! = 720$

3) Evalute $\frac{n!}{(n-r)!}$ where $n=6, r=2$

Sol:- $\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!} = \frac{6\times5\times\cancel{4\times3\times2\times1}}{\cancel{4\times3\times2\times1}} = 30$

4) If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$. Find x

Sol:- $\cancel{\frac{1}{6!}}\left[1+\frac{1}{7}\right] = \frac{x}{\cancel{6!}}\left[\frac{1}{8\times7}\right]$

$$\frac{8}{7} = \frac{x}{56} \Rightarrow \frac{8\times\cancel{56}^{8}}{\cancel{7}} = x$$

$$\boxed{x = 64}$$

5) Prove that ${}^{n}P_r = n \times {}^{n-1}P_{r-1}$

$$= n\left(\frac{(n-1)!}{(n-1-r+1)!}\right)$$

$$= n\left(\frac{(n-1)!}{(n-r)!}\right)$$

$$\frac{n!}{(n-r)!} \Rightarrow {}^{n}P_r$$

LHS = RHS

6) Find the value of n if $^nP_2 = 12$.

Sol:-

$$\frac{n!}{(n-2)!} = 12$$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 12$$

$$n^2 - n - 12 = 0$$

$$n^2 + 3n - 4n - 12 = 0$$

$$n(n+3) - 4(n+3) = 0$$

$n+3 = 0$, $n = -3$ | $n-4 = 0$, $n = 4$//

7) Find n if $^nP_3 = 5:12$, $^{n-1}P_3 : {}^{n+1}P_3 = 5:12$

8) Find n if $^{15}P_{r-1} : {}^{16}P_{r-2} = 3:4$

9) In how many ways the letter of the following word can be arranged i) RAM ii) COMBINE iii) EQUATION.

Sol:- i) There are 3-different letters in the word RAM

∴ no. of arrangements of the letters

$$^nP_3 = {}^3P_3 = 3! = 6//$$

ii) There are 7-different letters in the given word

∴ no. of arrangement $= {}^7P_7 = 7! = 5040$

iii) There are 8-different letters in the given word

∴ no. of arrangements $= {}^8P_8 = 8! = 40,320$

10) Find the no. of permutations of english vowels (a, e, i, o, u) taking two at a time also verify the result.

Sol:- no. of permutation of vowels taking two at a time

$$^nP_r = {}^5P_2 = \frac{5!}{(5-2)!} = \frac{5\times4\times\cancel{3!}}{\cancel{3!}} = 20//$$

11)

Sol

12)

Sol

13)

Verification → AE, AI, AO, AU

EA, EI, EO, EU

IA, IE, IO, IU

OA, OE, OI, OU

UA, UE, UI, UO

11) How many 3-digit numbers can be formed by using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if no digit is repeated.

> Note :- 1) If digits are repeated n^r
>
> 2) If not repeated ^nP_r

Sol:- There are 9-numbers & 3 digits numbers should be formed. $n=9$, $r=3$

∴ repetation is not allowed

$$^nP_r = {}^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = 9 \times 8 \times 7 = 504$$

12) How many 4-digit numbers can be formed by using the digits 1 to 9 if repetation of digit is allowed.

Sol:- There are 9 numbers and 4 digits number be formed. $n=9$ $r=4$

∴ repetation is allowed

$$n^r = 9^4 = 6561$$

13) 6 candidates are called for an interview to fill-up 4 posts in an office. Assuming that each candidate is fit for each post. Determine the no. of ways in which i) 1st & 2nd posts can be filled up.

ii) 1st 3 posts can be filled up.

iii) All 4 posts can be filled up.

Sol:- i) No. of candidates = 6

No. of ways of filling 1^{st} & 2^{nd} posts

$$6P_2 = \frac{6!}{(6-2)!} = 6 \times 5 = 30$$

ii) No. of ways of filling 3 posts are

$$6P_3 = \frac{6!}{(6-3)!} = 6 \times 5 \times 4 = 120$$

iii) No. of ways of filling 4 posts are

$$6P_4 = \frac{6!}{(6-4)!} = 6 \times 5 \times 4 \times 3 = 360$$

13) Find the no. of ways in which 8 boys & 5 girls can be arranged in a row so that no 2 girls are together.

Sol:- Since, no two girls sit together

First, we arrange the boys,

8 boys can be arranged in $8P_8 = 8! \Rightarrow 40320$ ways

$$\times B \times B \times B \times B \times B \times B \times B \times B$$

when boys occur seats corresponding to each sitting of boys there are of place marked "x" for 5-girls.

Girls can be arranged in the 9-places in $9P_5$

$$\frac{9!}{(9-5)!} \Rightarrow \frac{9!}{4!} \Rightarrow 9 \times 8 \times 7 \times 6 \times 5 = 15,120 \text{ ways}$$

the girls can be arranged in 15120 ways

the boys can be arranged in 40320 ways

$\therefore$ Total no. of arrangements $= 40320 \times 15120$

$= 60,96,38,400$ ways

14) In how many ways worlds with (or) without ~~repetation~~ meaning can be formed with the letters of vowels & consonents together?

Sol:- The word EQUATION consists of 8 - diff letters

The vowels = e, a, i, o, u

The consonents = Q, T, N

$\therefore$ Total no. of words $= 2 \times 5! \times 3!$

$= 2 \times 120 \times 6$

$= 1440$//

15) How many words with (or) without meaning can be formed using all the letters of the word GERMANY using each letter exactly once.

Sol:- $\because$ The given word has 7 - different letters

$^7P_7 = 7! = 5040$ ways//

16) How many diff ways can be the letter of word DELHI be arranged i) How many of these arrangement begin with D ends with 'I'?

ii) How many begin with 'D' & ends with 'I'?

iii) How many begin with 'D' & don't end with 'I'?

Sol:- No. of diff letters in DELHI = 5

$^5P_5 = 5! = 120$ ways

i) For the arrangement with D, D is kept fixed

D				

no. of arrangements $= 1 \times {}^4P_4$

$= 1 \times 4!$

$= 24$ ways

ii) For the arrangement with D & end with I, D & I fixed,

D				I

$\therefore$ no. of arrangements $= 1 \times 1 \times {}^3P_3$

$= 1 \times 1 \times 3!$

$= 6$ ways

iii) For the arrangement with D & don't end with I, D is fixed

D				

$\therefore$ no. of arrangements $= 24 - 6$

$= 18$ ways

17) Find the no.of 4-digit numbers that can be forced using the digits are 1, 2, 3, 4, 5 if no digit is repeated. How many of this will be even.

Sol:- ∴ The required no.of 4-digits numbers $= 5P_4 = \frac{5!}{(5-4)!}$

$5! \Rightarrow 120$ ways.

The number is even if the digit at unit place is 2, 4

No.of digits = 4

Number with 2 in units place $= 4P_3 = 24$ ways

Number with 4 in units place $= 4P_3 = 24$ ways

Required no.of 4 digit even numbers

$= 24 + 24 \Rightarrow 48$ ways

Permutation of object not all different

* the no.of permutations of n-~~things~~ taken all at a time when 'p' of them are a like and of one kind, q of them are a like and of a second kind rest of them are all ~~are~~ different r.

1) In how many ways can the letters of the following words be arranged 1) TALL 2) APPLE 3) FARIDABAD 4) KOMOKO

Sol:- 1) The word TALL there are 4 letters

'T' occurs once

'A' occurs once

'L' occurs Twice

no.of arrangements = $\frac{4!}{1!1!2!} = \frac{4 \times 3 \times \cancel{2!}}{\cancel{2!}}$

$= 12$ ways

2) The word APPLE there are 5 letters

'A' occurs once, 'P' occurs Twice.

'L' occurs once

'E' occurs once

no. of arrangement = $\frac{\cancel{9!}\ 5!}{1!\,2!\,1!\,1!}$

$= \frac{\cancel{9\times8\times7\times6\times5\times4\times3\times2\times1}}{}$

$= \frac{5\times4\times3\times\cancel{2!}}{\cancel{2!}} \Rightarrow 60$

3) word = FARIDABAD   Total numbers = 9

F = 1

A = 3

R = 1

D = 2

B = 1

no. of arrangements = $\frac{9!}{3!\,2!}$

$= \frac{9\times8\times7\times\cancel{6}\times5\times4\times3\times\cancel{2\times1}}{(3\times\cancel{2\times1})\ \cancel{(2\times1)}}$

$= 30,240$

4) word = KOMOKO

Total numbers = 6

K = 2

O = 3

M = 1

no. of arrangements = $\frac{6!}{3!\,2!}$

$= \frac{\cancel{6}\times5\times4\times3\times\cancel{2\times1}}{(3\times\cancel{2\times1})\ \cancel{(2\times1)}}$

$= 60$

2) Find how many arrangements can be made with the letter of the word MATHEMATICS in how many of them the vowels occurs together.

Sol:- No. of Letters = 11 (not all are same)

M = 2, A = 2, T = 2

H = 1, E = 1, I = 1

C = 1, S = 1

no. of arrangements = $\frac{11!}{2!\,2!\,2!}$

= 49,89,600 ways

The vowels in the given word are A, E, I, A

$= \frac{4!}{2!} \Rightarrow 12$

No. of arrangements of objects $= \frac{8!}{2!2!} = 10,080$

The 4-vowels in single objects can also arranged in

$\frac{4!}{2!} = 12$ ways

Required no. of arrangements $= 10,080 \times 12$

$= 1,20,960$ ways

3) In how many of the different permutations of the letters in the word "MISSISSIPPI" do the form I'not come together?

Sol:- No. of arrangements ~~of~~ of given word MISSISSIPPI =

$$\frac{11!}{4!4!2!} = \frac{39916800}{1152} \Rightarrow 34630$$

Taking all I's as one ~~complement~~ component & remaining all as one component $\quad$ IIII $\quad$ MSSSSPP

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ 1 $\qquad\qquad$ 1

$$\frac{8!}{4!2!} = \frac{40320}{48} = 840$$

No. of arrangements the all I's not come together

$= 34630 - 840$

$= 33810$//

4) How many different words can be formed with the letter of the word "HARYANA",

1) How many of these have H & N together.

2) Begin with H and End with N. $\qquad$ HARYANA = 7!

3) Have 3 vowels together.

Sol:- No. of arrangements $= \frac{7!}{3!1!1!1!1!} = \frac{5040}{6} = 840$

1) (HN) (ARYAA)
$\quad 1 \qquad 5$

$\Rightarrow \frac{n!}{p!\,q!} = \frac{6!}{3!} = \frac{720}{6} = 120$

H & N can also be arranged among themselves in $2! = 2$ ways

$\therefore$ Total no. of words in which H & N together

$= 120 \times 2 \Rightarrow 240$ ways

2) Begin with H & End with N.

$1! \times \frac{5!}{3!} \times 1! \Rightarrow \frac{5!}{3!} = \frac{120}{6}$

$\Rightarrow 20$ ways

3) (AAA) (HRYN)
$\quad 1 \qquad 4$

we've to arrange 5 objects $= \frac{5!}{1!} = 120$

$\therefore$ Total no. of words = 120 ways.

5) How many number grater than a million can be formed by using the digits (4, 6, 0, 6, 7, 4, 6).

Sol:- $\frac{n!}{p!\,q!} = \frac{7!}{3!\,2!} \Rightarrow \frac{7 \times \cancel{6} \times 5 \times 4 \times 3 \times \cancel{2 \times 1}}{(3 \times \cancel{2 \times 1})\,(\cancel{2 \times 1})}$

$\Rightarrow 420$ ways

The number formed up of this 7 digits begining with '0' are not grater than one million.. The remaining six places can be filled in

$\frac{6!}{2!\,3!} = \frac{\cancel{6} \times 5 \times 4 \times 3 \times \cancel{2 \times 1}}{(3 \times \cancel{2 \times 1})\,(\cancel{2 \times 1})} = 60$ ways

numbers greater than one million $= 420 - 60$

$= 360$ ways

6) If the different permutation of all the letters of the word "EXAMINATION" as listed as in a dictionary. How many items are there in the list before the first word starting with E.

Sol: The word is EXAMINATION, the letter in the assending order are A, A, E, I, I, M, N, O, T, X.

~~the~~ no. of words begin with $A = \frac{10!}{2!2!}$ ~~= 3628800~~

$= \frac{10 \times 9 \times \cancel{8} \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2} \times 1}{\cancel{(2 \times 1)(2 \times 1)}}$

$=$ ~~3628~~ $8\,453600$ ways//

∴ There are 453600 words before the first word starting with E.

7) Find the no. of arrangement of the letters of the word "INDEPENDENCE". In how many of this arrangement

1) Do the word start with 'P'.
2) Do all the vowel occur together.
3) Do the vowels occur together
4) Do the vowels begin with I & end in P.

e

7

~~ieo~~

ord

nent

Combinations :- It is a group of outcomes in which the order does not matter. The no. of combinations of n-objects taken 'r' at a time is denoted by "^nC_r" $\quad ^nC_r = \frac{n!}{(n-r)!\,r!}$

1) Evaluate the following i) $^6C_3$ ii) $\sum_{r=1}^{5} {^5C_r}$

Sol:- i) $^6C_3 = \frac{6!}{(6-3)!\,3!} = \frac{\cancel{6}\times 4\times 5\times \cancel{3!}}{\cancel{3!}\times \cancel{3!}} \Rightarrow 20$ //

ii) $\sum_{r=1}^{5} {^5C_r} = {^5C_1} + {^5C_2} + {^5C_3} + {^5C_4} + {^5C_5}$

$= \frac{5!}{(5-1)!\,1!} + \frac{5!}{(5-2)!\,2!} + \frac{5!}{(5-3)!\,3!} + \frac{5!}{(5-4)!\,4!} + \frac{5!}{(5-5)!\,5!}$

$= \frac{5!}{4!\,1!} + \frac{5!}{3!\,2!} + \frac{5!}{2!\,3!} + \frac{5!}{1!\,4!} + \frac{5!}{(0)!\,5!}$

$= 5 + \frac{5\times \overset{2}{\cancel{4}}\times \cancel{3!}}{\cancel{3!}\,\cancel{2!}} + \frac{5\times \overset{2}{\cancel{4}}\times \cancel{3!}}{\cancel{3!}\,\cancel{2!}} + 5 + 1$

$= 5 + 10 + 10 + 5 + 1 \Rightarrow 31$ //

2) Evaluate $2\cdot {^7C_4} = {^8C_4}$

Sol:- $2\left(\frac{7\times \cancel{6}\times 5\times \cancel{4!}}{\cancel{3!}\cdot \cancel{4!}}\right) \Rightarrow 35\times 2$

$\Rightarrow 70$

$^8C_4 = \frac{\overset{2}{\cancel{8}}\times 7\times \cancel{6}\times 5\times \cancel{4!}}{\cancel{4!}\,\cancel{4!}} \Rightarrow 2\times 7\times 5$

$\Rightarrow 70$

LHS = RHS

3) If $^{15}C_r : {^{15}C_{r-1}} = 11 : 5$. find r.

Sol:- $\frac{^{15}C_r}{^{15}C_{r-1}} = \frac{11}{5}$

$\frac{\frac{15!}{(15-r)!\,r!}}{\frac{15!}{(15-(r-1))!\,(r-1)!}} = \frac{11}{5}$

$\frac{\cancel{15!}}{(15-r)!\,r!} \times \frac{(15-(r-1))!\,(r-1)!}{\cancel{15!}} = \frac{11}{5}$

$\frac{(16-r)!\,(r-1)!}{(15-r)!\,r!} = \frac{11}{5}$

$\frac{(16-r)\cancel{(15-r)!}\,\cancel{(r-1)!}}{\cancel{(15-r)!}\,r\cancel{(r-1)!}} = \frac{11}{5}$

ch.

$$\frac{16-r}{r} = \frac{11}{5}$$

$$5(16-r) = 11r$$

$$80 - 5r = 11r$$

$$80 = 16r$$

$$\boxed{r = 5}$$

4) Find n if ${}^{2n}C_3 : {}^{n}C_3 = 12:1$

Sol:-

$$\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$$

$$\frac{\frac{2n!}{(2n-3)!\,3!}}{\frac{n!}{(n-3)!\,3!}} = \frac{12}{1}$$

$$\frac{2n!}{(2n-3)!\,\cancel{3!}} \times \frac{(n-3)!\,\cancel{3!}}{n!} = \frac{12}{1}$$

$$\frac{2n(2n-1)(2n-2)\cancel{(2n-3)!}}{\cancel{(2n-3)!}} \times \frac{\cancel{(n-3)!}}{n(n-1)(n-2)\cancel{(n-3)!}} = \frac{12}{1}$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{12}{1}$$

$$2n(2n-1)(2n-2) = 12[n(n-1)(n-2)]$$

$$\cancel{2n}(2n-1)(2n-2) = \cancel{2n}[6(n-1)(n-2)]$$

$$(2n-1)(2\cancel{(n-1)}) = 6\cancel{(n-1)}(n-2)$$

$$\cancel{2}(2n-1) = {}^{3}\cancel{6}(n-2)$$

$$2n-1 = 3n-6$$

$$6-1 = 3n-2n$$

$$\boxed{n = 5}$$

5) If ${}^{28}C_{2r} : {}^{28}C_{2r-4} = 225:11$

$\frac{11}{5}$

Sol:-

$$\frac{\frac{28!}{(28-2r)!\,2r!}}{\frac{24!}{(28-(2r-4))!\,(2r-4)!}} = \frac{225}{11}$$

$$\frac{28!}{(28-2r)!\,2r!} \times \frac{(28-2r)!\,(2r-4)!}{24!} = \frac{225}{11}$$

$$\frac{28\times27\times26\times25\times\cancel{24!}}{\cancel{(28-2r)!}\cdot 2r!} \times \frac{\cancel{(28-2r)!}\,(2r-4)!}{\cancel{24!}} = \frac{225}{11}$$

$$\frac{28\times27\times26\times25\times\cancel{(2r-4)!}}{2r(2r-1)(2r-2)(2r-3)\cancel{(2r-4)!}} = \frac{225}{11}$$

$$\frac{491400}{(2r^2-2r)(4r^2-6r-4r+6)} = \frac{225}{11}$$

$$\frac{491400}{8r^4-12r^3-8r^3+12r^2-8r^3+12r^2+8r^2-12r} = \frac{225}{11}$$

$$\cancel{8r^4-20r^3+4r^2+20r-12r}$$

$$\frac{491400}{8r^4-28r^3+32r^2-12r} = \frac{225}{11}$$

$$491400(11) = (8r^4-28r^3+32r^2-12r)(225)$$

$$5405400 = (8r^4-28r^3+32r^2-12r)(225)$$

$$\frac{5405400}{225} = 8r^4-28r^3+32r^2-12r$$

$$8r^4-28r^3+32r^2-12r = 24024$$

$$8r^4-28r^3+32r^2-12r-24024 = 0$$

$$2r^4-7r^3+8r^2-3r-6006 = 0$$

Q)

50

Q) A commity of 3-persons is to be constituted from a group of 2 men & 3 women in how many ways can this be done. How many of these commities would consist of 1 man and 2 women.

Sol:- There are combinations 5 different person taken 3 at a time hence the recurrence

No. of ways are ${}^5C_3$ $\qquad \frac{n!}{(n-r)!\,r!}$

$$\frac{5!}{(5-3)!\,3!} \Rightarrow \frac{5!}{2!\,3!} \Rightarrow \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{1 \times \cancel{2} \cdot \cancel{3!}} \Rightarrow 10$$

Now one man can be selected from 2 men in ${}^2C_1$ ways and 2 women can be selected from 3 womenn in ${}^3C_2$.

$$= {}^2C_1 \times {}^3C_2$$

$$= \frac{2!}{(2-1)!\,1!} \times \frac{3!}{(3-2)!\,2!}$$

$$= \frac{\cancel{2}}{1} \times \frac{3}{\cancel{2}} \Rightarrow 3$$

Q) In how many can a team of 2 boys & 2 girls be selected from 5 boys & 4 girls.

Sol:- No.of boys = 5

No.of girls = 4

$$= {}^5C_2 \times {}^4C_2$$

$$= \frac{5!}{(5-2)!\,2!} \times \frac{4!}{(4-2)!\,(2!)}$$

$$= \frac{5!}{3!\,(2)!} \times \frac{4!}{2!\,(2!)} \Rightarrow \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!}\,\cancel{2!}} \times \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2!}\,\cancel{2!}}$$

$$= 10 \times 6 \Rightarrow 60$$

Q) Find the no.of ways of selecting 9 balls from 6 Red balls, 5 white balls & 5 blue balls. If each selection consists of 3 balls of each colour.

Sol:- No.of red balls = 6

No.of white balls = 5

No.of blue balls = 5

No.of required balls from each color = 3

$$\Rightarrow {}^6C_3 \times {}^5C_3 \times {}^5C_3$$

$$\Rightarrow \frac{6!}{(6-3)!\,3!} \times \frac{5!}{(5-3)!\,3!} \times \frac{5!}{(5-3)!\,3!}$$

$$\Rightarrow \frac{\cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}\,\cancel{3!}} \times \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2!} \times \cancel{3!}} \times \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{2!} \times \cancel{3!}}$$

$$\Rightarrow 20 \times 10 \times 10 \Rightarrow 2000$$

Q) A commity of 7 has to be formed from 9 boys & 4 girls. In how many can this be done when the commity consists of,

i) Exactly 3 girls

ii) Atleast 3 girls

iii) At most 3 girls

selec

Sol:- i) Exactly 3 girls

Given

No. of boys = 9

No. of girls = 4

i) No. of required commity numbers = 7

$\therefore$ the required no. of ways $= {}^4C_3 \times {}^9C_4$

$$= \frac{4!}{(4-3)!\,3!} \times \frac{9!}{(9-4)!\,4!}$$

$$= \frac{4 \times \cancel{3!}}{1!\,\cancel{3!}} \times \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!}\ 4 \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 4 \times (9 \times 7 \times 2)$$

$$= 4 \times 126 \Rightarrow 504$$

ii) Atleast 3 girls

$\because$ Atleast 3 girls are to be there in every committee.

3 girls & 4 boys $= {}^4C_3 \times {}^9C_4$

3 boys & 4 girls $= {}^9C_3 \times {}^4C_4$

$\therefore$ the required no. of ways $= {}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$

$$= \frac{4!}{(4-3)!\,3!} \times \frac{9!}{(9-4)!\,4!} + \frac{4!}{(4-4)!\,4!} \times \frac{9!}{(9-3)!\,3!}$$

$$= \frac{4 \times \cancel{3!}}{1!\,\cancel{3!}} \times \frac{9 \times \cancel{8}^{2} \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!}\ 4 \times \cancel{3} \times \cancel{2} \times 1} + 1 \times \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!}\ \cancel{3} \times \cancel{2} \times 1}$$

$$= (4)(9 \times 7 \times 2) + 1\,(3 \times 4 \times 7)$$

$$= 504 + 84 \Rightarrow 588$$

iii) At most 3 girls

$${}^9C_7 \times {}^4C_0 + {}^9C_6 \times {}^4C_1 + {}^9C_5 \times {}^4C_2 + {}^9C_4 \times {}^4C_3$$

$$= \frac{9!}{(9-7)!\,7!} \times \frac{4!}{(4-0)!\,0!} + \frac{9!}{(9-6)!\,6!} \times \frac{4!}{(4-1)!\,1!} + \frac{9!}{(9-5)!\,5!} \times \frac{4!}{(4-2)!\,2!} + \frac{9!}{(9-4)!\,4!} \times \frac{4!}{(4-3)!\,3!}$$

Red

n.

4

ity

$$= \frac{9\times 8\times 7!}{2!\times 7!}\times 1^4 + \frac{9\times 8\times 7\times 6!}{3\times 2\times 6!}\times\frac{4\times 3!}{3!\,1!} + \frac{9\times 8\times 7\times 6\times 5!}{4\times 3\times 2\times 1\,(5!)}\times\frac{4\times 3\times 2}{2!\,2!}$$

$$+ \frac{9\times 8\times 7\times 6\times 5!}{5!\times 4\times 3\times 2\times 1}\times\frac{4\times 3!}{1!\,3!}$$

$$= (9\times 4\times 1) + (3\times 4\times 7\times 4) + (9\times 2\times 7\times 2\times 3) + (3\times 7\times 6\times 4)$$

$$= 36 + 336 + 756 + 504$$

$$= 1632$$

Q) It is required to sit 5 men & 4 women in a row so that the women occupy the even places. How many such arrangements are possible.

M W M W M W M W M

Sol:- No. of men = 5

No. of women = 4

Total no. of ways $= 5! \times 4!$

$= 120 \times 24 \Rightarrow 2880$

Q) What is the no. of ways choosing 4 cards from a pack of 52 playing cards. In how many of this,

i) 4 cards are of same suit

ii) 4 cards belongs to different suit

iii) 2 are red cards & 2 are black card

iv) cards are of same colour

Sol:- There are 52 playing cards out of which 4 cards are choosen.

$$^{52}C_4 = \frac{52\times 51\times 50\times 49\times 48!}{48!\;4\times 3\times 2\times 1}$$

$$= 2,70,725$$

i)

$$4\left(^{13}C_4\right) = 4\left(\frac{13\times 12\times 11\times 10\times 9!}{9!\,(4\times 3\times 2\times 1)}\right)$$

$$= 4(13\times 11\times 5) \Rightarrow 2860$$

ii) 4 cards are different from different suits

$$^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$$

$$\frac{13\times\cancel{12!}}{\cancel{12!}\times 1!} \times \frac{13\times\cancel{12!}}{\cancel{12!}\times 1!} \times \frac{13\times\cancel{12!}}{\cancel{12!}\times 1!} \times \frac{13\times\cancel{12!}}{\cancel{12!}\times 1!}$$

$$= 28,561 //$$

iii) 2 are red & 2 are black cards

$$^{26}C_2 \times {}^{26}C_2$$

$$\frac{\overset{13}{\cancel{26}}\times 25\times\cancel{24!}}{\cancel{24!}\times\cancel{(2\times1)}} \times \frac{\overset{13}{\cancel{26}}\times 25\times\cancel{24!}}{\cancel{24!}\times\cancel{(2\times1)}}$$

$$13\times25\times(13\times25) \Rightarrow 105,625$$

iv) 2 cards are of same colors

$$2\left[{}^{26}C_4\right] \Rightarrow 2\left(\frac{26\times25\times\cancel{24}\times23\times\cancel{22!}}{\cancel{22!}\,(\cancel{4\times3\times2\times1})}\right)$$

$$\Rightarrow 2(14950)$$

$$\Rightarrow 29,900 //$$

P) It is required to 5 men & 4 women in a row so that women occupy the even places. How many such arrangements are possible.

$M = 5!$ $\quad W = 4!$

no. of ways $= 5! \times 4! \Rightarrow 2880 //$

P) There are 15 points in a plane of which 5 are co-linear, find the no. of i) straight line ii) Triangles which can be formed by this points

Sol:- $\because$ We've given 5 co-linear points this 5 points gives only one line.

Now, the no. of straight lines with 15 points is given by $^{15}C_2$

i) No. of straight line $= {}^{15}C_2 - {}^{5}C_2 + 1$

$$= \frac{15 \times \cancel{14}^{7} \times \cancel{13!}}{\cancel{2}\cancel{13!} \times \cancel{2!}} - \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!}\,\cancel{2!}} + 1$$

$$= 105 - 10 + 1$$

$$= 96$$

ii) No. of Triangles $= {}^{15}C_3 - {}^{5}C_3$

$$= \frac{\cancel{15}^{5} \times \cancel{14}^{7} \times 13 \times \cancel{12!}}{\cancel{12!}\,(\cancel{3} \times \cancel{2} \times 1)} - \frac{5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2!}}{\cancel{2!}\,\cancel{3!}}$$

$$= (5 \times 7 \times 13) - (5 \times 2)$$

$$= 455 - 10 \Rightarrow 445$$

P) In how many ways can a foot ball team of 11-players, Students from 15 players. How many of there will
i) Include one particular player
ii) Exclude one particular player

Sol:- i) ~~14~~ ${}^{14}C_{10} = \frac{14!}{4!\ 10!}$

$$= \frac{\cancel{14}^{7} \times 13 \times \cancel{12} \times 11 \times \cancel{10!}}{\cancel{4 \times 3 \times 2} \times 1\ \cancel{10!}}$$

$$= 7 \times 13 \times 1 \Rightarrow 1001$$

ii) ${}^{14}C_{11} = \frac{14 \times 13 \times \cancel{12}^{2} \times \cancel{11!}}{\cancel{3!}\ \cancel{11!}}$

$$= 364$$

Recurance Relation

* A recurance relalation for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms $a_0, a_1, a_2, \ldots a_{n-1}$ for all integers 'n'.

1) 1
sol:-

2)
sol

1) Write the recurence relation for a factors of a no

Sol:- Always $0! = 1$

when $n=1$, $1! = 1$

when $n=2$, $2! = 2 \times 1!$

when $n=3$, $3! = 3 \times 2!$

when $n=4$, $4! = 4 \times 3!$

The initial value is $f(0)=1$, the second value is calculated based on first value, third value is calculated based on second value it is cleard that n^{th} value is called $n \times (n-1)!$

$\therefore$ The recurrance relation is

$$f(n) = n\, f(n-1)!$$

2) RR of the following positive integer 3, 9, 21, 45

Sol:- Let $f(0) = 3$

$f(1) = 9 \Rightarrow 2 f(0) + 3$

$f(2) = 21 \Rightarrow 2 f(1) + 3$

$f(3) = 45 \Rightarrow 2 f(2) + 3$

$$\boxed{f(n) = 2 f(n-1)! + 3}$$

Solution of recurrence relation.

A sequence is called a solution of a recurrence relation of its terms satisfy the recurrence relation.

example :- Let $\{a_n\}$ be a sequence that satisfies that recurrence relation $a_n = a_{n-1} - a_{n-2}$ for $n=2,3,$ -- and suppose that $a_0 = 3$ & $a_1 = 5$ what are ~~are~~ a_2 & a_3 ?

Sol:- Given that $a_n = a_{n-1} - a_{n-2}$

putting $n = 2, 3,$ -- so that

$\therefore a_2 = a_1 - a_0 = 5 - 3 = 2$

$a_3 = a_2 - a_1 = 2 - 5 = -3$

Linear Recurrence Relation with constant coefficients

A recurrence relation of the form

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} + \cdots + C_k a_{r-k} = f(r) \quad -(1)$$

where C's are constant, is called a linear recurrence relation with constant coefficients

The recurrence relation in eq (1) is known as a k^{th} order recurrence relation, provided that both $C_0 \neq 0$ and $C_k \neq 0$

2^{nd} order recurrence relation

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} = f(r)$$

3^{rd} order recurrence relation.

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} + C_3 a_{r-3} = f(r)$$

The solution of eq (1) is

$$a_r = a_r^{(k)} + a_r^{(p)}$$

where, $a_r^{(k)}$ = Homogeneous Solution.

$a_r^{(p)}$ = Particular Solution.

Homogeneous Recurrence Relation.

If $f(r) = 0$, the eq (1) is called homogeneous Recurrence relation.

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} + C_3 a_{r-3} + \cdots + C_k a_{r-k} = 0$$

Non-Homogeneous Recurrence Relation.

If $f(r) \neq 0$, then eq (1) is called non-homogeneous recurrence relation.

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} + \cdots + C_k a_{r-k} = f(r)$$

Homogeneous Linear Recurrence Relation with constant co-efficient

Suppose the 2^{nd} order homogeneous linear recurrence relation is

$$C_0 a_r + C_1 a_{r-1} + C_2 a_{r-2} = 0$$

The characteristic Equation is

$$C_0 m^2 + C_1 m + C_2 = 0$$

<u>case 1</u> :- If the roots of characteristic eq are real and inequal $(m_1 \neq m_2)$

$\therefore$ The general solution is

$$\boxed{a_r = c_1 m_1^r + c_2 m_2^r}$$

<u>case 2</u> :- If the roots of characteristic eq are Real & Equal $(m_1 = m_2 = m)$

$\therefore$ The general solution is

$$\boxed{a_r = (c_1 + r c_2) m^r}$$

<u>case 3</u> :- If the roots of characteristic eq are in complex numbers $(m = \alpha \pm i\beta)$

$\therefore$ The general solution is

$$\boxed{a_r = (C_1 \cos r\theta + C_2 \sin r\theta) R^r}$$

where, $R = \sqrt{\alpha^2 + \beta^2}$

$\theta = \tan^{-1}(\beta/\alpha)$.

1) What is the solution of the recurrence relation. Find its characteristics equations $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ & $a_1 = 7$

Sol:-

$r^2 = r + 2$ $\quad \boxed{r = -1, 2}$

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$a_n = c_1 r^n + c_2 r^n$

$a_n = c_1 (-1)^n + c_2 (2)^n \rightarrow$ (A)

Now, we find c_1 & c_2 using initial condition

$a_0 = 2$, $n=0 \Rightarrow a_0 = c_1 (-1)^0 + c_2 (2)^0$

$2 = c_1 + c_2 \rightarrow$ (1)

$a_1 = 7$, $n = 1 \Rightarrow a_1 = c_1 (-1)^1 + c_2 (2)$

$7 = -c_1 + 2c_2 \rightarrow$ (2)

substract eq (1) & eq (2)

$$2 = \cancel{c_1} + c_2$$
$$7 = \cancel{-c_1} + 2c_2$$
$$9 = 3c_2 \Rightarrow \boxed{c_2 = 3}$$

substitute, $c_2 = 3$ in eq (1), we get

$2 = c_1 + 3$

$\boxed{c_1 = -1}$

substitute, c_1 & c_2 values in eq (A)

$\therefore a_n = (-1)(-1)^n + 3(2)^n \rightarrow$ is a solution.

2) What is the solution of the recurrence relation $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with $a_0 = 8$, $a_1 = 6$ & $a_2 = 26$?

Sol:- Given

$a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$

Hence, it is a linear homogeneous recurrence relation. The characteristic equation is

$r^3 = -r^2 + 4r + 4$ $\qquad r = -1 \mid r = -2, +2$

$r^3 + r^2 - 4r - 4 = 0$

$r^2(r+1) - 4(r+1) = 0$

$(r+1)(r^2 - 4) = 0$

$$a_n = c_1 r^n + c_2 r^n + c_3 r^n.$$

$$a_n = c_1(-1)^n + c_2(-2)^n + c_3(2)^n \rightarrow A$$

Now, we find $c_1, c_2, \& c_3$ by using initial values

$a_0 = 8,\ n=0 \Rightarrow a_0 = c_1(-1)^0 + c_2(-2)^0 + c_3(2)^0$

$$8 = c_1 + c_2 + c_3 \rightarrow 1$$

$a_1 = 6,\ n=1 \Rightarrow a_1 = c_1(-1)^1 + c_2(-2)^1 + c_3(2)^1$

$$6 = -c_1 - 2c_2 + 2c_3 \rightarrow 2$$

$a_2 = 26,\ n=2 \Rightarrow a_2 = c_1(-1)^2 + c_2(-2)^2 + c_3(2)^2$

$$26 = c_1 + 4c_2 + 4c_3 \rightarrow 3$$

$1 + 2 \Rightarrow$

$$\cancel{c_1} + c_2 + c_3 = 8$$
$$-\cancel{c_1} - 2c_2 + 2c_3 = 6$$
$$\overline{-c_2 + 3c_3 = 14} \rightarrow 4$$

$2 + 3 \Rightarrow$

$$-\cancel{c_1} - 2c_2 + 2c_3 = 6$$
$$\cancel{c_1} + 4c_2 + 4c_3 = 26$$
$$\overline{2c_2 + 6c_3 = 32} \rightarrow 5$$

$4 \times 2 + 5 \Rightarrow$

$$-\cancel{2c_2} + 6c_3 = 28$$
$$\cancel{2c_2} + 6c_3 = 32$$
$$\overline{12c_3 = 60}$$

$$\boxed{c_3 = 5}$$

= 26?

put $c_3 = 5$ in eq 4

$$-c_2 + 3(5) = 14$$
$$-c_2 = 14 - 15$$
$$\boxed{c_2 = 1}$$

put $c_3 = 5 \& c_2 = 1$ in eq 1

$$c_1 + c_2 + c_3 = 8$$
$$c_1 + 1 + 5 = 8$$
$$\boxed{c_1 = 2}$$

substitute $c_1, c_2 \& c_3$ values in eq A

$$a_n = 2(-1)^n + 1(-2)^n + 5(2)^n$$

is a solution//

Module - 4 (Matrix)

11-12-24

Def :- Matrix is a rectangular array or arrangement of "m, n" numbers in m (rows) & n (columns) which is enclosed by a square brackets.

Order (or) Dimension of a matrix

The order (or) dimension of a matrix is a ordered pair having first component has no. of rows and second component has the no. of columns in matrix.

⇒ If there are 3-rows & 2-columns in matrix then its order is written as "3x2".

Types of Matrix

1) **NULL Matrix** :- A matrix in which each element is "zero", is called NULL (or) zero matrix.

$$ex :- A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2) **Row Matrix** :- A matrix in consisting of a single row is called row matrix. The order is $1 \times n$.

$$ex :- \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1 \times 3}$$

3) **Column Matrix** :- A matrix in which having single column is called column matrix. The order is "$m \times 1$".

$$ex :- \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1}$$

4) **Rectangular Matrix** :- A matrix in which no. of rows is not equal to no. of columns.

$$ex :- A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$$

5) **Square Matrix** :- A matrix 'A' having no. of rows

equal to no.of columns. ex :- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2}$

6) <u>Diagonal Matrix</u> :- A square matrix in which all elements are zero except those in the principle diagonal.

$$ex:- \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$$

7) <u>Scalar Matrix</u> :- A diagonal matrix in which all the values in diagonal are same is called scalar matrix.

$$ex :- \quad A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3\times3}$$

8) <u>Identity</u> (or) <u>Unit Matrix</u> :- It is a scalar matrix but diagonal value is 1 is called unit matrix.

$$I_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

* An identity matrix of order 'n' is denoted by 'I_n'.

9) <u>Triangular Matrix</u> :- A square matrix is said to be triangular matrix if all the elements have above or below of principle diagonal is zero.

i) If all the elements above the principle diagonal is zero, it is Lower triangular matrix.

$$ex :- \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 7 \end{bmatrix}_{3\times3}$$

ii) If all the elements below the principle diagonal is zero, is upper triangular matrix

$$ex :- \quad A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{bmatrix}_{3\times3}$$

10) Sub Matrix :- A matrix formed by deleting of some rows (or) some columns from a matrix.

11) Transpose of Matrix :- A matrix obtained by interchanging its rows & columns is called transpose of a matrix.

* It is denoted by A' (or) A^T.

12) Symmentric Matrix :- A square matrix which is equal to its transpose is known as symmentric.

* A square matrix - A is called symmentric

$$A = A^T.$$

13) Skew symmentric Matrix :- If the transpose of the matrix equals to its negative.

* A square matrix - A is called skew-symmentric if $A' = -A$.

* For a skew-symmentric matrix the diagonal elements are zero.

14) Equal Matrix :- Two matrices A & B are said to be equal if and only if they have same order each elements of matrix - A is equal to correspon-ding to matrix - B. (A = B)

$$ex :- \; A = B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

15) Negative Matrix :- If $A_{m\times n}$ is denoted by $-A_{m\times n}$ is a matrix formed by replacing each element in the matrix with its additive inverse

Operations on Matrix

1) **Addition of Matrix :-** Let A & B be two matrices of same order then their sum A+B is a matrix whose elements are sum of the corresponding elements A & B.

→ Two matrices can be added / substracted / multiplication if and only if they've same order

→ Let A & B are any two matrizer then addition of 2 matrices can be obtained by adding the corresponding elements of the given matrices.

ex :- $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \\ 4 & 2 \end{bmatrix}_{3\times 2}$ $B = \begin{bmatrix} -1 & 4 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}_{3\times 2}$

$$A+B = \begin{bmatrix} 2+(-1) & 1+4 \\ 3+2 & -1+3 \\ 4+1 & 2+(-1) \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 \\ 5 & 2 \\ 5 & 1 \end{bmatrix}$$

Q) $A = \begin{bmatrix} 3 & 6 & 0 & 9 \\ 4 & 2 & -1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 6 & 3 & 0 & 9 \\ 3 & -1 & 6 & 9 \end{bmatrix}$

Q) $A = \begin{bmatrix} 3 & 6 & 1 \\ 9 & -9 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -5 & 3 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

2) **Substraction of a Matrix**

Let A & B be any two matrices with same order then their difference A−B is defined as

Q) $A = \begin{bmatrix} 1 & 4 \\ 5 & 6 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}$ $\Rightarrow A-B = \begin{bmatrix} 1-2 & 4-4 \\ 5-0 & 6-2 \end{bmatrix}$

$$A-B = \begin{bmatrix} -1 & 0 \\ 5 & 4 \end{bmatrix}$$

Q) $A = \begin{bmatrix} 1 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -2 & 3 \\ 0 & -1 & 2 \\ -3 & 4 & 5 \end{bmatrix}$ & $C = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 4 & 5 \\ 7 & 8 & 3 \end{bmatrix}$

i) $$A+B+C = \begin{bmatrix} 1+4+2 & 5+(-2)+4 & 6+3+1 \\ 7+0+1 & 8+(-1)+4 & 9+2+5 \\ 10+(-3)+7 & 11+4+8 & 12+5+3 \end{bmatrix}$$

$$A+B+C = \begin{bmatrix} 7 & 7 & 10 \\ 8 & 11 & 16 \\ 14 & 23 & 20 \end{bmatrix}_{3\times 3}$$

11/13/

1)

Sol!

2)

Sol

11/13/24

1) If $A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 \\ 4 & 1 \end{bmatrix}$ & $C = \begin{bmatrix} -1 & 2 \\ 7 & 6 \end{bmatrix}$

Find $5A - 2B - 2C$.

Sol:- Let

$$5A - 2B - 2C = 5\begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 3 & 3 \\ 4 & 1 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ 7 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 10 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 6 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 14 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 25-6+2 & 10-6-4 \\ 0-8-14 & 5-2-12 \end{bmatrix}$$

$$5A - 2B - 2C = \begin{bmatrix} 21 & 0 \\ -22 & -9 \end{bmatrix}_{2\times 2}$$

2) If $A = \begin{bmatrix} -1 & 5 & 6 \\ 7 & 8 & 9 \\ 0 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -2 & 3 \\ 0 & 1 & 2 \\ 2 & 4 & 5 \end{bmatrix}$

Find i) $A + 2B$ ii) $2A - B$

Sol:- i) Let

$$A + 2B = \begin{bmatrix} -1 & 5 & 6 \\ 7 & 8 & 9 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 8 & -4 & 6 \\ 0 & 2 & 4 \\ 4 & 8 & 10 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} 7 & 1 & 12 \\ 7 & 10 & 13 \\ 4 & 9 & 12 \end{bmatrix}$$

ii)

$$2A - B = \begin{bmatrix} -2 & 10 & 12 \\ 14 & 16 & 18 \\ 0 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 8 & -4 & 6 \\ 0 & 2 & 4 \\ 4 & 8 & 10 \end{bmatrix}$$

$$2A - B = \begin{bmatrix} -10 & -14 & 6 \\ 14 & 14 & 14 \\ -4 & -6 & -6 \end{bmatrix}$$

3) solve for A & B for if $3A+2B = \begin{bmatrix} 21 & 16 & 1 \\ 21 & 2 & 12 \end{bmatrix}$,

$$2A-3B = \begin{bmatrix} -12 & -11 & 5 \\ 1 & -16 & 8 \end{bmatrix}$$

Sol:-

$$3A+2B = \begin{bmatrix} 21 & 16 & 1 \\ 21 & 2 & 12 \end{bmatrix} \rightarrow ① \qquad 2A-3B = \begin{bmatrix} -12 & -11 & 5 \\ 1 & -16 & 8 \end{bmatrix} \rightarrow ②$$

solving eq ① & eq ②

$$\text{eq ①} \times 3 \qquad 9A + \cancel{6B} = \begin{bmatrix} 63 & 48 & 3 \\ 63 & 6 & 36 \end{bmatrix}$$

$$\text{eq ②} \times 2 \qquad 4A - \cancel{6B} = \begin{bmatrix} -24 & -22 & 10 \\ 2 & -32 & 16 \end{bmatrix}$$

$$13A = \begin{bmatrix} 63-24 & 48-22 & 3+10 \\ 63+2 & 6-32 & 36+16 \end{bmatrix}$$

$$13A = \begin{bmatrix} 39 & 26 & 13 \\ 65 & -26 & 52 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 5 & -2 & 4 \end{bmatrix} \rightarrow ③$$

solve eq ③ in eq ①, we get

$$3\begin{bmatrix} 3 & 2 & 1 \\ 5 & -2 & 4 \end{bmatrix} + 2B = \begin{bmatrix} 21 & 16 & 1 \\ 21 & 2 & 12 \end{bmatrix}$$

$$+2B = \begin{bmatrix} 21 & 16 & 1 \\ 21 & 2 & 12 \end{bmatrix} - \begin{bmatrix} 9 & 6 & 3 \\ 15 & -6 & 12 \end{bmatrix}$$

$$2B = \begin{bmatrix} 12 & 10 & -2 \\ 6 & 8 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 5 & -1 \\ 3 & 4 & 0 \end{bmatrix} //$$

4) If $2A+B = \begin{bmatrix} 6 & 3 \\ 6 & -2 \end{bmatrix}$ & $3A+2B = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$ Find A & B

Sol:-

$$2A+B = \begin{bmatrix} 6 & 3 \\ 6 & -2 \end{bmatrix} \rightarrow ① \qquad 3A+2B = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} \rightarrow ②$$

$$eq① \times 2 \Rightarrow 4A + \cancel{2B} = \begin{bmatrix} 12 & 6 \\ 12 & -4 \end{bmatrix}$$

$$eq② \Rightarrow \underset{-}{3A} + \underset{-}{\cancel{2B}} = \underset{-}{\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}}$$

$$A = \begin{bmatrix} 12 & 6 \\ 12 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 11 & 6 \\ 12 & -9 \end{bmatrix} \rightarrow eq ③$$

sub eq ③ in eq ①

$$2\begin{bmatrix} 11 & 6 \\ 12 & -9 \end{bmatrix} + B = \begin{bmatrix} 6 & 3 \\ 6 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 3 \\ 6 & -2 \end{bmatrix} - \begin{bmatrix} 22 & 12 \\ 24 & -18 \end{bmatrix}$$

$-2+18$

$$B = \begin{bmatrix} -16 & -9 \\ -18 & 16 \end{bmatrix} //$$

5) If $A = \begin{bmatrix} 2 & 7 & 3 \\ 4 & -5 & 6 \end{bmatrix}$ show that $(A')' = A$

Sol:- Given

$$A = \begin{bmatrix} 2 & 7 & 3 \\ 4 & -5 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 4 \\ 7 & -5 \\ 3 & 6 \end{bmatrix} \qquad (A^T)^T = \begin{bmatrix} 2 & 7 & 3 \\ 4 & -5 & 6 \end{bmatrix}$$

$$\boxed{\therefore (A^T)^T = A}$$

5) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

Sol:-

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} \qquad B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -1-4 & 5+1 & -2+1 \\ 2+1 & 7+2 & 1+3 \\ 3-5 & 9+0 & 1+1 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & +4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1-4 & 2+1 & 3-5 \\ 5+1 & 7+2 & 9+0 \\ -2+1 & 1+3 & 1+1 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$(A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$\boxed{\therefore (A+B)' = A' + B'}$$

7) compute the following

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

sol:- $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

8) Find the values of x, y & z i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

ii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

sol:- By using equility of matrix

i) $x = 1 \quad y = 4 \quad z = 3$

ii) $x + y + z = 9 \rightarrow ①$

$x + z = 5 \rightarrow ②$

$y + z = 7 \rightarrow ③$

eq③ − eq② ⇒

$$\begin{aligned} y + \cancel{z} &= 7 \\ \underset{(-)}{x} \underset{(-)}{+} \cancel{z} &= \underset{(-)}{5} \\ \hline y - x &= 2 \rightarrow ④ \end{aligned}$$

eq② − eq④

$$\begin{aligned} \cancel{x} + z &= 5 \\ -\cancel{x} + y &= 2 \\ \hline y + z &= 7 \rightarrow ⑤ \end{aligned}$$

eq① − eq⑤ ⇒

$$\begin{aligned} x + y + z &= 9 \\ y + z &= 7 \\ \hline x &= 2 \end{aligned}$$

$\boxed{\begin{aligned} x &= 2 \\ y &= 4 \\ z &= 3 \end{aligned}}$

put $x = 2$ in eq ②

$2 + z = 5$

$\boxed{z = 3}$

Put $z = 3$ in ③

$y + 3 = 7$

$\boxed{y = 4}$

9) Find x & y if $x+y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$ & $x-y=\begin{bmatrix}3 & 6\\ 0 & 1\end{bmatrix}$

10) Find x, if $y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$ & $2x+y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$

11) If $x\begin{bmatrix}2\\ 3\end{bmatrix}+y\begin{bmatrix}-1\\ 1\end{bmatrix}=\begin{bmatrix}10\\ 5\end{bmatrix}$. Find the values of of x & y

9 sol:- Given

$$eq(1) \Rightarrow x+y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}$$

$$eq(2) \Rightarrow x-y=\begin{bmatrix}3 & 6\\ 0 & 1\end{bmatrix}$$

$$2x=\begin{bmatrix}8 & 8\\ 0 & 10\end{bmatrix}$$

$$x=\begin{bmatrix}4 & 4\\ 0 & 5\end{bmatrix}$$

sub x in eq(2)

$$\cancel{x=\begin{bmatrix}3 & 6\\ 0 & 1\end{bmatrix}+\begin{bmatrix}4 & 4\\ 0 & 5\end{bmatrix}}$$

$$\cancel{x=\begin{bmatrix}7 & 10\\ 0 & 6\end{bmatrix}}$$

$$y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}-\begin{bmatrix}4 & 4\\ 0 & 5\end{bmatrix}$$

$$y=\begin{bmatrix}1 & -2\\ 0 & 4\end{bmatrix}$$

10 sol:- Given

$$y=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix} \rightarrow (1)$$

$$2x+y=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix} \rightarrow (2)$$

sub eq(1) in eq(2)

$$2x+\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$2x=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}-\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}$$

$$2x=\begin{bmatrix}-2 & -2\\ -4 & -2\end{bmatrix}$$

$$x=\begin{bmatrix}-1 & -1\\ -2 & -1\end{bmatrix}$$

sol:- Given

$$x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$$

$$\begin{bmatrix}2x-y\\3x+y\end{bmatrix} = \begin{bmatrix}10\\5\end{bmatrix}$$

$2x - \cancel{y} = 10 \rightarrow ①$

$3x + \cancel{y} = 5 \rightarrow ②$

(2)

$5x = 15$

$\boxed{x = 3}$

sub $x=3$ in eq ①

$2x - y = 10$

$2(3) - y = 10$

$6 - y = 10$

$\boxed{y = -4}$

$$\boxed{\therefore x = 3 \text{ \& } y = -4}$$

11-19-24

Q) If $A = \begin{bmatrix}2 & 1 & 4\\7 & 3 & 6\end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix}6 & 4 & 3\\3 & 2 & 5\\7 & 3 & 1\end{bmatrix}_{3\times3}$ find AB

sol:

$$AB = \begin{bmatrix}12+3+28 & 8+2+12 & 6+5+4\\42+9+42 & 28+6+18 & 21+15+6\end{bmatrix}$$

$$AB = \begin{bmatrix}43 & 22 & 15\\93 & 52 & 42\end{bmatrix}_{2\times3}$$

Q) If $A = \begin{bmatrix}2 & -3 & 4\\0 & 5 & 1\end{bmatrix}$ $B = \begin{bmatrix}1 & 6\\-5 & 0\\5 & -2\end{bmatrix}$ & $C = \begin{bmatrix}8 & -1\\3 & 1\\0 & 6\end{bmatrix}$

show that $A(B+C) = AB + AC$

sol:

$$B + C = \begin{bmatrix}1 & 6\\-5 & 0\\5 & -2\end{bmatrix} + \begin{bmatrix}8 & -1\\3 & 1\\0 & 6\end{bmatrix} \Rightarrow \begin{bmatrix}9 & 5\\-2 & 1\\5 & 4\end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 2 & -3 & -4 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 9 & 5 \\ -2 & 1 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 18+6+20 & 10-3+16 \\ 0+(-10)+5 & 0+5+4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 44 & 23 \\ -5 & 9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ -5 & 0 \\ 5 & -2 \end{bmatrix}$$

$$= \begin{matrix} 2+15+20 & 12+0-8 \\ 0 \overset{-25}{\cancel{+10}} + 25 & 0+0-2 \end{matrix}$$

$$AB = \begin{bmatrix} 37 & 4 \\ 20 & -2 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 8 & -1 \\ 3 & 1 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 16-9+0 & -2-3+24 \\ 0+15+0 & 0+5+6 \end{bmatrix} = \begin{bmatrix} 7 & 19 \\ 15 & 11 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 37 & 4 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 7 & 19 \\ 15 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 44 & 23 \\ 20 & +9 \end{bmatrix}$$

$$\therefore A(B+C) = AB + AC$$

Q)

Sol

$A^2 -$

Q)

Sol:

Q) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. Find $A^2 - 4A - 5I = 0$

Sol

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$4A = 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$$5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 4A - 5I = 0$ // Hence proved

Q) The book shop of a particular shool has 10 dozen chemistry books & 8 dozen of physics books, 10 dozen of economics books. Then selling price are RS.80, RS.60 RS 40 respectively. Find the total amount of the book shop will receive from selling all the books using matrix.

Sol:- There are 10 dozen chemistry = 120

physics = 96

economic = 120

$$\begin{bmatrix} 120, & 96 & 120 \end{bmatrix} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} = 120 \times 80 + 96 \times 60 + 120 \times 40$$

$$= 20,160$$

Amt received by selling price = RS 20,160

Determinent :- Let 'A' be a square matrix of order 'n' with entries of real (or) complex number then it is associated with number called Determinent of matrix 'A'. (or). $|A|$, and is denoted $|A|$ / $\det A$ / Δ.

1) Evaluate $\begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$

Sol:- $|A| = \begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} = 3 - 10 = -7$

$|A| = -7$

2) $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ Find det A.

Sol:- $\det A = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1$

$\det |A| = 1$

3) Evaluate $\begin{bmatrix} x & x+1 \\ x-1 & x \end{bmatrix}$

Sol:- $\det A = \begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix} = (x+1)(x-1) - x^2$

$\det A = x^2 + 1 - x^2$

$\det A = 1.$

4) $A = \begin{bmatrix} 2 & 5 \\ -1 & 4 \end{bmatrix}$ find det A

Sol:- $\det A = \begin{vmatrix} 2 & 5 \\ -1 & 4 \end{vmatrix} = 8 - (-5) = 13$

5) Evaluate $A = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 0 & 4 \\ 3 & 2 & 9 \end{bmatrix}$. find $|A|$

Sol:- $|A| = 1(0 - 8) - 3(-9 - 12) + 5(-2 + 0)$

$= -8 - 3(-21) + 5(-2)$

ler
it
f
1.

$$|A| = -8 + 863 - 10$$

$$|A| = 63 - 18$$

$$|A| = 45$$

6) Evaluate $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

Sol:- $|A| = 3(1+6) + 4(1+4) + 5(3-2)$

$= 3(7) + 4(5) + 5(1)$

$= 21 + 20 + 5$

$|A| = 46$

7) $A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{bmatrix}$ find det A by expanding

i) first row ii) first column

Sol:- i) $|A| = 3(4+6) - 2(0+2) + 1(0-1)$

$|A| = 3(10) - 2(2) + 1(-1)$

$|A| = 30 - 4 - 1$

$|A| = 25$

ii) $|A| = 3(4+6) - 0 + 1(-4-1)$

$|A| = 30 - 5$

$|A| = 25$

8) If $\begin{vmatrix} a & 5 \\ -8 & 4 \end{vmatrix} = 0$ find a ?

$a4 + 40 = 0$

$4a = -40$

$\boxed{a = -10}$

9) Find the value of x if $\begin{vmatrix} 1 & 4 & 5 \\ 2 & x & 0 \\ 3 & 3 & 8 \end{vmatrix} = 0$. 12)

Sol:-

$1(8x-0)-4(16-0)+5(6-3x)=0$

$8x-64+30-15x=0$

$-7x=34$ 13)

$x=\dfrac{-34}{7}$

Sol:-

10) $\begin{vmatrix} x & 2 & 1 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = -12$ find x?

Sol:- $x(45-48)-2(36-42)+1(32-35)=-12$

$-3x-2(-6)+1(-3)=-12$

$-3x-12+-3=-12$

$-(3x+15)=-12$

$3x=12-15$

$x=-1$

$1(32-35)-6(8x-14)+9(5x-8)=-12$

$-3-48x+84+45x-72=-12$

$-3-3x+12=-12$

$-3x=-12-9$

$\boxed{x=7}$

11) So

11) find x if $\begin{vmatrix} x & 4 & 7 \\ 4 & 1 & x \\ -5 & -4 & 2 \end{vmatrix} = -40$

$x(2+4x)-4(8+5x)+7(-16+5)=-4$

-77

$2x+4x^2-32-20x \cancel{-112+35}=-40$

$4x^2-18x-109=-40$

$4x^2-18x$

$4x^2-18x-105=0$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\dfrac{18 \pm \sqrt{(18)^2-4(4)(105)}}{2(4)}=0$

$18 \pm \sqrt{}$

12) Find x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Sol:- $10 - 12 = 5x - 6x$

$-2 = -x$

$\boxed{x = 2}$

13) If $A = \begin{bmatrix} 1 & 2 \\ 5 & 9 \end{bmatrix}$ then show that $|2A| = 4 \cdot |A|$

Sol:- $|A| = 9 - 10 \Rightarrow |A| = -1$

$4|A| = -4$

$2A = \begin{bmatrix} 2 & 4 \\ 10 & 18 \end{bmatrix}$

$|2A| = 36 - 40$

$|2A| = -4$

$\boxed{|2A| = 4|A|}$ // Hence proved

8) = -12

= -12

11) Sol:- $x(2 + 4x) - 4(8 + 5x) + 7(-16 + 5) = -4$

$2x + 4x^2 - 32 - 20x + 7(-11) = -40$

$4x^2 - 18x - 32 - 77 = -40$

$4x^2 - 18x - 109 + 40 = 0$

$4x^2 - 18x - 69 = 0$

$\Rightarrow \dfrac{18 \pm \sqrt{18^2 - 4(4)(-69)}}{2(4)}$

$\Rightarrow \dfrac{18 \pm \sqrt{1428}}{8}$

$\Rightarrow \dfrac{18 \pm 37.7}{8}$

$\dfrac{18 + 37.7}{8}$ & $\dfrac{18 - 37.7}{8}$

$x = 6-2.4$ & $x = 6.9$

x

Minors :- Minor of an element a_{ij} of a determinent is a determinent obtained by deleting its i-row & j-column in which element a_{ij} lies.
Minor of an element a_{ij} is demoted by M_{ij}

Problem

1) $A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$M_{11} = 8 \qquad M_{12} = 5$

$M_{21} = 4 \qquad M_{22} = 3$

1) Find Minor of element 7 in the determinent

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Sol:-

$$M_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 12 - 15$$

$$M_{31} = -3$$

11-20-24

Co-Factor :- It is a signed minor, Let det A = det a_{ij} (or) $|A| = |[a_{ij}]|$ be a determinent of order 'n' then the co-factor of element a_{ij} and is denoted by A_{ij} and defined as $A_{ij} = (-1)^{i+j} M_{ij}$

1) Find the cofactor of elements of a matrix $\begin{bmatrix} 2 & 1 & \cancel{3} \\ 3 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

$(-1)^{i+j} M_{ij}$

Sol:-

$$A_{11} = 2 \Rightarrow (-1)^{1+1} \begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} \Rightarrow +(-1+2) \Rightarrow 1$$

$$A_{12} = 1 \Rightarrow (-1)^{1+2} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} \Rightarrow (-1)(3-4) \Rightarrow 1$$

$$A_{13} = -2 \Rightarrow (-1)^{1+3} \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} \Rightarrow (1)(-3+2) \Rightarrow -1$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix}1 & -3\\ -1 & 1\end{vmatrix} \Rightarrow (-1)(1-3) \Rightarrow 2$$

$$A_{22} = -1 \Rightarrow (-1)^{2+2}\begin{vmatrix}2 & -3\\ 2 & 1\end{vmatrix} \Rightarrow (1)(2+6) = 8$$

$$A_{23} = 2 \Rightarrow (-1)^{2+3}\begin{vmatrix}2 & 1\\ 2 & -1\end{vmatrix} \Rightarrow (-1)(-2-2) = 4$$

$$A_{31} = 2 \Rightarrow (-1)^{3+1}\begin{vmatrix}1 & -3\\ -1 & 2\end{vmatrix} \Rightarrow (1)(2-3) = -1$$

$$A_{32} = -1 \Rightarrow (-1)^{3+2}\begin{vmatrix}2 & -3\\ 3 & 2\end{vmatrix} \Rightarrow (-1)(4+9) = -13$$

$$A_{33} = 1 \Rightarrow (-1)^{3+3}\begin{vmatrix}2 & 1\\ 3 & -1\end{vmatrix} \Rightarrow (1)(-2-3) = -5$$

$$\text{cofactor }(A) = \begin{vmatrix}1 & 1 & -1\\ 2 & 8 & 4\\ -1 & -13 & -5\end{vmatrix}$$

2) Find the minors & cofactors of the elements of the determinent $\begin{vmatrix}2 & -3 & 5\\ 6 & 0 & 4\\ 1 & 5 & -7\end{vmatrix}$ and verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$

Sol:

$$A_{31} = 1 \Rightarrow (-1)^{3+1}\begin{vmatrix}-3 & 5\\ 0 & 4\end{vmatrix} \Rightarrow (1)(-12) \Rightarrow -12$$

$$A_{32} = 5 \Rightarrow (-1)^{3+2}\begin{vmatrix}2 & 5\\ 6 & 4\end{vmatrix} \Rightarrow (-1)(8-30) \Rightarrow 22$$

$$A_{33} = -7 \Rightarrow (-1)^{3+3}\begin{vmatrix}2 & -3\\ 6 & 0\end{vmatrix} \Rightarrow (1)(0+18) \Rightarrow 18$$

$$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$$

$$2(-12) + (-3)(22) + 5(18) = 0$$

$$-24 - 66 + 90 = 0$$

LHS = RHS //

Adjoint of a Matrix

* The adjoint of a matrix 'A' is the transpose of cofactor matrix and denoted by "Adj A"

Problems

1) Find the 'Adjoint of A' (or) 'Adj A' $= \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

Sol:

$$A_{ij} = (-1)^{i+j} M_{ij}$$

$M_{11} = 3$ $\quad A_{11} = (-1)^{1+1}(3) = 3$

$M_{12} = 5$ $\quad A_{12} = (-1)^{1+2}(5) = -5$

$M_{21} = 1$ $\quad A_{21} = (-1)^{2+1}(1) = -1$

$M_{22} = 2$ $\quad A_{22} = (-1)^{2+2}(2) = 2$

$$\text{cofactor}(A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

2) Find the adjoint of $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & -1 & 5 \\ 1 & 2 & 3 \end{bmatrix}$ and verify $|A|(\text{adj} A) = |A| I$

Sol:

$A_{11} = 2 \Rightarrow (-1)^{1+1} \begin{vmatrix} -1 & 5 \\ 2 & 3 \end{vmatrix} = (1)(-3-10) \Rightarrow -13$

$A_{12} = -1 \Rightarrow (-1)(12-5) \Rightarrow -7$

$A_{13} = 3 \Rightarrow (1)(8+1) \Rightarrow 9$

$A_{21} = 4 \Rightarrow (-1)(-3-6) \Rightarrow 9$

$A_{22} = -1 \Rightarrow (1)(6-3) \Rightarrow 3$

$A_{23} = 5 \Rightarrow (-1)(4+1) \Rightarrow -5$

$A_{31} = 1 \Rightarrow (1)(-5+3) \Rightarrow -2$

$A_{32} = 2 \Rightarrow (-1)(10-12) \Rightarrow 2$

$A_{33} = 3 \Rightarrow (1)(-2+4) \Rightarrow 2$

of

~~cofactor(A)~~ = ~~-7 9 9 3 -5~~

$$cofactor(A) = \begin{bmatrix} -13 & -7 & 9 \\ 9 & 3 & -5 \\ -2 & 2 & 2 \end{bmatrix} \qquad Adj(A) = \begin{bmatrix} -13 & 9 & -2 \\ -7 & 3 & 2 \\ 9 & -5 & 2 \end{bmatrix}$$

$$|A| = 2(\cancel{-3} \cancel{+} 10) + 1(12-5) + 3(8+1)$$
$$= 2(-13) + 1(7) + 3(9)$$
$$= -26 + 7 + 27$$
$$|A| = 8$$

$$(A)(adj\,A) = \begin{bmatrix} 2 & -1 & 3 \\ 4 & -1 & 5 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -13 & 9 & -2 \\ -7 & 3 & 2 \\ 9 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -26+7+27 & 18-3-15 & -4-2+6 \\ -52+7+45 & 36-3-25 & -8-2+10 \\ -13-14+27 & 9+6-15 & -2+4+6 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = (A)\,adj A$$

$$|A|\,I = 8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$\therefore (A)\,adj A = |A|(I)$ // Hence proved.

<u>Inverse of Matrix</u> :- $A^{-1} = \frac{Adj(A)}{|A|}$

* Let 'A' be a square matrix of order n and if there exists a matrix B of order n×n such that AB = BA = I where I is the identity matrix of

of ordered 'n'. Then a matrix B is called the inverse of A & is denoted by $A^{-1} = B$. If it exits & is given by $A^{-1} = Adj(A)$.

1) Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

sol:- $|A| = \begin{vmatrix} 2 & -1 \\ 3 & -2 \end{vmatrix} \Rightarrow -4+3$
$\Rightarrow -1$

coeff $A_{11} = -2$, $A_{12} = -3$, $A_{21} = -(-1) = 1$, $A_{22} = 2$.

coeff $= \begin{bmatrix} -2 & -3 \\ 1 & 2 \end{bmatrix}$

$Adj(A) = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$. $\quad A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$

2) Find the inverse of $\begin{bmatrix} 2 & -4 \\ -3 & 5 \end{bmatrix}$

sol:- Given $\begin{vmatrix} 2 & -4 \\ -3 & 5 \end{vmatrix} \Rightarrow 10-12$
$\Rightarrow -2$

cofactor $= \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$ $\quad Adj(A) = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \frac{Adj(A)}{|A|} \Rightarrow \begin{bmatrix} -5/2 & -2 \\ -3/2 & -1 \end{bmatrix}$

3) Find the inverse of $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix} = A$

sol:- $|A| = 1(-28+30) - 0 - 1(-18)$
$= 2+18 \Rightarrow 20$

$A_{11} = 1 \Rightarrow (-28+30) \Rightarrow 2$
$A_{12} = 0 \Rightarrow (-21-0) \Rightarrow +21$
$A_{13} = -1 \Rightarrow (-18-0) \Rightarrow -18$
$A_{21} = 3 \Rightarrow (0-6) \Rightarrow +6$
$A_{22} = 4 \Rightarrow (-7+0) \Rightarrow -7$
$A_{23} = 5 \Rightarrow (-6-0) \Rightarrow +6$
$A_{31} = 0 \Rightarrow (0+4) \Rightarrow 4$
$A_{32} = -6 \Rightarrow (5+3) \Rightarrow +8$
$A_{33} = -7 \Rightarrow (4-0) \Rightarrow 4$

$$\text{cofactor}(A) = \begin{bmatrix} 2 & +21 & -18 \\ +6 & -7 & -6 \\ 4 & 8 & 4 \end{bmatrix} \qquad Adj(A) = \begin{bmatrix} 2 & +6 & 4 \\ +21 & -7 & 8 \\ -18 & -6 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{20}\begin{bmatrix} 2 & +6 & 4 \\ +21 & -7 & 8 \\ -18 & -6 & 4 \end{bmatrix}$$

Rank of Matrix :- A matrices 'A' is said to be rank of 'r' when atleast one non-zero minor of order 'r'.

* The rank of matrix is denoted by $\rho(A)$

1) Find the rank of the matrix $A = \begin{bmatrix} 1 & 5 \\ 3 & 9 \end{bmatrix}$

Sol:-

$$|A| = \begin{vmatrix} 1 & 5 \\ 3 & 9 \end{vmatrix} \Rightarrow 9 - 15 \Rightarrow -6 \neq 0$$

$\because |A| \neq 0$, then row of matrix is 2

$\therefore \rho(A) = 2$ //

Problems

1) Illimitate the cemetric rule :- $x + 2y + 3z = 1$
$-x + 2z = 2$
$-2y + z = -2$

Sol:-

$$|\Delta| = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & -2 & 1 \end{vmatrix} = 1(4) - 2(-1) + 3(2) = 4 + 2 + 6 \Rightarrow 12$$

$$|\Delta x| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 2 \\ -2 & -2 & 1 \end{vmatrix} = 1(4) - 2(2+4) + 3(-4) = 4 - 12 - 12 = -20 //$$

$$|\Delta y| = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 2 & 2 \\ 0 & -2 & 1 \end{vmatrix} = 1(2+4) - 1(-1) + 3(2) = 6 + 1 + 6 = 13 //$$

$$x = \left|\frac{\Delta x}{\Delta}\right| = \frac{-20}{12} = \frac{-5}{3}$$

$$y = \left|\frac{\Delta y}{\Delta}\right| = \frac{13}{12}$$

$$|\Delta z| = \begin{vmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 0 & -2 & -2 \end{vmatrix} \quad \begin{aligned} &= 1(4) - 2(2) + 1(2) \\ &= 4 - 4 + 2 \\ &= 2. \end{aligned}$$

$$\Delta z = \left|\frac{\Delta z}{\Delta}\right| = \frac{20}{12} \Rightarrow \frac{1}{6}$$

P) Solve using matrix method,
$$4(x+y) = 5z - 22$$
$$3z + 4x = 6y + 2$$
$$z - 3y = 14 - 10x$$

Sol:-
$$-4x + 4y - 5z = -22$$
$$4x - 6y + 3z = 2$$
$$10x - 3y + z = 14$$

$$|A| = \begin{vmatrix} -4 & 4 & -5 \\ 4 & -6 & 3 \\ 10 & -3 & 1 \end{vmatrix} \quad \begin{aligned} &= -4(-6+9) - 4(4-30) - 5(-12+60) \\ &= -4(3) - 4(-26) - 5(48) \\ &= -12 + 104 - 240 \\ &= -148 \end{aligned}$$

$$|\Delta x| = \begin{vmatrix} -22 & 4 & -5 \\ 2 & -6 & 3 \\ 14 & -3 & 1 \end{vmatrix} \quad \begin{aligned} &\Rightarrow -22(-6+9) - 4(2-42) - 5(-6+84) \\ &\Rightarrow -22(3) - 4(40) - 5(78) \\ &\Rightarrow -66 - 160 - 390 \\ &\Rightarrow -616 \end{aligned}$$

$$|\Delta y| = \begin{vmatrix} -4 & -22 & -5 \\ 4 & 2 & 3 \\ 10 & 14 & 1 \end{vmatrix} \quad \begin{aligned} &\Rightarrow -4(2-42) + 22(4-30) - 5(56-30) \\ &\Rightarrow -4(40) + 22(-26) - 5(26) \\ &\Rightarrow -160 + 572 - 130 \\ &\Rightarrow 572 - 290 \\ &\Rightarrow 282 \end{aligned}$$

$$|\Delta y| = \begin{vmatrix} -4 & 4 & -22 \\ 4 & -6 & 2 \\ 10 & -3 & 14 \end{vmatrix} \Rightarrow -4(-84+6)-4(56+60) -22(-12+60)$$

$$\Rightarrow -4(-78)-4(116)-22(48)$$

$$\Rightarrow 312 - 464 - 1056$$

$$\Rightarrow -1208$$

$$x = \left|\frac{\Delta x}{\Delta}\right| = \frac{\cancel{+616}^{\cancel{38}\,19}}{\cancel{+148}_{\cancel{74}\,37}} = \frac{19}{37}$$

$$y = \left|\frac{\Delta y}{\Delta}\right| = \frac{\cancel{282}^{141}}{-148} = \frac{141}{-74}$$

$$z = \left|\frac{\Delta z}{\Delta}\right| = \frac{\cancel{-}\cancel{+1208}^{\cancel{604}\,302}}{\cancel{-}\cancel{148}_{\cancel{78}\,39}} = \frac{302}{39}$$

Matrix Method :-
1) Consistent – one (or) more solutions
2) Inconsistent – No solution.

Q) Examine the consistency

i) $\left.\begin{aligned} x + 2y &= 2 \\ 2x + 3y &= 3 \end{aligned}\right\} \rightarrow$ Inconsistent

ii) $\left.\begin{aligned} 5x - y + 4z &= 5 \\ 2x + 3y + 5z &= 2 \\ 5x - 2y + 6z &= -1 \end{aligned}\right\} \rightarrow$ Consistent

ii) Sol:- Above the given equation written in matrix form.

$$\boxed{Ax = B}$$

$$\begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix} = 5(18+10)+1(12-25)+4(-4-15)$$

$$= 5(28) + (-13) + 4(-19)$$

$$= 140 - 13 - 76 \Rightarrow 51 \neq 0$$

Since, $|A| \neq 0$, A is non-sigular

Hence the given system of equation is consistancy.

Q) Solve the matrix method $5x+2y=4$
$7x+3y=5$

Sol:- Above the eq written in matrix ~~m~~ form

$$Ax = B$$

$$\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} \Rightarrow 15-14 \Rightarrow 1 \neq 0$$

$\therefore |A| \neq 0$, A is non-sigular matrix

A^{-1} is exists

Solving the matrix method $x = A^{-1}B$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$\text{Adj } A = (\text{co-fact})^T$

$A_{11} = 3$ $\quad A_{21} = -2$

$A_{12} = -7$ $\quad A_{22} = 5$

$$\text{cof} = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$X = \begin{bmatrix} 12 & -10 \\ -28 & +25 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

$\therefore x = 2$ & $y = -3$

Q)

Sol:-

Eg. Q) Solve

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

Sol:- Above eq is written in matrix form $\boxed{AX = B}$

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix} \begin{aligned} &= 3(2-3) + 2(4+4) + 3(-6-4) \\ &= 3(-1) + 2(8) + 3(-10) \\ &= -3 + 16 - 30 \\ &= -17 \neq 0 \end{aligned}$$

$$|A| = 17 \neq 0$$

A is non-sigular

A^{-1} is exists

Solve the matrix method $X = A^{-1}B$

$$A^{-1} = \frac{Adj\,A}{|A|} \qquad Adj\,A = (cofact)^T$$

$A_{11} = 3 \Rightarrow (2-3) = -1$

$A_{12} = -2 \Rightarrow (4+4) = 8 \Rightarrow -8$

$A_{13} = 3 \Rightarrow (-6-4) = -10$

$A_{21} = 2 \Rightarrow (-4+9) = 5 \Rightarrow -5$

$A_{22} = 1 \Rightarrow (6-12) = -6$

$A_{23} = -1 \Rightarrow (-9+8) = -1$

$A_{31} = 4 \Rightarrow (2-3) = -1$

$A_{32} = -3 \Rightarrow (-3-6) = -9$

$A_{33} = 2 \Rightarrow (3+4) = 7$

$$cof = \begin{vmatrix} -1 & 8 & -10 \\ 5 & -6 & -1 \\ -1 & -9 & 7 \end{vmatrix} \Rightarrow (cof)^T = \begin{vmatrix} -1 & 5 & -1 \\ -8 & -6 & +9 \\ -10 & -1 & 7 \end{vmatrix} = Adj\,A$$

$$A^{-1} = \frac{1}{-17}\begin{bmatrix} 1 & 5 & -1 \\ 8 & -6 & -9 \\ -10 & -1 & 7 \end{bmatrix}$$

$$X = \frac{1}{-17}\begin{bmatrix} 1 & 5 & -1 \\ 8 & -6 & -9 \\ -10 & -1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$X = \frac{1}{-17}\begin{bmatrix} 8+5-4 \\ 64-6-36 \\ -80-1+28 \end{bmatrix}$$

$$X = \frac{1}{-17}\begin{bmatrix} 9 \\ 22 \\ -53 \end{bmatrix}$$

$$\therefore x = \frac{-9}{17}, \quad y = \frac{-22}{17} \quad \& \quad z = \frac{53}{17}$$

Q) Solve $2x_1 + 3x_2 = 3$

$5x_1 + 4x_2 = 11$

Gause Elimination Method

11-27-24

System of L.E Using Gause Elimination Method

Def :- It is a systematic procedure for solving system of Linear equation, it transforms a given system of equation into an equivalent one in triangular form, Make it easier to solve by back substution, it's a fundamental technique in Linear algebra and numerical analysis.

1) Solve the following system of L.E by using the gause elimination method $x+y+2z=2$, $x+2y+3z=5$, $2x+3y+4z=11$

sol:- Given L.E are

$$x+y+z=2 \rightarrow (1)$$
$$x+2y+2z=5 \rightarrow (2)$$
$$2x+3y+4z=11 \rightarrow (3)$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 11 \end{bmatrix} R_2 = R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 2 & 3 & 4 & 11 \end{bmatrix} R_3 = R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 7 \end{bmatrix} R_3 = R_3 - R_2$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$\therefore$ There is no solution it is inconsistence

2) Solve the following system of L.E by using the gause elimination method $4y + 2z = 1$

$$2x + 3y + 5z = 0$$

$$3x + y + z = 11$$

Sol:-

$$\begin{bmatrix} 0 & 4 & 2 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 1 & 1 & 11 \end{bmatrix} \quad R_2 \leftrightarrow R_1$$

$$\begin{bmatrix} 2 & 3 & 5 & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{bmatrix} \quad R_1 \Rightarrow \frac{1}{2} R_1$$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{bmatrix} \quad R_3 \to R_3 - 3R_1$$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 4 & 2 & 1 \\ 0 & -\frac{7}{2} & -\frac{13}{2} & 11 \end{bmatrix} \quad R_3 \Rightarrow 4R_3 + \frac{7}{2} R_2$$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 4 & 2 & 1 \\ 0 & 0 & -19 & \frac{95}{2} \end{bmatrix}$$

$$x + \frac{3}{2} y + \frac{5}{2} z = 0$$

$$4y + 2z = 1$$

$$-19z = 95/2$$

$$z = \frac{95}{2 \times -19}$$

$$z = \frac{-5}{2}$$

$\frac{19 \times 2}{38}$

$\frac{95}{38}$

$eq - 2 \Rightarrow 4y + 2z = 1$

$$4y + 2\left(\frac{-5}{2}\right) = 1$$

$$4y + (-5) = 1$$

$$4y = 1 + 5$$

$$\boxed{y = \frac{3}{2}}$$

put $y = \frac{3}{2}$, $z = \frac{-5}{2}$ in eq (1)

$$x + \frac{3}{2} \cdot \frac{3}{2} + \frac{5}{2}\left(\frac{-5}{2}\right) = 0$$

$$x + \frac{9}{2} - \frac{25}{2} = 0$$

$$x = \frac{25}{2} - \frac{9}{2}$$

$$\boxed{x = 8}$$

$\therefore$ $x = 8$, $y = \frac{3}{2}$ & $z = \frac{-5}{2}$ //

3) Solve the system of equation by using the gauss elimination method $2x + y + z = 7$, $3x + 2y + z = 12$, $x - 3y + 2z = -5$.

sol:-

$$\begin{bmatrix} 2 & 1 & 1 & 7 \\ 3 & 2 & 1 & 12 \\ 1 & -3 & 2 & -5 \end{bmatrix} R_1 \Leftrightarrow R_3$$

$$\begin{bmatrix} 1 & -3 & 2 & -5 \\ 3 & 2 & 1 & 12 \\ 2 & 1 & 1 & 12 \end{bmatrix} R_2 \to R_2 - 3R_1$$

$$\begin{bmatrix} 1 & -3 & 2 & -5 \\ 0 & 11 & -5 & 27 \\ 2 & 1 & 1 & 12 \end{bmatrix} R_3 \to R_3 - 2R_1$$

$$\left[\begin{array}{cccc} 1 & -3 & 2 & -5 \\ 0 & 11 & -5 & 27 \\ 0 & 7 & -3 & 22 \end{array}\right] \quad R_3 \to 11R_3 - 7R_2$$

$-33+35$

$242-189$

$$\left[\begin{array}{cccc} 1 & -3 & 2 & -5 \\ 0 & 11 & -5 & 27 \\ 0 & 0 & 2 & 53 \end{array}\right]$$

$$x - 3y + 2z = -5$$

$$11y - 5z = 27$$

$$2z = 53$$

$$\boxed{z = 53/2}$$

put $z = 53/2 \Rightarrow 11y - 5(53/2) = 27$

$$11y - \frac{265}{2} = 27$$

$$11y = 27 + \frac{265}{2}$$

$$11y = \frac{54+265}{2}$$

$$y = \frac{54+265}{2 \times 11} \Rightarrow \frac{319}{22}$$

$$\boxed{y = \frac{29}{2}}$$

put $y = \frac{29}{2}$ & $z = \frac{53}{2}$ in eq (1)

$$x - 3y + 2z = -5$$

$$x - 3\left(\frac{29}{2}\right) + \cancel{2}\left(\frac{53}{\cancel{2}}\right) = -5$$

$$x - \frac{87}{2} + 53 = -5$$

$$x = \frac{87}{2} - 58$$

$$x = \frac{87-116}{2}$$

$$\boxed{x = \frac{-29}{2}}$$

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Probability :- The probability is the measure of the (in head of an event to happen if measure of the certainity of the event. The formula for probability is given by,

$$\boxed{P(E) = \frac{n(E)}{n(S)}}$$

where, $n(E)$ = No. of event for countable for event
$n(S)$ = Total no of outcomes.

Sample Space :- It is a collection of all possible outcomes of a randam experiment mathemalically the sample space is denoted by the symbol 'S'.

Events :- Events are the possible outcomes of a trail/ experiment which are generally denoted by capital letters A, B, C etc.

Forex :- Appearance of the outcomes 1 (or) 2 (or) 3 (or) 4 (or) 5 (or) 6 are called events on the rolling of a fair dice.

Types of Events :- There are various kind of events in probability. ~~They~~ are defined below;

Independent & Dependent Events

* If the occurance of any event is completely unaffected by the occurance of any other event. Such events are known as an Independent event.
* In probability and the events which are affected by other events are known as dependent events.

Module 5 :- Discrete Probability

1) In a ~~through~~ throw of a coin find the probability of getting head.

Sol:- $S = \{H, T\}$ $P(E) = \frac{P(B)}{P(S)}$

$E = \{H\}$ $= \frac{1}{2}$

2) Two unbiased coins are tossed. What is the probability of getting atmost one head?

Sol:- $S = \{(H,T)\ (H,H)\ (T,H)\ (T,T)\}$

$E = \{(H,T)\ (T,H)\ (T,T)\}$

$P(E) = \frac{P(E)}{P(S)} = \frac{3}{4}$

3) Unbiased dye is tossed. Find the probability of getting a multiple of '3'.

Sol:- $S = \{1, 2, 3, 4, 5, 6\}$ $E = \{3, 6\}$

$P(E) = \frac{nP(E)}{nP(S)} = \frac{2}{6} = \frac{1}{2}$

4) In a simultaneous throw a pair of ~~dies~~ dice find the probability of getting a total more than 7

Sol:- $n(S) = 6 \times 6 = 36$

$n(E) = \{(2,6)\ (3,5)\ (3,6)\ (4,4)\ (4,5)\ (4,6)\ (5,3)\ (5,4)\ (6,2)\ (5,5)\ (5,6)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$P(E) = \frac{n(S)}{n(E)} = \frac{15}{36} = \frac{5}{12}$ //

5) Two dice are thrown together, what is the probability that then sum of the numbers on the two dices is divisible by 4 (or) 6

Sol:- $n(S) = 6 \times 6 = 36$

$E = \{ (1,3)\ (1,5)\ (2,2)\ (2,4)\ (2,6)\ (3,1)\ (3,3)\ (3,5)$
$(4,2)\ (4,4)\ \cancel{(4,6)}\ (5,1)\ (5,3)\ (6,2)\ (6,6) \}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}$$

6) Two unbiased coins are ~~tossed~~ simultaneously solve to find the probability of getting;

i) Exactly one head

ii) No Tail

iii) Two Tails

iv) Atleast one tail

v) Atmost one tail

$$P(E) = \frac{n(E)}{n(S)}$$

Sol:- $n(S) = \{ (H,H)\ (H,T)\ (T,H)\ (T,T) \}$

i) $\frac{2}{4} = \frac{1}{2}$ ii) $\frac{1}{4}$ iii) $\frac{1}{4}$ iv) $\frac{3}{4}$ v) $\frac{3}{4}$

7) A bag contains 6-white balls & 4-black balls two balls are drawn at random find the probability that they are of same colour.

Sol!- S = sample space

$n(S)$ = no. of balls $= {}^{10}C_2 = \frac{10!}{(10-2)!\,2!} = \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!} \times \cancel{2!}}$

$= 45$ //

the

$n(E) = $ (2 balls out of 6) $\cancel{\times}$ + (2 balls out of 4)

$= {}^6C_2 \ \cancel{\times} + \ {}^4C_2$

(5,4)

$= 15 + 6$

$= 21$

$$P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15} //$$

8) Two cards are drawn at random from 52-pack, what is the probability either both are black or both are queens.

$n(S) = {}^{52}C_2 = 1326$

${}^{52}C_2$

${}^{26}C_2 = n(A)$

${}^{4}C_2 = n(B)$

${}^{2}C_2$

Sol:- $n(A)$ = Event of getting both black cards $= {}^{26}C_2$

$n(B)$ = Event of getting 2 queens $= {}^{4}C_2$

$n(A \cap B)$ = Event of getting 2 black queens $= {}^{2}C_2$

ty

divisibl

$= {}^{26}C_2 + {}^{4}C_2 - {}^{2}C_2$ $\quad n(A) = 325$

$= \frac{\cancel{26}^{13} \times 25 \times \cancel{24!}}{\cancel{24!}\,\cancel{2!}} + \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{2!\,\cancel{2!}} - 2$ $\quad n(B) = 6$

$n(A \cap B) = 1$

$= 325 + 6 - 2 \Rightarrow 329$

$$p(A) = \frac{n(A)}{n(S)} = \frac{325}{1326}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326} \qquad P(C) = \frac{n(C)}{n(S)} = \frac{1}{1326}$$

$$P(A \cup B) = \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}$$

$$P(A \cup B) = \frac{330}{1326} = \frac{55}{221}$$

12-03-24

P) Suppose 3-bulbs are selected at random from a lawn. each bulb is tested and classified as defective ~~and~~ or non-defective, illustrate the sample space of this experiment. $\boxed{2^3 = 8}$ $\Rightarrow$ The total no. of possible outcome

Sol:- The sample space,

$S = \{DDD, DDN, DND, NDD, NND, NDN, DNN, NNN\}$

$\boxed{\text{Note:- should explain all}}$

DDD = All 3-bulbs are defective.

DDN = 2-bulbs are defective & 1 - non. defective

NNN = All 3 bulbs are non-defective.

NND = 2-bulbs are non-defective & 1 - defective.

P) A letter is chosen at random from the word "ASSASSINATION". Solve to calculate the probability that letter is 1) A vowels 2) A consonent.

Sol:- Total letters of the given word = 13

vowels = $\frac{6}{13}$ $\qquad$ consonents = $\frac{7}{13}$ //

vowels = A, A, I, A, I, O $\qquad$ consonents = S, S, S, S, N, T, N

Probability of chosing a vowels = $\frac{6}{13}$

" " " Consonents = $\frac{7}{13}$

$$\boxed{\frac{\text{Total no. of vowels}}{\text{Total no. of letters}}}$$

Addition Theorem

St :- For any two events 'A' & 'B' if there are not mutually exclusive, then probability of "$A \cup B$"

A union B $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

P) A card is drawn from a well shuffled playing cards of playing cards what is the probability that it is either spade (or) king.

<u>Sol:-</u> Let, A be the event card is spade $\left[P(A) = \frac{13}{52}\right]$

B = the event card is king $\left[P(B) = \frac{4}{52}\right]$

$$P(A \cap B) = \frac{1}{52}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} \Rightarrow \frac{\cancel{16}\ \cancel{8}\ 4}{\cancel{52}\ \cancel{26}\ 13} = \frac{4}{13}$$

P) The probability that a student will pass the final exam in both English & Hindi is 0.5 and the probability of passing in neither is 1. If the probability passing in english exam is 0.75. Solve to find what is the probability of passing the hindi is ?

<u>Sol:-</u> Given

$P(E \cap H) = 0.5$ $\qquad$ $P(E) = 0.75$

$P(E^c \cap H^c) = 0.1$

$P(E \cup H) = P(E) + P(H) - P(E \cap H) \rightarrow$ probability of union of Two events.

$\therefore P(E \cup H) + P(\overline{E \cup H}) = 1 \rightarrow$ <u>Total probabily rule</u>

$P(E \cup H) + P(E^c \cap H^c) = 1$

$P(E \cup H) + 0.1 = 1$

$P(E \cup H) = 0.9$

Now

$0.9 = 0.75 + P(H) - 0.5$

$P(H) = 0.9 - 0.75 + 0.5$

$P(H) = 0.65$

P) A card is drawn at random from a standard pack of 52 - cards find the probability of getting,

i) A Jack (or) A Queen ~~of~~ (or) A ace.

ii) ~~Two~~ 2's of spades or diamonds.

Sol:- i) $\text{Jack} = \frac{4C_1}{52C_1} = \frac{4}{52} = \frac{1}{13}$

$$\text{Queen} = \frac{4C_1}{52C_1} = \frac{4}{52} = \frac{1}{13}$$

$$\text{Ace} = \frac{4C_1}{52C_1} = \frac{4}{52} = \frac{1}{13}$$

$$P(J \cup Q \cup A) = P(J) + P(Q) + P(A)$$

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} \Rightarrow \frac{3}{13}$$

ii) $\frac{1}{52} + \frac{1}{52} = \frac{2}{52}$

$$\Rightarrow \frac{1}{26} \;//$$

Multiplication Theorem

If "A & B" are any two events which are not independent.

$P(A \cap B) = P(A) \cdot P(B/A)$ if 'A' preceeds 'B'

(or)

$P(A \cap B) = P(B) \cdot P(A/B)$ if 'B' preceeds 'A'

Proof :- Let n = Total no. of outcomes of a random experiment

$A, B, A \cap B$ = The no. of favaurable outcomes

It is denoted by $M_A, M_B, M_{A.B}$

i) for A/B

M_B = Total no. of outcomes

M_{AB} = no. of favourable outcomes

$$\therefore P(A/B) = \frac{M_{AB}}{M_B}$$

simplify $P(B/A) = \frac{M_{AB}}{M_A}$

Consider $P(A\cap B) = \frac{m_{AB}}{n}$

$= \frac{m_{AB}}{n} \cdot \frac{m_A}{m_A}$

$= \frac{m_{AB}}{m_A} \cdot \frac{m_A}{n}$

$P(A\cap B) = P(B/A) \cdot P(A)$

Further $P(A\cap B) = \frac{m_{AB}}{n} \cdot \frac{m_B}{m_B}$

$= \frac{m_{AB}}{m_B} \cdot \frac{m_B}{n}$

$P(A\cap B) = P(A/B) \cdot P(B)$

Hence proved

put $n = 3$ - events

$$\boxed{P(A\cap B\cap C) = P(A) \cdot P(B/A) \cdot P(C/A\cap B)}$$

P) Consider the experiment of rolling a dice let 'A' be the event getting a prime number and 'B' be the event getting an odd number. Write the sets representing the events, i) A (or) B ii) A & B iii) A but not B iv) not A

Sol:- Sample space

$S = \{1, 2, 3, 4, 5, 6\}$

prime Numbers $A = \{2, 3, 5\}$

odd Numbers $B = \{1, 3, 5\}$

i) $A\cup B = \{1, 2, 3, 5\}$

ii) $A\cap B = \{3, 5\}$

iii) $A - B = \{2\}$

iv) $A^c / \bar{A} = S - A = \{1, 4, 6\}$

12/04/24

Conditional Probability

* Let the probability of events for all ~~be~~ 'B' ~~then~~ be +ve of the conditional probability 'A' they provided 'B' as occured equal the probability of 'AB' divided by the probability of 'B'.

$$P(A/B) = \frac{P(AB)}{P(B)} \quad \text{where } (B > A)$$

$$P(AB) = P(B) \cdot P(A/B)$$

$$P(B/A) = \frac{P(AB)}{P(A)}$$

$$P(AB) = P(A) \cdot P(B/A)$$

If A_1, A_2, A_3 events

$$P(A_1, A_2, A_3) = P(A_1)\; P(A_2/A_1)\; P(A_3/A_2 \cdot A_1)$$

$$P(A_1, A_2 \ldots A_n) = P(A_1)\; P(A_2/A_1) \ldots P(A_n/A_1 \cdot A_2 \ldots A_{n-1})$$

be

pres

not B

P) A servey reveals that 70% of population watch sports and 40% of population watch both sports & news, if a person is selected randomly from this population. What is the probability that they watch news given that they watch sports.

Sol:

$P(S)$ = Probability of watching sports = 0.7

$P(S \cap N)$ = Probability " " sports & News = 0.4

probability of N/S

$$P(N/S) = \frac{P(S \cap N)}{P(S)} = \frac{0.4}{0.7}$$

$$P(N/S) = 0.571$$

Probability of watch ~~sp~~ News is 0.571 (or) 57.1%

P) A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike and 0.32 that the construction job will be completed on time if there is a strike. Solve to find the probability that the construction job will be completed on time.

Sol:- To find the probility that the construction job will be completed on time = $P(c)$

we use the Low of total probability

$P(c) = P(c/s)\,P(s) + P(c/s')\,P(s')$

$P(s) = 0.65$

$P(\bar{s}) = 1 - P(s) = 1 - 0.65 = 0.35$

$P(c/s)$ = probability that the job will be completed on time given as time.

$\therefore P(c/s) = 0.32$

$P(c/\bar{s})$ = Probability that the job will be completed on time given no since.

$P(c/\bar{s}) = 0.80$

$P(c) = P(c/s)\,P(s) + P(c/\bar{s})\,P(\bar{s})$

$P(c) = (0.32)(0.65) + (0.80)(0.35)$

$= 0.488$

$\therefore$ The probability that the construction job will be completed on time is 0.488 (or) 48.8%.

P) If $P(A) = 0.2$, $P(B/A) = 0.8$ and $P(A \cup B) = 0.3$ then find $P(B)$ value (or) $P(B) = ?$

Sol:- Given $P(A) = 0.2$

$P(B/A) = 0.8$

$P(A \cup B) = 0.3$

P)

Sol:-

$$P(B/A) = \frac{P(A \cup B)}{P(A)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A)\, P(A) = P(A \cap B)$$

$$(0.8)(0.2) = P(A \cap B)$$

$$\cancel{1.6}$$

$$0.16 = P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.3 = 0.2 + P(B) - 0.16$$

$$0.3 - 0.2 + 0.16 = P(B)$$

$$\boxed{P(B) = 0.26}$$

P) A coin is tossed 3 - times find the conditional probability $P(C/B)$ where C = head on 1st toss and D = Tail on 2nd Toss.

Sol:

Baye's theorem

* If $E_1, E_2, --- E_n$ are ~~evalu~~ exhaustive & mutually disjoint event with $P(E_i) \neq 0$., $i = 1, 2, ---n$ then for any arbitary event A which is a subset of $\bigcup_{i=1}^{n} E_i$ Each that $P(A) > 0$ then
$$P(E_i/n) = \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} (P(E_i) \cdot P(A/E_i)}$$

Proof :- Given $A \subset \bigcup_{i=1}^{n} E_i$

we can write it as

$$A = A \cap \left(\bigcup_{i=1}^{n} E_i\right)$$

$$A = \bigcup_{i=1}^{n} A \cap E_i$$

Taking probability on both sides

$$P(A) = P\left[\bigcup_{i=1}^{n} A \cap E_i\right]$$

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

$$P(A) = \sum_{i=1}^{n} P(E_i)\,P(A/E_i)$$

||'y $$P(B) = \sum_{i=1}^{n} P(E_i)\,P(B/E_i)$$

Consider $P(A \cap E_i) = P(A) \cdot P(E_i/A)$

$$P(E_i/A) = \frac{P(A \cap E_i)}{P(A)}$$

$$= \frac{P(E_i)\,P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

Consider, $P(B \cap E_i) = P(B) \cdot P(E_i/B)$

$$P(E_i/B) = \frac{P(B \cap E_i)}{P(B)}$$

$$= \frac{P(E_i)\, P(B/E_i)}{\sum_{i=1}^{n} P(E_i)\, P(B/E_i)}$$

Hence Proved

P) In a factory which manifactures bolts mechain A, B, C manifactures respectively 25%, 35% & 40% of the bolts, of their output, 5%, 4%, & 2% are respectively defective bolts. A bolt is draw random from the product and it further to be defective. Solve to find what is the probability that it is manifacture by the machine B?

Sol:- $E_1 = \frac{25\%}{100}$ $P(E_1)$ = Bolts from 'A' $= \frac{25}{100}$

$P(E_2)$ = Bolts from B $= \frac{35}{100}$

$P(E_3)$ = Bolts from C $= \frac{40}{100}$

P(Defective bolts from 'A') $= P(A/E_1) = 5/100$

P(Defective bolts from 'B') $= P(A/E_2) = 4/100$

$= P(A/E_3) = 2/100$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1)\, P(A/E_1) + P(E_2)(P(A/E_2)) + P(E_3)\, P(A/E_3)}$$

$$= \frac{35/100 \times 4/100}{\frac{25}{100}\left(\frac{5}{100}\right) + \frac{35}{100}\left(\frac{4}{100}\right) + \frac{40}{100}\left(\frac{2}{100}\right)}$$

$$= \frac{140/10000}{125+140+180/10000}$$

$$= \frac{140}{\cancel{10000}} \times \frac{\cancel{10000}}{345}$$

$$= \frac{140}{345} \Rightarrow \frac{28}{69}$$

$\therefore$ The probability by that is manifactured by machine - B is " $\frac{28}{69}$ " //

P) A man is known to speak truth 3 out of 4 times, he throws a dice and report that its 6. Find the proba-bility that it's ~~ex~~ actually a '6'.

Sol:- In a throw of a dice, E_1 = Event of getting a 6

E_2 = Event of not getting a 6

$$P(E_1) = \frac{1}{6} \qquad P(E_2) = 1 - \frac{1}{6} \Rightarrow \frac{5}{6}$$

$P\left(\frac{E}{E_1}\right)$ = Probability ~~of getting~~ that man reparts 6 occurs, when 6 is actually occured.

Probability that the man speaks true $= \frac{3}{4}$

$P\left(\frac{E}{E_2}\right)$ = The probability that the man report 6 occurs, when 6 has not actually occured.

$$= 1 - \frac{3}{4} \Rightarrow \frac{1}{4}$$

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)}$$

$$= \frac{\frac{1}{6}\left(\frac{3}{4}\right)}{\frac{1}{6}\left(\frac{3}{4}\right) + \frac{5}{6}\left(\frac{1}{4}\right)} \Rightarrow \frac{3/24}{\frac{3}{24} + \frac{5}{24}}$$

$$\Rightarrow \frac{3}{\cancel{24}} \times \frac{\cancel{24}}{8}$$

$$\Rightarrow \frac{3}{8} \;//$$

12-09-24

1) A person has undertaken a construction job the probability are 0.65 that there will be a stright the construction job will be combleled on time 0.80 if there is no straight and 0.32 that the construction job will be completed on. if there is strick. the construction job will Solve to find probability that the construction job will be completed.

; he
oba

a 6

a b

2) In answering a question multiple choose test a student either knows the answer or guses. Let 3/4 be the probability that he know the answer & 1/4 be the probabitity that he gusses, assuming that a the student. who gusses that answer will correct the probability. 1/4. Solve to find what is the probability that std knows the answers given that he answers is correct.

Sol:- By using baye's theorem.

$$p(A/B) = \frac{P(B/A)\,P(A)}{P(B)}$$

P(A) = The student knows the answer = 3/4

P(A^c) = Student knows the answers correctly = 1/4

P(B)/A = If the student knows the answer the correctly & efficiently = 1.