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Q1 $$f(x_1, \theta_1, \theta_2) = \frac{1}{\sqrt{\theta_2} \times \sqrt{2\pi}} e^{-\frac{(x_i - \theta_1)^2}{2\theta_2}}$$

Likelihood function

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} (x_i, \theta_1, \theta_2) = \theta_2^{-n/2} (2\pi)^{-n/2} e^{\left[\frac{-1}{2\theta_2} \Sigma (x_i - \theta_1)^2\right]}$$

$$\log L(\theta_1, \theta_2) = \frac{-n}{2} \log \theta_2 - \frac{n}{2} \log 2\pi - \Sigma \frac{(x_i - \theta_1)^2}{2\theta_2}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_1} = -2 \Sigma \frac{(x_i - \theta_1)(-1)}{2\theta_2} = 0$$

$$\Sigma x_i - n\theta_1 = 0$$

$$\hat{\theta}_1 = \hat{\mu} = \frac{\Sigma x_i}{n} = \bar{x}$$

$$\frac{\partial \log L(\theta_1, \theta_2)}{\partial \theta_2} = \frac{-n}{2\theta_2} + \Sigma \frac{(x_i - \theta_1)^2}{2\theta_2^2} = 0$$

$$-n\theta_2 + \Sigma (x - \theta_1)^2 = 0$$

$$\hat{\theta}_2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

$$\theta_1 = \frac{\Sigma x_i}{n}, \quad \theta_2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

Q2 $P(y|\theta) = Bin(y, m, \theta)$

$= {}^{m}C_{y} \times \theta^{y} [1-\theta]^{m-y}$

- log likelyhood function = $\log \theta(y|\theta)$

$$ll(\theta) = \log {}^{m}C_{y} + y\log\theta + (m-y) \log(1-\theta)$$

~~w.r.t~~ Differentiating w r t θ

$$\frac{d\, ll(\theta)}{d\theta} = \frac{y}{\theta} - \left(\frac{m-y}{1-\theta}\right) = 0$$

$$\frac{y}{\hat{\theta}} - \frac{m-y}{1-\hat{\theta}} = 0$$

$$(m-y)(\hat{\theta}) = y(1-\hat{\theta})$$

$$m\hat{\theta} - y\hat{\theta} = y - y\hat{\theta}$$

$$m\hat{\theta} = y$$

$$\hat{\theta} = \frac{y}{m}$$