Linear algebra [see from pg 4]

* vectors [Lists of numbers]
* The process of stretching, reversing a vector. ex:- $1.8\vec{v}$, $\frac{1}{3}\vec{v}$, $-2\vec{v}$ is called Scaling.

$\hat{i}$ and $\hat{j}$ are the "basic vectors" of the xy co-ordinate system.

* linear combination of $\vec{v}$ & $\vec{w}$
  $a\vec{v} + b\vec{w}$

The "span" of $\vec{v}$ & $\vec{w}$ is the set of all their linear combinations.

$\vec{u} = a\vec{v} + b\vec{w}$ [linearly dependent]
$\vec{u} \neq a\vec{v} + b\vec{w}$ [linearly independent]

* Two rules ~~from~~ for linear transformation:-
1) line shoud remain line.
2) origin fixed

$$\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

where $\hat{i}$ lands   where $\hat{j}$ lands

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \text{shear matrix}$$

* Associativity:-

$$(AB)C = A(BC)$$

* $|AB| = |A||B|$
* Rank:- number of dimensions in the output.

3 basis vector

$$\begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \end{bmatrix}$$ } 2 coordinates for each landing spots

3d ~~output~~ input & output in 2d.

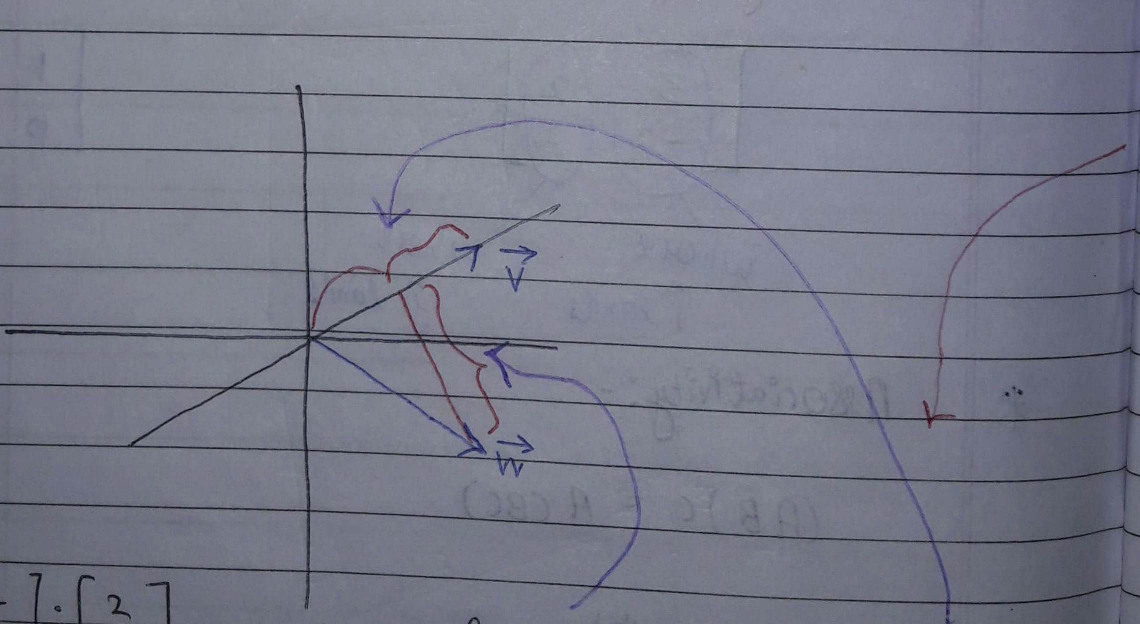
2 basis vector

$$\begin{bmatrix} 2 & 0 \\ -1 & 1 \\ -2 & 1 \end{bmatrix}$$ } 3 coordinates for each landing spots

2D space & 3D output

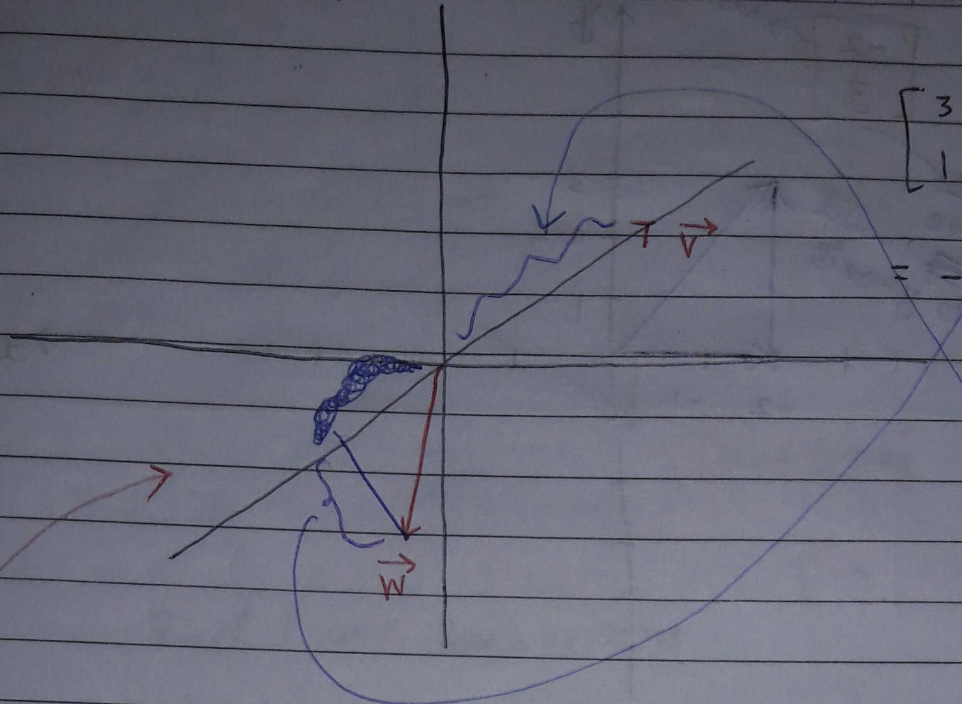
* $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 1\cdot 3 + 2\cdot 4 = 11$

Dot product



$\underbrace{\begin{bmatrix} 4 \\ 1 \end{bmatrix}}_{\vec{v}} \cdot \underbrace{\begin{bmatrix} 2 \\ -1 \end{bmatrix}}_{\vec{w}}$ = (length of projected $\vec{w}$)(length of $\vec{v}$)

$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$



$\begin{bmatrix} 3 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$= -$ (length of projected $\vec{w}$) $\cdot$ (length of $\vec{v}$)

$\vec{v} \cdot \vec{w} > 0$ [They are in same direct^n]

$\vec{v} \cdot \vec{w} = 0$ [Perpendicular]

$\vec{v} \cdot \vec{w} < 0$ [Opposite direc^n]

* Cross product

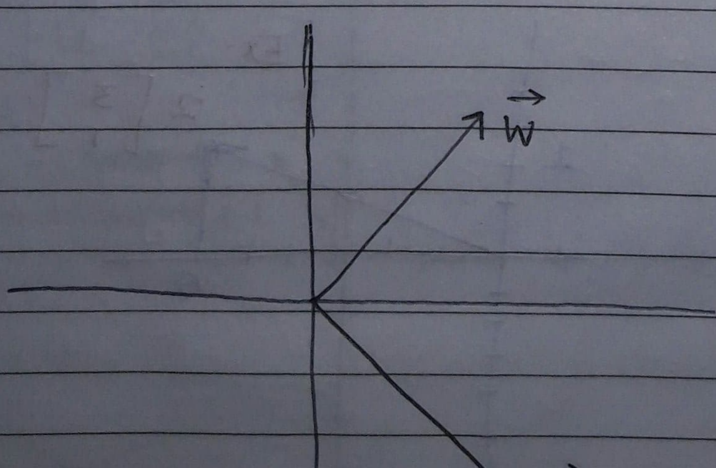
$\vec{v} \times \vec{w} = - \vec{w} \times \vec{v}$

$(3\vec{v}) \times \vec{w} = 3(\vec{v} \times \vec{w})$

[Direc^n of cross product using right hand thumb rule]

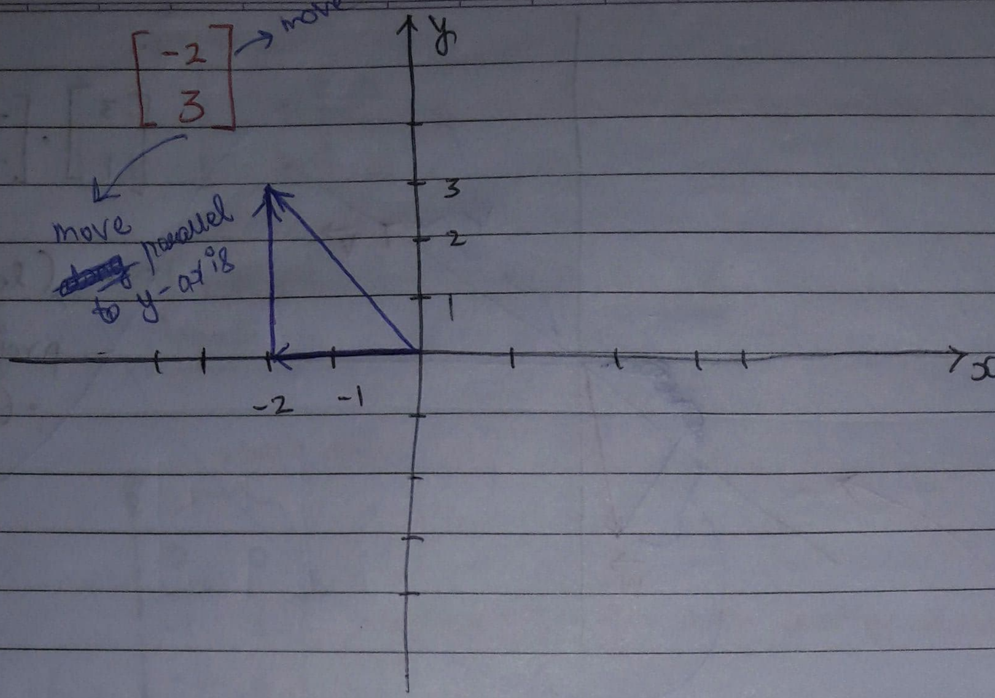
[Two vectors as forefinger & middle finger]

Direct of cross product



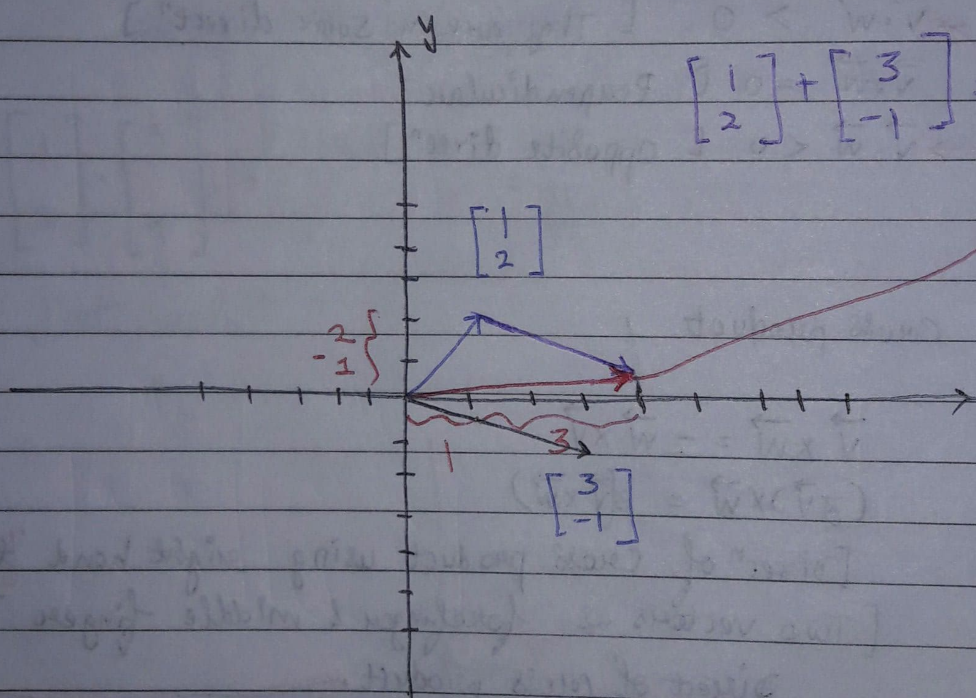
$\vec{v} \times \vec{w}$ = +ve as $\vec{v}$ is on left of $\vec{w}$

*



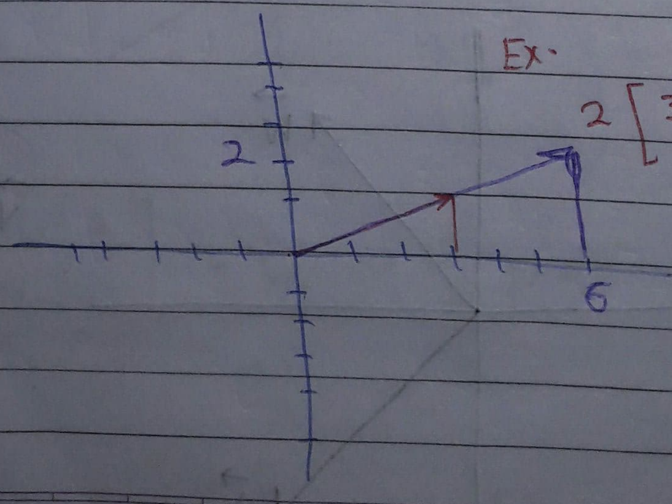
Vector addition

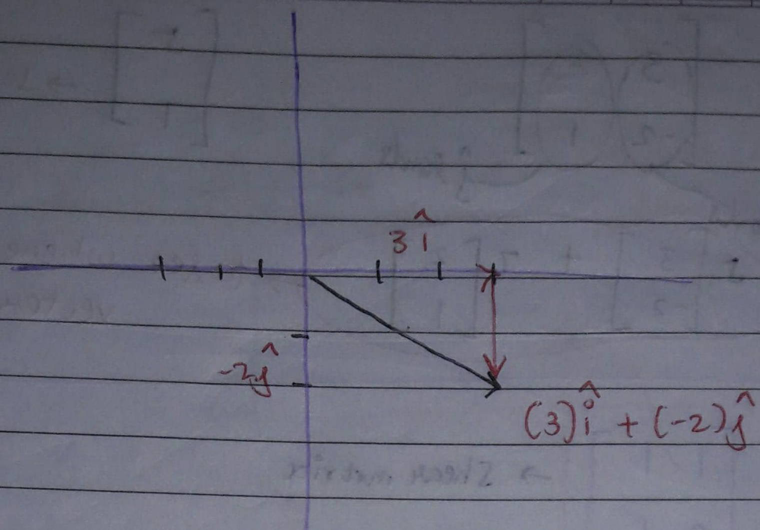
$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$



Vector multiplication.

Ex. $2\begin{bmatrix} 3 \\ 1 \end{bmatrix}$



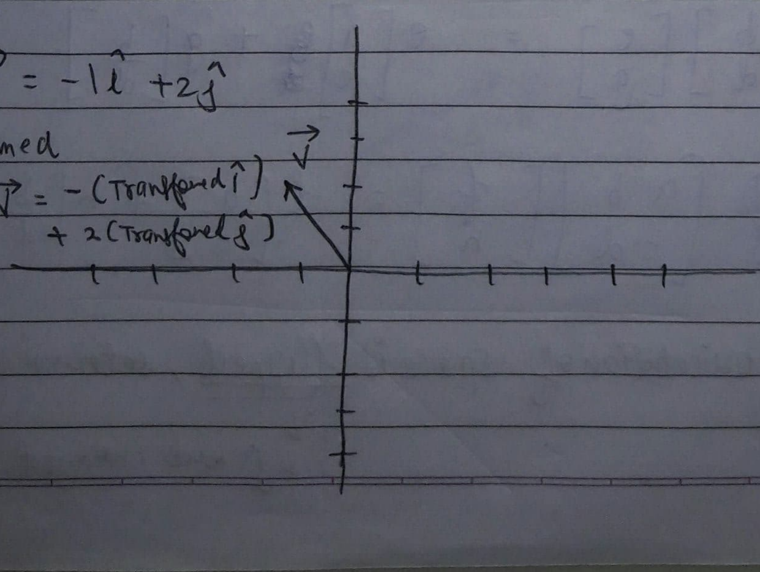


$\hat{i}$ and $\hat{j}$ are basis vectors.

* If one vector is fixed and other is movable tip of resulting vector draws straight line
* If both vectors are free, every possible pt you can reach.
* when two vectors are in same line, you can move in that line only.
* Linear transformation
  (function) [Recieves input & gives output]

$\vec{V} = -1\hat{i} + 2\hat{j}$

Transformed $\vec{V}$ = $-$(Transformed $\hat{i}$) + 2(Transformed $\hat{j}$)



$$\begin{bmatrix} 3 & 2 \\ -2 & 1 \end{bmatrix}$$

$\hat{i}$ lands $\hat{j}$ lands

$\begin{bmatrix} 5 \\ 7 \end{bmatrix}$ → Some specific vector

$5\begin{bmatrix} 3 \\ -2 \end{bmatrix} + 7\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ → to see where vector lands.

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ → Shear matrix

* If you first apply rotation then shear ←

$$\underset{\text{Shear}}{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}} \underset{\text{Rotation}}{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}} = \underset{\text{Composition}}{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}$$

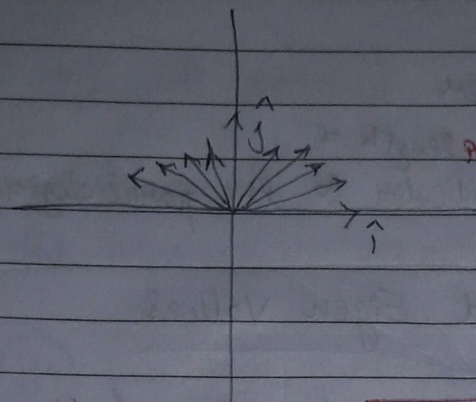
$$\underset{M_2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \underset{M_1}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \left[e\begin{bmatrix} a \\ c \end{bmatrix} + g\begin{bmatrix} b \\ d \end{bmatrix} \quad f\begin{bmatrix} a \\ c \end{bmatrix} + h\begin{bmatrix} b \\ d \end{bmatrix} \right]$$

$\hat{i}$

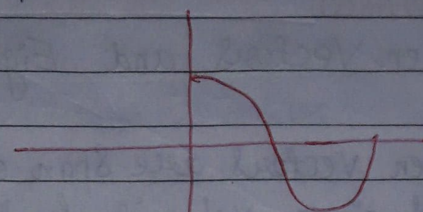
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e \\ g \end{bmatrix} = e\begin{bmatrix} a \\ c \end{bmatrix} + g\begin{bmatrix} b \\ d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} f \\ h \end{bmatrix} = f\begin{bmatrix} a \\ c \end{bmatrix} + h\begin{bmatrix} b \\ d \end{bmatrix}$$

* Whenever orientation of space is flipped, determinant is negative
↓
$\hat{i}$ & $\hat{j}$ are changed.



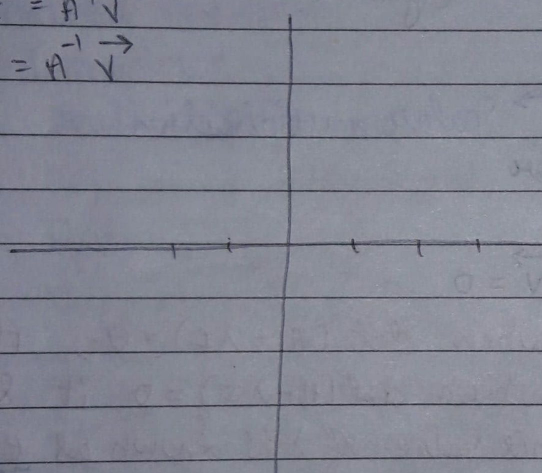
As $\hat{i}$ becomes closer to $\hat{j}$ determinant value decreases



* $A\vec{x} = \vec{v}$

$A^{-1}A\vec{x} = A^{-1}\vec{v}$

$\vec{x} = A^{-1}\vec{v}$

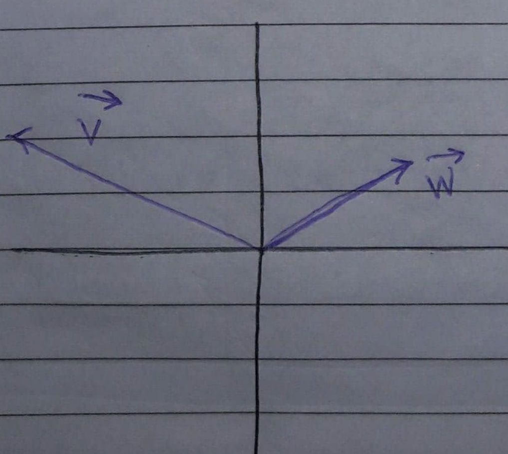


Set of all possible outputs of $A\vec{v}$

column space

$$\underbrace{\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}}_{\text{span of columns}}$$

column space



$\vec{v} \times \vec{w} = \det\left(\begin{bmatrix} -3 & 2 \\ 1 & 1 \end{bmatrix}\right)$

$= \underline{-5}$

($\because \vec{v}$ is on right of $\vec{w}$)

(Right hand thumb rule)

$$\vec{V} \times \vec{W} = \vec{P}$$

vector
with length x
perpendicular to the parallelogram

* Eigen Vectors and Eigen Values

→ Eigen vectors are span of vectors of the transformation and eigen value is factors by which it is stretched or squished during transformation.

$$A\vec{v} = \lambda\vec{v}$$

eigen value

Transformatⁿ matrix

Eigen vector

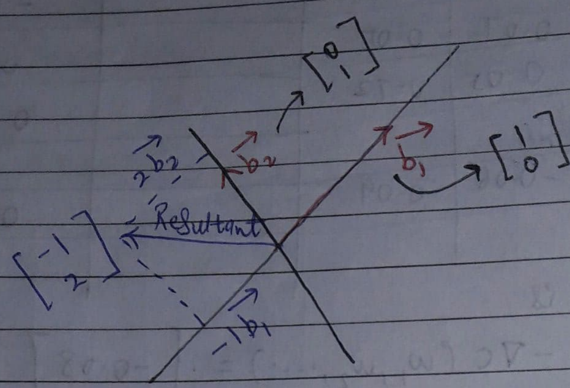
Scalar multiplication

$$(A - \lambda I)\vec{v} = 0$$

This is possible when $\det(A - \lambda I) = 0$, There is a value λ for which $\det(A - \lambda I) = 0$, it squishes space into a line. This value of λ is known as eigen value.

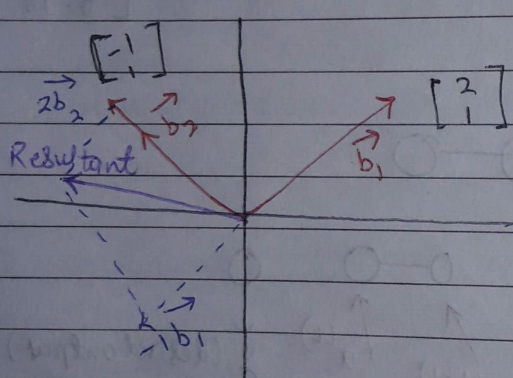
* Change of basis

In different system



⇓

In our system



$$-1\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}-1\\1\end{bmatrix}$$
$$=\begin{bmatrix}-4\\1\end{bmatrix}$$

This is same as,

Matrix multiplic^n :- $\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}-1\\2\end{bmatrix}$

Our grid $\longrightarrow$ other system's grid

Our language $\longleftarrow$ Different system's language

Now, Taking inverse

other system's grid $\longrightarrow$ our grid

other system's languge $\longleftarrow$ our language

$$\underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1}}_{\text{Inverse change of base matrix}} \underbrace{\begin{bmatrix} 3 \\ 2 \end{bmatrix}}_{\text{Any vector in our system}} = \underbrace{\begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix}}_{\text{Same vector in different system}}$$

* Now, To transform the vector in other co-ordinate system & vice versa.

$$\underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}^{-1} \overbrace{\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Transform}^n\text{ matrix in our lang.}} \underbrace{\underbrace{\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}}_{\text{change of base matrix}} \underbrace{\begin{bmatrix} -1 \\ 2 \end{bmatrix}}_{\text{Any vector in other system}}}_{\text{Some vector in our language}}}^{\text{Transformed vector in our language}}}_{\text{Transformed vector in other language}}$$

Inverse change of base matrix