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B.Tech Project - EP 3085

Leptonic Drell-Yan Process

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Aug-Nov 2019

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1 Introduction

The electron-positron annihilation process involves the collision of the two particles to produce a muon-antimuon pair. These processes typically happen at the TeV scale.
This process is analogous to the Drell-Yan process that occurs in hadron colliders where the colliding beams are protons(quark-gluon plasma) and they produce muon-antimuon pairs as a result. It could be talked of as a leptonic Drell-Yan process.

$$q + \bar{q} \to \gamma^* \to l^+ + l^-$$

$$e^+ + e^- \to \mu^+ + \mu^-$$

The process takes place through a virtual photon as is explained through the Feynmann Diagram for the same.

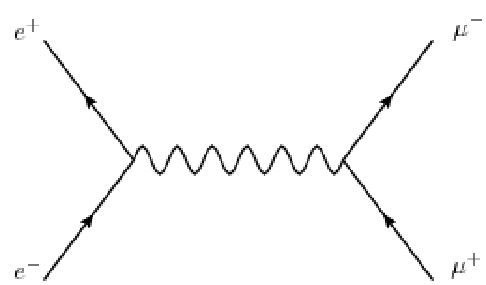

Figure 1: Feynman Diagram

2 Mandelstam Variables

For the process

$$A + B \rightarrow C + D$$

We have the Mandelstamm variables which capture some information of the collision

$$s = (\frac{p_a + p_b}{c})^2$$

$$t = (\frac{p_a - p_c}{c})^2$$

$$u = (\frac{p_a - p_d}{c})^2$$

... where $p_{a,b,c,d}$ are the four-momenta. These variables are Lorentz Invariants for any collision processes.

3 Cross Section

The cross-section of a process is essentially the probability of reaction between two colliding particles in a certain way.

$$\dot{N} = \mathcal{L}\sigma$$

Where $\dot{N}$ is the reaction rate, $\mathcal{L}$ is the luminosity of the collider and σ is the cross section of the reaction. The cross section can be mathematically derived for a certain process using the Feynman Rules for calculating the amplitude of the process from its Feynman Diagram(s). While the luminosity is a machine characteristics, the cross section is determined by the fundamental interaction properties of the particles in the initial and final states. Determining the reaction cross section and studying the scattering properties as a function of energy, momentum, and angular variables will be of ultimate importance to uncover new dynamics at higher energy thresholds[1].

Differential Cross Section

The differential cross section is the only experimentally measurable information from a process. It is a function of the final-state particles' variables.

4 Electron-Positron Case[2]

The only significant diagram for the process is:

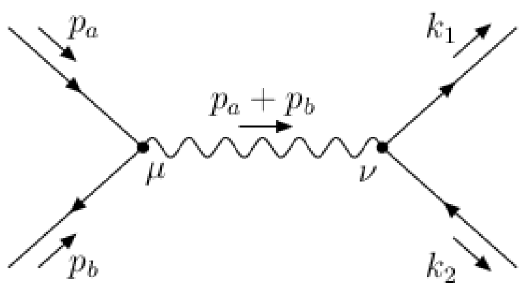

Figure 2: Electron-Positron Annihilation

Particle	Momentum	Spin	Spinor
e^-	p_a	s_a	$u(p_a, s_a)$
e^+	p_b	s_b	$\bar{v}(p_b, s_b)$
μ^-	k_1	s_1	$\bar{u}(k_1, s_1)$
μ^+	k_2	s_2	$v(k_2, s_2)$

The Feynman rules can then be applied to this QED process to find the reduced matrix element ($\mathcal{M}$) that can be used to find the cross section of the process.

$$d\sigma = |\mathcal{M}|^2 \frac{\left|\vec{k_1}\right|}{32\pi {E_{CM}}^2 \left|\vec{p_a}\right|} d(\cos\theta)$$

Using the rules, for the above process we find

$$\mathcal{M} = (\bar{v}(p_b, s_b)(ie\gamma^\mu)u(p_a, s_a)(\frac{-ig_{\mu\nu}}{s})\bar{u}(k_1, s_1)(ie\gamma^\nu)v(k_2, s_2))$$

where s is the mandelstam variable

$$s = (p_a + p_b)^2 = {E_{CM}}^2$$

Using tensor contraction and properties of the Dirac Spinors(u,v)

$$|\mathcal{M}|^2 = \frac{e^4}{s^2}(\bar{v_b}\gamma_\mu u_a)(\bar{u_a}\gamma_\nu v_b)(\bar{u_1}\gamma^\mu v_2)(\bar{v_2}\gamma^\nu u_1)$$

Here now if the final and initial state spins are not measured we are required to average over all possible spins and their combinations ($4^2 = 16$). Using the general form of the cross section equation and averaging in the CM frame

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{4}\sum_{s_a}\sum_{s_b}\sum_{s_1}\sum_{s_2} |\mathcal{M}|^2 \frac{\left|\vec{k_1}\right|}{32\pi s \left|\vec{p_a}\right|}$$

, whereby in the approximation for the electron-positron mass being negligible this equates to

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\sqrt{1-\frac{4m_\mu^2}{s}}\left[1+\frac{4m_\mu^2}{s}+(1-\frac{4m_\mu^2}{s})\cos^2\theta\right]$$

where *alpha* is the fine structure constant.

Polarized Scattering Cross Sections

An advantage of using the electron-positron scattering process is that the beams can be polarised by the collider to unveil different reactions. The hadron collision process can not be controlled this way since the proton is a composite mass and melts into its quark-gluon constituents. These can not be polarised independently. Thus the cross section derived previously may take different forms if we were to consider helicities/spin-states.
QED rules do not allow processes where the muon-antimuon pair and/or the electron-positron pair hold the same polarizations (e_L^+, e_L^- or μ_R^+, μ_R^-). Here, projection matrices ($P_{R/L}$) are used to force the spinors into being left/right handed.
Using the same working we can derive for $e_L^- e_R^+ \to \mu_L^- \mu_R^+$

$$|\mathcal{M}|^2 = \frac{e^4}{s^2}(\bar{v_b}\gamma^\mu P_L u_a)(\bar{u_a}P_R\gamma^\nu v_b)(\bar{u_1}\gamma_\mu P_L v_2)(\bar{v_2}P_R\gamma_\nu u_1)$$

and this can be summed over the spins since we have forced handedness on the particles and then substituted in the differential cross section formula,

$$\frac{d\sigma_{e_L^- e_R^+ \to \mu_L^- \mu_R^+}}{d(\cos\theta)} = \frac{\pi\alpha^2}{2s}(1+\cos\theta)^2$$

The same cross section is developed for $e_R^- e_L^+ \to \mu_R^- \mu_L^+$.
The cross section for $e_L^- e_R^+ \to \mu_R^- \mu_L^+$ and $e_R^- e_L^+ \to \mu_L^- \mu_R^+$ is then derived similarly to be

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{2s}(1-\cos\theta)^2$$

The rest of the combinations of polarizations vanish and thus the average cross section that remains is taken over the above four possibilities.

$$\frac{d\bar{\sigma}}{d\cos\theta} = \frac{\pi\alpha^2}{2s}(1+\cos^2\theta)$$

5 Verification of Cross Section

This result of the cross section can be verified through simulations. As had been pointed out earlier, the cross section is an experimentally measured quantity as well, being a function of the particle variables' (four-momentum).

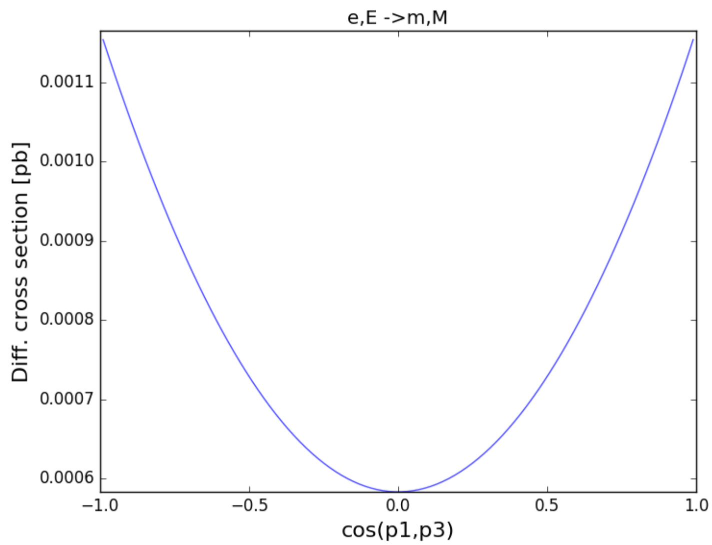

Figure 3: Cross Section Through CalcHEP

This is the accurate cross-section showing a $1+\cos^2\theta$ generated using CalcHEP at 4 TeV. The Large Hadron Collider runs at 14 TeV while the Tevatron ran at 1 TeV.

$$\cos\theta = \frac{\vec{p_a}.\vec{k_1}}{|p_a|\,|k_1|}$$

This relation can be used to plot the differential cross section.

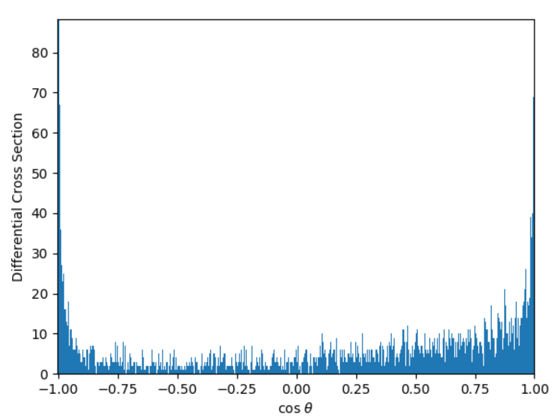

Figure 4: Cross Section Through PYTHIA

The above plot has been generated using events from PYTHIA[4] for 12 TeV with no beam polarization, and then using the dot product formula to find the 1-3 angle correlation with cross-section.

6 Conclusion

The Muon Pair Production through the Electron–Positron annihilation is a well-studied process for the very reasons shown above. The ability to polarize the beam in the collider can let us control the kind of reactions that take place and have different cross sections for the same too. It can also be interesting to study the points where the cross section vanishes non-trivially. This is being done at the SLAC and will serve as the basis for the ILC[5] Having symmetric beams between the electron and positron allows for the CM frame to be the same as the laboratory frame and thus allows maximum usage of the process.

References

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[3] A. Pukho. *CalcHEP 2.3: MSSM, structure functions, event generation, batchs, and generation of matrix elements for other packages (2004)*

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[5] G. Moortgat-Pick et. al *The role of polarized positrons and electrons in revealing fundamental interactions at the Linear Collider*arXiv:hep-ph/0507011