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Assignment - Parameter Estimation

Q-1) mean $\to \theta_1$
Variance $\to \theta_2$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} \cdot e^{-\frac{(x_i - \theta_i)^2}{2\theta_2}}$$

Take Logarithm

$$\log L(\theta_1, \theta_2) = -\frac{n}{2} \log(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_i)^2$$

for θ_1 differentiate $\log(L(\theta_1, \theta_2))$ w.r.t θ_1 and se[t] equal to zero.

$$\frac{\partial(\log(L))}{\partial \theta_2} = \frac{-n}{2\theta_2} + \frac{1}{2(\theta_2)^2} \sum_{i=1}^{n} (x_i - \theta_1)^2 = 0$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

Q2) $B<m, \theta>$ Binomial distribution
$m \to$ number of trials
$\theta \in (0,1)$ Prob of Success

$$L\theta = \prod_{i=1}^{n} f(x_i; n, \theta)$$

Pmf

$$f(x, n, \theta) = {}^{n}C_{x} \cdot \theta^{x} (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i} \cdot \theta^{x_i} (1-\theta)^{n-x_i}$$

Take logarithm

$$\log(L(\theta)) = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \sum_{i=1}^{n} x_i \log(\theta)$$
$$+ \sum_{i=1}^{n} (n-x_i) \log(1-\theta)$$

$$\frac{\partial \log(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i - \frac{1}{1-\theta} \sum_{i=1}^{n} (n-x_i) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{n} (n-x_i)$$

multiply both sides by θ & $(1-\theta)$

$$(1-\theta) \sum_{i=1}^{n} x_i = \theta \sum_{i=1}^{n} (n-x_i)$$

$$\sum_{i=1}^{n} x_i - \theta \sum_{i=1}^{n} x_i = \theta n - \theta \sum_{i=1}^{n} x_i$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{m}$$

MLE of θ for $B(n, \theta)$ is $\bar{x}$ where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$