EE2703 : Applied Programming Lab
Assignment 9
The Digital Fourier Transform

Dhruv Goyal, EE19B077

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1. Aim

The aim of this assignment is to analyse signals by computing their Discrete Fourier Transform (DFT) using the Fast Fourier Transform (FFT) algorithm.For this we can use the fft function offered by numpy.

2. The Accuracy of the `numpy.fft` Package

A random vector is taken, and it's Fourier transform is computed and then the result's inverse Fourier transform is computed. The maximum error between the actual and the result is obtained.

The maximum error came out to be of the order of $4.344095589800383e - 16$.

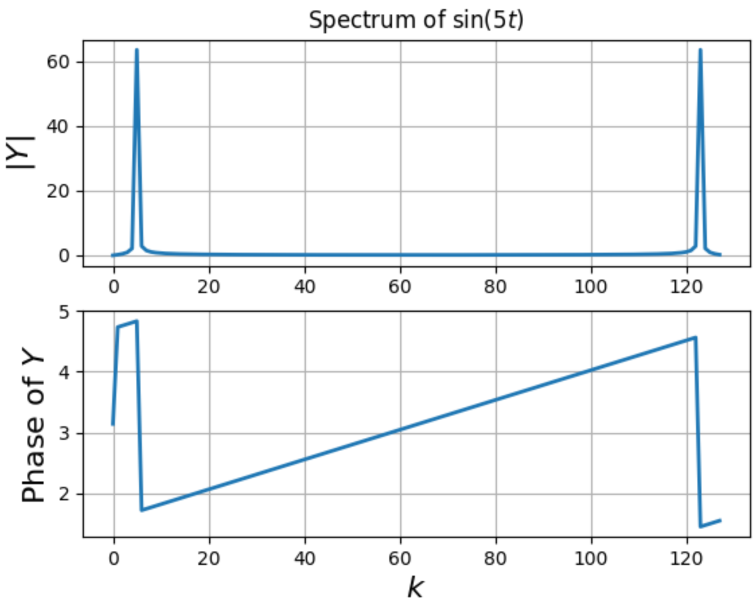
3. Spectrum of $\sin(5t)$

3.1 Without using fftshift():

For obtaining the expected phase plot using FFT, it is required to shift the phase appropriately using the fftshift() method. If we do not do this shift, we obtain a plot which resembles the expected plot, but the plots do not match exactly. The corresponding code is:

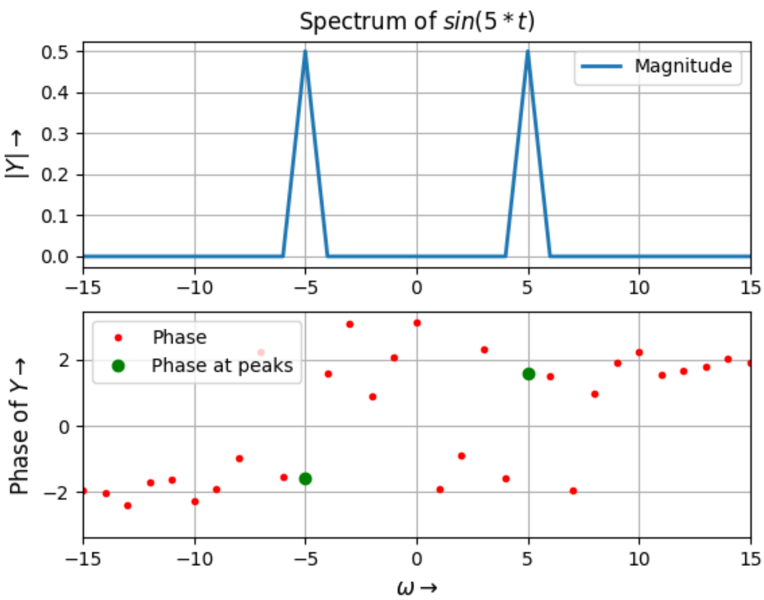
```
x = linspace(0, 2*pi, 129)[:-1]
y = sin(5*x)
Y = fft(y)

figure(0)
subplot(2, 1, 1)
plot(abs(Y), lw = 2)
grid()
ylabel(r"$|Y|$")
title('Spectrum of sin(5t)')
subplot(2, 1, 2)
plot(unwrap(angle(Y)), lw = 2)
ylabel(r"Phase of $Y$")
xlabel(r"$k$")
grid()
```

The corresponding plot:



3.2 Using fftshift():

Using fftshift() function, we obtain plots which are identical to the expected plots:

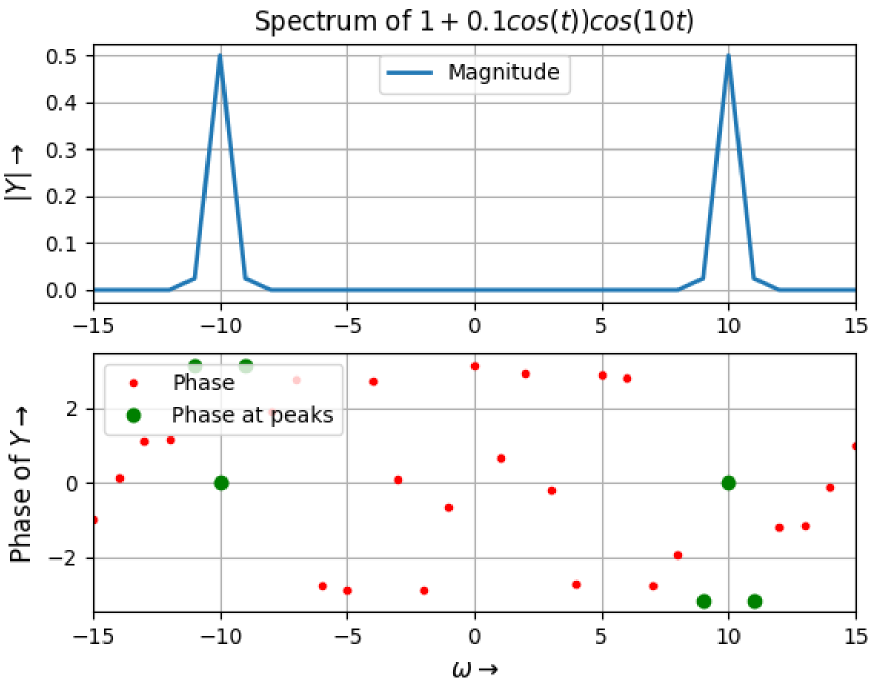


4. Spectrum of Amplitude Modulated Wave

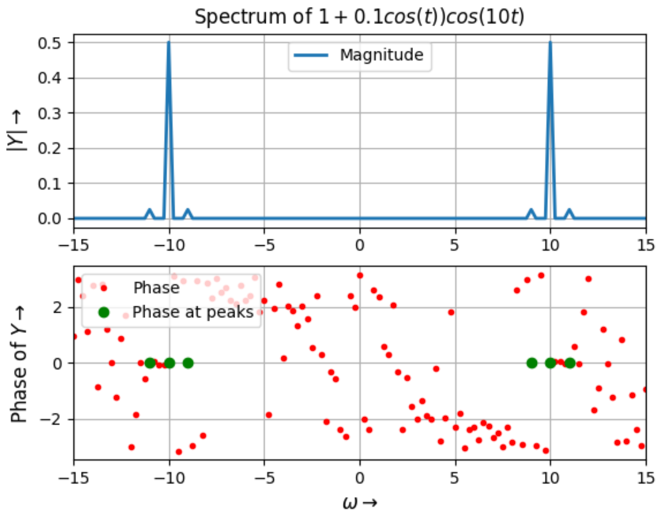
We consider the signal:

$$f(t) = (1 + 0.1cos(t))cos(10t)$$

If we use the same time window and number of samples as before, we will see that two of three expected peaks in the output will merge with the larger peak in the middle. That is, we obtain the following plot:



Now, this overlap happens because we did not consider an adequate number of frequencies. For solving this problem, we scale the time window as well as the number of samples by a factor of 4. This ensures that the sampling frequency remains the same and at the same time more frequencies are accommodated for. We can see that in this plot, we obtain 3 distinct peaks with phase equal to zero for these peaks. The phase points corresponding to these peaks are marked in green.

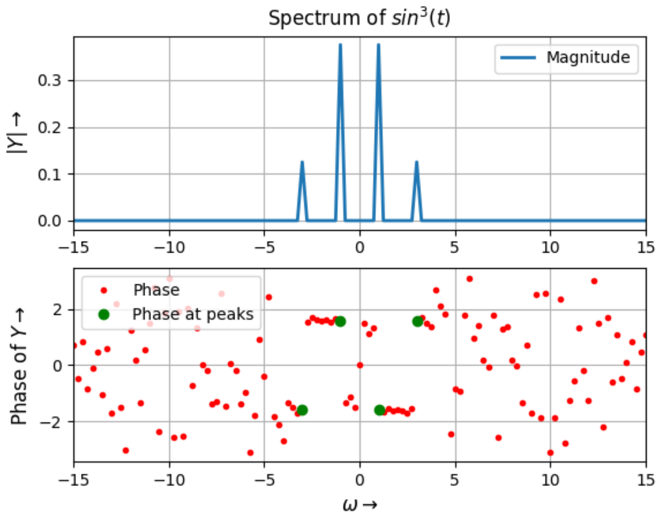


5. Spectrum of $sin^3(t)$

The signal can be expressed as:

$$sin^3(t) = \frac{3}{4}sin(t) - \frac{1}{4}sin(3t)$$

Clearly, the transform is expected to have two peaks, one at frequency 1 and 3. Also, the phases at these peaks will either be $\frac{\pi}{2}$ or $-\frac{\pi}{2}$. Again, the phase corresponding to the peaks are marked in green. The obtained plot is:

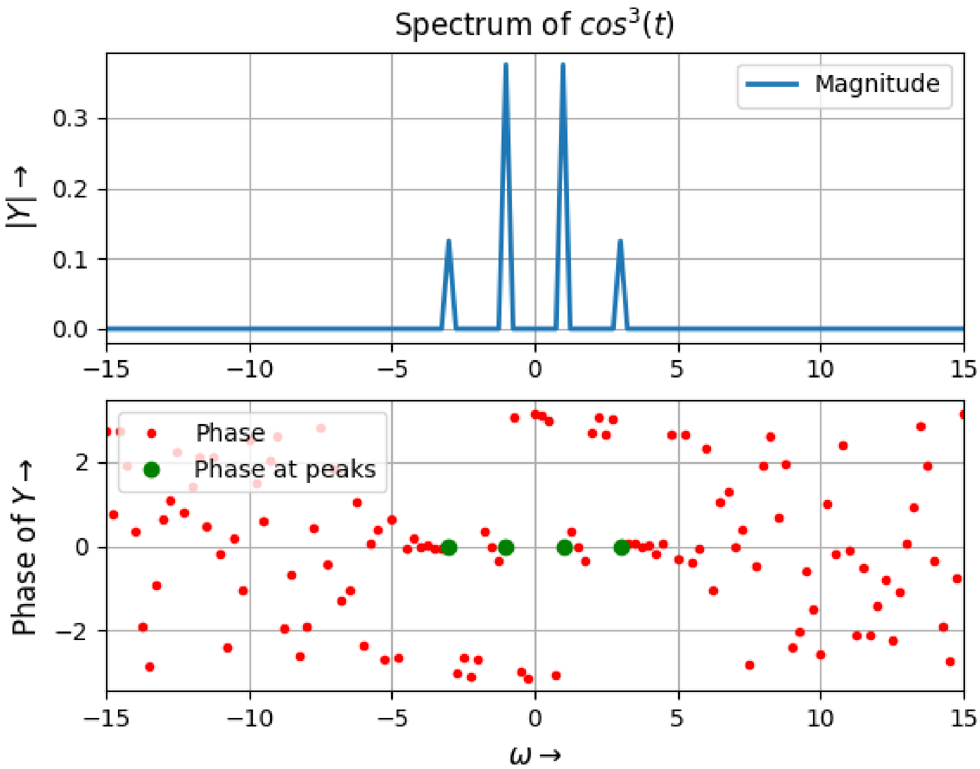


6. Spectrum of $cos^3(t)$

The signal can be expressed as:

$$cos^3(t) = \frac{3}{4}cos(t) + \frac{1}{4}cos(3t)$$

Again, the transform is expected to have two peaks, one at frequency 1 and frequency 3. Also, the phases at these peaks will be 0. Again, the phase corresponding to the peaks are marked in green. The obtained plot is:

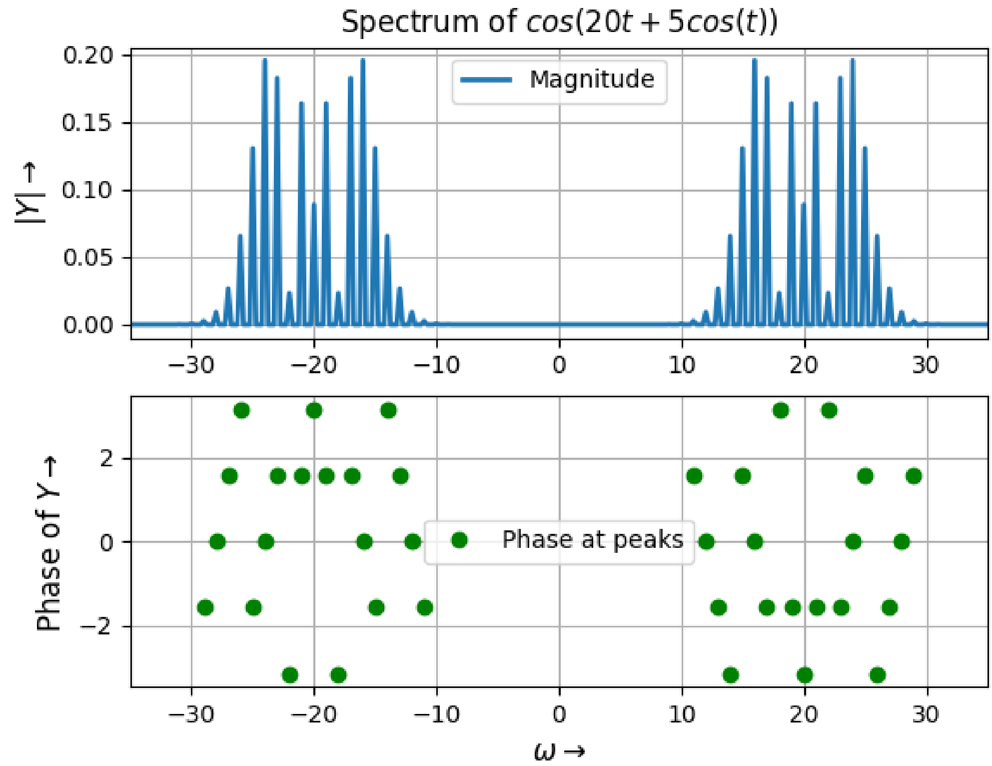


7. Spectrum of Frequency Modulated Wave

We consider the signal:

$$f(t) = cos(20t + 5cos(t))$$

. We calculate the DFT of this signal and plot the Magnitude and Phase spectrum. We plot the phase points corresponding to frequencies which have a corresponding magnitude above 10^{-3} to improve the clarity of the plot.



8. DFT of Gaussian Distribution

Let $x(t)$ represent the Gaussian. Then the CTFT is given by:

$$X(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

The expression for the Gaussian is :

$$x(t) = e^{\frac{-t^2}{2}} \tag{1}$$

The CTFT is given by:

$$X(j\omega) = \frac{1}{\sqrt{2\pi}}e^{\frac{-\omega^2}{2}} \tag{2}$$

The code snippet for calculation of the phase and magnitude spectrum while changing the time range in a loop so that maximum error goes below $10^(- 14)$

```
def estimate(N, T):
t = linspace(- T/2, T/2, N + 1)[:-1]
w = linspace(- N*pi/T, N*pi/T, N + 1)[:-1]
y = exp(-0.5 * t**2)
Y_true = exp(-0.5 * w**2)*sqrt(2*pi)
Y = fftshift(abs(fft(y)))/N
Y = Y*sqrt(2*pi)/max(Y)

return max(abs(Y - Y_true)), w, Y, Y_true

i = 1
while estimate(N = 512, T = i*pi)[0] > 1e-14:
i += 1

print('Time range for accurate spectrum : ' + str(i) + 'pi')
print('Accuracy : ' + str(estimate(N = 512, T = i * pi)[0]))
w, Y, Y_true = estimate(N = 512, T = i*pi)[1:]
```

We obtain the plot for the gaussian by taking different number of sampling points and the limits for time. The true gaussian spectrum is also plotted. It is observed that for a given time range if the number of sampling points increases, the width of the gaussian reduces (i.e it approaches its true value).However, if for the same number of sampling points the time range increases then the width of the gaussian spectrum increases (i.e it goes away from it's true value.). This is happening because in the first case the sampling frequency increases, while in the second case the sampling frequency falls.

After iterating, the final value of N is 512, T is 8π and the error is of the order 10^{-15}. The plot of estimated CTFT is given below. Again, the green dots represent regions where magnitude is greater than 10^{-3}.

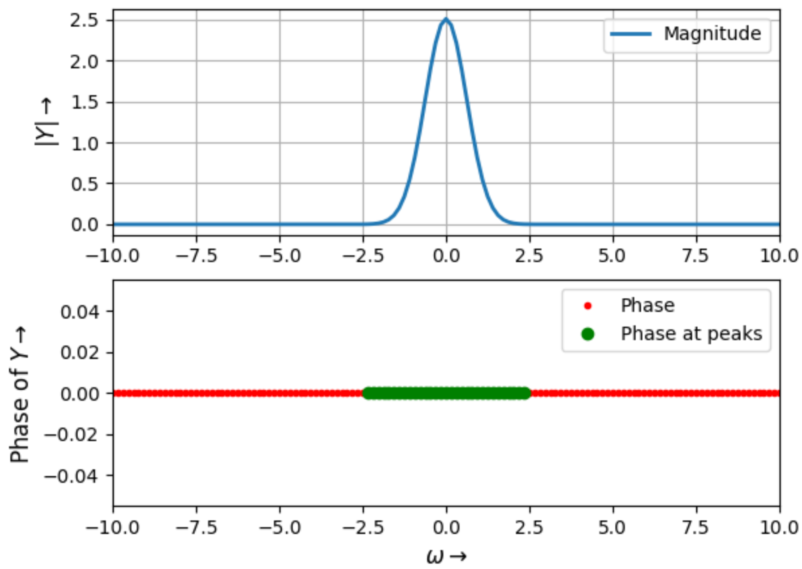


Figure 1: Estimated CTFT of Gaussian for N= 512, tlim= 4π

It is clear that a very good approximation of the CTFT is obtained using this approach.

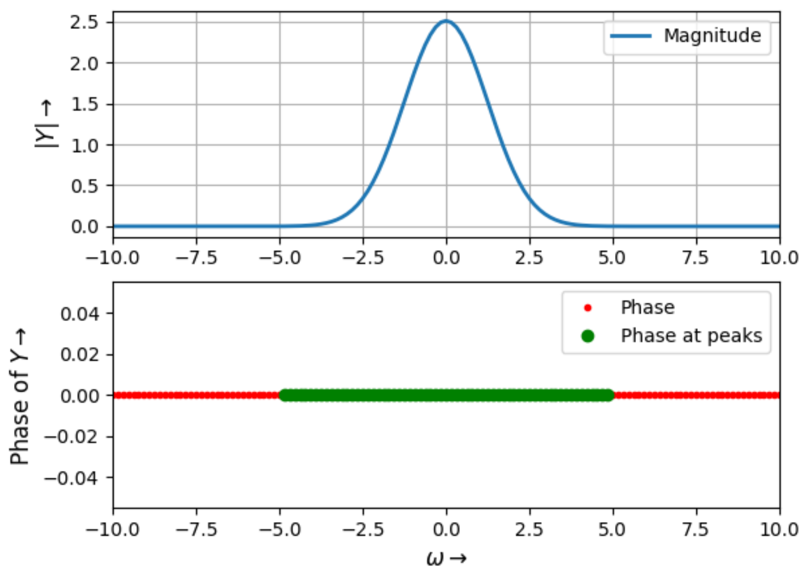


Figure 2: Estimated CTFT of Gaussian for N= 512, tlim= 8π

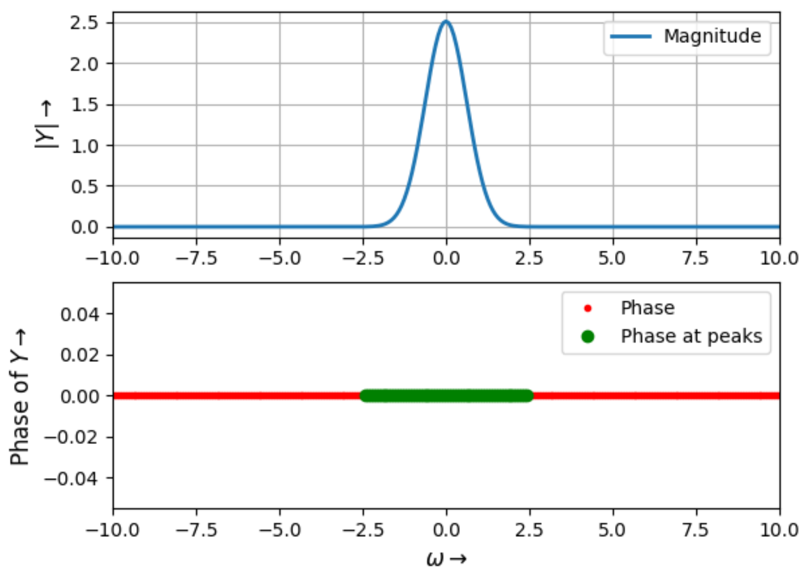


Figure 3: Estimated CTFT of Gaussian for N= 1024, tlim= 8π

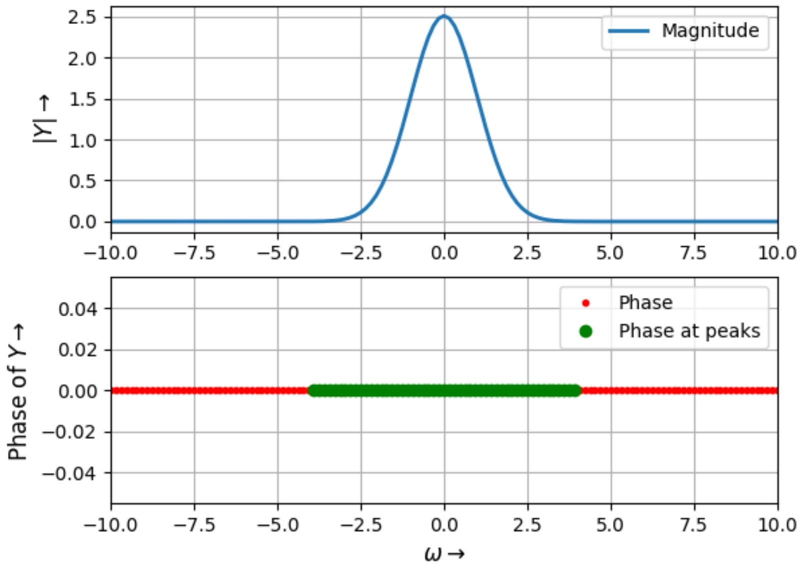


Figure 4: True Gaussian spectrum

9. Conclusion

- The difference between a signal and it's IFFT of it's FFT was obtained.It was found to be negligible.
- We used fft to find the DFT of the signal sin(5t). fftshift was used to center the 0 frequency component.
- The spectrum for (1 + 0.1cos(t))(cos(10t) (amplitude modulated) was analyzed for different for sampling points. The spectrum becomes more accurate for higher number of sampling points.
- Similarly spectrum for cos(20t+ 5cos(t)) (frequency modulated) was analyzed.
- Then we estimated the CTFT of a Gaussian distribution using FFT. This was achieved through an iterative approach in which we calculated values of T and N such that the error in the estimated CTFT is below the set threshold of 10^{-15}.
- For DFT, it is observed that for a given time range if the number of sampling points increases, the width of the gaussian reduces (i.e it approaches its true value).

- However, if for the same number of sampling points the time range in-creases then the width of the gaussian spectrum increases (i.e it goes away from it's true value).
- This is happening because in the first case the sampling frequency increases, while in the second case the sampling frequency falls.