EE2703 : Applied Programming Lab
Assignment 10
Spectra Of Non-Periodic signal

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1. Aim

The aim is to compute the Discrete Fourier transforms for non-periodic functions, and also those functions which are not periodic in integer multiples of 2π.
We also extract the frequency and phase of an unknown cosine function using DFT techniques.

2. Given Examples

The following examples have been worked out in the assignment:

Spectrum of $sin(\sqrt{2}t)$ is given below

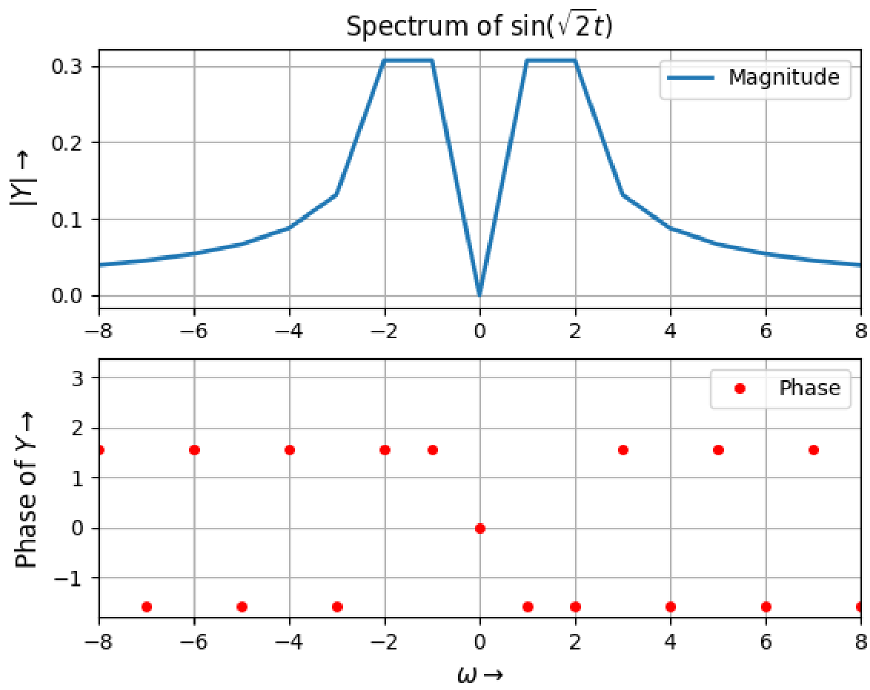

Figure 1: Spectrum of $sin(\sqrt{2}t)$

The DFT is supposed to be calculated for the function given below:

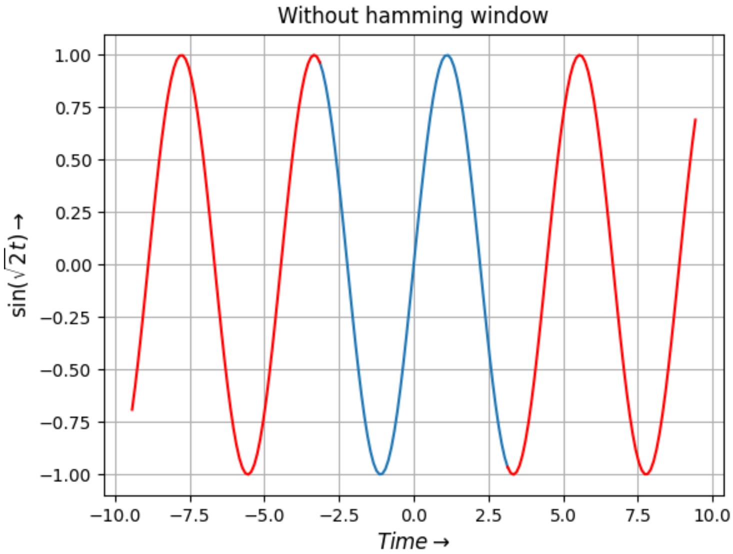

Figure 2: $sin(\sqrt{2}t)$

Since the DFT is computed over a finite time interval, we actually plotted the DFT for this function

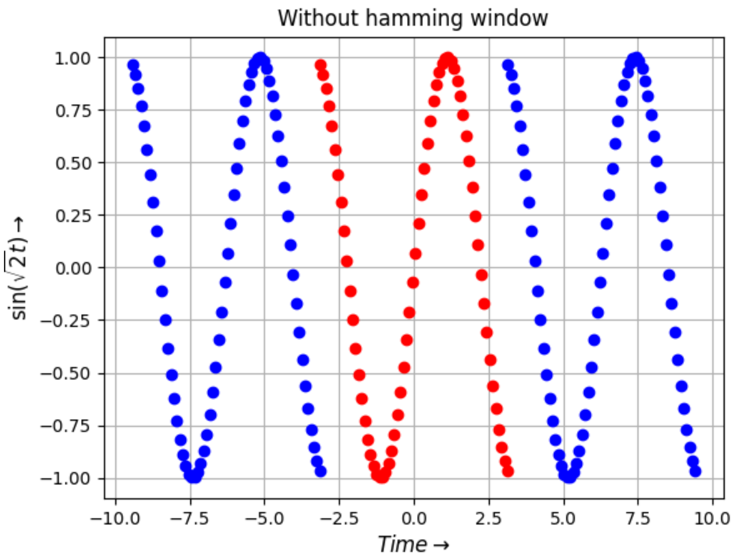

Figure 3: $sin(\sqrt{2}t)$

These discontinuities lead to non-harmonic components in the FFT which decay as $\frac{1}{\omega}$.
To reduce the effect of these discontinuities, we use the hamming window which is given by

$$x[n] = 0.54 + 0.46cos(\frac{2\pi n}{N-1}) \tag{1}$$

Now, to obtain the expected graph we multiply with the Hamming window to reduce the discontinuities due to high frequency components and this results in peaks at the expected frequency as shown below:

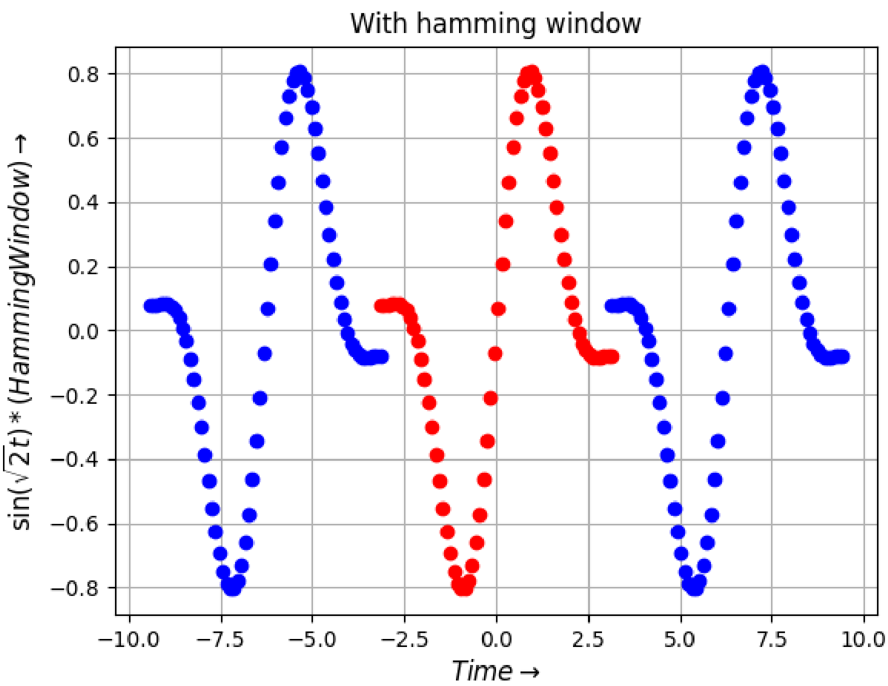

Figure 4: Spectrum of $sin(\sqrt{2}t) * w(t)$

The spectrum that is obtained with a time period 2π and windowing is given below:

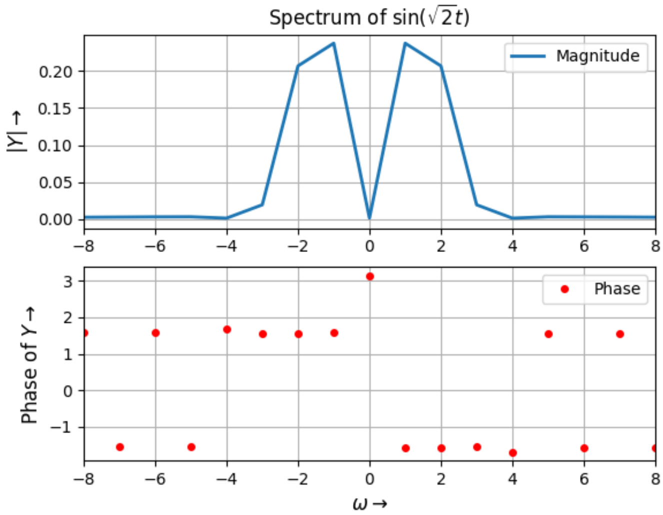

Figure 5: Spectrum of $sin(\sqrt{2}t) * w(t)$

The spectrum with greater number of sampling points has sharper peaks as shown

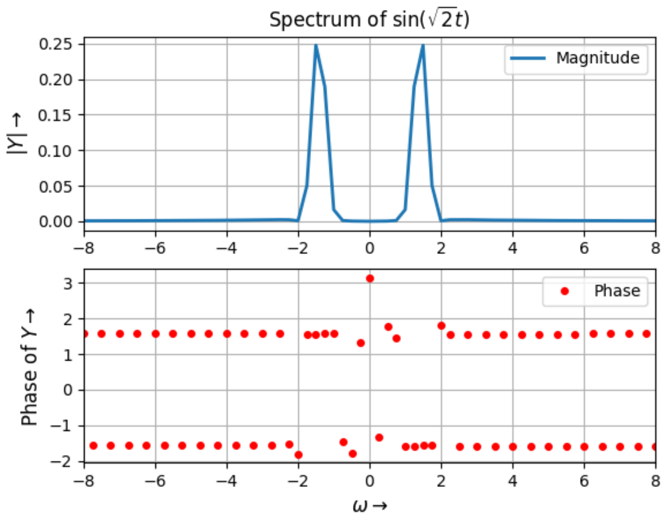

Figure 6: Spectrum of $sin(\sqrt{2}t) * w(t)$ with period 8π

3. Assignment questions

3.1 Spectrum of $\cos^3(\omega_0 t)$

We are computing the DFT of the $\cos^3(\omega_0 t)$ where $\omega_0 = 0.86$.
Without hamming window:

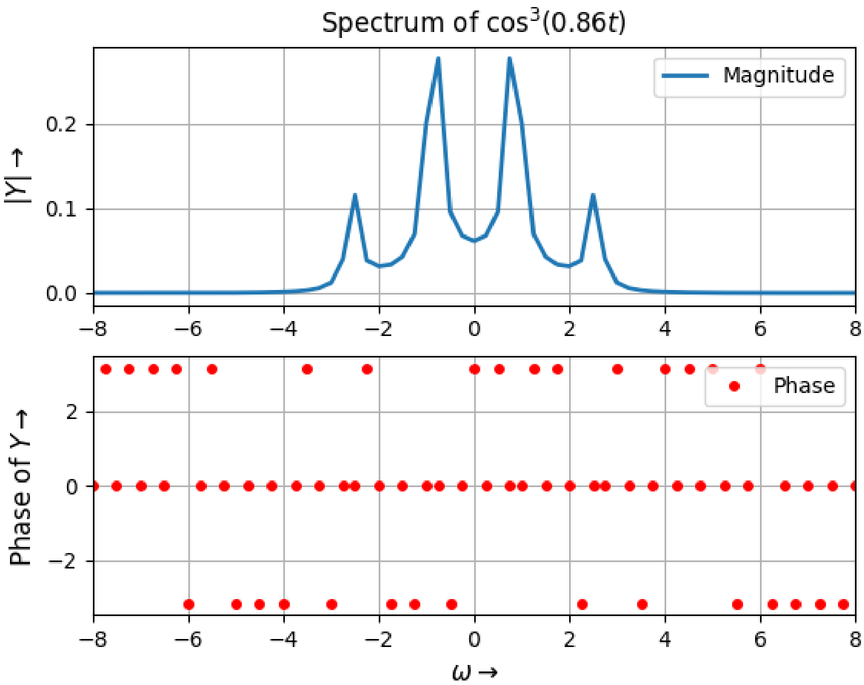

Figure 7: Spectrum of $\cos^3(0.86t)$) without hamming windowing

Now for sharp peaks at expected frequency, we are using the Hamming Window. This gives as the following DFT Spectrum:

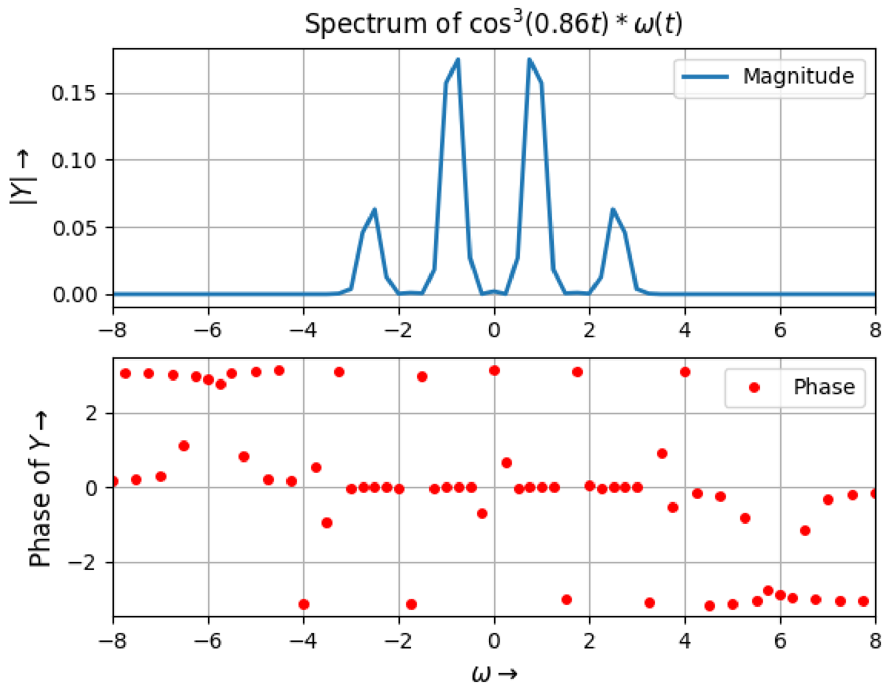

Figure 8: Spectrum of $\cos^3(0.86t))$ with hamming windowing

3.2 Estimation using DFT

We need to estimate ω and δ for a signal $\cos(\omega t + \delta)$ for 128 samples between $[-\pi, \pi)$. This is done for arbitrary δ and $0.5 < \omega_0 < 1.5$, we try to estimate the value of ω_0 and δ using DFT techniques.

3.2.1 Frequency Estimation

For less sampling points, we can not rely on the location of peak or maximum for estimating our frequency. Hence, we take weighted average or expected value of ω_0 over $|Y|$ should produce results close to the true value shown below:

$$\omega_{0,est} = \frac{\sum |Y|^2 * \omega}{\sum |Y|^2} \tag{2}$$

3.2.2 Phase Estimation

The phase can be obtained using the phase plot of the spectra. By knowing the fact that:

$$\cos(\omega_0 t + \delta) \xrightarrow{\mathscr{F}} \frac{1}{2}(e^{j\delta}\delta(\omega - \omega_0) + e^{-j\delta}\delta(\omega + \omega_0)) \tag{3}$$

This gives us intuition of getting the phase at ω_0, which in turn gives phase at the peak of magnitude of spectra. For delta, we consider a window on each half(positive and negative).

We are using the `numpy.random.uniform` to get the arbitrary values of ω_0 and δ in the specified period, which gives us the following graph:

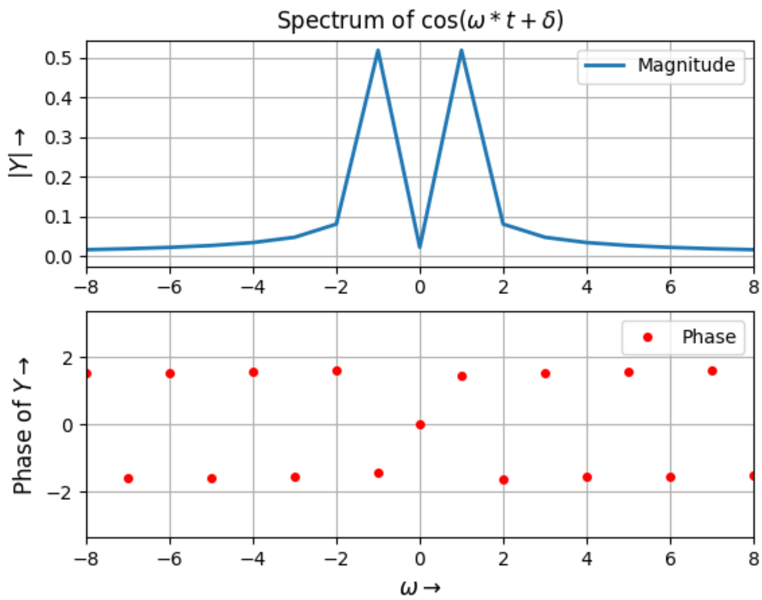

Figure 9: Spectrum of $\cos(\omega_0 t + \delta)$

```
Input frequency = 0.8628274954256683, delta = 1.4301048597258292
Estimated frequency = 0.8677060399508644, delta = 1.4374182507469238
```

The estimate function defined is shown below:

```
def estimate(w,y):
ind = where(w>0)[0]
y = y*hamming(128)
y[0]=0
y = fftshift(y)
Y = fftshift(fft(y))/128.0
w0 = sum((abs(Y[ind])**2)*w[ind])/sum(abs(Y[ind])**2)
delta = angle(Y[::-1][argmax(abs(Y[::-1]))])
return w0,delta
```

3.3 Estimation when White Gaussian noise is added

We repeat the exact same process as question 3 but with White Gaussian noise added to the original signal.

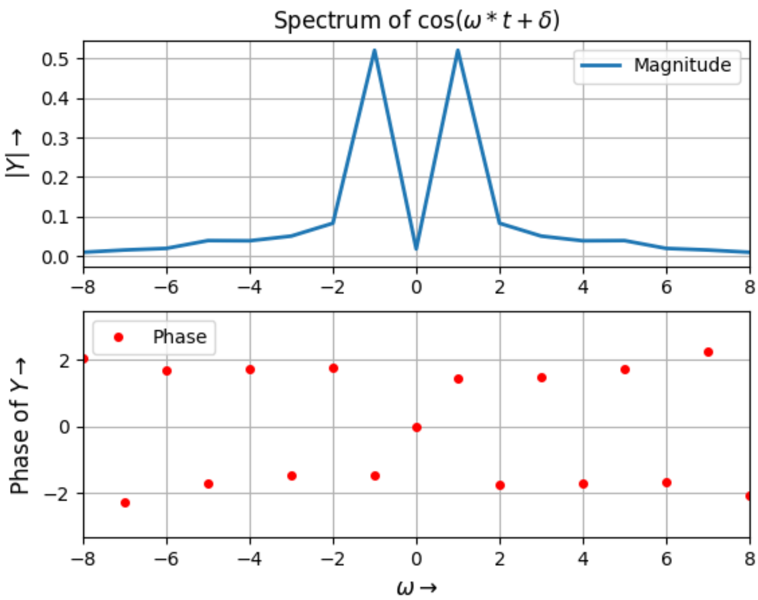

Figure 10: Spectrum of $\cos(\omega_0 t + \delta)$ with noise

```
Input frequency = 0.8628274954256683, delta = 1.4301048597258292
With noise, Estimated frequency = 1.2795803398187473, delta = 1.4313047790214335
```

3.4 Chirped Signal

Now we analyze the chirp signal in which with frequency is modulated with time as it is directly proportional to it. The chirp signal considered here is:

$$f(t) = cos(16(1.5 + \frac{t}{2\pi})t) \tag{4}$$

On windowing, the majority frequencies are spread from from 16 to 32 radians per second as we move from $-\pi$ to π. The signal can be seen as:

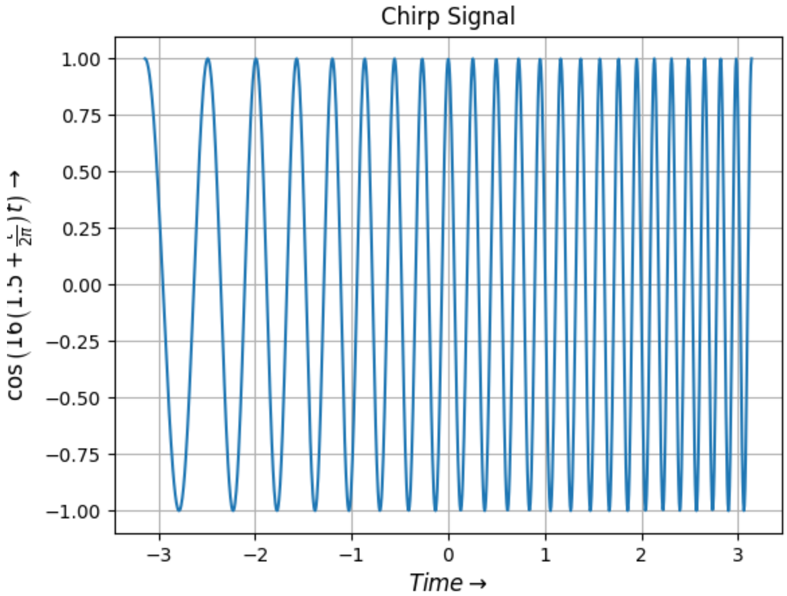

Figure 11: Chirp Signal

The spectrum of the Chirp Signal is as follows:

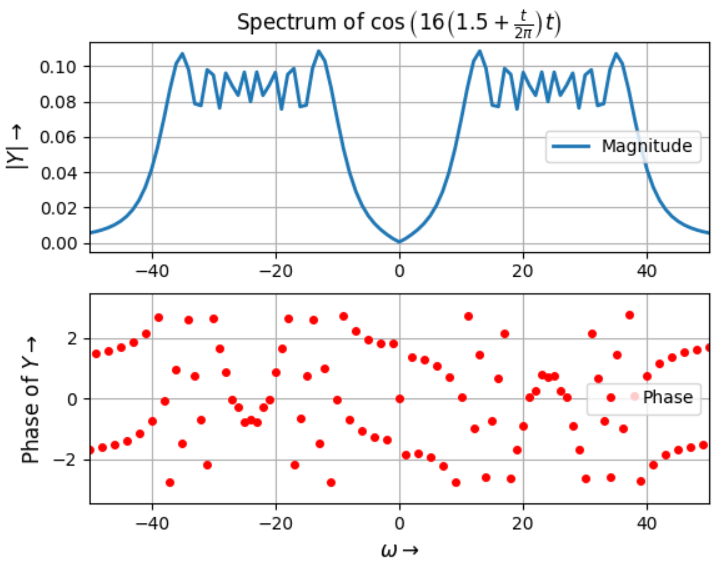

Figure 12: Spectrum of Chirp Signal without windowing

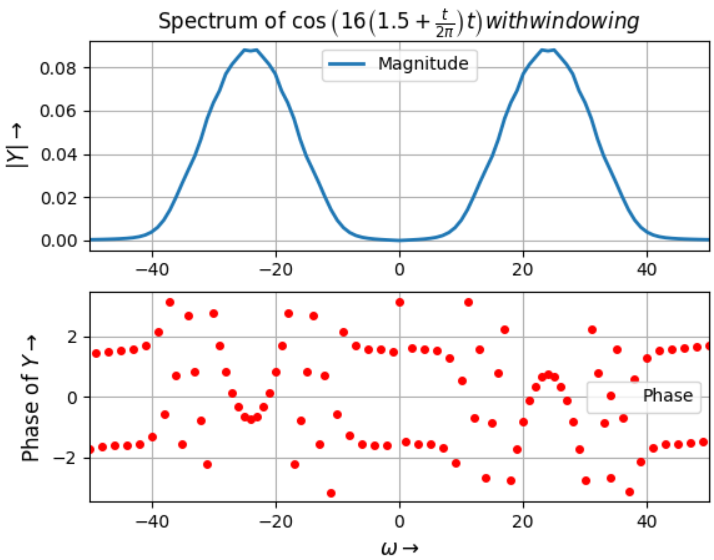

Figure 13: Spectrum of Chirp Signal with windowing

3.5 Spectrum of Chirp Signal for varying frequency and time

For the same chirped signal, we break the 1024 vector into pieces that are 64 samples wide thus having 16 different signals.
Their DFT is evaluated and stored as a column in a 2D array. We then plot the array as a surface plot to show variation of magnitude and how the frequency of the signal varies with time.

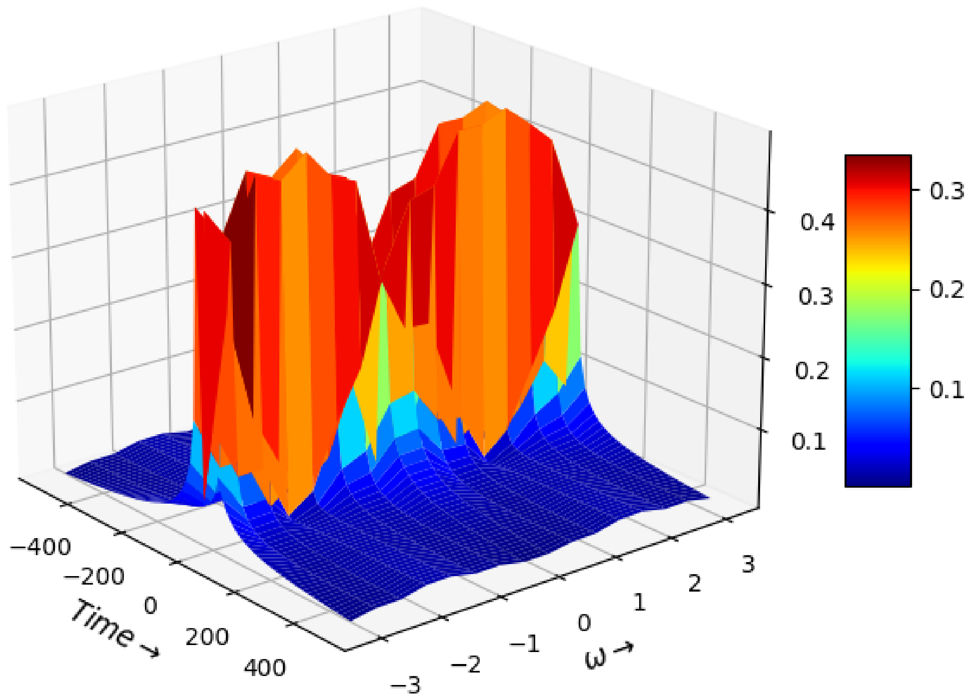

Figure 14: Surface Magnitude Plot of Potential

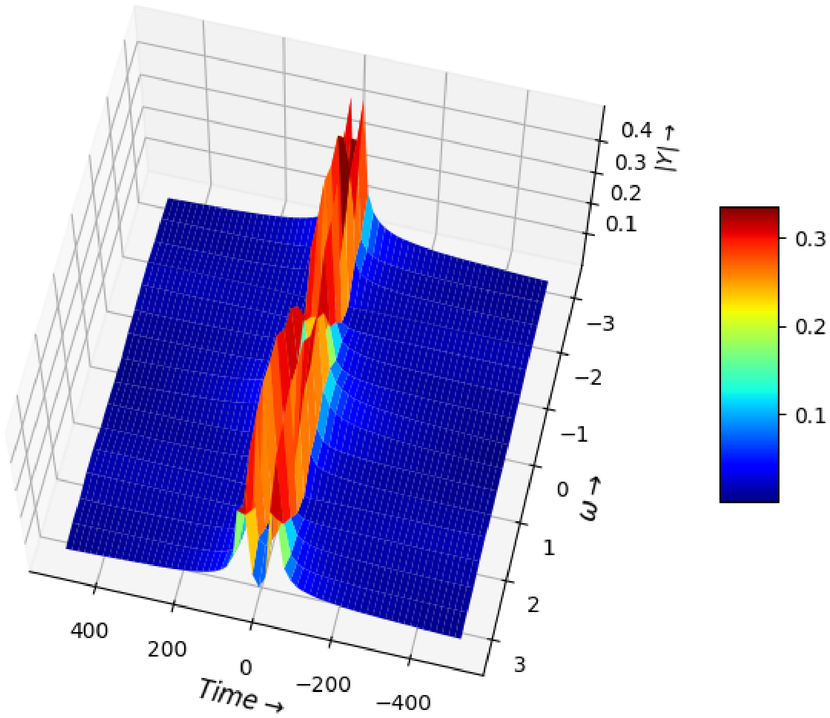

Figure 15: Surface time-frequency plot of Chirp Signal

4. Conclusion

We tried to find and analyze the Digital Fourier Transform of non-periodic signals and came to following conclusions:

- DFT is based on 2π periodicity of functions, and hence non 2π periodic functions have discontinuities and don't get sharp peaks in the spectrum.
- To get rid of this discontinuities, we multiply the original function by hamming window, which attenuates the higher frequencies or reduce discontinuity. We analyzed $\sin\left(\sqrt{2}t\right)$ and $\cos^3(0.86t)$ to confirm this.
- In the third question, we estimate ω and δ by using the square of the magnitude of the frequency spectrum as the probability distribution for weighted average of ω as estimate, and δ is calculated by finding the phase at the peaks in the spectrum. The results are reasonably accurate.
- The above was also repeated when the input cosine signal had a white Gaussian noise added to it.
- We plotted and analyzed the spectrum of the frequency modulated "chirped" signal. The graph showed two peaks =, each consisting of 4 local peaks or maximas as we moved from left to right.
- In problem 6. we plotted the surface plot of the above mentioned Chirp signal by dividing the signal into 16 parts, which varied with time and frequency both.
- From the surface plot, we come to know that the peak frequency increases with time and the magnitude spectra showed two gradual peaks.