Homework 7 for *Optimal Control*

Spring 2020

New Homework Requirement

- From now on, all homework submissions should be typed in a word processor (such as MS Word or LaTeX). Scanned handwritten work will not be accepted.
- Programming codes should be included as appendix at the end of the submitted file.

Homework 7

The discrete approximation to a nonlinear continuous system is given by

$$x(k+1) = x(k) - 0.4x^2(k) + u(k).$$

The state and control values are constrained by

$$\begin{aligned} 0 \leq x(k) &\leq 1 \\ -0.4 \leq u(k) &\leq 0.4. \end{aligned}$$

The performance measure to be minimized is

$$J = 4|x(2)| + \sum_{k=0}^{1} |u(k)|.$$

1. Quantize the state into the levels 0, 0.5, 1, and the control into the levels -0.4, -0.2, 0, 0.2, 0.4. Use any programming language (such as Matlab, Python, Java, C, ...) to implement dynamic programming with linear interpolation to calculate $J^*_{1,2}(x(1))$ and $J^*_{0,2}(x(0))$.

$x(1)$	$J^*_{1,2}(x(1))$
0.0	0
0.5	0.4
1.0	1.2

$x(0)$	$J^*_{0,2}(x(0))$
0.0	0
0.5	0.32
1.0	0.52

2. Quantize the state into the levels 0, 0.002, 0.004, ..., 1, and the control into the levels -0.4, -0.398, -0.396, ..., 0.4. Implement dynamic programming.

   (a) Plot $J^*_{1,2}$ as a function of $x(1)$, and plot $J^*_{0,2}$ as a function of $x(0)$.

   (b) Find the optimal control sequence $u^*(0)$, $u^*(1)$ and the minimum cost if the initial state is 1.0.

3. With the new performance measure to be minimized

$$J = 4|x(20)| + \sum_{k=0}^{19} |u(k)|,$$

   quantize the state into the levels 0, 0.002, 0.004, ..., 1, and the control into the levels -0.4, -0.398, -0.396, ..., 0.4. Implement dynamic programming.

   (a) Plot $J^*_{0,20}$ as a function of $x(0)$, plot $J^*_{1,20}$ as a function of $x(1)$, plot $J^*_{18,20}$ as a function of $x(18)$, and plot $J^*_{19,20}$ as a function of $x(19)$. What do you observe about these figures?

   (b) If the initial state is 1.0, find the minimum cost just using $J^*_{0,20}$ (not by calculating the optimal trajectory).

   (c) Plot the optimal feedback control policy at step 19, which is $u^*(19)$, as a function of $x(19)$, and plot the optimal feedback control policy at step 18, which is $u^*(18)$, as a function of $x(18)$. What do you observe and can you guess why it occurs?

   (d) If the initial state is 1.0, plot the optimal trajectory $x^*(k)$, $k = 0, 1, ..., 20$ as a function of k, and plot the optimal control $u^*(k)$, $k = 0, 1, ..., 19$ as a function of k. Calculate the performance measure J using the trajectory of x^* and u^*.

   J should be very close to your answer in (b). Otherwise, you may want to double check your code.