Extra Problems on Coordinate Frame Transformations

Note that $M_{ext} = {}^{c}T_{W}$

1. A camera matrix can be expressed as $M_{int}M_{ext}$ (or $K[R|\ t]$) . In Figure 1, what is M_{ext}? The coordinate frame transformation matrix (rotation and translation) can be obtained by noting how the camera frame is orientated and positioned with respect to the world frame. By inspection, we can write down the transformation from camera to world coordinates, then use matrix inverse to find the transformation from world to camera M_{ext}. The first column is the x unit vector of the camera frame with respect to the world frame, ${}^{w}\hat{x}_c = [-0.707\ 0.707\ 0]$. The second column is the y unit vector of the camera frame with respect to the world frame ${}^{w}\hat{y}_c = [0\ 0\ 1]$. The third column is the z unit vector of the camera frame with respect to the world frame ${}^{w}\hat{z}_c = [0.707\ 0.707\ 0]$. The last column is the origin of the camera frame with respect to the world frame ${}^{w}\hat{t}_c = [9\ 3\ 0]$. Therefore

$$M_{ext} = {}^{c}T_{w} = ({}^{w}T_{c})^{-1} = \begin{bmatrix} -0.707 & 0.707 & 0 & 4.24 \\ 0 & 0 & 1 & 0 \\ 0.707 & 0.707 & 0 & -8.48 \end{bmatrix}. \quad (1)$$

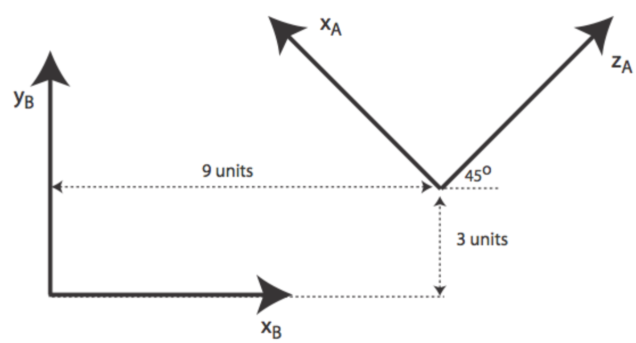

Figure 1: Coordinate Frames: let A represent the camera coordinate frame and B represent the world coordinate frame.

2. Consider a camera coordinate frame C initially aligned with a world coordinate frame W. Rotate C around $\hat{z}_W$ by 90° and then rotate about $\hat{x}_W$ by 180 degrees. Then rotate about $\hat{y}_W$ by 90 degrees. Then translate along $\hat{y}_W$ by 5 units.

   (a) Where is the camera? (describe as a numeric vector with respect to the world coordinate frame) This position of the camera is its origin with respect to (w.r.t) the world frame, i.e. the vector ${}^{w}t_c = [0\ 5\ 0]$

   (b) What is the camera orientation? (describe as the unit vectors with respect to the world coordinate frame) The camera orientation is the unit vectors of the camera frame w.r.t. the world frame. We can find the rotation matrix ${}^{w}R_c = \text{Rot}(\hat{y},90)\text{Rot}(\hat{x},180)\text{Rot}(\hat{z},90)$ where

$$Rot(\hat{y}, 90) = \begin{bmatrix} c90 & 0 & s90 \\ 0 & 1 & 0 \\ -s90 & 0 & c90 \end{bmatrix} \quad (2)$$

$$Rot(\hat{x}, 180) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c180 & -s180 \\ 0 & s180 & c180 \end{bmatrix} \quad (3)$$

$$Rot(\hat{z}, 90) = \begin{bmatrix} c90 & -s90 & 0 \\ s90 & c90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$^{w}R_{c} = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{5}$$

Therefore, the camera orientation is given by $^{w}x_{c} = [0\ -1\ 0]$, $^{w}y_{c} = [0\ 0\ 1]$,$^{w}z_{c} = [-1\ 0\ 0]$.

(c) Given a point P whose coordinates with respect to the w coordinate frame are $^{w}P = [10\ 1\ 4]$, what is ^{c}P?

$$^{c}P =^{c} T_{w}(^{w}P) = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} . \begin{bmatrix} 10 \\ 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ -10 \end{bmatrix} \tag{6}$$