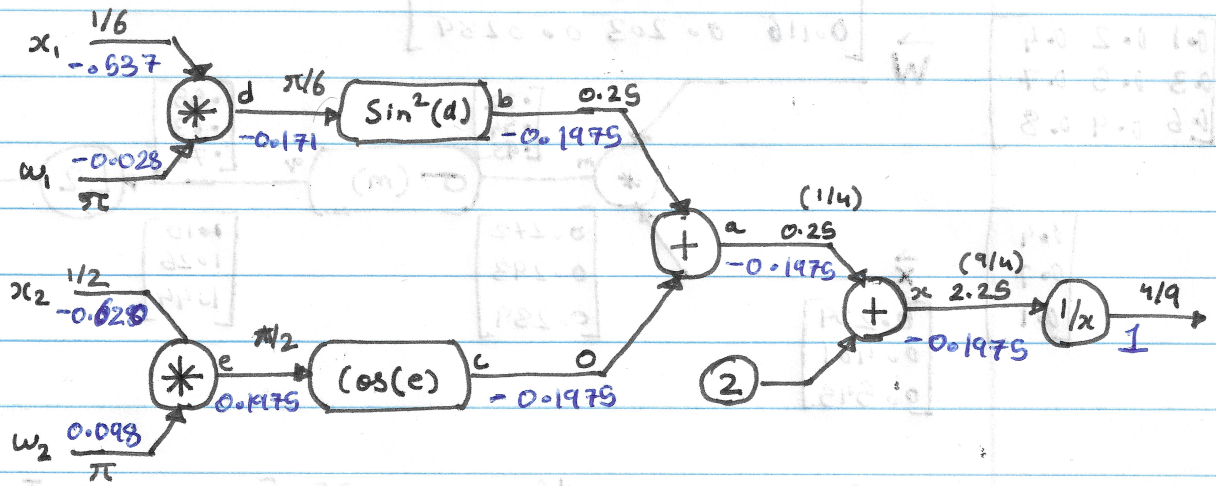
Dhruv Rana

# HW2

1. a)




$f(x) = \frac{1}{x} \rightarrow \frac{\partial f}{\partial x} = -\frac{1}{x^2}$, $x = a+2 \rightarrow \frac{dx}{da} = 1$, $a = b+c \rightarrow \frac{\partial a}{\partial b} = 1 \quad \frac{\partial a}{\partial c} = 1$

$b = \sin^2(d) \rightarrow \frac{\partial b}{\partial d} = 2\sin(d)\cos(d)$, $c = \cos(e) \Rightarrow \frac{\partial c}{\partial e} = -\sin(e)$

$d = x_1 * w_1 \rightarrow \frac{\partial d}{x_1} = w_1 \quad \frac{\partial d}{w_1} = x_1$, $e = x_2 * w_2 \rightarrow \frac{\partial e}{\partial x_2} = w_1 \quad \frac{\partial e}{\partial w_2} = x_2$

$$\frac{\partial f}{\partial w_1} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial a} \cdot \frac{\partial a}{\partial b} \cdot \frac{\partial b}{\partial d} \cdot \frac{\partial d}{\partial w_1} = -\frac{2x_1 \sin(x_1 w_1)\cos(x_1 w_1)}{[\sin^2(x_1 w_1) + \cos(x_2 w_2) + 2]^2} = -0.0285$$

$$\frac{\partial f}{\partial x_1} = -\frac{2w_1 \sin(x_1 w_1)\cos(x_1 w_1)}{[\sin^2(x_1 w_1) + \cos(x_2 w_2) + 2]^2} = -0.537$$

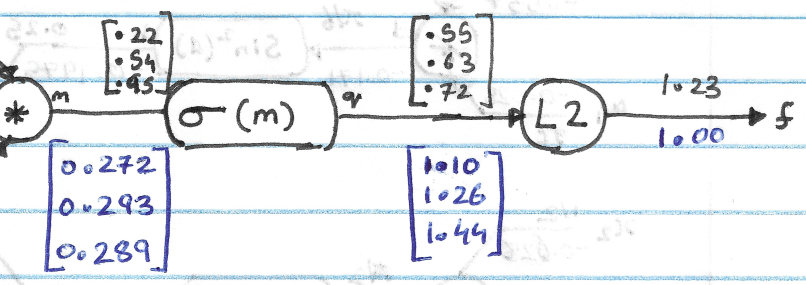
$$\frac{\partial f}{w_2} = \frac{x_2 \sin(x_2 w_2)}{[\sin^2(x_1 w_1) + \cos(x_2 w_2) + 2]^2} = 0.09876$$

$$\frac{\partial f}{\partial x_2} = \frac{w_2 \sin(x_2 w_2)}{[\sin^2(x_1 w_1) + \cos(x_2 w_2) + 2]^2} = 0.62056$$

2. a)

$$\begin{bmatrix} 0.1 & 0.2 & 0.4 \\ 0.3 & 0.5 & 0.7 \\ 0.6 & 0.9 & 0.8 \end{bmatrix} \quad \vec{W} \qquad \begin{bmatrix} 0.109 & 0.190 & 0.0272 \\ 0.117 & 0.205 & 0.0293 \\ 0.116 & 0.203 & 0.0289 \end{bmatrix}$$

$$\begin{bmatrix} 1.4 \\ 0.7 \\ 0.1 \end{bmatrix} \quad \vec{x} \qquad \begin{bmatrix} 0.289 \\ 0.461 \\ 0.545 \end{bmatrix}$$




$$\begin{bmatrix} .22 \\ .54 \\ .95 \end{bmatrix} \quad \begin{bmatrix} 0.272 \\ 0.293 \\ 0.289 \end{bmatrix} \qquad \begin{bmatrix} .55 \\ .63 \\ .72 \end{bmatrix} \quad \begin{bmatrix} 1.10 \\ 1.26 \\ 1.44 \end{bmatrix}$$

$$f = \|q\|^2 = q_1^2 + q_2^2 + q_3 \qquad \frac{df}{\partial q_k} = 2q_k \rightarrow \frac{\partial f}{\partial q} = \begin{bmatrix} 2q_1 & 2q_2 & 2q_3 \end{bmatrix}^T$$

$$q_k = \sigma(m_l) \qquad \frac{\partial q_k}{\partial m_l} = [1-\sigma(m_l)]\sigma(m_l) = [1-q_k]q_k$$

$$m_l = W_{l,j} \cdot x_j \qquad \frac{\partial m_l}{\partial W_{i,j}} = W_{i,j} \cdot x_j = 1_{(i=l)} \cdot x_j \qquad \frac{\partial m_l}{\partial x_j} = W_{l,j}$$

$$\frac{\partial f}{\partial W_{i,j}} = \sum_k \frac{\partial f}{\partial q_k} \cdot \frac{\partial q_k}{\partial m_l} \cdot \frac{\partial m_l}{\partial W_{i,j}} = 2q_k \cdot (1-q_k) q_k \cdot x_j = \begin{bmatrix} 0.109 & 0.190 & 0.0272 \\ 0.117 & 0.205 & 0.0293 \\ 0.116 & 0.203 & 0.0289 \end{bmatrix}$$

$$\frac{\partial f}{\partial x_j} = \sum_k \frac{\partial f}{\partial q_k} \cdot \frac{\partial q_k}{\partial m_l} \cdot \frac{\partial m_l}{\partial x_j} = 2q_k \cdot (1-q_k) q_k \cdot W_{k,j} = \begin{bmatrix} 0.289 \\ 0.461 \\ 0.545 \end{bmatrix}$$