Date: / / Linear Differential Equations-10ty Differential Equations A differential equation can be defined a derivatives of various order and the derivatives · Ordinary Differential Equation (OOE):d differential equation which involve one independent variable is called ordinary differential equation. X -> independent variable y → dependent variable. dy , rate of change. y=y(n) En D dy + 4y = 0 y'= dy

Date: Order: The corder of a SE is the corder of highest devilative occurring in the equation. Ex. D y" + Ey)2 + 4 y = 0 Ordn = 2 (1) x2y" + x(y')" + 3y2 = 0 ordy = 2 @ (y")2 + 2y +4y = 3n Order - 3 Degree: - The degree of a DE is the egree of the highest order occurring in the equation after the equation has been made feel of radical and fractions in its derivative. 9. It (y") = (y')4 (1+14")2)3 = (4)} 1+(y") = (y') Degrue = 2 1+ y = 2x Cogree = 2 y") + y = .2x(g")2

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Date: · Linear Differential Equation (LOE): ofn: - d differential equation is linear when derivative occur only in the first degree and no products of the derivative or of various order derivative occur. linear D(y") + 2y + 3yu - Smx Linear (2) n'yr + ny+2y=3x (3(y")2+ 2y' + link y= 0 Non linear 9 44 + 24 = Com Non linear Non linear (1) cosy y + 2y = 0 linear ( Sinn y + y = 2n Solution = y = f(x) - explicit form 44 y = K J(y,y',2) =0 y== ny+ n+) ) - implicit form री क्या बा बेरी पदाओ

The form of linear differential equation: ao(n) yn + a,(n) yn-1+ --- +a(n) y'+an(n) y'=r(n) 2nd order LOF (10(m), a, (m), -a, (n) & r(n).

are called coefficients of

O(n) y = +a(n)y + a/(n)y = r(n) DE when n=1, It corder LOE 90(x) y' + 9, (x) y = 1(x) I when  $q_0(n)$ ,  $q_1(n)$ ; ...,  $q_n(n)$  are constants

then eqn (\*) is called LOE with

constant coefficients.

En (D) y''' + 2y' + y' = 3n. (D) y''' + y'' + 2y' + 4y = c. 2 When ap(x), 9, (x), 9, (x) -- , (1n (x)) are variables then egn & is called LOE usith variable coefficients. En. n-y" + ny +2y-0 (n-1)y" + n2y, +ny - 2x

Date: Theorem: - If the function dolar), (1,(n), 9,(n), --, an(n) and 2(n) are continous ova the interval I and acht to on interval I, then there exists a unique solution to the initial value problem a. (n) yn + a, (w) yn-1+ - - - + 9, (n) y = 1(2) y(x)=y, y'(n)=y, y"(n)=y, - 1 yn (no) = yn 4" + 4' +24=0 on [0,1] 4(0)=0, y(1)=1)=> Boundary cond? sol =0, y'(1)=0 3) no sol 4) unique seem nornal interval on (-00,00)

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(x2-10) y"+ y + 2y = 2n I-1-0

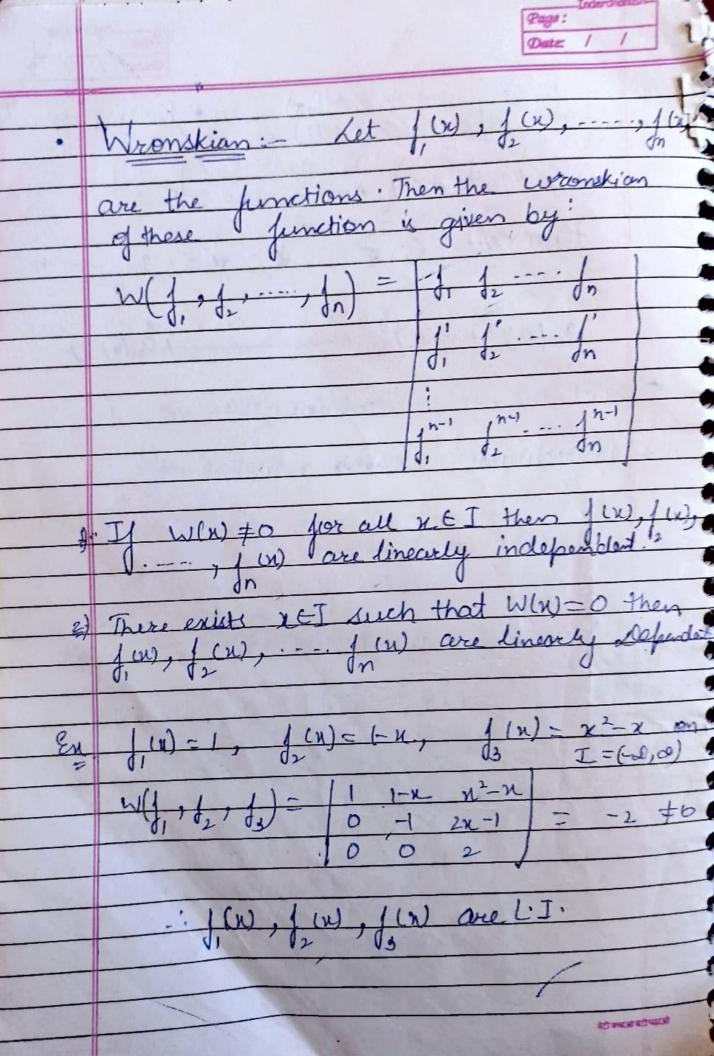
 $\chi^{2}/0 = 0$   $\chi^{2} = 10 = \sqrt{10}$   $\chi = 10 = \sqrt{10}$ 

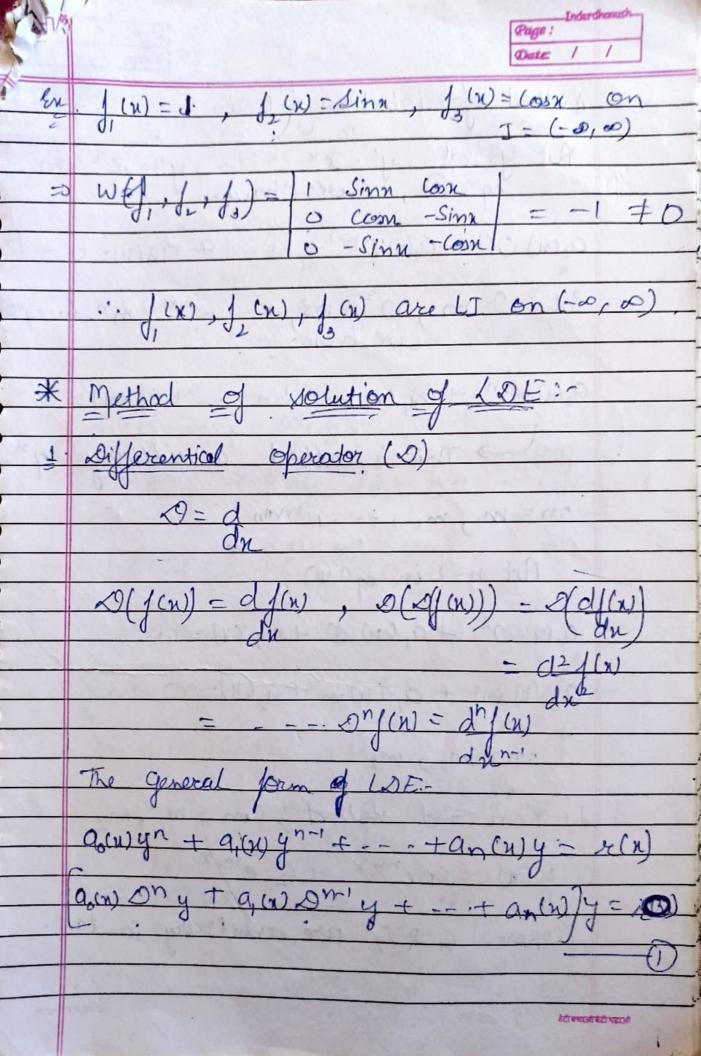
 $\frac{(-\infty,\infty)^{-10}}{(-\infty,\infty)^{-10}} = \frac{(-\infty,\infty)^{-10}}{(-\infty,\infty)^{-10}}$ 

Let 4, (W), 42 (M), --, 4n(M) are finition the be of these of are given by C14, TC42 + C243 + - + C240 \_\_ Cn are arbitrary cust. where 9,6,9, Unear Independence (LI) C14, + C24, + C343+ + Cnyn = 0 after compaining both sides, we have 9=0=9=-==G y= 1, y= 2-x, y= 3+n Ep. C14, +C14, + Sy, =0 (1.1 + (2.(2-x2) + (3(3+x) = 0  $(C_1 + 2C_2 + 3C_3) + (3C_3) \cdot \chi - 2C_2 \chi^2 = 0$ · confare both sides, -29 =0 79 =0 6=1)

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Linear. Dependence (LD). $GY_1 + C_2Y_3 + C_3Y_3 + \cdots + C_nY_n = 0$ There exist $C_i \neq 0$ $\forall j \neq 1 = 1, 2, 1 = n$
90 [x) y" + 9, (x) y" + - + 9n(n) y
C total total down less only with total and the state of
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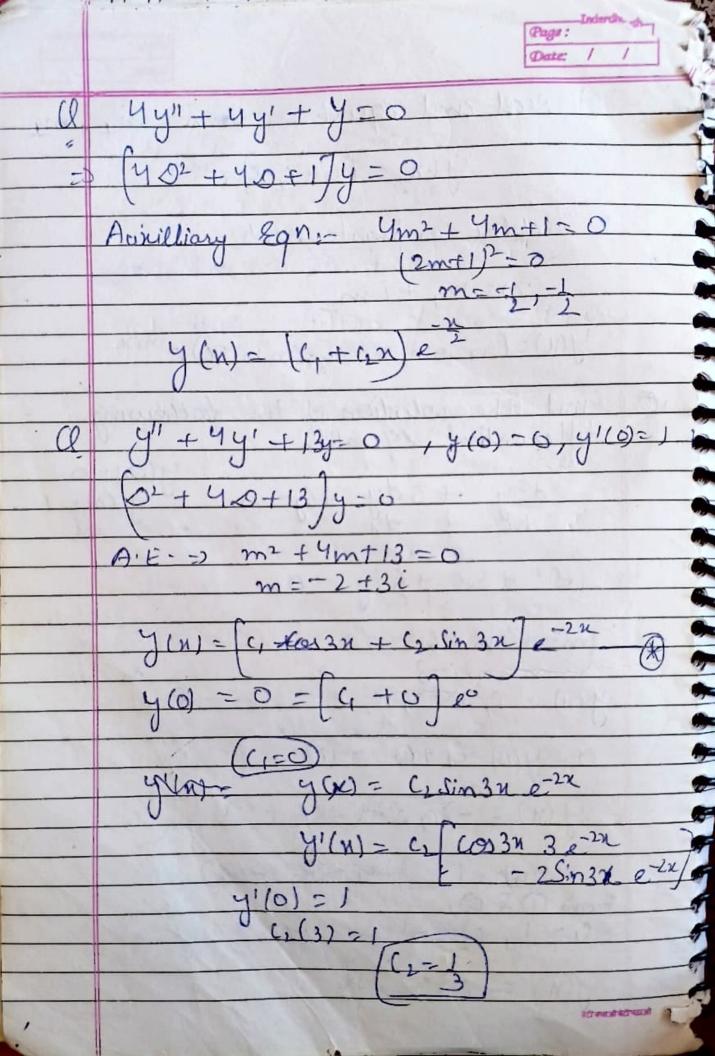
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Date: / / method of solution of LOE. Put y=ex; y'=ex -, yn=ex in ann) on + annon-1+ --- + ann)=0-2 Put D=m, 02=m², --- 20n mn in eque 900mm + 9, (n) mn-1 + - - + 9n (n) = 0 This is called auxilliary egn. m=m, m, --- 1 mm Put n- 2 in egn D, ao(n) 02 + a, (n) 0 + a, (n) = 0 = 00 (n) m2 + a, (u) m + q, (u) = 0 m=m,, m2 1) Real and distinct, m=m, , m2 y (n) = c, em, x + c, em2x where G&G are arbitrary constant.

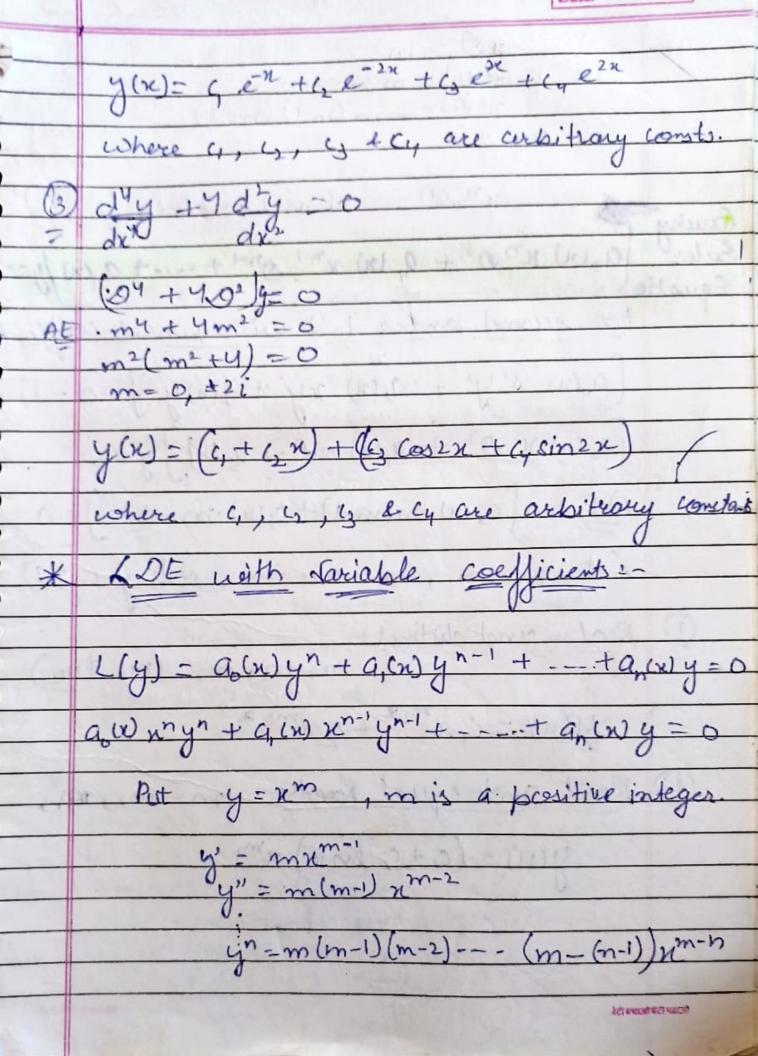
Date: / 2) real and equal: m=m, m, y(x) = (4+6x) emix 3) complere conjugate 4001: m=m, +im\_ You = ( (com x + ( sin m, x) emix Find the solution of the following differential equation. d2y +5dy +6y-0 y(0)=0 = (02+50+6)y=0 m2+5m+6-0 y(n) = C, e-2n+ C, e-3x 0= y(0) -C1+C2 -D y(n) = -2c, e-2n -3c, e-3n y'(0) = 1 = -2(1 - 3(1 - 2))From  $0 \neq 0$ , (1 = 1 + (1 = -1))y(n) = e-2x-e-3x



Put c, and co in eg 1 0, we have y(x) = 1 sin3x e-2x \* Higher order LOE with constant cofficient any + a, yn+1 + --- + any =0 and + a, on + - - - + an y = 0 AE = ) ao mn + ao mn + -- - + an - o DRoots: Real & Distinct.:
Let m=m,, m,,---m, are real and

distinct roots. y(n) = c, emin + C, emin + C3 emin + -- + Cnemin where (,, (,, --- (n are arbitrary constants 2.) Real and Equal roots: J(N) - (4+62+63 h2+6413+--+(n11m) emix 3) Complex conjugate ecook: m= m, + im2, m3 + imu + ms + im6, mn-1 + imn, ACCUPATION AND ACCUPANTED IN

Page : Date y(n)=(c, ccom2 n+(cinm2 n) emin\_ (3 Comy 2 +65 in my 2) eman --Cn-, Codom ne + Consin mone) emny u Find the solution of the following differential equations: y" - 2y" -5y' + 6y = 0 (03-202-50-fb) y=0  $\frac{AE \Rightarrow m^3 - 2m^2 - 5m + 6 - 0}{(m-1)(m^2 - 4) = 0}$ m=1, == -2, 3 y(x) = 4 ex + 6 e-2x + 6 e+3x where c, , (, and c, are arbitrary constants y" - 54" + 44 =0 09-502 +4) 4=0 AE > m4 -5 m2 + 4 = 0 my - 4m2 - m2 +4 =0 m2(m2-4)-1(m2-4)=0 m2-1)(m2-4)=0 m= =1, ±2

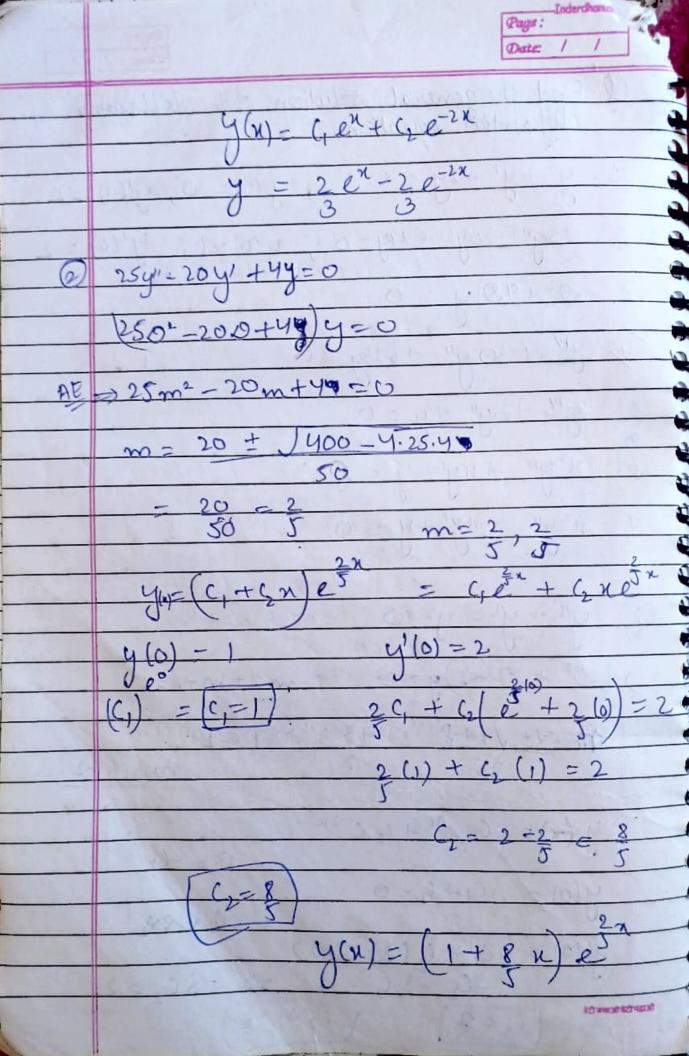


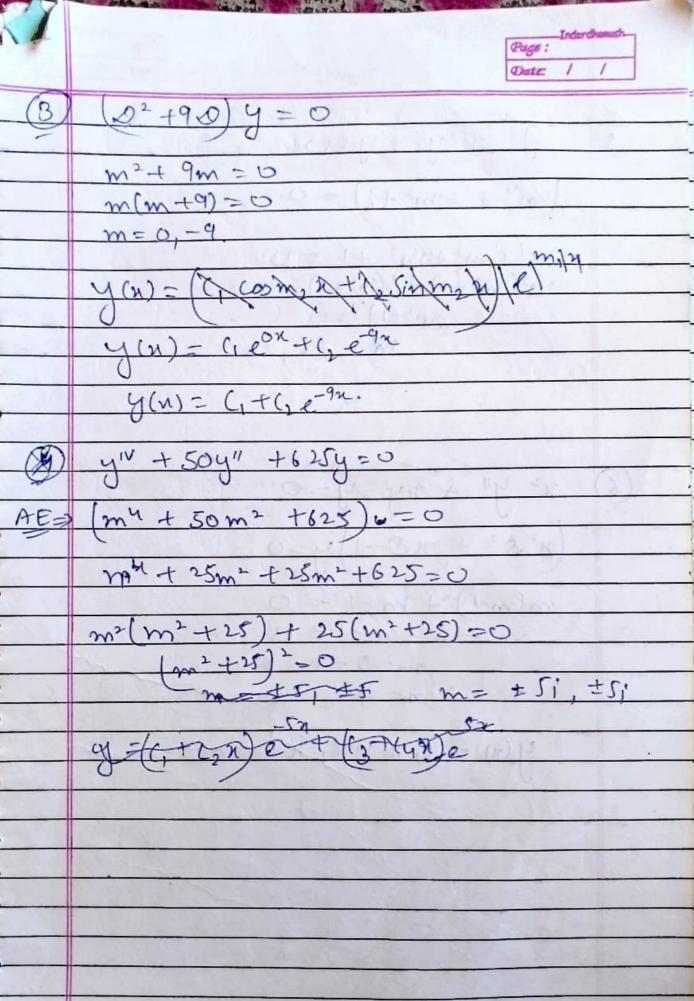
Date: 2203 - m (m-1) (m-2) xinon - m (m) (m-2) - ... (m-(n-1)) Euler aoin x on + a, in x n-1 on-1 + .... + an(n) /400 For second order LOE with variable welfsich [ao(n) n2y2 + 9,(n) xy + 9,(n) y ] -0-(a, (n) x202+ a, (n) 20+ a, (n) y=0 A.E. - [a, w m (m-1) + 9, m) · m + 0, (w) = 0 Let m= m, m2 are the roots of eq. 0) 1 Real and distinct in m=m, m2 (m, +m) y(n) = 4xm, + 1, xm2 (2) Real and equal Rook = m=m, m, y(n) = (4+ 2 lnx) xm,

(3) complex conjugate roots: m= m, + ima y(u) = [acosm, lnx+6 Sinm, lnx] xm, Q. Find the solution of the following differental 1 x2y" + 2xy' - 2y - 0 => x202 + 2n0 -2 y=0 A.F.= (m(m-1)+2m-2)=0 y(x) = qx + c2 0x-2 2. 2x2y" + xy'-6y=0 2n2 02 + KO -6/4=0 2m(m-1) + m - 6 = 02m2-2m+m-6-0 2m--m-6=0 m= 1+ 11+48 = 1+7 - 8 + y(x)=1, x2+ (2x-+4

Date: / / Q. 4x2y"+y=0 Yn202 + y=0 4m(m-1)+1=0 y(x)-(++ (2 lnx) n2 4n2yn +8ny1 +17y-0 (4x202+8n0+17) 4=0 4m(m-1) + 8m+17=0 4m2-4m +8m+1720  $4m^2 + 4m + 17 = 0$   $m = -4 + \sqrt{16 - 16.17} - -4 + 4\sqrt{-16}$ -- 4 ± 16i - - 1 ± 4i m=-1+41, -1-4i y(n) - (, cos2 lnn + 4 Sin2 lnn) n 2

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Q	Find the general solution of the following differential equation.
	differential equation.
1	The state of the s
3	y'' + y' - 2y = 0, $y(0) = 0$ , $y'(0) = 2$
2	25y"-20y'+4y=0, y(0)=1, y'(0)=2
3.	$\left( \mathcal{O}^{2} + 9 \mathcal{O} \right) y = 0$
8	
7	y" + 50 y" + 625y = 0
(5)	A CONTRACTOR OF THE PROPERTY O
2	$y^{N} + 2y'' + y = 0$
6.	$x^2y'' + xy' - y = \emptyset$
7	
7.	$x^2y'' - xy' + y = 0$
	0 0
0	The state of the s
(7)	y" + y' -2y = 0
	(D2+0-2)y=0 m2+m-2=0
7	$(D^2 + D - 2)y = 0$ $m^2 + m - 2 = 0$
	OF-1+ 11+8 = -1+3 - 1, -2
	$2 \cdot 1 \cdot 2  m = 1, -2$
	y (n) = (1 ex + 12 e-2x
	C1=-()
	y(0) = C1+(1=0
-	410) - C +1 (2) - 22
	y'(0) = (+4(-2) = 0) -(-26-26-2) -3(-2)
-	
	(1=2) (2=-1) Moreover with the second





y" + 2y" + y = 0 (m4+2m2+1)=0  $m^{4}+m^{2}+m^{2}+1=0$   $m^{2}(m^{2}+1)+1(m^{2}+1)=0$  $(m^2+1)^2=0$   $m = \pm i, \pm i$ 22 411 + xy-y=0 (n202+ no -1) y=0 m(m-1) + m-1 = 0  $m^2 - m + m-1 = 0$ m2-1=0 m= 1,-1 y(n) = (, x + (, x) वेटी बन्दर्भ बेटी पदाजी

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\* higher order LDE with variable (Cauchy-Euler DE) ao(x) 2myn+ a, (n) 2m-1 yn-1+ -----+ +an-1(x) xy' + an(x)y = x(x), a(x) +0 where r(n) =0 ao(n) x yn + a, (n) x n-1 yn-1 + --- + an(x) y =0  $\mu S = m$   $\mu^{2} S^{2} - m(m-1)$   $\mu^{3} S^{3} = m(m-1)(m-2)$ 2non = m(m-1) (m-2) \_\_\_\_ (m-(n-1)) Auxiliary: egn - (m-(n-1)) ] + +0,(x) [m(m-1) (m-2). --- (m-(n-2))] + . \_ - . + an(n)=0 1 Real and distinct roots: m= m, m2, ----, m, are distinct then the scaln of equito is given by

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3	
2	y (x) = (, xm, + c, xm2+ (3 xm3 ++ + Cn xmn
3	constants. Cn are curbitrary
2	Real and Equal roots:
	$m = m$ , $m_1$ , $$ , $m_1$ then soin.
	y(n)=[(+ (2lnx + (3lnx)2.+ Cy(lnx)3+
1	+ cn (lnn) n-1 xm,
3	Complex soots:
,	m= m, ± im, m3 ± imy, m ± imn
9	are complex roots then
9	y(n)=[c, cosm, (enn) + c, sin m, (enn)] nm,+
9	(GCos my (lnx) + Cy Sin my (lnx)) nm3 +
	+ 7 mn-1 [cn-1 contm, (lnx) + cn, Sinm, (nx)]
	THE THE PARTY OF THE PARTY.

Date: / / Find the general xix1" of the following differential eq": (i) x2y" +2xy'-2y=0 (n202+2n0-2)y=0 Att 7m(m-1) + 2m-2-6  $m^2 + m - 2 = 0$ m=1,-2J(n)= c, 21+c2x-2 = c, x + c2 (2) · 2n2y" + ny' -64 = 0 => [2n202+in0-6]=0 A-E-= 2m(m-1) + 2m-6=0. 2m2 - 1/m + 2m-6=0  $m^2 = 3$   $2m^2 + 4m - 3m - 6 = 0$   $m = \pm (3)$  2m(m+2) - 2(m+2) = 0y(x) = c/x = (2m-3) (m+2)=0  $y(n) = c_1 x^2 + c_2 x^2$ वेटी बनाउसे बेटी पडाओ

(3) 4n2y" +y-0 => (4x2 02 + 1)4=0 AE => 4m(m-1) +1=0 4m2-4m+120 (m-1)2=0 m=1 1 y(n) = (c, +c,(enx)) x2 4x2y" +8ny, +17y=0 4n202 + 8n0 +17)4=0 4m(m-1) + 8m+17=0 4m2 -4m + 3m +17=0 4m2 +4m +17=0 m=-4+ \(\int\_{16}-4.4.17 - -4+4\)\(\int\_{16} - 4+16\)\\
\(\mathref{m}=-4+\)\(\sigma\_{16}-4.4.17 - -4+16\)\\
\(\mathref{m}=-4+\)\(\mathref{m}=-4+16\)\\
\(\mathref{m}=-4+16\)\(\mathref{m}=-4+16\)\(\mathref{m}=-4+16\)\\
\(\mathref{m}=-4+16\)\( 4(n) = (c, cos2 lnx + G. Sin 2 enx) 2

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(5) 2x2 y" +3ny -3y=0 =) (2 n2 02 + 3n0 -3) y=0 2m(m-1) +3m-3=0 2m2 + m-3=0 2m2 +3m-2m-3=0 m(2m+3)-1(2m+3)=0 (m-1) (2m+3)=0 y(x)=c, x'+c, x== (1 x3y" + 5n2 y" + 5ny1+y=0 7 (x303 + 5n202 + 5n0 +1) y=0 AE=> m(m-1) (m-2) + 5 m (m-1) + 5m+1=0 m3 + 2m2 + 2m+1=0 (m+1) (m2 + m+1)=6 m = -1 m++m+ = 0 m=-1 + J-4 -- 1+ Bi y(x) = Gx-1+ Gcos (Blook) + GSin (3 long) x=