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# Unit - 3.
## Some Special Discrete Distribution

## * Binomial Distribution

- **Bernouli Trial** : It is a random experiment with exactly two outcomes success and failure, where the probability of each outcome remains constant every time the experiment is conducted.

Ex: Toss a coin, Head to be succes

$P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$

- ⇒ **Bernouli process** : A sequence of independent bernouli trials.

Properties :-

① The experiment consists of repeated trials

② Each trial results in an outcome that can be classified as success or failure

③ The probability of success is denoted by $P$ and probability of failure is denoted by $q = 1-p$.

④ The repeated trials are independent.

The number $X$ of success in $n$ Bernouli trials is called Binomial Random Variable.

The probability distribution of this discrete random variable is called Binomial distribution and is denoted by $P(X=x) = b(x, n, p)$ and is defined as

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$$P(X=x) = b(x, n, p) = \binom{n}{x} p^x q^{n-x}$$

[ $n, p \Rightarrow$ Parameters of distribution ]

$x = 0, 1, 2, \ldots n$

$q = 1-p$.

$$\sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x}$$

$$= \binom{n}{0} q^n + \binom{n}{1} p q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \ldots$$
$$\ldots + \binom{n}{n} p^n$$

$$= (p+q)^n$$

Example:- Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.

$\Rightarrow$ Here, $n = 10$, $p = \frac{1}{2}$, $q = \frac{1}{2}$ $(q = 1-p)$

$$P(X=x) = b(x, n, p) = \binom{n}{x} p^x q^{n-x}$$

$$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= \binom{10}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^3 + \binom{10}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 + \binom{10}{9}\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)$$

$$+ \binom{10}{10}\left(\frac{1}{2}\right)^{10} = \frac{176}{1024}$$

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Example: A and B play a game in which their chances of winning is in the ratio of 3:2. Find A's chance of winning at least three games out of five games.

Sol$^n$ Here $n = 5$, $p = \frac{3}{5}$, $q = 1-p = \frac{2}{5}$

$\Rightarrow P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= \binom{5}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2 + \binom{5}{4}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^1 + \binom{5}{5}\left(\frac{3}{5}\right)^5$

$= \frac{2133}{3125}$

Example: In a binomial distribution consists of 5 independent trials probability of 1 and 2 success are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution.

$\Rightarrow$ $n = 5$,

$P(X=1) = 0.4096$ $\quad P(X=2) = 0.2048$

$P(X=x) = \binom{n}{x} p^x q^{n-x} = \binom{n}{x} p^x (1-p)^{n-x}$

$\binom{5}{1} p(1-p)^4 = 0.4096$ — (1)

$\binom{5}{2} p^2(1-p)^3 = 0.2048$ — (2)

Divide (1) by (2),

$\frac{\binom{5}{1} p(1-p)^4}{\binom{5}{2} p^2(1-p)^3} = \frac{\overset{2}{\cancel{0.4096}}}{\underset{1}{\cancel{0.2048}}}$

$p = \frac{1}{5}$.

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## * Moment generating function (mgf):

The moment generating function of a random variable X having probability distribution function $f(x)$ is given by

$$M_X(t) = E\left[e^{tX}\right] = \begin{cases} \sum_x e^{tx} f(x) & \text{for discrete distribution} \\ \int e^{tx} f(x)\, dx & \text{for continous distribution} \end{cases}$$

**NOTE** :- Mgf of a distribution is Unique.

## * Mgf of binomial distribution :-

Let $X \sim b(x, n, p)$

$$f(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

$$M_X(t) = E\left[e^{tx}\right]$$

$$= \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \binom{n}{x} \left(e^t\right)^x p^x q^{n-x}$$

$$M_X(t) = \sum_{x=0}^{n} \binom{n}{x} \left(pe^t\right)^x q^{n-x} = \left(q + pe^t\right)^n$$

$$\boxed{M_X(t) = \left(1 - p + pe^t\right)^n}$$

* Mean, Variance and Standard deviation :-

$M'_x(t) = n(1-p+pe^t)^{n-1} * pe^t$

$M''_x(t) = n(n-1)(1-p+pe^t)^{n-2} \cdot pe^t \cdot pe^t +$

$n(1-p+pe^t)^{n-1} \cdot pe^t$

Now, mean $\Rightarrow \mu = M'_x(0)$

$= n(1-p+pe^0)^{n-1} * pe^0$

$= n(1\cancel{-p}\cancel{+p})^{n-1} \cdot p$

$\boxed{\mu = np}$

$\sigma^2 = M''_x(0) - (M'_x(0))^2$

$= n(n-1)p^2 + np - n^2p^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$\sigma^2 = np - np^2$

$\sigma^2 = np(1-p)$

$\sigma^2 = npq$

$\boxed{\sigma = \sqrt{npq}}$

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Example: The mean and variance of binomial dist. are 4 and $\frac{4}{3}$ respectively. Find $P(X \geq 1)$

$\Rightarrow \quad np = 4 \quad , \quad npq = \frac{4}{3}$

$q = \frac{1}{3} \qquad p = 1-q$
$= 1 - \frac{1}{3} = \frac{2}{3}$

$n \times \frac{2}{3} = 4$

$n = 6.$

$f(x) = \binom{6}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{6-x}$

$P(X \geq 1) = 1 - P(X < 1)$
$= 1 - P(X=0)$
$= 1 - \binom{6}{0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6$
$= 1 - \frac{1}{3^6} = 1 - \frac{1}{729} = \frac{728}{729}$

Q: ~~If X has~~

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Q. If X has mgf, $m(t) = \left(\frac{1}{2} + \frac{1}{2}e^t\right)^5$
Find pdf of X,
mean of X, variance of X, Find $P(|X| \leq 1)$

$\Rightarrow$ $m(t) = \left(\frac{1}{2} + \frac{1}{2}e^t\right)^5$

$m(t) = (q + pe^t)^n$

$q = \frac{1}{2}$, $p = \frac{1}{2}$, $n = 5$.

$\mu = np = \frac{5}{2}$

$\sigma^2 = npq = 5 \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{4}$

$\sigma = \frac{\sqrt{5}}{2}$

$P(|X| \leq 1) = P(-1 \leq X \leq 1)$

$= P(X=0) + P(X=1)$

$= \binom{5}{0}$ $f(x) = \binom{n}{x} p^x q^{n-x}$

$= \binom{5}{0}\left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^5 + \binom{5}{1}\left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4$

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Q. The mgf of r.v. X $\left(\frac{2}{3}+\frac{1}{3}e^t\right)^9$.

Find $P(\mu-2\sigma < X < \mu+2\sigma)$

$\Rightarrow$ $m(t)=(q+pe^t)^n$

$q=\frac{2}{3}$, $p=\frac{1}{3}$, $n=9$

$\mu=np=\frac{2}{3}\times 9=6$ $\sigma^2=npq=9\times\frac{1}{3}\times\frac{2}{3}=2$

$P(\mu-2\sigma < X < \mu+2\sigma)$ $(\sigma=\sqrt{2})$

$P(3-2\sqrt{2} < X < 3+2\sqrt{2})$

$P(0.18 < X < 5.8)$

$= P(1)+P(2)+P(3)+P(4)+P(5)$

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## * Negative Binomial Distribution :-

The number X of trials required to produce k success in a negative binomial experiment is called Negative binomial random variable and probability distribution of this discrete random variable is called negative binomial ~~random variable~~ distribution and is denoted by $b^*(x, k, p)$ and is given by:

$$b^*(x, k, p) = \binom{x-1}{k-1} p^k q^{x-k}, \quad x = k, k+1, k+2 \ldots$$

NOTE :- $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \ldots$

$$(1+q)^k = 1 + kq + \frac{k(k-1)}{2!} q^2 + \ldots$$

Replace, $q$ by $-q$, $k$ by $-k$

$$(1-q)^{-k} = 1 + kq + \frac{k(k+1)}{2!} q^2 + \ldots$$

K successes in x trial with probability $p$.

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$$\sum_{x=k}^{\infty} \binom{x-1}{k-1} p^k q^{x-k} = p^k \sum_{x=k}^{\infty} \binom{x-1}{k-1} q^{x-k}$$

$$= p^k \left[ \binom{k-1}{k-1} + \binom{k}{k-1} q + \binom{k+1}{k-1} q^2 + \cdots \right]$$

$$= p^k \left[ 1 + kq + \frac{k(k+1)}{2!} q^2 + \cdots \right]$$

$$= p^k (1-q)^{-k}$$

$q = 1-p$

$$= p^k p^{-k}$$

$$= 1$$

Ex. Find the probability that a person fliping a coin gets

(a). the third head on the seventh flip.

(b). first head on the fourth flip.

Solⁿ (a). Here, $x = 7$, $k = 3$, $p = \frac{1}{2}$

$$\therefore \quad b^*(7, 3, \tfrac{1}{2}) = \binom{7-1}{3-1} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{7-3}$$

$$= \binom{6}{2} \left(\frac{1}{2^3}\right) \left(\frac{1}{2^4}\right) = \frac{15}{128}$$

$b^*(x, k, p) = \binom{x-1}{k-1} p^k q^{x-k}$

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$x = k, k+1, k+2, \ldots$ Page. |

(b) Here, $x = 4, k = 1, p = \frac{1}{2}$

$b^*(4, 1, \frac{1}{2}) = \binom{4-1}{1-1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1}$

$= \frac{1}{16}.$

Ex. The probability that a person living in a city owns a dog is ~~estim~~ estimated to be 0.3. Find the probability that the tenth person randomly interviewed in that city is the fifth one to own a dog.

Sol$^n$. Here, $p = 0.3$, & $x = 10$, $k = 5$, $q = 0.7$ $(q = 1-p)$

$b^*(10, 5, 0.3) = \binom{10-1}{10-5} \left(\frac{3}{10}\right)^5 \left(\frac{7}{10}\right)^5$

$= \binom{9}{5} \frac{3^5 . 7^5}{10^{10}}$

$= \frac{9!}{5!\,4!} \frac{(21)^5}{(10)^{10}}$

$=$

# * Geometric Distribution

For $k=1$ in negative binomial distribution it reduces to following distribution.

$$b^*(x, 1, p) = g(x, p) = pq^{x-1}, \quad x = 1, 2, \ldots$$

this distribution is called Geometric distribution

$$\sum_{x=1}^{\infty} p\, q^{x-1}$$

$$= p \sum_{x=1}^{\infty} q^{x-1}$$

$$= p(1+q+q^2+q^3+ \text{---})$$

$$= p \times \frac{1}{1-q} = p \times \frac{1}{p} = 1$$

Ex. For a certain manufacturing process, it is known that on the average 1 in 4 every 100 items is defective. What is probability that fifth item inspected is the first defective item found.

Sol$^n$ $\Rightarrow$ $p = \frac{1}{100}$, $q = 1 - \frac{1}{100} = \frac{99}{100}$

$$\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{5-1} = \frac{(99)^4}{(100)^5}$$

$$= 0.0096$$

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## * Mgf of Negative binomial dist. and Geometrical dist.

Let $X \sim b^*(x, k, p)$,

then $f(x) = \binom{x-1}{k-1} p^k q^{n-k}$

then,

$$M_x(t) = E[e^{tx}]$$

$$= \sum_{x=k}^{\infty} e^{tx} f(x)$$

$$= \sum_{x=k}^{\infty} e^{tx} \binom{x-1}{k-1} p^k q^{x-k}$$

$$M_x(t) = p^k \sum_{x=k}^{\infty} \binom{x-1}{k-1} \frac{e^{tx} q^x}{q^k}$$

$$= \frac{p^k}{q^k} \sum_{x=k}^{\infty} \binom{x-1}{k-1} (q e^t)^x$$

$$= \frac{p^k}{q^k} \left[ \binom{k-1}{k-1} (q e^t)^k + \binom{k}{k-1} (q e^t)^{k+1} + \binom{k+1}{k-1} (q e^t)^{k+2} + \cdots \right]$$

$$= \frac{p^k}{q^k} (q e^t)^k \left[ 1 + k(q e^t) + \frac{k(k+1)}{2!} (q e^t)^2 + \cdots \right]$$

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$$M_x(t) = \frac{p^k q^k e^{tk}}{q^k} (1-qe^t)^{-k}$$

$$M_x(t) = \frac{(pe^t)^k}{(1-qe^t)^k}$$

$$M_x(t) = p^k e^{tk} (1-qe^t)^{-k}$$

$$M_x'(t) = p^k k e^{tk} (1-qe^t)^{-k} + p^k e^{tk} \times -k(1-qe^t)^{-k-1} \times -qe^t$$

$$= kp^k e^{tk} \left[ \frac{1}{(1-qe^t)^k} + \frac{qe^t}{(1-qe^t)^{k+1}} \right]$$

$$M_x'(t) = \frac{kp^k e^{tk}}{(1-qe^t)^k} \left[ 1 + \frac{qe^t}{1-qe^t} \right]$$

$$M_x'(t) = \frac{k(pe^t)^k}{(1-qe^t)^{k+1}}$$

then mean $\mu = m'(0) = \frac{kp^k}{(1-q)^{k+1}}$

$$\mu = \frac{kp^k}{p^{k+1}}$$

$$\boxed{\mu = \frac{k}{p}}$$

$$M'_X(t) = k p^k e^{tk} (1-qe^t)^{-k-1}$$

$$M''_X(t) = k p^k \left[ k e^{tk} (1-qe^t)^{-k-1} + e^{tk} (-k-1) \times (1-qe^t)^{-k-2} \times (-qe^t) \right]$$

$$M''_X(0) = k p^k \left[ \frac{k}{(1-q)^{k+1}} + \frac{(k+1)q}{(1-q)^{k+2}} \right]$$

$$= k p^k \left[ \frac{k}{p^{k+1}} + \frac{(k+1)q}{p^{k+2}} \right]$$

$$= \frac{k p^k}{p^{k+1}} \left[ k + \frac{(k+1)}{p} q \right]$$

$$= \frac{k}{p} \left[ k + \frac{(k+1)}{p} q \right]$$

$$M''_X(0) = \frac{k^2}{p}$$

(Above calculation is wrong in some way

Answer should be

$$\sigma^2 = \frac{kq}{p^2}$$

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NOTE: $b^*(x, k, p) = \binom{x-1}{k-1} p^k q^{x-k}$, $x = k, k+1, \ldots$

mgf $\Rightarrow$ $M(t) = \dfrac{(pe^t)^k}{(1-qe^t)^k}$

$\boxed{\mu = \frac{k}{p}}$ $\boxed{\sigma^2 = \frac{kq}{p^2}}$

* for Geometric Distribution,

$$g(x, p) = p q^{x-1} , \quad x = 1, 2, \ldots$$

mgf $\Rightarrow$

$$M_x(t) = \frac{pe^t}{(1-qe^t)}$$

$\boxed{\mu = \frac{1}{p}}$ , $\boxed{\sigma^2 = \frac{q}{p^2}}$

**Example:** If $X$ has a mgf,

$$M_X(t) = \frac{(e^t)^3}{8\left(1-\frac{e^t}{2}\right)^3}$$

Find the pdf of $X$, mean and variance of $X$.

**Example:** $X$ has mgf,

$$M_X(t) = \frac{(1+2e^t)^3}{27}$$

Find pdf of $X$, mean & variance of $X$

$$\Rightarrow M_X(t) = \left(\frac{1}{3} + \frac{2}{3}e^t\right)^3$$

mgf.

$$M_X(t) = (q + pe^t)^3$$

$$q = \frac{1}{3}, \ p = \frac{2}{3}, \ n = 3$$

pdf $\Rightarrow$

$$f(x) = \binom{3}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{3-x}$$

$x = 0, 1, 2, 3.$

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## * Poisson Distribution :-

A random variable X is said to follow Poisson if it assumes only non-negative values and its probability distribution function is given,

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

where $\lambda$ is parameter.

Now, $$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\sum_{x=0}^{\infty}\frac{\lambda^x}{x!}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \ldots\right]$$

$$= e^{-\lambda} e^{\lambda} = 1$$

Poisson distribution is the limiting case of binomial distribution under following conditions :-

1. n, the number of trials is indefinitely large i.e. $n \to \infty$.
2. p, the constant probability of success of each trial is indefinitely small i.e., $p \to 0$
3. $np = \lambda$.

- **Mgf of Poisson distribution :-**

Let $X \sim P(\lambda)$ then,

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

then $$M_X(t) = E\left[e^{tX}\right]$$

$$= \sum_{x=0}^{\infty} e^{tx} f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{e^{tx}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda e^t}{1!} + \frac{(\lambda e^t)^2}{2!} + \frac{(\lambda e^t)^3}{3!} + \cdots\right]$$

$$= e^{-\lambda} e^{\lambda e^t}$$

$$\boxed{M_X(t) = e^{\lambda(e^t - 1)}}$$

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$$M_X(t) = e^{\lambda(e^t-1)}$$

$$M'_X(t) = e^{\lambda(e^t-1)} \cdot \lambda e^t$$

$$M''_X(t) = e^{\lambda(e^t-1)} \cdot \lambda e^t \cdot \lambda e^t + e^{\lambda(e^t-1)} \cdot \lambda e^t$$

$\therefore$ mean $\Rightarrow$ $\mu = M'_X(0)$.

$$= e^{\lambda(e^0-1)} \times \lambda e^0$$

$$\boxed{\mu = \lambda}$$

$$\therefore \sigma^2 = M''(0) - (M'(0))^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$\boxed{\sigma^2 = \lambda}$$

Standard deviation $(\sigma)$

$$\boxed{\sigma = \sqrt{\lambda}}$$

NOTE: In case of Poisson distribution,

Mean = Variance

Thus Poisson distribution can be written as

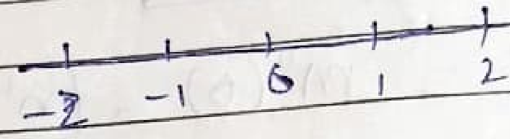
$$f(x) = P(X=x) = \frac{e^{-\mu} \mu^x}{x!}$$

Example: If $X$ has mgf,

$$m(t) = e^{4(e^t-1)} \quad \forall t$$

Find $P(X=1)$, $P(X=1,2)$, $P(|X| \leq 2)$, $P(|X| \geq 1)$. Also find the pdf of $X$.

$\Rightarrow$ $P(|X| \leq 2) = P(-2 \leq X \leq 2)$




Poisson only take, $P(X=0,1,2)$

$= P(X=0) + P(X=1) + P(X=2)$

$P(|X| \geq 1) = 1 - P(|X| < 1)$

$= 1 - P(-1 < X < 1)$

$= 1 - P(X=0)$

$$\boxed{P(X=x) = \frac{e^{-4}\, 4^x}{x!}}$$

$= 1 - \dfrac{e^{-4} \cdot 4^0}{0!}$

$= 1 - e^{-4}$

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For pdf :-

$\lambda = 4$,

$\therefore \; P(X = x) = \dfrac{e^{-4}\, 4^x}{x!}$

Example: If X has a poisson distribution such that
$$P(X=1) = P(X=2)$$
Find $P(X=4)$.

$\Rightarrow$ Since, $P(X=1) = P(X=2)$

$$\frac{e^{-\lambda}\lambda}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

$$\lambda = 2$$

$\lambda = 0$
$P(X=x) = 0$
$\therefore$ Probability of every event is zero which is not possible

$$P(X=4) = \frac{e^{-2}\,2^4}{4!} = \frac{e^{-2}\cdot 16}{24}$$
$$= \frac{2}{3}e^{-2}$$

Example: Let the pdf $f(x) \geq 0$ on non negative integers given that
$$f(x) = \frac{4}{x} f(x-1) \qquad x = 1, 2, \ldots$$

Find $f(x)$.

$\Rightarrow$
$$f(1) = \frac{4}{1} f(0)$$
$$f(2) = \frac{4}{2} f(1) = \frac{4}{2} \times 4 f(0) = \frac{4^2}{2!} f(0)$$
$$f(3) = \frac{4}{3} f(2) = \frac{4}{3} \times \frac{4^2}{2!} f(0) = \frac{4^3}{3!} f(0)$$
$$\vdots$$
$$f(x) = \frac{4^x}{x!} f(0)$$

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$$f(0) + f(1) + f(2) + f(3) + \cdots \cdots = 1$$

$$f(0) + 4f(0) + \frac{4^2}{2!} f(0) + \frac{4^3}{3!} f(0) + \cdots = 1$$

$$f(0)\left[1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \cdots\right] = 1$$

$$f(0)\, e^4 = 1$$

$$\underline{f(0) = e^{-4}}$$

$$\therefore f(x) = \frac{4^x}{x!} f(0)$$

$$\boxed{f(x) = \frac{4^x}{x!} e^{-4}}$$

① Binomial distribution:

$$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x} \quad , x = 0,1,2,\dots,n$$

$$M_X(t) = E\left[e^{tX}\right] = \begin{cases} \sum_x e^{tx} f(x) & , D.D. \\ \int e^{tx} f(x)\,dx & , C.D. \end{cases}$$

$$\mu = M_X'(0)$$
$$\sigma^2 = M_X''(0) - \left(M_X'(0)\right)^2$$

mgf of binomial dist. $\Rightarrow$ $M_X(t) = (q + pe^t)^n$

$$\mu = np$$
$$\sigma^2 = npq$$

② . Negative Binomial distribution:

$$b^*(x, k, p) = \binom{x-1}{k-1} p^k q^{x-k} \quad , x = k, k+1, \dots$$

mgf $\Rightarrow$ $M_X(t) = \dfrac{(pe^t)^k}{(1-qe^t)^k}$

$$\mu = \frac{k}{p}$$

$$\sigma^2 = \frac{kq}{p^2}$$

③ Geometric

$$g(x, p) = pq^{x-1} \quad , x = 1, 2, \dots$$

$$M_X(t) = \frac{pe^t}{1-qe^t} \qquad \boxed{k=1}$$

$$\mu = \frac{1}{p} \qquad \sigma^2 = \frac{q}{p^2}$$

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(4.) **Poisson Distribution:**

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots$$

Mgf $\Rightarrow$

$$M_X(t) = e^{\lambda(e^{t}-1)}$$

$\mu = \lambda$ $\sigma^2 = \lambda$