University of Mumbai

Bachelor of Science in Computer Science 2016 - 2017 March

Semester 4 (SYBSc)

Mathematics 1

Regular Examination March-2017
SYBSc[CS] Sem-IV
Sub- Maths-II

CSR405

09/03/17

Time: 2½ hrs. Marks:75

Note:
1. **All questions are compulsory with internal options.**
2. **Figures to the right indicate full marks.**

Q. 1 (A) Answer the following: (Attempt any one) **(08)**

(p) Let (x_m) be a sequence in $\mathbb{R}^n$ given by $x_m=(x_m^{(1)}, x_m^{(2)}, \ldots, x_m^{(n)})$. Show that (x_m) is convergent in $\mathbb{R}^n$ if and only if the co-ordinate sequences $(x_m^{(1)})$, $(x_m^{(2)})$, ..., $(x_m^{(n)})$ are convergent in $\mathbb{R}$.

(q) Let $(\emptyset \neq)S\subseteq\mathbb{R}^n$. Let f, g : $S \to \mathbb{R}$ be two scalar fields. For $a\in S$ suppose $\lim_{x\to a} f(x)=l$ & $\lim_{x\to a} g(x)=m$. Show that

(i) $\lim_{x\to a}(f+g)(x)=l+m$

(ii) $\lim_{x\to a}(kf)(x)=kl$

(iii) $\lim_{x\to a}(kf+g)(x)=kl+m$

Q. 1 (B) Answer the following: (Attempt any three) **(12)**

(p) Let $x, y \in\mathbb{R}^n$ then show that

(i) $||x||=0$ if and only if $x=0$.

(ii) $||x+y|| \leq ||x|| + ||y||$.

(q) Show that $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x-y)^2}$ does not exists.

(r) Find partial derivative of $f(x,y)=x^2y + y^2x$, at a=(2,3) by using definition.

(s) Show that limit of scalar field at a point, if exists, is unique.

Q. 2 (A) Answer the following: (Attempt any one) **(08)**

(p) Let $(\emptyset \neq)S\subseteq\mathbb{R}^n$. Let $f : S \to \mathbb{R}$ be a scalar field. If f is differentiable at $a \in S$. Show that total derivative of f at a is unique.

(q) Let $(\emptyset \neq)S\subseteq\mathbb{R}^n$. Let $f : S \to \mathbb{R}$ be differentiable function at $a \in S$. Let $g(t)=f(a+tv)$, $v \in\mathbb{R}^n$ & $t \in\mathbb{R}$, then show that g is differentiable at 0 & $g'(0)=Df(a)(v)$.

Q. 2 (B) Answer the following: (Attempt any three) **(12)**

(p) Let $f:\mathbb{R}^n\to \mathbb{R}$ be a linear transformation. Prove that total derivative $T(h)=f(h)$, for all $h \in\mathbb{R}^n$.

(q) Define gradient of scalar field f at $a \in S \subseteq\mathbb{R}^n$.
Find ∇f of $f(x,y,z)=4x^2-9x^2yz+3zy^2$ at (1,2,-3).

(r) Define tangent plane. Find equation of tangent plane to the surface $z=9x^2+y^2+6x-3y+5$ at (1,2,3).

(s) If f is homogeneous function of degree n in three variable x, y, z possessing first order partial derivative then show that $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}+z\frac{\partial f}{\partial z}=n\cdot f(x,y,z)$.

Q. 3 (A) **Answer the following: (Attempt any one)** **(08)**

(p) Let $(\emptyset \neq) S \subseteq \mathbb{R}^n$.
Let $f : S \to \mathbb{R}^m$ be a vector valued function differentiable over S. Let $a, b \in S$ & the line segment joining a & b, which is the set $\{a + (b-a)t | o \leq r \leq 1\}$ lies in so. Then show that
$f(b)-f(a)=\left(\int_0^1 J(f(a+(b-a)t))dt\right)(b-a)$.

(q) Let $(\emptyset \neq) S \subseteq \mathbb{R}^n$. Let $f : S \to \mathbb{R}$ be a scalar field. If f is differentiable at $a \in S$ and it has local minimum & local maximum at a then show that $\nabla f(a)=0$.

Q. 3 (B) **Answer the following: (Attempt any three)** **(12)**

(p) Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be define by $f(x,y,z)= (x+y,\ y+z,\ z+x)$. Find total derivative of f at (1,1,1).

(q) Find Hessian matrix of $f : \mathbb{R}^3 \to \mathbb{R}$ define by $f(x,y,z)=x^3+2xyz+y^2z$ at (1,1,1).

(r) Find local maxima, local minima & saddle points of the function $f(x,y)=x^2+xy+y^2+3x-3y+4$.

(s) Define Hessian matrix. Find Hessian matrix of $f : \mathbb{R}^2 \to \mathbb{R}$ define by $f(x,y)=x^2+4xy-y^2$ at (-1,2).

Q. 4 **Answer the following: (Attempt any three)** **(15)**

(p) Define convergence of sequence in $\mathbb{R}^n$. Prove that the sequence $\left(\frac{1}{n}\right)$ converges to 0 as $n \to \infty$.

(q) Find $\theta \in (0,1)$ such that $f(a+u)-f(a)=D_u f(a+\theta u)$, where $f(x,y)=x^2+y$, $a=(1,0)$, $u=(-1,1)$.

(r) Find f_{xy} & f_{yx} at (0,0) check whether they are equal or not, where

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{(x^2+y^2)}, & (x,y) \neq (0,0) \\ 0, & \text{otherwise} \end{cases}$$

(s) Find the points on the ellipsoid $x^2+2y^2+3z^2=1$, where the tangent plane is parallel to the plane $3x-y+3z=1$.

(t) Define Jacobian matrix of vector field. Find Jacobian matrix of $f : \mathbb{R}^2 \to \mathbb{R}^3$ define by $f(x,y)= (2x^2+3y, 4x-2y, x^3+y^3)$ at (-1,1).

(u) If $z=f(x,y)$ where $x=r\cos\theta$ & $y=r\sin\theta$. Find $\frac{\partial z}{\partial r}$ & $\frac{\partial z}{\partial \theta}$. Also prove that $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\left(\frac{\partial z}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$.

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