CUED - Engineering Tripos Part IIB 2024-2025 **Module Coursework**

Module	4G10	Title of report	Coursework #1

Date submitted: 18/11/2024	Assessment for this module is ☒ 100% / ☐ 25% coursework of which this assignment forms 50 %

UNDERGRADUATE and POST GRADUATE STUDENTS		
Candidate number:	5657B	☒ Undergraduate ☐ Post graduate

Feedback to the student ☐ **See also comments in the text**		Very good	**Good**	Needs improvmt
CONTENT	**Completeness, quantity of content:** Has the report covered all aspects of the lab? Has the analysis been carried out thoroughly?			
	Correctness, quality of content Is the data correct? Is the analysis of the data correct? Are the conclusions correct?			
	Depth of understanding, quality of discussion Does the report show a good technical understanding? Have all the relevant conclusions been drawn?			
	Comments:			
PRESENTATION	**Attention to detail, typesetting and typographical errors** Is the report free of typographical errors? Are the figures/tables/references presented professionally?			
	Comments:			

Marker: Date:

Report

All code can be found at the following link: GitHub Repository

1.

a.

Based on the subset of data that I observed over a multiple iterations of the code used to generate Figure 1. I found that the PSTHs showed a peak in activity during or just before the movement offset, the peak during the movement offset is likely due to neurons firing to move the arm. The peaks just before the movement offset, may be due to the monkey planning/predicting the move or those neurons may be attributed to sensory inputs. Well before the movement offset, many PSTHs are stable with little activity, however some exhibit high activity before the movement offset, these are likely before the go cue and in some cases may be noise or latent activity. In some cases the neurons may be attributed to sensory input from seeing the map. I also noticed some neurons had more activity than others, with consistent peaks of up to 30-40Hz in neuron 150 (Figure 1), this may be due to that neuron being more relevant to the movement.

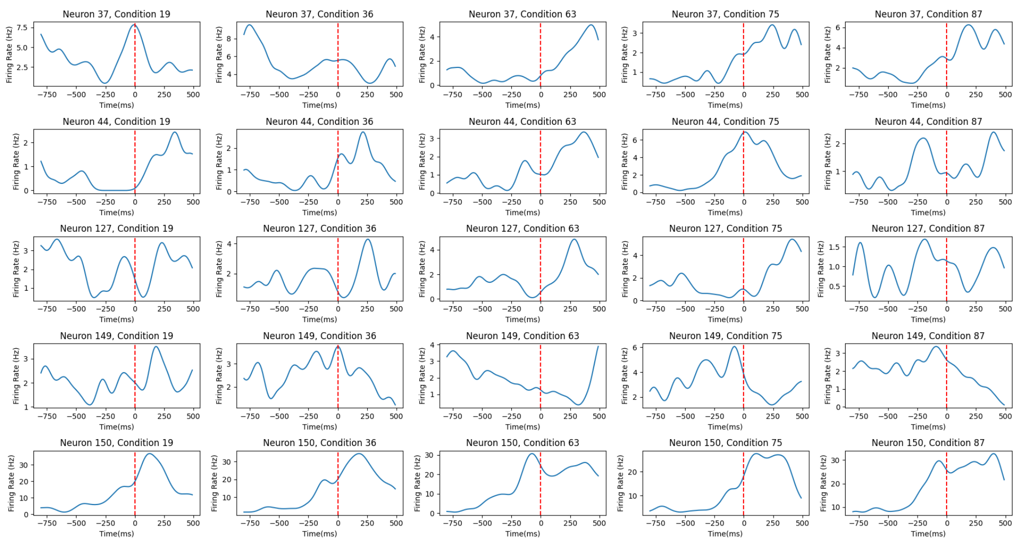

Figure 1: PSTHs of 5 random neurons and 5 random conditions with Movement Offset line

b.

Figure 11 shows the plot of the population average firing rate as a function of time. It begins a significant rise at approximately -300ms, peaking at -40ms. The rise beginning at -300ms is likely due to the monkey reacting to the go onset and the peak at -40ms is showing the high firing required for movement. The movement peak being at -40ms instead of 0ms is possible due to delays in movement being recorded vs the firing of the neurons to begin the movement.

2.

a.

The normalisation step, will be helpful in comparison of neurons, as well as PCA, as we want to be able to meaningfully compare the variances and eigenvalues of the neurons. This is because firing rates can vary due to intrinsic biological differences between neurosn and are not representative to their contribution to the movement.

b.

This is shown in the GitHub code (appendix)

c.

The V used in lectures is not the V used in standard SVD notation where $X = U\Sigma V^T$, and when using SVD, I had to add in the scale factor of $\frac{1}{T}$

3.

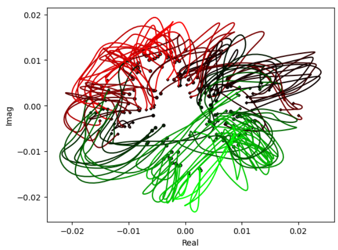

Figure 2: Plot of the trajectories in the PC1-PC2 plane

4.

a.

Detailed Derivation in appendix. (6)

$$\frac{\partial LL}{\partial A} = -(\Delta Z - AZ_{past})Z_{past}^T \tag{1}$$

[0] $p(z_0|A)$ is known/independent of A so can be ignored for our calculation of the log likelihood

b.

For an antisymmetric matrix, we know that the diagonal is 0 and that the upper right and lower left sectors are dependant, so the number of constraints(K):

$$K_M = \frac{M(M-1)}{2}$$
$$K_{12} = \frac{12(12-1)}{2} = 66 \tag{2}$$

The H matrix has to be a KxMxM matrix made of stacked MxM matrices with 0 on the diagonal, with one (+1) value in each position of the upper right traingle in each layer and one (-1) value in each position of the lower left triangle in each layer (this is easier explained by looking at the code).

c.

Detailed derivation in appendix (10)

$$\frac{\partial LL(\beta)}{\partial \beta_b} = \mathbf{b} - \beta Q \tag{3}$$

$$\mathbf{b}_a = \sum_{i,j}^{M} \Delta Z_{i,j} W_{a,i,j} \tag{4}$$

$$Q_{a.b} = \sum_{i,j}^{M} W_{a,i,j} W_{b,i,j} \tag{5}$$

d.

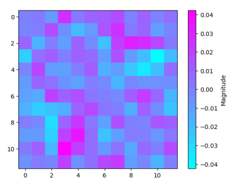

Figure 3: Colour Plot of A

e.

The test used np.allclose and passed with a tolerance of 10e-8.

5.

a.

This is shown in the GitHub code (appendix)

b.

This is shown in the GitHub code (appendix)

c.

Figure 4 shows the trajectories for the plane of fastest rotation. These trajectories seem to capture the overall trend shown in Figure 2 (the overarching circle shape that can be seen in Figure 2. While Figure 2 seems to be a combination of many simple trajectories, Figure 4 seems to represent a simple circular trajectory.

d.

Figure 5 and Figure 6 show the plots of the 2nd and 3rd fastest trajectories, respectively. These seem slightly more complex, this makes intuitive sense as that would explain why they aren't as fast. This can be seen with Figure 5 being more complex than Figure 4 and Figure 6 being more complex than Figure 5.

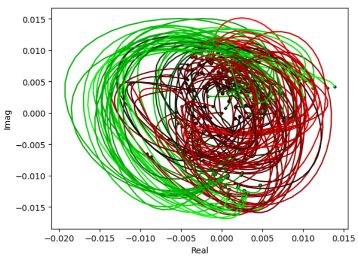

Figure 4: Trajectories for the plane of fastest rotation -150ms,+200ms

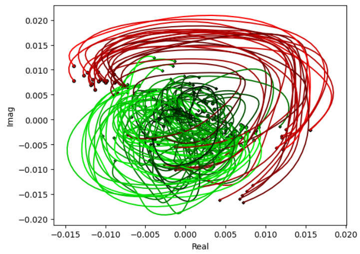

Figure 5: Trajectories for the plane of 2nd fastest rotation -150ms,+200ms

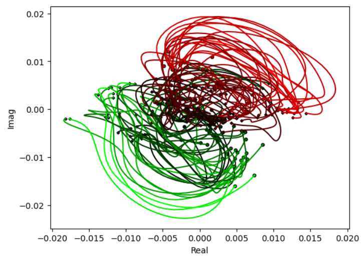

Figure 6: Trajectories for the plane of 3rd fastest rotation -150ms,+200ms

6

As shown in Figure 7 the trajectories for the pre-movement data are very clustered, however they seem to 'flick off' towards their ends. This makes sense, as the clustered data likely represents the data from before the map is displayed (target onset) and the flicks are after the target onset, with the monkey knowing, which direction to move and visualising it, connecting with the post movement onset trajectories.

7

The sort of distortion that is described in the handout, would be expected to destroy the cross condition correlation, as only half the conditions are flipped per neuron. We would expect to lose rotational dynamics from the data. We would also lose smoothness of data across time. We expect the mean firing rate at t_0 to be preserved, as well as the continuity of firing rates across time.

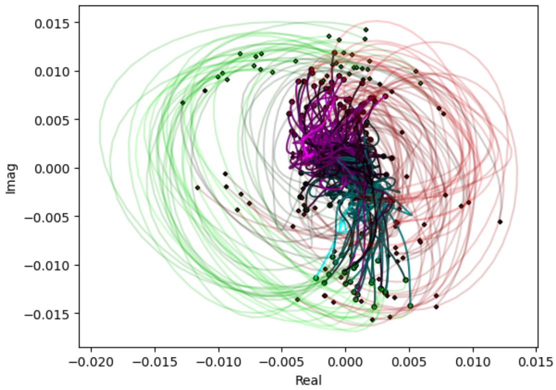

Figure 7: Trajectories -800ms,-150ms over-layed on plot of Trajectories -150ms,200ms

Figure 8, Figure 9 and Figure 10 show the new planes of fastest, 2nd and 3rd fastest rotation. These new trajectories are quite different from Figures fig. 4, fig. 5 and fig. 6. They seem quite random and appear to have no set direction or structure. They do seem to keep a similar shape to the undistorted trajectories, with Figure 8 have a very similar overall shape to Figure 4. This leads to the conclusion that the rotational dynamic we have observed are indeed real and not 'hallucinated'.

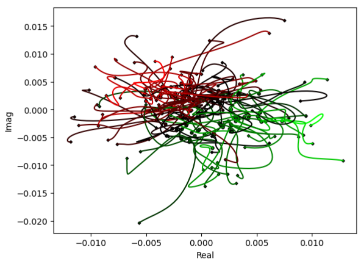

Figure 8: Trajectories for the plane of fastest rotation -150ms,+200ms, post distortion

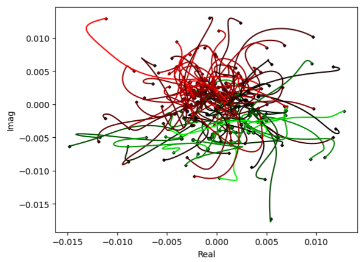

Figure 9: Trajectories for the plane of 2nd fastest rotation -150ms,+200ms, post distortion

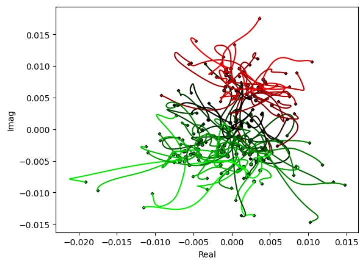

Figure 10: Trajectories for the plane of 3rd fastest rotation -150ms,+200ms, post distortion

Appendix

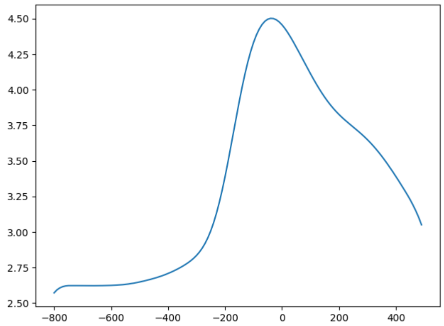

Figure 11: Population Average Firing Rate

Derivation of gradient of Log-Likelihood:

$$\begin{aligned} Likelihood &= p(Z|A) \\ p(\Delta z_{t+1}|z_t, A) &\sim N(Az_t, \sigma I_{m\times m}) \\ &= \frac{1}{(2\pi\sigma^2)^{M/2}} \exp(\frac{1}{2\sigma^2}\|\Delta z_{t+1} - Ax_t\|^2) \end{aligned} \tag{6}$$

$$\begin{aligned} p(Z|A) &= p(z_0|A)p(z_1, z_2, z_3...z_t|z_0, A) \\ &= p(z_0|A)p(\Delta z_1, \Delta z_2...\Delta z_t|z_0, A) \\ &= p(z_0|A)p(\Delta Z|z_0, A)^1 \\ &= p(\Delta Z|z_0, A) \\ &= \prod_{t=0}^{T} p(\Delta z_{t+1}|z_t, A) \end{aligned} \tag{7}$$

$$\begin{aligned} LL(A) = \log(p(Z|A)) &= \sum_{t=0}^{T} \log(p(\Delta z_{t+1}|z_t, A)) \\ &= \sum_{t=0}^{T} \log(\frac{1}{2\pi\sigma^{\frac{M}{2}}}) - \frac{1}{2}\|\Delta z_{t+1} - Az_t\|^2 \\ &= -\frac{1}{2}\sum_{t=0}^{T} \|\Delta z_{t+1} - Az_t\|^2 + const \\ &= -\frac{1}{2}\|\Delta Z - AZ_{past}\|_F^2 \end{aligned} \tag{8}$$

$$\frac{\partial LL}{\partial A} = -(\Delta Z - AZ_{past})Z_{past}^T \tag{9}$$

Detailed Derivation of Log Likelihood in terms of β

$$
\begin{aligned}
A(Z_{past})_{i,j} &= \sum_{k}^{M} A_{i,k}(Z_{past})_{k,j} \\
&= \sum_{k}^{M} \sum_{a}^{K} \beta_a H_{a,i,k}(Z_{past})_{k,j} \\
&= \sum_{a}^{K} \beta_a W_{a,i,j}
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
LL(A) &= -\frac{1}{2}\|\Delta Z - AZ_{past}\|_F^2 \\
&= -\frac{1}{2}\sum_{i,j}(\Delta Z_{i,j} - A_{i,j}(Z_{past})_{i,j})^2 \\
LL(\beta) &= -\frac{1}{2}\sum_{i,j}^{M}(\Delta Z_{i,j} - \sum_{a}^{K}\beta_a W_{a,i,j})^2
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
\frac{\partial LL(\beta)}{\partial \beta_b} &= \sum_{i,j}^{M}(\Delta Z_{i,j} - \sum_{a}^{K}\beta_a W_{a,i,j})W_{b,i,j} \\
&= \sum_{i,j}^{M}\Delta Z_{i,j}W_{b,i,j} - \sum_{a}^{K}\beta_a \sum_{i,j}^{M} W_{a,i,j}W_{b,i,j} \\
&= \mathbf{b} - \beta Q
\end{aligned} \tag{12}
$$

$$
\mathbf{b}_a = \sum_{i,j}^{M}\Delta Z_{i,j}W_{a,i,j} \tag{13}
$$

$$
Q_{a.b} = \sum_{i,j}^{M} W_{a,i,j}W_{b,i,j} \tag{14}
$$