Assignment 6

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Q3

Note $P(S_n > n) = 0$, and $P(S_n = i, (i+n) \mod 2 = 0) = 0$

For even n

$$\begin{aligned} E[e^{\alpha S_n}] =& \sum_i P(S_n = i) e^{\alpha i} \\ =& \sum_{i=2,\text{step}=2}^{n} \binom{n}{i} \left(p^{\frac{n+i}{2}} (1-p)^{\frac{n-i}{2}} e^{\alpha i} + p^{\frac{n-i}{2}} (1-p)^{\frac{n+i}{2}} e^{-\alpha i}\right) + p^{\frac{n}{2}} (1-p)^{\frac{n}{2}} \end{aligned}$$

For odd n

$$\begin{aligned} E[e^{\alpha S_n}] =& \sum_i P(S_n = i) e^{\alpha i} \\ =& \sum_{i=1,\text{step}=2}^{n} \binom{n}{i} \left(p^{\frac{n+i}{2}} (1-p)^{\frac{n-i}{2}} e^{\alpha i} + p^{\frac{n-i}{2}} (1-p)^{\frac{n+i}{2}} e^{-\alpha i}\right) \end{aligned}$$

If SSRW,

$p = 0.5$

For even n

$$E[e^{\alpha S_n}] = 0.5^n \left(\sum_{i=2,\text{step}=2}^{n} \binom{n}{i} \left(e^{\alpha i} + e^{-\alpha i}\right) + 1 \right)$$

For odd n

$$E[e^{\alpha S_n}] = 0.5^n \left(\sum_{i=1,\text{step}=2}^{n} \binom{n}{i} \left(e^{\alpha i} + e^{-\alpha i}\right) \right)$$

5 (b)

As proved in class,

$$
\begin{aligned}
P_0(N_0 \geq k) = 1 - P_0(N_0 < k) &= 1 - \sum_{i=1}^{k-1} (1 - f_{00}) f_{00}^i + (1 - f_{00}) \\
&= 1 - (1 - f_{00}) \frac{f_{00}(1 - f_{00})^{k-1}}{1 - f_{00}} - (1 - f_{00}) \\
&= 1 - f_{00}(1 - f_{00})^{k-1} - 1 + f_{00} \\
&= f_{00} \left(1 - (1 - f_{00})^{k-1}\right)
\end{aligned}
$$

where $f_{00} = P_0(T_0 < \infty) = \min(p, 1-p)$, and $1 - \min p, 1 - p = \max p, 1 - p$

Thus,

$P_0 = 2 \min p, 1 - p^k$