Q5 Let C be closed communicating and finite.

Let j be transient.

$\Rightarrow \lim_{n\to\infty} p_{ij}^{(n)} = 0$

But transience is a class property

$\Rightarrow \lim_{n\to\infty} \sum_{k\in C} p_{ik}^{(n)} = \sum_{k\in S} \lim_{n\to\infty} p_{ik}^{(n)}$ since sum over finite elements.

$= 0.$

But $P(X_n \in C \mid X_0 = i) = 1$ since we have a closed communicating

Hence we have a contradiction.

Q6 $T_i = \inf\{n \geq 0 \mid X_n \neq i\}$

This is a ~~stopping~~ time, therefore from SMP, Z is also MC.

$\Rightarrow \tilde{p}_{ij} = P(Z_n = j \mid \text{PATH}) = P(Z_n = j \mid Z_{n-1} = i)$

$= P(X_n = j \mid X_n \neq i \mid X_{n-1} = i)$

$= \dfrac{P(X_n = j \mid X_{n-1} = i)}{P(X_n \neq i \mid X_{n-1} = i)} = \dfrac{p_{ij}}{\sum_{k\neq i} p_{ik}}$