

# Summary before Midsem

Dhruva Sambrani

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## Can a Markov Chain be reversible but not symmetric

Does  $\pi_i p_{ij} = \pi_j p_{ji}$  imply  $p_{ji} = p_{ij}$ ?

No. Consider symmetric 1D chain.

## Markov Chains (Homogeneous)

1.  $\{X_n\}$  is  $MC(\lambda, P, S)$  if  $P(X_0 = i) = \lambda_i$  and  $P(X_{n+1}|PATH) = P(X_{n+1}|X_n)$
2. if  $\forall n \geq 0, P(PATH) = \lambda_{i_0} p_{i_0 i_1} \cdots p_{i_{n-1} i_n}$
3. Chapman Kolmogorov  $\Rightarrow p_{ij}^{n+m} = \sum_k p_{ik}^n p_{kj}^m$
4. Relations
  - a.  $i \rightarrow j$  if exists  $PATH(i,j)$  st  $P(PATH(i,j)) > 0$
  - b.  $i \leftrightarrow j$  if  $i \rightarrow j$  and  $j \rightarrow i$ . This is an equivalence relation and partitions  $S$  into communicating classes
  - c. communicating class is closed if  $\sum_{j \in C} p_{ij} = 1 \forall i \in C$
5. Absorbing State -
  - a.  $i$  st  $p_{ii} = 1$
  - b.  $\{i\}$  is closed communicating class
6. Irreducible chain -  $S$  is a communicating class
7. Inessential state  $i$ : If  $i$  st  $i \rightarrow j$  but  $j \not\rightarrow i$
8. Period of  $i$ 
  - a.  $d(i) = \gcd\{n \geq 0; p_{ii}^n > 0\}$
  - b. is a class property
  - c.  $i$  is aperiodic then exists  $N_0$  st  $p_{ii}^n > 0 \forall n \geq N_0$
  - d. Irreducible and aperiodic  $\Rightarrow N_0$  st  $p_{ij}^n > 0 \forall n \geq N_0$
  - e. New relation  $i \leftrightarrow^d j$  iff  $p_{ij}^{(nd)} > 0$  for some  $n \geq 0$ . Equivalence.
    1.  $S$  decomposed to classes, st chain moves from one  $C$  to next.
9. A rv  $\tau$  :  $S \rightarrow \mathbb{R}^+$ , st  $\{\tau=n\}$  in  $\sigma(X_0, \dots, X_n)$  is called a stopping time.
10. SMP - For  $T, Z_n = X_{\{T+n\}}$  is also a MC.

11.  $T_i^{(r)} = \inf \{n > T_i^{(r-1)}; X_n = i\}$
12.  $\text{Chi}_i^{\wedge}(r) = \{X_n : T_i^{\wedge}(r-1) \leq n < T_i^{\wedge}(r)\}$ 
  - a.  $\{\text{Chi}_i^{\wedge}(r)\}$  are iid from the SMP
13. Hitting Time
  - a.  $h_i^A = P_i(T_A < \infty)$
  - b.  $h_i^A = 1$  for  $i$  in  $A$  and  $= \sum_j p_{ij} h_j^A$  if  $i$  not in  $A$
  - c.  $\psi_i^A = E_i[T_A]$
  - d.  $\psi_i^A = 0$  for  $i$  in  $A$  and  $= 1 + \sum_j p_{ij} \psi_j^A$  if  $i$  not in  $A$
14. Recurrence and Transience
  - a. Both are class properties
15. Stability theorem: irr, + recurrent -  $P(X_n = i) \xrightarrow{n \rightarrow \infty} \pi_i$
16. detailed balance  $\iff$  KLC
17. Metropolis algorithm