

Summary before Midsem

Dhruva Sambrani

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Can a Markov Chain be reversible but not symmetric

Does $\pi_i p_{ij} = \pi_j p_{ji}$ imply $p_{ji} = p_{ij}$?

No. Consider symmetric 1D chain.

Markov Chains (Homogeneous)

1. $\{X_n\}$ is $MC(\lambda, P, S)$ if $P(X_0 = i) = \lambda_i$ and $P(X_{n+1}|PATH) = P(X_{n+1}|X_n)$
2. if $\forall n \geq 0, P(PATH) = \lambda_{i_0} p_{i_0 i_1} \cdots p_{i_{n-1} i_n}$
3. Chapman Kolmogorov $\Rightarrow p_{ij}^{n+m} = \sum_k p_{ik}^n p_{kj}^m$
4. Relations
 - a. $i \rightarrow j$ if exists $PATH(i,j)$ st $P(PATH(i,j)) > 0$
 - b. $i \leftrightarrow j$ if $i \rightarrow j$ and $j \rightarrow i$. This is an equivalence relation and partitions S into communicating classes
 - c. communicating class is closed if $\sum_{j \in C} p_{ij} = 1 \forall i \in C$
5. Absorbing State -
 - a. i st $p_{ii} = 1$
 - b. $\{i\}$ is closed communicating class
6. Irreducible chain - S is a communicating class
7. Inessential state i : If i st $i \rightarrow j$ but $j \not\rightarrow i$
8. Period of i
 - a. $d(i) = \gcd\{n \geq 0; p_{ii}^n > 0\}$
 - b. is a class property
 - c. i is aperiodic then exists N_0 st $p_{ii}^n > 0 \forall n \geq N_0$
 - d. Irreducible and aperiodic $\Rightarrow N_0$ st $p_{ij}^n > 0 \forall n \geq N_0$
 - e. New relation $i \leftrightarrow^{(nd)} j$ iff $p_{ij}^{(nd)} > 0$ for some $n \geq 0$. Equivalence.
 1. S decomposed to classes, st chain moves from one C to next.
9. A rv τ : $S \rightarrow \mathbb{R}^+$, st $\{\tau=n\}$ in $\sigma(X_0, \dots, X_n)$ is called a stopping time.
10. SMP - For $T, Z_n = X_{\{T+n\}}$ is also a MC.

11. $T_i^{(r)} = \inf \{n > T_i^{(r-1)}; X_n = i\}$
12. $\chi_i^{\wedge}(r) = \{X_n : T_i^{\wedge}(r-1) \leq n < T_i^{\wedge}(r)\}$
 - a. $\{\chi_i^{\wedge}(r)\}$ are iid from the SMP
13. Hitting Time
 - a. $h_i^A = P_i(T_A < \infty)$
 - b. $h_i^A = 1$ for $i \in A$ and $= \sum_j p_{ij} h_j^A$ if $i \notin A$
 - c. $\psi_i^A = E_i[T_A]$
 - d. $\psi_i^A = 0$ for $i \in A$ and $= 1 + \sum_j p_{ij} \psi_j^A$ if $i \notin A$
14. Recurrence and Transience
 - a. Both are class properties
15. Stability theorem: irr, + recurrent - $P(X_n = i) \xrightarrow{n \rightarrow \infty} \pi_i$
16. detailed balance \iff KLC
17. Metropolis algorithm