Summary before Midsem

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October 11, 2022

Can a Markov Chain be reversible but not symmetric

Does $\pi_i p_{ij} = \pi_j p_{ji}$ imply $p_{ji} = p_{ij}$?

No. Consider symmetric 1D chain.

Markov Chains (Homogeneous)

1. $\{X_n\}$ is $MC(\lambda, P, S)$ if $P(X_0 = i) = \lambda_i$ and $P(X_{n+1}|PATH) = P(X_{n+1}|X_n)$
2. if $\forall n >= 0$, $P(PATH) = \lambda_{i_0} p_{i_0 i_1} \dots p_{i_{n-1} i_n}$
3. Chapman Kolmogorov => $p_{ij}^{n+m} = \sum p_{ik}^n p_{kj}^m$
4. Relations
   a. i –> j if exists PATH(i,j) st P(PATH(i,j)) > 0
   b. i <-> j if i –> j and j –> i. This is an equivalence relation and partitions S into communicating classes
   c. communicating class is closed if $\sum_{j \in C} p_{ij} = 1 \forall i \in C$
5. Absorbing State -
   a. i st $p_i i = 1$
   b. {i} is closed communicating class
6. Irreducible chain - S is a communicating class
7. Inessential state i: If i st i -> j but j $\nrightarrow$ i
8. Period of i
   a. d(i) = gcd{n>=0; $p_i i^n > 0$}
   b. is a class property
   c. i is aperiodic then exists N_0 st $p_i i^n > 0 \forall n \geq N_0$
   d. Irreducible and aperiodic => N_0 st $p_{ij}^n > 0 \forall n >= N_0$
   e. New relation i <-d-> j iff $p_{ij}^{(nd)} > 0$ for some n >= 0. Equivalence.
      1. S decomposed to classes, st chain moves from one C to next.
9. A rv tau : S –> RR, st {tau=n} in sigma(X_0, …, X_n) is called a stopping time.
10. SMP - For T, Z_n_ = X_{T+n}_ is also a MC.

11. T_i_$^{(r)}$ = inf {n> T_i_$^{(r-1)}$; X_n_=i}
12. Chi_i^(r) = {X_n : T_i^(r-1) <= n < T^(r)}
    a. {Chi_i^(r)} are iid from the SMP
13. Hitting Time
    a. h_i^A = P_i(T_A < infty)
    b. h_i^A = 1 for i in A and = sum_j pij h_j^A if i notin A
    c. psi_i^A = E_i[T_A]
    d. psi_i^A = 0 for i in A and = 1 + sum_j pij psi_j^A if i notin A
14. Recurrence and Transience
    a. Both are class properties
15. Stability theorem: irr, + recurrent - P(X_n = i) –n to infty–> pi_i
16. detailed balance <==> KLC
17. Metropolis algorithm