# Assignment 7

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## Q2

**To Show -** $S_n = \Pi_{i=1}^{n} X_i$ **is a martingale**

For this, it is enough to show that $E\left[S_{n+1}|F_n\right] = S_n$.

**Proof:**

$$\begin{aligned} E\left[S_{n+1}|F_n\right] &= E\left[S_n|F_n\right]E\left[X_{n+1}|F_n\right] \rightarrow \text{ from independence} \\ &= S_n E\left[X_{n+1}\right] \rightarrow \text{ property of expectation} \\ &= S_n E\left[X_0\right] \rightarrow \text{ identically distributed} \\ &= S_n \end{aligned}$$

Thus, $S_n$ is a martingale.

## Q7

$$E\left[\left(B\left(0\right)+\int_0^t B\left(m\right)dm\right)\left(B\left(0\right)+\int_0^s B\left(m\right)dm\right)\right]-E\left[B\left(0\right)+\int_0^t B\left(m\right)dm\right]E\left[B\left(0\right)+\int_0^s B\left(m\right)dm\right]$$
$$=Var\left[B\left(0\right)+\int_0^t\right]+E\left[\left(B\left(0\right)+\int_0^t\right)\left(\int_t^s\right)\right]-E\left[B\left(0\right)+\int_0^t\right]E\left[B\left(0\right)\right]-E\left[B\left(0\right)+\int_0^t\right]E\left[\int_t^s\right]$$
$$=Var\left[X\left(t\right)\right]-E\left[X\left(t\right)\right]E\left[B\left(0\right)\right]$$