Dhanvina Sambrani
MS18163.

HSS646 - Quiz 2.

Q1 $U(x_1, x_2) = 2\log(x_1) + 3\log(x_2)$

$= \log x_1^2 x_2^3$

Since log is monotonic transformation,
$u'(x_1, x_2) = x_1^2 x_2^3$ is also a valid U.

$20x_1 + 30x_2 = 200$

$\Rightarrow x_1 = 10 - \frac{3}{2}x_2$

$\Rightarrow U = \left(100 + \frac{9}{4}x_2^2 - 30x_2\right)(x_2^3)$

$\Rightarrow \frac{\partial U}{\partial x_2} = x_2^3\left(\frac{9}{2}x_2 - 30\right) + 3x_2^2\left(100 + \frac{9}{4}x_2^2 - 30x_2\right)$

$= x_2^2\left(\frac{9}{2}x_2^2 - 30x_2 + 300 + \frac{27}{4}x_2^2 - 90x_2\right).$

$= x_2^2\left(\frac{45}{4}x_2^2 - 120x_2 + 300\right).$

$= \frac{15}{4}x_2^2\left(3x_2^2 - 32x_2 + 80\right).$

$\Rightarrow x_2 = 6.67$ or $x_2 = 4$

$\Rightarrow x_1 = 0$ or $x_1 = 4$

Clearly, $U(x_1^*, x_2^*)$ is highest for
$\boxed{x_1^* = 4, \quad x_2^* = 4}$

Dhruva Sambrani
MS18163.

Q2. $f(L,K) = L^2 K. = q = 32.$

$C = 2(L+K).$

Therefore,

$\min \; C := 2(L+K).$

over constraint $L^2 K = q.$

$\Rightarrow \mathcal{L} = 2(L+K) + \lambda(L^2K - q)$

$\frac{\partial \mathcal{L}}{\partial L} = 2 + 2\lambda LK = 0 \Rightarrow -1 = \lambda LK$. — ①

$\frac{\partial \mathcal{L}}{\partial K} = 2 + \lambda L^2 = 0 \Rightarrow -L^2 = \frac{\lambda}{2}.$ — ②

$\frac{\partial \mathcal{L}}{\partial \lambda} = L^2 k - q = 0 \Rightarrow L^2 K = q.$ — ③

From ① and ②,

$2L^3 K = 1$ — ④

from ④ and ③

$2L^3 K = 2q\,L = 1 \Rightarrow L = 1/64$

$\Rightarrow K = \frac{64^3}{2} = 131072.$ → from ④

$\Rightarrow \lambda = 2 \times L^2 = 2 \times \frac{1}{64^2} = \frac{1}{2048}$ → from ②

$\Rightarrow \boxed{L^* = \frac{1}{64},\; K^* = 131072. \Rightarrow C^* = 262144.03125.}$

Dhruva Sambrani
MS18163.

Q3. $f = L^{\alpha} K^{\beta}$

a) $C = wL + rK$.

min f on constraint C.

$\Rightarrow \mathcal{L} = L^{\alpha} K^{\beta} - \lambda(wL + rK)$

$$\frac{\partial \mathcal{L}}{\partial L} = \alpha L^{\alpha-1} K^{\beta} - \lambda w = 0 = MP_{L/w} = \lambda \quad -①$$

$$\frac{\partial \mathcal{L}}{\partial K} = \beta L^{\alpha} K^{\beta-1} - \lambda r = 0 = \frac{MP_K}{r} = \lambda \quad -②$$

$$\boxed{\Rightarrow \frac{MP_L}{MP_K} = \frac{w}{r} = MRTS.}$$

b) $f = L^{1/3} K^{2/3}$.

on solving ① and ②,

$$\frac{\alpha L^{\alpha-1} K^{\beta}}{w} = \frac{\beta L^{\alpha} K^{\beta-1}}{r}$$

$$\Rightarrow L^{-1} K^{1} = \frac{\beta w}{\alpha r}$$

$$\Rightarrow K = \frac{\beta w}{\alpha r} L = \frac{2/\cancel{3}\, w}{1/\cancel{3}\, r} L = \frac{2wL}{r}.$$

$$\Rightarrow \boxed{K = \frac{2wL}{r}}$$