PHY102 : Assignment 3

1. (Purcell 2.15) Compute the curl and divergence of each of the following vector fields. Which of these could be **E** fields ? Find the corresponding potential function ϕ.

   (a) $F_x = x + y; F_y = -x + y; F_z = -2z$.

   (b) $G_x = 2y; G_y = 2x + 3z; G_z = 3y$.

   (c) $H_x = x^2 - z^2; H_y = 2; H_z = 2xz$.

2. (Purcell 2.16) If **A** is any vector field with continuous derivatives, $\nabla \cdot (\nabla \times \mathbf{A}) = 0$. Prove this in two ways :

   (a) Prove explicitly by using the formula for ∇ in cartesian coordinates.

   (b) Consider a surface S, a balloon almost cut in two which is bounded by the closed curve C. Think about the line integral, over a curve like C, of any vector field. Then invoke Stokes' theorem and the divergence theorem with suitable arguments.

3. (Purcell 2.31) A flat nonconducting sheet lies in the xy plane. The only charges in the system are on this sheet. In the half-space above the sheet, $z > 0$, the potential is $\phi = \phi_0 e^{-kz} \cos kx$, where ϕ_0 and k are constants.

   (a) Verify that ϕ satisfies Laplace's equation in the space above the sheet.

   (b) What do the electric field lines look like ?

   (c) Describe the charge distribution on the sheet.

4. (a) A ring with radius R has charge Q uniformly distributed on it. It lies in the xy plane, with its center at the origin. Find the electric field at all points on the z axis. For what value of z is the field maximum ?

   (b) Make a rough sketch of the equipotential curves everywhere in a plane containing the z axis. The ring can be represented by two dots where it intersects the plane.

5. A point charge q is located at an arbitrary position inside a neutral conducting spherical shell. Explain why the electric field outside the shell is the same as the spherically symmetric field due to a charge q located at the center of thes shell.

6. (Purcell 3.1) A spherical conductor A contains two spherical cavities. The total charge on the conductor itself is zero. However, there is a point charge q_b at the center of one cavity and q_c at the center of the other. A considerable distance r away is another charge q_d. What force acts on each of the four objects, A, q_b, q_c, q_d ? Which answers, if any, are only approximate, and depend on r being relatively large ?