

Matrices Problem Sheet

Maths Workshop 2020

December 18, 2020

Problems

1. Find the order and evaluate the transpose of the following matrices :-

$$(i) \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \quad (ii) \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} \quad (iii) [2 \quad 5 \quad 7]$$

2. Given matrices $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and the equation $(A + B)^2 = A^2 + B^2$. Evaluate the values of a and b.

3. If $P(x) = \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix}$, then check that $P(x)P(y) = P(x+y) = P(y)P(x)$.

4. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 7 \\ 3 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$. Evaluate AB and verify that $(AB)^t = B^t A^t$.

5. Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Find the value of A^2 and A^3 and guess the value of A^n . Can you prove your guess using mathematical induction, or otherwise ?

6. Given matrix $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ show that $A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$ for some natural number n .