Tutorial-6 (PHY201) Due on Wednsday

1. Explain key features of the phenomenon of Rayleigh scattering of a plane EM radiation. Discuss by making careful diagram, how and when the Rayleigh scattering converts unpolarized light into a perfectly linear polarized light.

   Discuss five daily life phenomena that exploit the Rayleigh scattering of EM radiation.

2. A copper box with dimensions as shown in the figure acts as a cavity resonator. The electric field

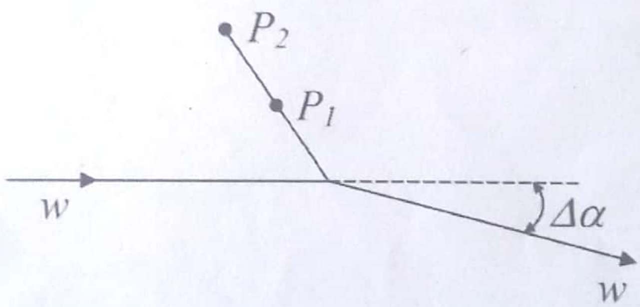
$$E_z = E_0\ sin(k_x x)\ sin(k_y y)\ sin(\omega t),\ E_x = E_y = 0$$

   is a possible solution of the wave equation for this case.

   (a) Find the lowest resonance frequency ω_1 and the corresponding free space wavelength λ_1.

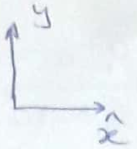
   (b) Find the next-to-lowest resonance frequency ω_2 and the corresponding free space wavelength λ_2.

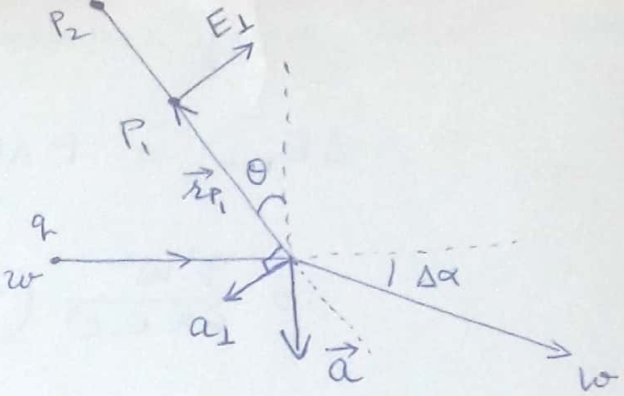
3. A point charge q has been moving with constant velocity w along a straight line until the time $t=t_0$. In the short time interval from time t_0 to $t_0+\Delta t$, a force perpendicular to the trajectory changes the direction without changing the magnitude of the velocity. After the time $t= t_0+\Delta t$ the charge again moves with the velocity w along a straight line making a small angle $\Delta\alpha$ with the initial trajectory.



   (a) What is the direction of E-field caused by the acceleration, at the distant point P_1.
   (b) In what direction is the radiation intensity of the accelerated charge the most intense?
   (c) Where is it least intense
   (d) Point P_2 is twice as far from the bend of trajectory as P_1. By what fraction does the amplitude of magnetic disturbance decrease as the radiation pulse move from P_1 to P_2?
   (e) What is the total energy radiated?

3





(a) Acceleration along $-y$ direction

$$\vec{a} = \frac{\Delta V_y}{\Delta t}\,\hat{y}$$

for $\Delta\alpha << 1 \Rightarrow \Delta v_y = w \sin\Delta\alpha \simeq w\,\Delta\alpha$

$$\vec{a} \simeq w \frac{\Delta\alpha}{\Delta t}\,\hat{y}$$

- Component $a_\perp$ $\perp$ to position vector to $\vec{r}_{P_1}$

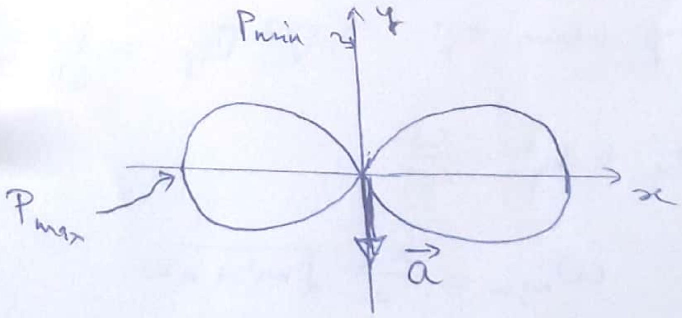
$$a_\perp = a_y \sin\theta = w\frac{\Delta\alpha}{\Delta t}\sin\theta$$

- E-field at P_1 is in the plane of $a_\perp$ & $\vec{r}_{P_1}$ & antiparallel to $a_\perp$

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r}\,\frac{a_\perp}{c^2}\,(\hat{r}_{P_1} \times \hat{z})$$

$$= \frac{q}{4\pi\epsilon_0 r}\,\frac{w\,\Delta\alpha\,\sin\theta}{c^2\,\Delta t}\,(\cos\theta\,\hat{x} + \sin\theta\,\hat{y})$$

(b) Radiation intensity $P \propto \sin^2\theta$



Most intense XZ plane

$P_{min} \equiv YZ$-plane

(d) Associated $\vec{B}$-field

$$\vec{B}(\vec{r},t) = \hat{r} \times \frac{\vec{E}(\vec{r},t)}{c}$$

$$\vec{B} \propto \frac{E_\perp}{c} \propto \frac{1}{r}$$

Amp. of B field reduces by a factor 2 at P_2

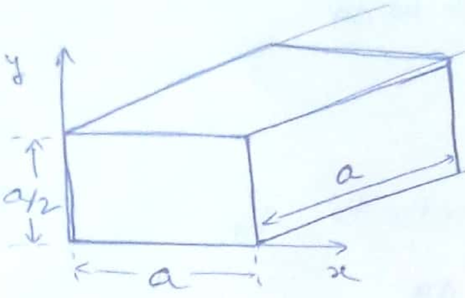
(e) Total energy radiated in Δt interval

$$\Delta E_{rad.} = P\,\Delta t = \frac{q^2 a^2}{6\pi\epsilon_0 c^3}\,\Delta t$$

$$= \frac{q^2 \omega^2}{6\pi\epsilon_0 c^3}\left(\frac{\Delta\alpha}{\Delta t}\right)^2 \Delta t$$

{ Radiated energy is integrated for entire sphere.

(2)



Box of dimension $a \times a \times \frac{a}{2}$

$$\vec{E} = E_0 \sin(k_x x)\sin(k_y y)\sin(\ldots)\,\hat{z}$$

Boundary condition

$$E_z(x=0) = E_z(x=a) = E_z(y=0) = E_z(y=a) = 0$$

$k_x = n\frac{\pi}{a}$, $k_y = m\frac{\pi}{a}$ & $k_z = 0$

$m, n \geq 1$ & integers

Z - component of wave eq^n. $\nabla^2 E_z = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$

$$k_x^2 + k_y^2 = \frac{\omega^2}{c^2}$$

Resonances $\omega_{m,n} = \frac{\pi c}{a}\sqrt{m^2+n^2}$

(a) Lowest freq. $m = n = 1 \Rightarrow \omega_1 = \frac{\pi c\sqrt{2}}{a}$; $\lambda_1 = \sqrt{2}\,a$

(b) Next freq. $m = 1,\ n = 2$ OR $m = 2,\ n = 1$ } $\omega_2 = \frac{\pi c}{a}\sqrt{5}$; $\lambda_2 = \frac{2a}{\sqrt{5}}$

$\boxed{Q-1}$ Tut. -6 : Solutions

Rayleight scattering!

(a) size of scatterer $<< \lambda$

(b) Power $\propto \omega^4 \propto \frac{1}{\lambda^4}$

(c) for $\vec{r} \perp$ to incoming beam light is perfectly linearly polarized for unpolarized radiation

Phenomena: Blue sky, Red sunset sunrise, blue smoke Scattering in water of laser, Red traffic sign etc.