Homework - 1

1. (i) Check all the incomplete proofs discussed in class.

   (ii) Solve all the problems mentioned in class.

2. Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ is continuous at $(a, b, c) \in \mathbb{R}^3$. Show that

$$g(t) = f(t, b, c) \text{ is continuous at}$$

$t = a$.

3. Check if the following functions are continuous:

(a) $$f(x, y) = \frac{xy}{x^2 + y^2 + 1}$$

(b) $$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^3 + y^3}, & (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0) \end{cases}$$

(c) $$f(x, y) = \begin{cases} \dfrac{x^4 + y^4}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

4. Suppose $f : A \to \mathbb{R}$ ~~and~~ is continuous. Show that $|f|$ is continuous where $|f|(a) = |f(a)| \; \forall a \in A$.

5. Let C be the parabola $x^2 = y^2$, i.e. $C = \{(x,y) : x = y^2\}$

Show that C is homeomorphic to $\mathbb{R}$.

6. Show that the unit disk $\mathbb{D} = \{(x,y) : x^2+y^2 < 1\}$ is homeomorphic to $\mathbb{R}^2$.

7. Let A be square in $\mathbb{R}^2$ with vertices $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$. Show that A is homeomorphic to the circle $C = \{(x,y) : x^2+y^2 = 1\}$.

8. Show by an example that a bijective continuous map is not necessarily a homeomorphism.