NFA

Dhruva Sambrani

February 12, 2020

Formal Defn of an NFA

M = (Q, Σ, δ, q0, F)

δ : Q $\times$ ($\Sigma \cup \{\epsilon\}$) $\rightarrow \mathcal{P}$(Q)

An NFA N accepts a string s = w1w2w3...wn iff $\exists$ a sequence of r0r1...rn $\in$ Q such that rn $\in$ F and ri+1 $\in$ δ(ri, wi)

Equivalence of NFA and DFA

Theorem: L is regular iff it can be recognized by an NFA.

Proof:

If L is recognisable by a NFA (let) N = (Q, Σ, δ, q0, F)

Consider the DFA, D = (Q', Σ, δ', q0', F') where -

- Q' = $\mathcal{P}$(Q)
- δ' : Q' $\rightarrow$ Q' and
  - δ'(A, c) $\rightarrow \bigcup$ E(δ(a,c)), where
    - a$\in$A
    - E(A) = the set of states connected to some q$\in$A by ϵ. Trivially, some qs map to themselves via a ϵ.
- q0' = E(q0)
- F' = {A'$\in$Q' | A$\cap$F $\neq \emptyset$}

If S is accepted by N iff $\exists$ r0...rn $\in$ Q r0=q0 and ri+1 $\in$ δ(ri, wi)

$\implies$ ri+1 $\in$ Ri = E(δ(ri, wi)) and rn $\in$ Rn $\in$ F.