

## Homework - 5

- 1) Solve the exercises mentioned in class.
- 2) For a regular, smooth <sup>plane</sup> curve  $\alpha$  show that
- (i)  $\frac{d\vec{n}_s}{ds} = -\kappa_s \vec{T}$

(ii) Show that  $\kappa_s$  is a smooth function of  $s$ .

- 3) Suppose  $\alpha: [a, b] \rightarrow \mathbb{R}^2$  is a smooth regular curve and  $\beta: [a, b] \rightarrow \mathbb{R}^2$  be defined by  $\beta(t) = \alpha(a+b-t)$ . Show that  $\beta$  is regular. (i) Show that their signed curvature functions are  $\kappa_1$  and  $\kappa_2$  respectively. Show that

$$\kappa_2(t) = -\kappa_1(a+b-t)$$

Interpret this geometrically.

- 4) Given an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b > 0$  find a parametrization. Using this find the points where the curvature is maximum and minimum respectively.

- 5) Draw the following curves and find curvature wherever defined.

(i)  $p = 2 + \cos \theta$

(ii)  $p = 1 + 2\cos \theta$

(iii)  $p = \sin 2\theta$

- 6) Find curvature of the following curves at any point.

(i)  $y = e^x$

(ii)  $y = \frac{1}{x}$ ,  $x > 0$

(iii)  $y = \log x$ .