Philosophy of Science: Solutions to Exercises

1 Exercises from 'PoS Notes 1'

Exercise 1.1. *Let A and B denote the statements 'Anil will be elected' and 'Mukesh will be elected' respectively. Write the following statements in the form of symbols A, B, their negations and/or conjunction:*

(i) Anil and Mukesh will not both be elected.

(ii) Anil and Mukesh will both not be elected.

Solution:

(i) "Anil and Mukesh will not both be elected" can be symbolised as $\sim (A \cdot B)$.

(ii) "Anil and Mukesh will both not be elected" can be symbolised as $(\sim A) \cdot (\sim B)$.

Exercise 1.2. *For statements p, q, let $A =\sim (p \cdot q)$ and $B = (\sim p) \cdot (\sim q)$. Verify that A and B are not same by writing a truth table.*

Solution: We consider the following truth table.

p	q	$\sim p$	$\sim q$	$\sim (p \cdot q)$	$(\sim p) \cdot (\sim q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	T

2 Exercises from 'PoS Notes 2'

Exercise 2.1. *If X, Y are false statements, is $X \implies (X \implies Y)$ true?*

Solution: Yes. Let's verify this. Suppose $Z = X \implies Y \equiv\sim (X\cdot \sim Y)$. To see that truth value of Z if T, check that truth values of $\sim (X\cdot \sim Y)$ is T when X, Y are false (F). Similarly for $X \implies Z \equiv\sim (X\cdot \sim Z)$. The truth table is as follows:

X	Y	Z	$X \implies Z$
F	F	T	T

Exercise 2.2 (Fallacy of Affirming the Consequent). *Show that the following argument is invalid.*

$$\begin{array}{l} p \implies q \\ q \\ \therefore \quad p \end{array}$$

Solution: We will show that the above argument and the following argument ('Denying the Antecedent') are both invalid.

$$\begin{array}{l} p \implies q \\ \sim p \\ \therefore \quad \sim q \end{array}$$

Let us look at the following truth table. Recall that an argument is invalid if any of the substitution

p	q	$\sim p$	$\sim q$	$p \implies q$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

instances are such that it has have true premisses and false conclusions. For the first argument i.e. "Fallacy of Affirming the Consequent", observe that in the third row of the truth table above we have 'T' for both. $p \implies q$ and q but 'F' for p. This shows that the argument is invalid. Argue similarly for "the Fallacy of Denying the Antecedent".

Exercise 2.3. *Show that the statements* $\sim (p \cdot q)$ *and* $\sim p \vee \sim q$ *are logically equivalent.*

Solution: We consider the following truth table (and check that last column contains only 'T's).

p	q	$\sim p$	$\sim q$	$p \cdot q$	$\sim (p \cdot q)$	$(\sim p) \vee (\sim q)$	$\sim (p \cdot q) \equiv (\sim p) \vee (\sim q)$
T	T	F	F	T	F	F	T
T	F	F	T	F	T	T	T
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	T

3 Exercises from 'PoS Notes 3'

Exercise 3.1. *State the rule of inference/replacement by which its conclusion follows from the premiss in the arguments given below:*

(i)

$$(\sim A \implies B) \cdot (C \vee \sim D)$$
$$\therefore (\sim A \implies B) \cdot (\sim D \vee C)$$

(ii)

$$(\sim E \vee F) \cdot (G \vee \sim H)$$
$$\therefore (E \implies F) \cdot (G \vee \sim H)$$

(iii)

$$M \implies \sim (N \vee \sim O)$$
$$\therefore M \implies (\sim N \cdot \sim\sim O)$$

Solution:

(i) Commutation: $(C \vee \sim D) \equiv (\sim D \vee C)$.

(ii) Material Implication: $(\sim E \vee F) \equiv (E \implies F)$.

(iii) De Morgan's Theorem: $\sim (N \vee \sim O) \equiv (\sim N \cdot \sim\sim O)$.

4 Exercises from 'PoS Notes 4'

Exercise 4.1. *Fill in the details of the proof in example 3.1 by specifying the elementary valid arguments used to deduce steps 6, 7, 8, 9, 10 and 13.*

Solution:

1. $A \implies (B \cdot C)$
2. $(B \vee D) \implies E$
3. $D \vee A$ $\quad$ ($\therefore E,$ this is the conclusion of the argument)
4. $\sim E$ $\quad$ (adding the negation - Indirect Proof (I.P.))
5. $\sim (B \vee D)$ $\quad$ $(M.T.\ 2, 4)$
6. $\sim B \cdot \sim D$ $\quad$ $(De.M.\ 5)$
7. $\sim D \cdot \sim B$ $\quad$ $(Comm.\ 6)$

8. $\sim D$ $\quad (Simp.\ 7)$

9. A $\quad (D.S.\ 3, 8)$

10. $B \cdot C$ $\quad (M.P.\ 1, 9)$

11. B $\quad (Simp.\ 10)$

12. $\sim B$ $\quad (Simp.\ 6)$

13. $B \cdot \sim B$ $\quad (Conj.\ 11, 12)$

This completes the indirect proof of the following argument:

$$\begin{array}{l} A \implies (B \cdot C) \\ (B \vee D) \implies E \\ D \vee A \\ \therefore \quad E \end{array}$$

Exercise 4.2. *Construct both a formal proof of validity and an indirect proof for the following argument:*

$$\begin{array}{l} (H \implies I) \cdot (J \implies K) \\ (I \vee K) \implies L \\ \sim L \\ \therefore \quad \sim (H \vee J) \end{array}$$

Solution: We first give the **direct proof:**

1. $(H \implies I) \cdot (J \implies K)$

2. $(I \vee K) \implies L$

3. $\sim L$ $\quad (\therefore \sim (H \vee J),$ this is the conclusion of the argument)

4. $\sim (I \vee K)$ $\quad (M.T.\ 2, 3)$

5. $\sim I \cdot \sim K$ $\quad (DeM\ 4)$

6. $\sim I$ $\quad (Simp.\ 5)$

7. $H \implies I$ $\quad (Simp.\ 1)$

8. $\sim H$ $\quad (M.T.\ 6, 7)$

9. $(J \implies K) \cdot (H \implies I)$ $\quad (Comm\ 1)$

10. $J \implies K$ $\quad (Simp.\ 9)$
11. $\sim K \cdot \sim I$ $\quad (Comm.\ 5)$
12. $\sim K$ $\quad (Simp.\ 11)$
13. $\sim J$ $\quad (M.T.\ 10, 12)$
14. $\sim H \cdot \sim J$ $\quad (Conj.\ 8, 13)$
15. $\sim (H \vee J)$ $\quad (DeM\ 14)$

We now give the **indirect proof:**

1. $(H \implies I) \cdot (J \implies K)$
2. $(I \vee K) \implies L$
3. $\sim L$ $\quad (\therefore \sim (H \vee J),$ this is the conclusion of the argument)

4. $\sim\sim (H \vee J)$ (adding the negation - Indirect Proof (I.P.))
5. $(H \vee J)$ $\quad (D.N.\ 4)$
6. $I \vee K$ $\quad (C.D.\ 1, 5)$
7. L $\quad (M.P.\ 2, 6)$
8. $L \cdot \sim L$ $\quad (Conj.\ 3, 7)$

which is a contradiction.