Sets and Functions

Maths Workshop 2020

Problems

1 Find the cardinalites of the following sets:

(a) $\{n \in \mathbb{Z} \mid n^2 \leq 16\}$

(b) $\{1, \{2, 3\}, 4\}$

(c) $A \cap B$, where $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 5, 7\}$

(d) $A \cup B$, where A is the set of multiples of 3 that are less than or equal to 50, and B is the set of multiples of 2 that are less than or equal to 50.

(e) $A - B$, where $A = \{1, 2, 3, 4, 5, 6\}$, and $B = \{2, 5, 7\}$

2 Show that the following 4 statements are equivalent

(a) $A \subset B$

(b) $A - B = \phi$

(c) $A \cup B = B$

(d) $A \cap B = A$

3 For a natural number n, let $n\mathbb{Z}$ denote the set of all integer multiples of n. Let $a, b \in \mathbb{N}$

(a) When does $a\mathbb{Z} \subseteq b\mathbb{Z}$ hold?

(b) What is $a\mathbb{Z} \cap b\mathbb{Z}$?

4 Define a relation $\sim$ on $\mathbb{Z}$ as $a \sim b$ iff 2 divides $a - b$.

(a) Show that $\sim$ is an equivalence relation.

(b) What are the set of all elements related to 1?

5

(a) Show that the function $f : \mathbb{R} \to \mathbb{R}, f(x) = x$ is both one-one and onto

(b) Show that the function $g : \mathbb{R} \to \mathbb{R}, g(x) = x^2$ is neither one-one nor onto

6 Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 4x + 3$

(a) Show that f is a bijection

(b) Find a function $g : \mathbb{R} \to \mathbb{R}$ such that $f(g(x)) = g(f(x)) = x$, for all $x \in \mathbb{R}$ (g is called the inverse of f).

7 Find a bijection between $\mathbb{Z}$ and the set of odd integers