PHY102 : Assignment 2

1. (Purcell 2.1) The vector function which follows represents a possible electrostatic field :

$$E_x = 6xy \quad E_y = 3x^2 - 3y^2 \quad E_z = 0$$

Calculate the line integral of **E** from the point $(0, 0, 0)$ to the point $(x_1, y_1, 0)$ along the path which runs straight from $(0, 0, 0)$ to $(x_1, 0, 0)$ and thence to $(x_1, y_1, 0)$. Make a similar calculation for the path which runs along the other two sides of the rectangle, via the point $(0, y_1, 0)$. Is the above assertion that the function represents an electrostatic field true ? With the potential function $\phi(x, y, z)$, get back the components of the given field.

2. (Purcell 2.4) Describe the electric field and the charge distribution that go with the following potential

$$\phi = x^2 + y^2 + z^2 \qquad \text{for} \quad x^2 + y^2 + z^2 < a^2$$

$$\phi = -a^2 + \frac{2a^3}{\sqrt{x^2 + y^2 + z^2}} \qquad \text{for} \quad a^2 < x^2 + y^2 + z^2$$

3. (Purcell 2.5) A sphere the size of a basketball is charged to a potential of -1000 volts. About how many extra electrons are on it, per m^2 of surface ?

4. (Purcell 2.12) A right angled triangle with vertex P at the origin of coordinates, base length b and height a has a uniform density of surface charge σ. Determine the potential at the vertex P. You can first find the contribution of a vertical strip of width dx at x and then integrate. Find the potential at P.

5. In Assignment 1, problem 6, you have shown that the self energy of a spherical shell of radius R with charge Q uniformly distributed over it is $Q^2/8\pi\epsilon_0 R$. What if we put two such shells right on top of each other, to make a shell with charge $2Q$? Since we now just have two copies of the original system, it seems like the energy should be twice as large, or $Q^2/4\pi\epsilon_0 R$. However, by the above formula, the energy is $(2Q)^2/8\pi\epsilon_0 R = Q^2/2\pi\epsilon_0 R$. Which answer is correct and why ?

6. (a) A spherical shell with charge Q uniformly distributed throughout its volume has inner radius R_1 and outer radius R_2. Calculate and plot the electric field as a function of r for $0 \leq r \leq \infty$.

   (b) What is the potential at the center of the shell ? You can let $R_2 = 2R_1$ in this part of the problem, to avoid complications. Give your answer in terms of $R \equiv R_1$.