MTH202: Assignment 12

April 11, 2019

1. The mean or expected score of a test is 75, with a variance of 25. What is the probability that the test score is between 50 and 100?

2. The expected age of a worker in a factory is 40 years old, with variance of 64. What is the probability that the age of a particular worker lies between 20 and 60?

3. Computers from a particular company are found to last on average for three years without any hardware malfunction, with a standard deviation of two months. At least what percent of the computers last between 31 months and 41 months?

4. You have invited 64 guests to a party. You need to make sandwiches for the guests. You believe that a guest might need $0, 1$ or 2 sandwiches with probabilities $1/4, 1/2$ and $1/4$ respectively. You assume that the number of sandwiches each guest needs is independent from other guests. How many sandwiches should you make so that you are 95 percent sure that there is no shortage?
   (**Hint:** Let Y be the total number of sandwiches needed. You need to determine y (or a lower bound on y) such that $P(Y \leq y) \geq 0.95$. For a good approximation of y, use CLT. Assume $\phi^{-1}(0.95) \approx 1.65$.)

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $Exp(1)$ random variables. Let
   $$Y = \frac{X_1 + X_2 + \ldots + X_n}{n}$$
   How large n should be such that $P(0.9 \leq Y \leq 1.1) \geq 0.9$?

6. Let X and Y be independent random variables such that $Var(X) = 2$ and $Y \sim Poi(1)$. Given $E[X^2Y + Y^2X] = 3$, compute $E[X]$.

7. * Using weak law of large numbers, prove that for every continuous function $f : [0, 1] \to \mathbb{R}$,
   $$\sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}x^k(1-x)^{n-k} \longrightarrow f(x)$$
   uniformly in $x \in [0, 1]$ as $n \to \infty$.