INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI

Probability and Statistics (MTH202)

Practice Sheet

Academic Session 2019-20

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1. Three dice are rolled and the three outcomes are added. What is more likely to be this sum, 9 or 10? Motivate yourself to solve the following: Discover[1] a formula for $P(E_n)$, where E_n is the event that the sum of three outcomes is n.

   > The first part of this problem was solved by Galileo in a paper titled *Sopra le Scoperte dei Dadi* (On a discovery concerning dice). And this was almost 35 years before Pascal's interaction with Chevalier de Méré, the gentleman gambler who is immortal in the history of probability theory! A more general problem for n dice was solved a century later by de Moivre.

2. **(Monmort's Problem)** In a class of size 200, assume that the registration numbers are in a sequence from MS18001 to MS18200, without gaps. A teacher writes down these registration numbers on 200 chits, mixes them well, and distributes in the class. A student reports a *surprise* when (s)he receives a chit with her or his registration number on it. What is the probability of a surprise being reported? Hint : Let E_i be the event that i^{th} students reports a surprise. Use inclusion-exclusion principle.

   > This problem first occured in a classic book *Essay d'analyse sur les jeux de hazard* (1708) by the French mathematician Pierre Raymond de Montmort. "No surprise" is actually a big surprise!

3. A fair die is rolled 15 times. What is the probability that the die will not show some number at all during these 15 rolls? Hint : Let E_i be the event (well, what's the sample space) that i is never shown by the die. Use inclusion-exclusion principle.

4. **(Prosecutor's Fallacy)** Let I be the event that an accused person is actually innocent and E be the event that some evidence/testimony presented to the court is actually genuine. Let us accept that $P(E|I)$ is negligible. Argue that $P(I|E)$ need not be negligible. Convince yourself that $P(E|I) = P(I|E)$ is true if and only if $P(E) = P(I)$.

   > Justify that often lawyers argue assuming that $P(E|I) = P(I|E)$. This way of arguing assuming $P(E|I) = P(I|E)$ is called *prosecutor's fallacy*. You may like to know the case of Sally Clark who became a victom of prosecutor's fallacy and was wrongly convicted of killing her two sons. Read about Meadow's Law which are based on this fallacy. Also read about the case of Trupti Patel.

5. **(False Positives)** Suppose just one person in 500 has a rare disease, and its test is 99% accurate; that is, if the test is conducted on someone who has the disease then this test would catch the disease 99% of the time. However, if someone doesn't have a disease then the person is falsely tested positive in 1% cases. What is the probability that someone who tests positive actually has the disease?

   > If something is rare and you are tested positive, don't worry. Probability theory is there to rescue.

6. You and your friend play the following game. You roll a fair die and note down the number $i \in \{1, 2, 3, 4, 5, 6\}$ that appears on the die. You then spell i in English language (e.g. if $i = 5$ then you spell '`FIVE`') and note down the number ℓ_i of distinct letters in the spelling. If i is even then you give $i + \ell_i$ many chocolates to your friend, else you receive as many chocolates from her.

   (a) Write the probability space (S, P), depicting the roll of die.

   (b) Write the random variable X signifying your gain or loss in the game.

   (c) What is the probability mass function for X?

   (d) What is the clever thing to do in this game - to throw the die, or watch your friend throwing it?

7. The fire alarm in a hostel is very accurate. If there is some fire, then it raises an alert with 0.99 probability. However, when there is no fire, it still raises an alert with 0.02 probability (false alarm). Assume that, on average, there is actual fire on 1 out of 200 days. One fine day, you hear the alarm ringing. What is the probability that the fire is actually there?

8. A fair coin is tossed 10 times. Given that there is `HEAD` on first 5 tosses, what is the probability that sixth toss will also be a `HEAD`?

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[1] You may ignore if it takes too much of your time!

9. **(Bertrand's Three Boxes)** There are three boxes - A, B and C, each having two drawers. The box A has one gold coin in each of its drawers, the box B has one silver coin in each of its drawers, while the box C has one gold coin in one drawer and one silver coin in the other. A box is chosen at random and a drawer is opened randomly. It is observed that the drawer contains gold coin. What is the probability that the box chosen is C.

10. Conduct the following experiment : Keep tossing a coin and stop only when a `HEAD` occurs. Let X be the random variable signifying the number of tosses in such an experiment. Plot the probability mass function of X.