IISER Mohali
MTH102: Analysis in One Variable
Homework No. 03
To be discussed during tutorial on January 29, 2016

- Please solve all the problems.
- **Tutorial Problems** will be discussed during tutorial sessions.
- If time permits, tutors may also discuss **extra problems** during tutorial sessions.

Tutorial Problems:

(1) Prove that $\lim \frac{12n^5+73n^4-18n^2+9}{25n^5+2n^3} = \frac{12}{25}$.
(2) Suppose that $\lim x_n = 5$, $\lim y_n = 9$ and that $y_n \neq 0$ for all $n \in \mathbb{N}$. Determine $\lim(x_n + y_n)$ and $\lim \frac{3y_n - x_n}{y_n^3}$.
(3) Let $s_1 = 1$ and $s_{n+1} = \sqrt{s_n + 1}$ for all $n \in \mathbb{N}$. Assuming that the sequence (s_n) converges, prove that $\lim s_n = \frac{1+\sqrt{5}}{2}$.
(4) Let (s_n) and (t_n) be two sequences. Suppose that there exists $N_0 \in \mathbb{N}$ such that $s_n \leq t_n$ for all $n > N_0$.
  (a) Prove that if $\lim s_n = +\infty$, then $\lim t_n = +\infty$.
  (b) Prove that if $\lim t_n = -\infty$, then $\lim s_n = -\infty$.
  (c) Prove that if $\lim s_n = +\infty$ and $k > 0$ a real number, then $\lim ks_n = +\infty$.
  (d) Prove that $\lim s_n = +\infty$ if and only if $\lim -s_n = -\infty$.
(5) Prove that

$$\lim a^n = \begin{cases} 0 & \text{if } 0 \leq |a| < 1 \\ 1 & \text{if } a = 1 \\ +\infty & \text{if } a > 1 \\ \text{Does not exist} & \text{if } a \leq -1. \end{cases}$$

Extra Problems:

(1) Let (s_n) and (t_n) be two sequences such that $\lim s_n = +\infty$ and $\lim t_n > 0$ (it could be finite or $+\infty$). Then prove that $\lim s_n t_n = +\infty$.
Hint: It is Theorem 9.9 of the book.
(2) Let (s_n) be a sequence of positive real numbers. Prove that $\lim s_n = +\infty$ if and only if $\lim \frac{1}{s_n} = 0$.
Hint: It is Theorem 9.10 of the book.
(3) Prove that $\lim \frac{n^4+8n}{n^2+9} = +\infty$.
(4) Prove that $\lim \frac{2^n}{n^2} = +\infty$.