MTH-201 / Homework 3

* Draw figures wherever possible.

1. Find equations of a line passing through $(1,-1,2)$ and $(-1,1,3)$.

2. (i) If the equation of a line in $\mathbb{R}^2$ is $ax+by+c=0$ show that the line is perpendicular to (a,b), i.e. $a\vec{i}+b\vec{j}$.

(ii) Similarly show that the plane $ax+by+cz+d=0$ is perpendicular to (a,b,c).

3. Check if the following lines L_1, L_2 are parallel.

(i) L_1: $x = 2t+3$, $y = 3t+4$ $\quad$ L_2: $x = 4-t$, $y = 1-3t/2$

(ii) L_1: $x = -t+1$, $y = 2t-3$, $z = -3t+4$ $\quad$ L_2: $x = t-2$, $y = 1-t$, $z = 2t+3$

4. Two lines L_1, L_2 in $\mathbb{R}^3$ are called skew lines if they do not intersect and they are not parallel to each other.

Consider the lines

L: $x = t-1$, $y = 2t-3$, $z = 3t-4$

L_2: $x = t$, $y = -2t$, $z = at$

Determine a so that L_1, L_2 are skew lines.

5. Find parametric equations of the straightline passing through $(1,2,3)$ and perpendicular to the plane $2x-3y-4z=5$.

6. Find parametrized curves whose images are the following curves:

Draw these curves!

(i) $\frac{x^2}{4}+\frac{y^2}{9}=1$

(ii) $x^2-4y^2=1$

(iii) $x^2+y^2+z^2=1$

$x=2y$

(iv) $x^2+y^2+z^2=1$

$z=a\sqrt{x^2+y^2}$, a is a constant.

7. Check that the following give surfaces in $\mathbb{R}^3$:

(i) $x^2+y^2+z^2=1$

(ii) $x^2-y^2+z^2=1$

(iii) $y=x^2$

(iv) $x^2+y^2=1$

(v) $z=x^2+y^2$

Draw these surfaces!

8. Draw the following curves:

(i) $\alpha: \mathbb{R}\to\mathbb{R}^3$ $\alpha(t)=(\cos t, \sin t, t)$

(ii) $\alpha: \mathbb{R}\to\mathbb{R}^2$ $\alpha(t)=e^t(\cos t, \sin t)$

Find the equations of the tangent lines to these curves at $\alpha(0)$, $\alpha(1)$.