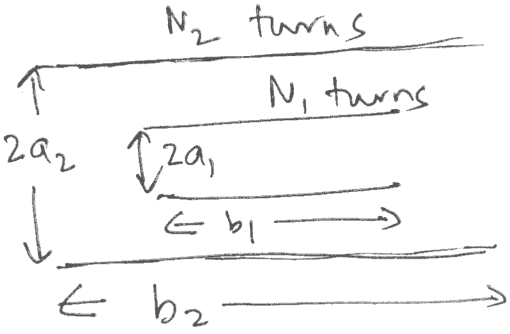
2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5$cm ? [2.5]

ASSIGNMENT 9 (SOLS.)

1. Let current flowing through outer solenoid be I_2.



Assuming field inside the solenoid to be uniform, & assuming the magnitude to be the same as that of an infinite solenoid, we have,

$$B_2 = \mu_0 n_2 I_2 = \mu_0 \left(\frac{N_2}{b_2}\right) I_2$$

since, $n_2 = \frac{N_2}{b_2}$

∴ Flux through inner solenoid,

$$\phi_{12} = N_1 B_2 A_1 \quad (A_1 : \text{area of inner solenoid})$$

$$= N_1 \mu_0 \frac{N_2}{b_2} I_2 \cdot \pi a_1^2$$

$$= \left(\frac{\mu_0 \pi a_1^2 N_1 N_2}{b_2}\right) I_2$$

∴ Mutual inductance,

$$M = \frac{\mu_0 \pi a_1^2 N_1 N_2}{b_2}$$

PHY102 : Quiz 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]

You may also have calculated the flux through the outer solenoid due to the inner solenoid.

Try it & see if they match!

2. Magnetic field inside solenoid,

$$B = \mu_0 n_1 I_1 = \mu_0 \frac{N_1 I_1}{l}.$$

Flux, $\phi = BA = \dfrac{\mu_0 N_1 I_1 A}{l}$.

$\therefore$ Self Inductance, $L_1 = \dfrac{N_1 \phi}{I_1} = \dfrac{\mu_0 N_1^2 A}{l}.$

Now, ϕ' is the magnetic flux through each turn of the outer coil.

$\phi' = BA'$, B : mag. field due to inner solenoid.
A': area of outer coil $= A$.

$$\therefore \phi' = \frac{\mu_0 N_1 I_1}{l} . A$$

$\therefore$ Mutual inductance,

$$M = \frac{N_2 \phi'}{I_1} = \frac{\mu_0 N_1 N_2 A}{l}$$

Now, self inductance of inner solenoid, $L_1 = \dfrac{\mu_0 N_1^2 A}{l}$,

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$? [2.5]

Self inductance due to outer coil, $L_2 = \frac{\mu_0 N_2^2 A}{l}$.

$\therefore M = \sqrt{L_1 L_2}$.

(This is an idealization $\rightarrow$ all of the magnetic flux produced by solenoid passes through the outer coil).

3. $E_x = 0, \quad E_y = E_0 \sin(kx + \omega t), \quad E_z = 0$

$B_x = 0 \quad B_y = 0, \quad B_z = -\frac{E_0}{c}\sin(kx + \omega t)$

$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0.$

Similarly, $\vec{\nabla}\cdot\vec{B} = 0$.

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \hat{z}\, E_0 k \cos(kx + \omega t)$$

$$-\frac{\partial \vec{B}}{\partial t} = \hat{z}\,\frac{E_0 \omega}{c}\cos(kx + \omega t)$$

To require, $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow k = \frac{\omega}{c} \Rightarrow \boxed{\omega = ck}$

Use this to show that, $\vec{\nabla}\times\vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$.

PHY102 : Quiz 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]

4. $\vec{\nabla}\cdot\vec{E} = 0.$

$\vec{\nabla}\cdot\vec{B} = \left(-B_0 k \sin kx \sin ky + k B_0 \sin kx \cos ky\right) \sin\omega t$

$= 0.$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_0 \cos kx \cos ky \cos\omega t \end{vmatrix}$$

$$= \hat{x}\left[\frac{\partial}{\partial y}\left(E_0 \cos kx \cos ky \cos\omega t\right) - \frac{\partial}{\partial z}(0)\right].$$

$$- \hat{y}\left[\frac{\partial}{\partial x}\left(E_0 \cos kx \cos ky \cos\omega t\right) - \frac{\partial}{\partial z}(0)\right].$$

$$+ \hat{z}\left[\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(0)\right]$$

$$= -\hat{x} E_0 k \cos kx \sin ky \cos\omega t + \hat{y} E_0 k \sin kx \cos ky \cos\omega t$$

$$-\frac{\partial\vec{B}}{\partial t} = \hat{x} B_0 \omega \cos kx \sin ky \cos\omega t - \hat{y} B_0 \sin kx \cos kx \cos\omega t$$

Now for $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ to hold,

(1) — $\boxed{E_0 k = B_0 \omega}$ (equating components).

Now, lets look at the other Maxwells eqn.

$$\vec{\nabla}\times\vec{B} = \mu_0 \epsilon_0 \frac{\partial\vec{E}}{\partial t}.$$

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$? [2.5]

$$\frac{\partial \vec{E}}{\partial t} = -\hat{z}\, \omega E_0 \cos kx \cos ky \sin \omega t.$$

$$\vec{\nabla} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_0 \cos kx \sin ky \sin\omega t & -B_0 \sin kx \cos ky \sin \omega t & 0 \end{vmatrix}$$

$$= \hat{x}(0-0) - \hat{y}(0-0) + \hat{z}\left(-B_0 k \cos kx \cos ky \sin\omega t - B_0 k \cos kx \cos ky \sin \omega t\right).$$

$$= -\hat{z}\, 2B_0 k \cos kx \cos ky \sin \omega t.$$

$\therefore$ For $\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ to hold,

$$2B_0 k = \mu_0 \epsilon_0 . \omega E_0 , = \frac{\omega E_0}{c^2} \quad \left(\because c^2 = \frac{1}{\mu_0 \epsilon_0}\right).$$

$$\Rightarrow \boxed{2B_0 k = \frac{\omega E_0}{c^2}} \quad \text{——(2)}$$

$$\therefore (1) \div (2) \Rightarrow \frac{E_0}{2B_0} = \frac{B_0}{E_0/c^2} \Rightarrow E_0^2 = 2B_0^2 c^2$$

$$\therefore E_0 = \sqrt{2}\, c B_0.$$

Substituting for E_0 in (1) $\Rightarrow k\sqrt{2}\, c \cancel{B_0} = \cancel{B_0}\, \omega.$

$$\therefore \omega = \sqrt{2}\, ck.$$

PHY102 : Quiz 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]

5. Proof of $\vec{E}'\cdot\vec{B}' = \vec{E}\cdot\vec{B}$ given in text.

Basically, for a frame F' moving with speed v in the $\hat{x}$ direction relative to F, the transformation equations are,

$$E_x' = E_x \; ; \; E_y' = \gamma(E_y - vB_z) \; ; \; E_z' = \gamma(E_z + vB_z)$$

$$B_x' = B_x \; ; \; B_y' = \gamma\left(B_z + \frac{v}{c^2}E_z\right) \; ; \; B_z' = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

$$\vec{E}'\cdot\vec{B}' = E_x'B_x' + E_y'B_y' + E_z'B_z'$$

Using the transformations given above it is easy to see that, $\vec{E}'\cdot\vec{B}' = \vec{E}\cdot\vec{B}$.

To show $E'^2 - cB'^2 = E^2 - cB^2$, we can use the same transformations as above & do it.

Or, as suggested by Purcell in prob 9.12, we can break $\vec{E}$ & $\vec{B}$ into $\parallel^{l}$ & $\perp^{r}$ vectors

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp} \quad \& \quad \vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp}$$

$$\therefore E'^2 - cB'^2 = \vec{E}'\cdot\vec{E}' - c\vec{B}'\cdot\vec{B}'$$

$$= (\vec{E}_{\parallel}' + \vec{E}_{\perp}')\cdot(\vec{E}_{\parallel}' + \vec{E}_{\perp}')$$

$$- c\,(\vec{B}_{\parallel}' + \vec{B}_{\perp}')\cdot(\vec{B}_{\parallel}' + \vec{B}_{\perp}')$$

~~2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of **E**, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5$cm ? [2.5]~~

Now, $\vec{E}'_{\parallel}\cdot\vec{E}'_{\perp} = 0$, $\vec{B}'_{\parallel}\cdot\vec{B}'_{\perp} = 0 = \vec{B}'_{\perp}\cdot\vec{B}'_{\parallel}$

$= \vec{E}'_{\perp}\cdot\vec{E}'_{\parallel}$

$\therefore \; E'^2 - c^2B'^2 = \left(\vec{E}'_{\parallel}\cdot\vec{E}'_{\parallel} + \vec{E}'_{\perp}\cdot\vec{E}'_{\perp}\right) - c^2\left(\vec{B}'_{\parallel}\cdot\vec{B}'_{\parallel} + \vec{B}'_{\perp}\cdot\vec{B}'_{\perp}\right)$

Now, in vector form, the transformation eqs. are,

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} \quad ; \quad \vec{E}'_{\perp} = \gamma\left(\vec{E}_{\perp} + \vec{v}\times\vec{B}_{\perp}\right)$$

$$\vec{B}'_{\parallel} = \vec{B}_{\parallel} \quad ; \quad \vec{B}'_{\perp} = \gamma\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2}\times\vec{E}_{\perp}\right).$$

$$\therefore \; \vec{E}'_{\parallel}\cdot\vec{E}'_{\parallel} - c^2\vec{B}'_{\parallel}\cdot\vec{B}'_{\parallel} = \vec{E}_{\parallel}\cdot\vec{E}_{\parallel} - c^2\vec{B}_{\perp}\cdot\vec{B}_{\perp}$$

$$\vec{E}'_{\perp}\cdot\vec{E}'_{\perp} - c^2\vec{B}'_{\perp}\cdot\vec{B}'_{\perp}$$

$$= \gamma^2\left(E_{\perp} + \vec{v}\times\vec{B}_{\perp}\right)\cdot\left(\vec{E}_{\perp} + \vec{v}\times\vec{B}_{\perp}\right)$$

$$- \gamma^2c^2\left(B_{\perp} - \frac{\vec{v}}{c^2}\times\vec{E}_{\perp}\right)\cdot\left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2}\times\vec{E}_{\perp}\right).$$

$$= \gamma^2\Big[\vec{E}_{\perp}\cdot\vec{E}_{\perp} + \vec{E}_{\perp}\cdot(\vec{v}\times\vec{B}_{\perp}) + (\vec{v}\times\vec{B}_{\perp})\cdot\vec{E}_{\perp}$$

$$+ (\vec{v}\times\vec{B}_{\perp})\cdot(\vec{v}\times\vec{B}_{\perp}) - c^2\Big\{\vec{B}_{\perp}\cdot\vec{B}_{\perp}$$

$$- \frac{1}{c^2}\vec{B}_{\perp}\cdot(\vec{v}\times\vec{E}_{\perp}) - \frac{1}{c^2}(\vec{v}\times\vec{E}_{\perp})\cdot\vec{B}_{\perp}$$

$$+ \frac{1}{c^4}(\vec{v}\times\vec{E}_{\perp})\cdot(\vec{v}\times\vec{E}_{\perp})\Big\}\Big].$$

Now, $\vec{E}_{\parallel}$ is parallel to $\vec{v}$ by definition.

PHY102 : Quiz 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R? [2.5]

$\therefore \vec{E}_\perp$ is $\perp^r$ to $\vec{v}$ $\quad \therefore \vec{v}.\vec{E}_\perp = \vec{E}_\perp.\vec{v} = 0.$

$\therefore$ ~~$\vec{E}_\perp.(\vec{v}\times\vec{B}_\perp) = (\vec{v}\times\vec{B}_\perp).\vec{E}_\perp$~~ Similarly, $\vec{v}.\vec{B}_\perp = \vec{B}_\perp.\vec{v} = 0$

$$\therefore \underbrace{(\vec{v}\times\vec{E}_\perp)}_{\vec{A}}.\underbrace{(\vec{v}}_{B}\times\underbrace{\vec{E}_\perp)}_{C} = \vec{v}.[\vec{E}_\perp\times(\vec{v}\times\vec{E}_\perp)].$$

$$\left(\because \vec{A}.(\vec{B}\times\vec{C}) = \vec{B}.(\vec{C}\times\vec{A})\right]$$

$$\text{Now, } \underbrace{\vec{E}_\perp}_{A}\times(\underbrace{\vec{v}}_{B}\times\underbrace{\vec{E}_\perp}_{C}) = \vec{v}(\vec{E}_\perp.\vec{E}_\perp) - \vec{E}_\perp\cancel{(\vec{v}.\vec{E}_\perp)}^{0}$$

$$\left(\because \vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})\right]$$

$$= \vec{v}\,E_\perp^2$$

$$\therefore (\vec{v}\times\vec{E}_\perp).(\vec{v}\times\vec{E}_\perp) = (\vec{v}.\vec{v})E_\perp^2 = v^2E_\perp^2.$$

$$\Rightarrow (\vec{v}\times\vec{E}_\perp)^2 = v^2E_\perp^2.$$

Similarly, $(\vec{v}\times\vec{B}_\perp).(\vec{v}\times\vec{B}_\perp) = v^2B_\perp^2.$

$$\therefore E_\perp'^2 - c^2B_\perp'^2 = \gamma^2\left[E_\perp^2 + 2\vec{E}_\perp.(\vec{v}\times\vec{B}_\perp) + v^2B_\perp^2 - c^2B_\perp^2 + 2\vec{B}_\perp.(\vec{v}\times\vec{E}_\perp) - \frac{1}{c^2}.v^2E_\perp^2\right].$$

But, $\vec{E}_\perp.(\vec{v}\times\vec{B}_\perp) = -\vec{B}_\perp.(\vec{v}\times\vec{E}_\perp)$ $\left(\because \vec{A}.(\vec{B}\times\vec{C}) = -\vec{C}.(\vec{B}\times\vec{A})\right)$

$$\therefore E_\perp'^2 - c^2B_\perp'^2 = \gamma^2\left[E_\perp^2\left(1-\frac{v^2}{c^2}\right) - c^2B_\perp^2\left(1-\frac{v^2}{c^2}\right)\right]$$

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9} C$ and $l = 5\text{cm}$? [2.5]

$$\therefore E_1'^2 - c^2 B_2'^2 = \gamma^2 \left[\frac{E_1^2}{\gamma^2} - \frac{c^2 B_2^2}{\gamma^2} \right] = E_1^2 - c^2 B_2^2$$

$$\therefore E'^2 - c^2 B'^2 = E^2 - c^2 B^2 .$$ Hence proved.

6. $P = 60$ watt, $V = 120$ volts.

$\therefore$ Current, $I = \frac{P}{V} = \frac{60}{120} = 0.5$ A.

Resistance of filament, $R = \frac{V}{I} = \frac{120}{0.5} = 240\,\Omega$.

To have same I when bulb is connected in series with an inductance L, we have,

impedence, $Z = \sqrt{R^2 + (\omega L)^2}$

$\therefore$ Current, $I = \frac{V'}{\sqrt{R^2 + (\omega L)^2}}$ with $V' = 240$ volts

$\Rightarrow 0.5 = \frac{240}{\sqrt{R^2 + (\omega L)^2}} \Rightarrow (\omega L)^2 = (480)^2 - (240)^2$

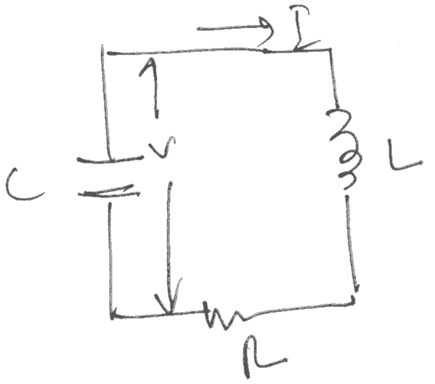
$\Rightarrow \omega L \simeq 415.7\,\Omega$ $\qquad \omega = 2\pi f = 2\pi \cdot (60)$

$\therefore L \simeq \frac{415.7}{2\pi \cdot 60} \simeq 1.10$ henry.

PHY102 : Quiz 1

1. A spherical charge distribution has a density ρ that is constant from $r = 0$ out to $r = R$ and is zero beyond. What is the electric field for all values of r, both less than and greater than R ? [2.5]

7. Let's do the serial RLC ckt the Purcell way.



V is +ve if upper capacitor plate is +vely charged. +ve current direction defined by the arrow. Then,

$$I = -\frac{dQ}{dt} \; ; \; Q = CV \; ; \; V = L\frac{dI}{dt} + IR$$

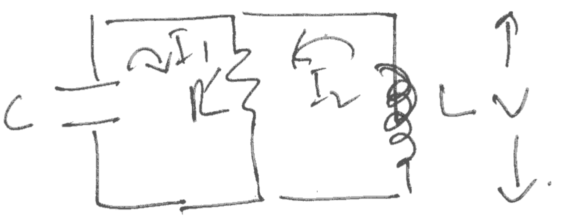
$$\therefore I = -\frac{d}{dt}(CV) = -C\frac{dV}{dt}$$

$$\therefore V = -LC\frac{d^2V}{dt^2} - CR\frac{dV}{dt}$$

$$\Rightarrow \frac{d^2V}{dt^2} + \frac{R}{L}\frac{dV}{dt} + \left(\frac{1}{LC}\right)V = 0 \quad \text{—— (1)}$$

We can proceed to solve this ala Purcell (see text)

Now consider the parallel LCR ckt.



We still have, $Q = CV$

& $I_1 = -\frac{dQ}{dt} = -C\frac{dV}{dt}$.

Now, $V = R'(I_1 + I_2)$ Also, $V = -L\frac{dI_2}{dt}$ (Note the sign)

$\therefore$

2. Designate the corners of a square, l on a side, in clockwise order, A, B, C, D. Put charges $2q$ at A and $-3q$ at B. Determine the value of the line integral of $\mathbf{E}$, from point C to point D. (No actual integration needed!) What is the numerical answer if $q = 10^{-9}C$ and $l = 5\text{cm}$? [2.5]

$$\therefore \frac{dV}{dt} = R'\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right)$$

$$= R'\left[-C\frac{d^2V}{dt^2} - \frac{V}{L}\right].$$

$$\Rightarrow \frac{dV}{dt} = -R'C\frac{d^2V}{dt^2} - \frac{R'V}{L}.$$

$$\Rightarrow \frac{d^2V}{dt^2} + \frac{1}{R'C}\frac{dV}{dt} + \frac{V}{LC} = 0. \quad \text{—— (2)}$$

Comparing eq. (1) & (2), they are same

if,

$$\frac{R}{L} = \frac{1}{R'C} \Rightarrow \boxed{R' = \frac{L}{RC}}.$$