

MTH102: Analysis in One variable
Home Work No. 01
13 January 2018

- Please do as many problems as possible.
 - Please maintain a separate notebook for home work problems.
 - Tutors will discuss some of these problems during tutorial sessions.
 - \mathbb{N} denote the set of natural numbers.
 - \mathbb{Z} denote the ring of integers.
 - \mathbb{Q} denote the field of rational numbers.
 - \mathbb{R} denote the field of real numbers.
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- (1) Use the principle of mathematical induction to prove the following:
 - (a) $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \in \mathbb{N}$.
 - (b) $n^2 > n + 1$ for all $n \in \mathbb{N}$ such that $n \geq 2$.
 - (c) $1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$ for all $n \in \mathbb{N}$.
 - (2) Prove that $(2 + 5^{1/3})^{1/2}$, $(2 + 2^{1/2})^{1/2}$ and $(5 - 3^{1/2})^{1/3}$ are not rational numbers.
 - (3) Prove that $||a| - |b|| \leq |a - b|$ for all $a, b \in \mathbb{R}$.
 - (4) Prove that $|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| + \cdots + |a_n|$ for any n real numbers.
 - (5) Prove that $|a - b| \leq c$ if and only if $b - c \leq a \leq b + c$.
 - (6) Let $a, b \in \mathbb{R}$. Prove that if $a \leq c$ for all $c > b$, then $a \leq b$.
 - (7) Prove that the set of irrational numbers is dense in the set of real numbers.
 - (8) Determine whether the following sets are bounded or not. If so, then determine their supremums and infimums. Do these numbers lie in the given sets?
 - (a) $A = \{r \in \mathbb{Q} \mid r^2 < 4\}$.
 - (b) $B = \{1 - \frac{1}{3^n} \mid n \in \mathbb{N}\}$.
 - (c) $C = \{n^{(-1)^n} \mid n \in \mathbb{N}\}$.
 - (d) $D = \{\frac{1}{n} \mid n \in \mathbb{N}\}$.
 - (9) Let A be a subset of \mathbb{R} and $b \in \mathbb{R}$ a fixed real number. Suppose that $a < b + \epsilon$ for all $a \in A$ and each $\epsilon > 0$. Then prove that b is an upper bound for A .
 - (10) Suppose that A, B are non-empty sets of real numbers such that $x \leq y$ for all $x \in A$ and $y \in B$. Then prove that $\sup A \leq \inf B$.
 - (11) Write down a proof of the Binomial Theorem using the principle of mathematical induction.