Sets

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Sets

Set is a well defined collection of objects. Well defined here means that an element of the Domain x should be unambiguously belong to or not belong to set S.

$x \in A \equiv$ "x belongs to set A"

$x \notin A \equiv$ "x does not belong to set A"

Properties

- All elements unique.
- Unordered.

Types

Sets need not be of a particular "type" - $\{\{1,2,3\},6,7\}$ May also be infinite. $\{1,2,3,4....\}$ Null/Empty set = $\{\} = \emptyset$

Operations on sets

- Subset: $A \subseteq B$. "A is a subset of B." $x \in A \implies x \in B$.
- Equality: $A=B$ iff $A \subseteq B \wedge B \subseteq A$. "A is equal to B." $x \in A \implies x \in B \wedge x \notin A \implies x \notin B$
- Proper Subset: $A \subset B$ $x \in A \implies x \in B \wedge A \neq B$.
- Complement: $U \setminus A$ OR Ac. Exactly all x's in U NOT in A.
- Union: $A \cup B$. "A union B". Exactly all elements in $A \vee B$.
  - If $\{A\alpha\}$ is a collection of sets indexed by I, then $\bigcup A\alpha$ = set of x st $x \in \alpha 0$ for some $\alpha \in I$.
- Intersection: $A \cap B$ "A intersection B". Contains exactly all elements in A AND B.
  - $x \in \bigcap A\alpha$ iff x $\forall \alpha \in I$, $x \in A\alpha$

- Cartesian Product: $A \times B$ "Cartesian Product of A and B". Its the collection of all 2 element sequences (a,b) st $a \in A$, $b \in B$
  - $A1 \times A2 \times ... \times An$ is the collection of all n element sequences (ai) st $ai \in Ai$
- Relation operator: aRb "a is related to b". A relation is a subset of $A \times B$.
- Function: f: $A \to B$. It is a relation such that
  1. $\forall a \in A$, $\exists b$ st aRb.
  2. aRb and aRc $\implies$ b=c.
  - Equality of functions - $f=g \implies f \subseteq g \wedge g \subseteq f$.

Languages

Σ is a finite set called an "alphabet".

The finite sequence is called a string.

A language over Σ is a set of strings.