

ROLL NO : MS NAME :

PHY202 Jan-Aug 2019: Midsem 1 Dated February 4, 2019: Inst: Dipanjan Chakraborty

- Time : 60 minutes
- Max Marks : 40
- Attempt all questions. No aids (Books/Notes/Gadgets).

Question	1	2	3	4	T
Marks					

1. Each of the following question has 1 marks for the correct tick mark and the rest for justification.

(a): The differential $dz = 2xydx + (x^2 + 2y)dy$ is not an exact differential

[4]
YES NO ✓

Justification

$$dz = A(x,y)dx + B(x,y)dy \quad \text{--- ①}$$

$$\frac{\partial A}{\partial y} = 2x \quad \frac{\partial B}{\partial x} = 2x \quad \text{--- ①}$$

$$dz = d(x^2y + y^2) \quad \text{--- ①}$$

(b): For an ideal gas, the internal energy per unit volume is $P/(\gamma - 1)$, where P is the pressure and γ is the ratio of specific heats.

[4]
✓ YES NO

Justification

$$U = \frac{3}{2} nRT$$

$$PV = nRT$$

$$C_v = \frac{3}{2} nR$$

$$C_p = \frac{5}{2} nR$$

$$\frac{C_p}{C_v} = \gamma$$

$$C_p - C_v = nR$$

$$C_v(\gamma - 1) = nR$$

$$U = C_v T = \frac{nRT}{(\gamma - 1)} = \frac{PV}{(\gamma - 1)} \quad \text{--- ①}$$

$$U/V = \frac{P}{(\gamma - 1)} \quad \text{--- ①}$$

(c): For n moles of an ideal gas, initially confined in volume V and at temperature T is expanded to a volume αV (i) by a reversible isothermal expansion (ii) removing a partition and allowing a free expansion. The change in entropy is the same in the two cases.

[4]
☒ YES ☐ NO

Justification

Entropy is a state function. Change in
 entropy is $\Delta S = nR \ln \alpha$.

(b): An ideal monoatomic gas in three dimensions is initially at a temperature T . The temperature is changed to $4T$. The root-mean square velocity $v_{\text{rms}} = \sqrt{\langle \vec{v}^2 \rangle}$ changes by a factor 3.

[4]
☐ YES ☒ NO

Justification

$$v_{\text{rms}}^1 = \sqrt{\langle \vec{v}^2 \rangle} = \sqrt{\frac{3k_B T}{m}} \quad (1) \quad v_{\text{rms}}^2 = \sqrt{\frac{12k_B T}{m}} = 2 \sqrt{\frac{3k_B T}{m}} \quad (1)$$

$$\therefore \frac{v_{\text{rms}}^2}{v_{\text{rms}}^1} = 2 \quad (1)$$

Ideal monoatomic Gas. & specific heat constant. ³

2. Consider a hydrostatic system with fixed particle number. The system undergoes a change in state going from a state-1 (P_1, V_1) to a state-2 (P_2, V_2) by an isothermal expansion. Now consider another process in which we first go from (P_1, V_1) to (P_2, V_1) and then to (P_2, V_2). Estimate the entropy change in going from state 1 to state 2 along each of these paths. What do you conclude?

[Marks=12]

For isothermal expansion from (P_1, V_1) to (P_2, V_2)

$$\Delta S = nR \int_{V_1}^{V_2} \frac{dV}{V} = nR \ln V_2/V_1 \quad \text{--- (2)}$$

Change in entropy T_1 (P_1, V_1) to (P_2, V_1)

$$\Delta S = C_p \int_{T_1}^{T_1} \frac{dT}{T} = C_p \ln \frac{T_1}{T_1} \quad \text{--- (2)}$$

Change in entropy for (P_2, V_1) to (P_2, V_2)

$$\Delta S = C_p \int_{T_1}^{T_2} \frac{dT}{T} = C_p \ln \frac{T_2}{T_1} \quad \text{--- (2)}$$

$$\therefore \text{Total Change } \Delta S = C_p \ln \frac{T_1}{T_1} + C_p \ln \frac{T_2}{T_1}$$

$$C_p - C_v = nR$$

$$\text{and } T_1 = T_2 = T \quad \text{--- (1)}$$

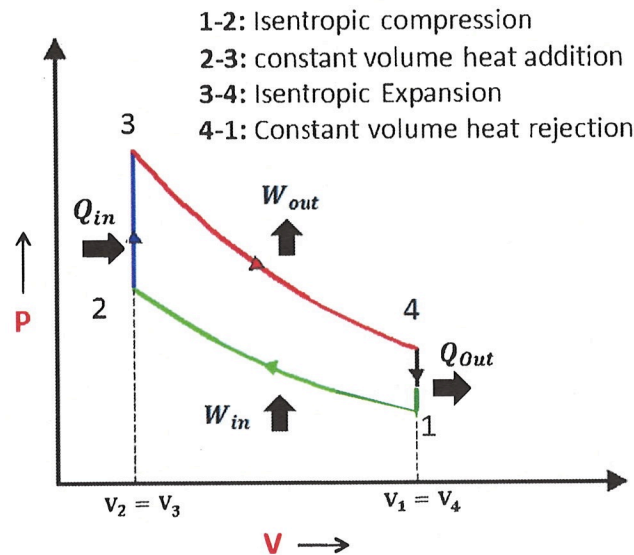
$$\Delta S = (C_v + nR) \ln \frac{T_1}{T_1} + C_v \ln \frac{T_1}{T_1} + nR \ln \frac{T_1}{T_1} \quad \text{--- (2)}$$

$$= nR \ln \frac{T_1}{T_1} = nR \ln \frac{P_1}{P_2}$$

$$\Delta S = nR \ln \frac{V_2}{V_1} \quad \text{--- (2)}$$

Entropy change is a state function and its change depends only on the initial and final points. (1)

3. Consider an ideal heat engine operated in a cycle using two adiabatic processes and two isochoric processes. The cycle in the $p - V$ diagram is shown below: The heat engine is operated with an ideal gas as a working substance. Calculate



the efficiency of the heat engine and show that this efficiency is less than a Carnot engine working between the same hot and cold reservoirs.

[Marks=8+4]

$$Q_{in} = C_v (T_3 - T_2) \quad \text{and} \quad Q_{out} = C_v (T_4 - T_1) \quad \text{--- (2)}$$

$$\therefore \eta = \frac{W}{Q_{in}} \quad \text{where} \quad W = W_{in} - W_{out}$$

$$= 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \quad \text{--- (4)}$$

$$\left. \begin{aligned} T_3 V_2^{r-1} &= T_4 V_1^{r-1} \\ T_2 V_2^{r-1} &= T_1 V_1^{r-1} \end{aligned} \right\} \Rightarrow \frac{(T_3 - T_2) V_2^{r-1}}{T_2 V_2^{r-1}} = \frac{(T_4 - T_1) V_1^{r-1}}{T_1 V_1^{r-1}}$$

$$\frac{T_4 - T_1}{T_3 - T_2} = \left(\frac{V_2}{V_1} \right)^{r-1} = \frac{1}{\alpha^{r-1}}$$

$$\therefore \eta = 1 - \frac{1}{\alpha^{r-1}} = 1 - \alpha^{1-r} \quad \text{--- (2)} \quad \text{where } \alpha = \frac{V_1}{V_2}$$

Let us compare the temperatures.

$$\left. \begin{aligned} \frac{P_2}{T_2} &= \frac{P_3}{T_3} & T_3 &= T_2 \frac{P_3}{P_2} & P_3 &> P_2 & \Rightarrow T_3 &> T_2 \\ \frac{P_4}{T_1} &= \frac{P_1}{T_1} & T_4 &= T_1 \frac{P_4}{P_1} & P_4 &> P_1 & \Rightarrow T_4 &> T_1 \end{aligned} \right\} \quad \text{--- (1)}$$

$$T_3 V_2^{r-1} = T_4 V_1^{r-1} \quad \left\{ \begin{array}{l} \text{Even for a monoatomic gas } r = 5/3 \\ r-1 = 2/3 \\ \left(\frac{V_1}{V_2}\right)^{r-1} > 1 \end{array} \right. \quad 5$$

$$T_3 = T_4 \left(\frac{V_1}{V_2}\right)^{r-1}$$

$$T_3 > T_4$$

Similarly $T_2 > T_1$. ① Hence we have

$$T_3 - T_1 > T_4 - T_2$$

$$\text{This means } T_3 - T_1 > T_2 - T_1$$

Now, the maximum Carnot efficiency is obtained when we operate a ~~max~~ heat engine between T_1 & T_3 . The efficiency is

$$\eta_c = 1 - \frac{T_1}{T_3}$$

For the given heat-engine the efficiency is

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \left. \right\} \text{①} = 1 - \frac{T_1}{T_2} \frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)} = 1 - \frac{T_1}{T_2}$$

$$\text{Since } T_3 V_2^{r-1} = T_4 V_1^{r-1}$$

$$T_2 V_2^{r-1} = T_1 V_1^{r-1}$$

$$\text{Hence since } T_3 - T_1 > T_2 - T_1$$

$$\frac{T_3}{T_1} > \frac{T_2}{T_1} \quad \Rightarrow \quad \frac{T_1}{T_3} < \frac{T_1}{T_2}$$

$$\therefore 1 - \frac{T_1}{T_3} > 1 - \frac{T_1}{T_2}$$

$$\boxed{\therefore \eta_c > \eta} \quad \text{--- ①}$$

