$$\boxed{f(v_x)\,dv_x = A\,e^{-b v_x^2}\,dv_x}$$ 1 D velocity distribution

↓

probability (fraction of molecules) having velocity in the range v_x to $v_x + dv_x$

3 3 3.

$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$

Head Head Head

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$$f(v_x)dv_x \times f(v_y)dv_y \times f(v_z)dv_z$$

$$= A e^{-b v_x^2} dv_x \times A e^{-b v_y^2} dv_y \times A \cdot e^{-b v_z^2} dv_z$$

$$= A^3 e^{-b(v_x^2 + v_y^2 + v_z^2)} dv_x dv_y dv_z$$

$$= A^3 e^{-b v^2} dv_x dv_y dv_z$$

$$= f(v)\,\underbrace{dv_x dv_y dv_z}$$

↓

probability of having velocity simultaneously v_x to $v_x + dv_x$

v_y to $v_y + dv_y$

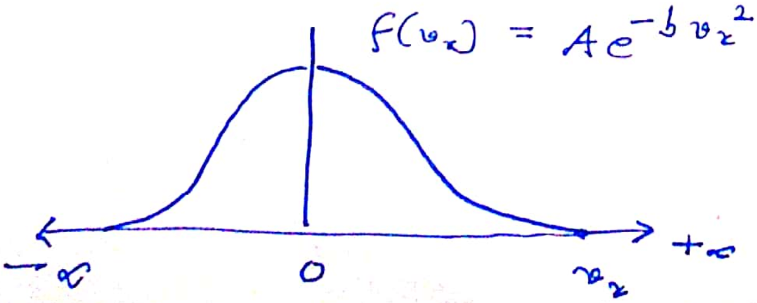
....

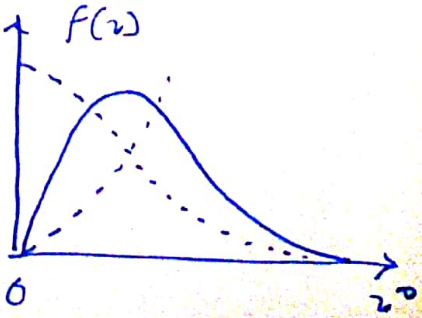
$$\boxed{f(v)\,dv = 4\pi A^3 e^{-b v^2} v^2 dv}$$ 3 D speed distribution

↓

probability of having speed in the range v to $v + dv$

$$f(v_x) = A e^{-b v_x^2}$$





$$f(v)\,dv = 4\pi A^3\, e^{-bv^2}\, v^2\, dv \qquad A = ? \quad b = ?$$

$$\int_0^\infty f(v)\,dv = 1$$

$$= 4\pi A^3 \int_0^\infty e^{-bv^2}\, v^2\, dv$$

$$= 4\pi A^3 \frac{1}{2}\int_0^\infty e^{-bv^2} \times v \times 2v\,dv$$

$$= 4\pi A^3 \frac{1}{2}\int_0^\infty e^{-bv^2} (v^2)^{3/2-1}\, d(v^2)$$

$$= 2\pi A^3 \int_0^\infty e^{-bx}\, x^{3/2-1}\, dx \qquad \Big|\; v^2 \equiv x$$

$$= 2\pi A^3\, \frac{\Gamma(3/2)}{b^{3/2}}$$

$$= \cancel{2}\pi A^3 \times \frac{1}{\cancel{2}}\pi^{1/2} \times \frac{1}{b^{3/2}}$$

$$= A^3\, \frac{\pi^{3/2}}{b^{3/2}} = 1 \implies \boxed{A^3 = \left(\frac{b}{\pi}\right)^{3/2}}$$

$$\int_0^\infty e^{-ax}\, x^{n-1}\, dx = \frac{\Gamma n}{a^n}$$

$$\Gamma n = (n-1)! = (n-1)(n-2)!$$

Formula

$$\Gamma(1/2) = \pi^{1/2}$$

$$\Gamma n = (n-1)\,\Gamma(n-1)$$

$$\Gamma(3/2) = \frac{1}{2}\Gamma(1/2) = \frac{1}{2}\pi^{1/2}$$

$$f(v)\,dv = 4\pi\left(\frac{b}{\pi}\right)^{3/2} e^{-bv^2}\, v^2\, dv$$

$$\langle \epsilon \rangle = \langle \epsilon_x + \epsilon_y + \epsilon_z \rangle$$

$$\left\langle \frac{1}{2} m v^2 \right\rangle = \left\langle \frac{1}{2} m v_x^2 \right\rangle + \langle \cdots \rangle + \langle \quad \rangle$$

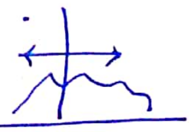
$$= \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T$$

$$= \frac{3}{2}k_B T$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2}k_B T \implies \boxed{\langle v^2 \rangle = \frac{3k_B T}{m}}$$

RHS

x



$$\sqrt{\langle (v - \langle v \rangle)^2 \rangle}$$

$$= \sqrt{\langle v^2 \rangle}$$

$$= v_{rms}$$

$$f(v)\,dv = 4\pi A^3\, e^{-bv^2}\, v^2\, dv$$

$$\downarrow \quad \int_0^\infty f(v)\,dv = 1$$

$$f(v)\,dv = 4\pi\left(\frac{b}{\pi}\right)^{3/2} e^{-bv^2}\, v^2\, dv$$

$$\downarrow \quad \langle v^2\rangle = \frac{3k_BT}{m}$$

?

$$\langle v^2\rangle = \int_0^\infty \underline{f(v)\,dv}\ \underset{m}{v^2}$$

$$= 4\pi\left(\frac{b}{\pi}\right)^{3/2} \times \int_0^\infty v^2\, e^{-bv^2}\, v^2\, dv$$

$$= \times \int_0^\infty e^{-bv^2}\, v^4\, dv$$

$$= \times \frac{1}{2}\int_0^\infty e^{-bv^2} \times v^3 \times (2v\,dv)$$

$$= \times \frac{1}{2}\int_0^\infty e^{-bv^2}\, (v^2)^{5/2-1}\, d(v^2)$$

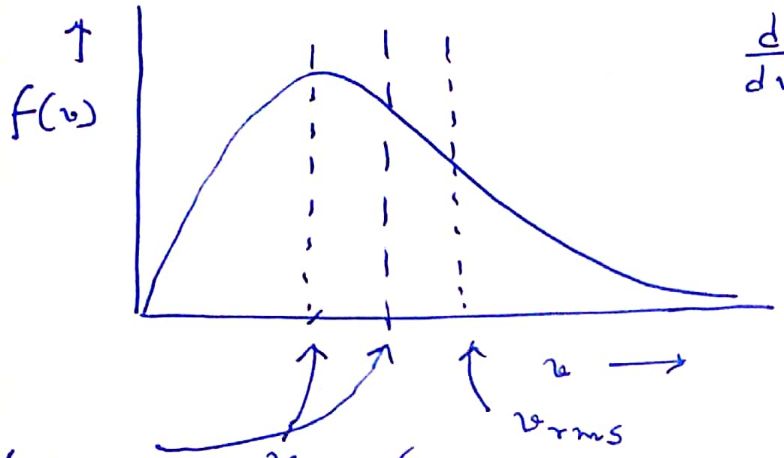
$$= 4\pi\left(\frac{b}{\pi}\right)^{3/2} \times \frac{1}{2}\ \frac{\Gamma(5/2)}{b^{5/2}}$$

$$= \cancel{4\pi}\left(\frac{b}{\cancel{\pi}}\right)^{3/2} \times \frac{1}{\cancel{2}} \times \underbrace{\frac{3}{\cancel{2}} \times \frac{1}{2} \times \cancel{\pi^{1/2}}} \times \frac{1}{b^{5/2}}$$

$$= \frac{3}{2b} = \frac{3k_BT}{m} \Rightarrow$$

$$\boxed{b = \frac{m}{2k_BT}}$$

$$\boxed{f(v)\,dv = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} e^{-\frac{mv^2}{2k_B T}}\, v^2\,dv}.$$



$\langle v\rangle$ v_{mp} (most-probable speed)

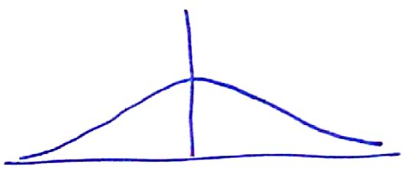
$= \left(\frac{8k_BT}{\pi m}\right)^{1/2} \qquad = \left(\frac{2k_BT}{m}\right)^{1/2}$

$$\frac{d}{dv} f(v) = 0 \begin{cases} v = 0 \\ v = \infty \end{cases} \text{min} \qquad v = \left(\frac{2k_BT}{m}\right)^{1/2}$$

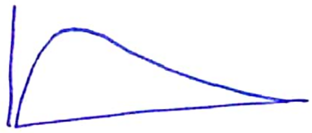
HW

$$v_{rms}^2 = \langle v^2\rangle = \frac{3k_BT}{m}$$

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$



$\langle v_x/v_y/v_z\rangle = 0$



$\langle v\rangle \neq 0$

$$\langle v\rangle = \int_0^\infty v\, f(v)\, dv$$

$$= 4\pi\left(\frac{b}{\pi}\right)^{3/2} \int_0^\infty e^{-bv^2} v^3\, dv$$

$$= \frac{1}{2}\int_0^\infty e^{-bv^2} (v^2)^{2-1}\, d(v^2)$$

$$= \frac{1}{2}\,\frac{\Gamma 2}{b^2}$$

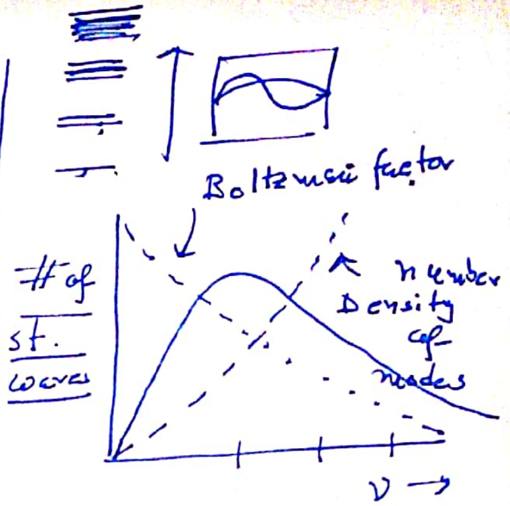
$$= \overset{2}{\cancel{4}}\pi\left(\frac{b}{\pi}\right)^{3/2} \cancel{\frac{1}{2}} \times \frac{1}{b^2} \qquad \Big|\ \Gamma 2 = 1\,\Gamma 1 = 1$$

* CHECK * $= (8k_BT/\pi m)^{1/2}$

$$= 2 \frac{1}{b^{1/2}} \frac{1}{\pi^{1/2}}$$

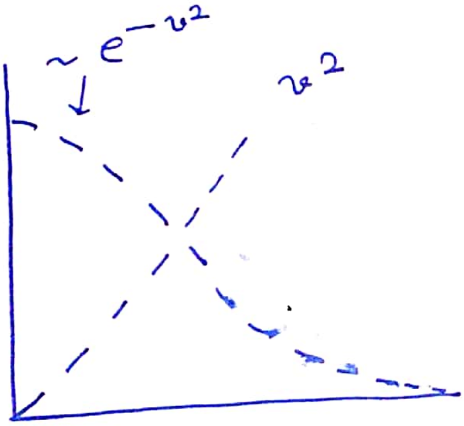
$$= 2 \left(\frac{2k_BT}{m}\right)^{1/2} \frac{1}{\pi^{1/2}}$$

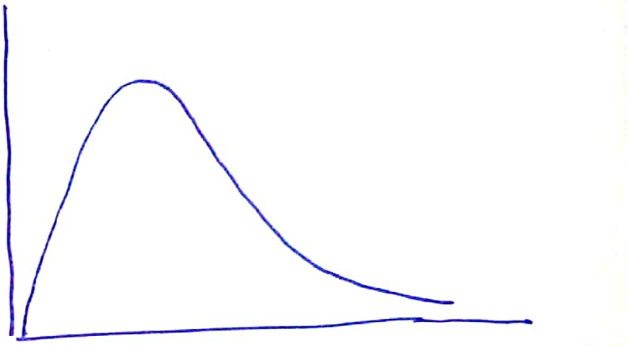
$$= \left(\frac{8\, k_B T}{\pi m}\right)^{1/2}$$

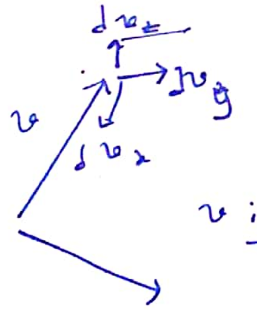


Blackbody radiation

MB distribution

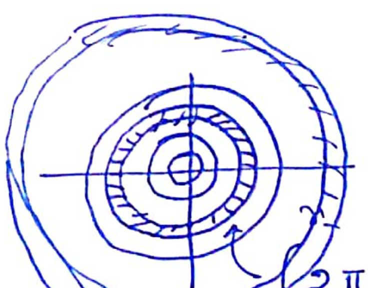






$\rightarrow 4\pi v^2 dv$

v independent of direction



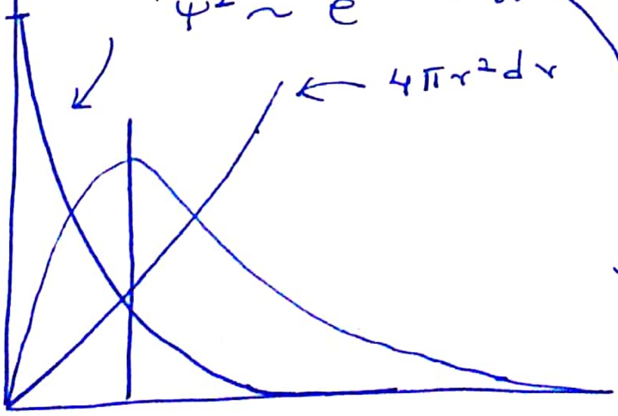
$$\int_0^r 2\pi r' dr' = \pi r^2$$

$$|\psi|^2 \times dv$$

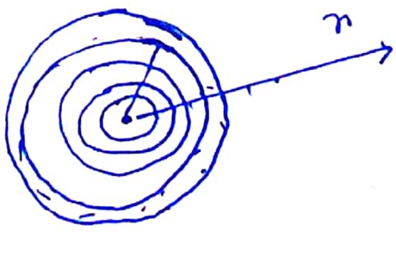
$$\psi \sim e^{-r/a_0}$$

electron prob distribution

$$\psi^2 \sim e^{-2r/a_0}$$



$r = a_0$



Coulomb $V \sim \frac{1}{r}$

$$\frac{4}{3}\pi(r+dr)^3 - \frac{4}{3}\pi r^3$$

$$= 4\pi r^2 dr$$