Solutions to be given in class by tutors

1. What is the probability that the energy E_3 is observed in an experiment given that the system is prepared as the following state:

$$\Psi(x,t) = 0.2\psi_1 e^{-iE_1 t/\hbar} + 2i\psi_2 e^{-iE_2 t/\hbar} + 1i\psi_3 e^{-iE_3 t/\hbar} - 3\psi_4 e^{-iE_4 t/\hbar}$$

   Here $\{\psi_i\}_{i=1}^{i=4}$ are eigenfunctions with corresponding eigenvalues of E_1, E_2, E_3 and E_4. Remember that the probabilities should sum up to 1.

2. Let us say that a particle in an infinite box were to be described by a wavefunction:

$$f(x) = \left(\frac{30}{a^5}\right)^{1/2} x(a-x) \;\; 0 \leq x \leq a$$

   What is the probability of measuring the energy value $E_n = \frac{n^2 h^2}{8ma^2}$?

3. Is the operator $\hat{A} = i\left(\hat{x}^2 + 1\right)\frac{d}{dx} + i\hat{x}$ Hermitian?

4. For a system in a non-stationary state, given by $\Psi(x,t) = \sum_j c_j \psi_j e^{-iE_j t/\hbar}$ show that the average energy is independent of time but the expectation values of other properties vary with time.

5. Construct a 3 by 3 matrix representation of the following operators in a particle in an infinite box basis set.

   (a) $\hat{x}$

   (b) $\hat{x}^2$

   (c) $\hat{H}$

   (d) $\hat{p}_x$

   The *matrix elements* of an operator $\hat{A}$ in a basis $\{\psi_i\}$ are given by

$$a_{ij} = \int \psi_i^* \hat{A} \psi_j dx$$

   Is there anything special that you notice about the matrix representation for the Hamiltonian operator? What about the other operators?

6. Match the following eigenfunctions in Column B to their operators in Column A.

(a) $\left(1-x^2\right)\frac{d^2}{dx^2} - x\frac{d}{dx}$	(i)$4x^4 - 12x^2 + 3$
(b) $\frac{d^2}{dx^2}$	(ii)$5x^4$
(c) $x\frac{d}{dx}$	(iii)$e^{3x} - e^{-3x}$
(d) $\frac{d^2}{dx^2} - 2x\frac{d}{dx}$	(iv)$x^2 - 4x + 2$
(e) $x\frac{d^2}{dx^2} + (1-x)\frac{d}{dx}$	(v)$4x^3 - 3x$

7. For any dynamical quantity represented by operator $\hat{A}$, show that

$$\frac{d\langle\hat{A}\rangle}{dt} = \int \Psi^* \frac{i}{\hbar}\left[\hat{H}, \hat{F}\right]\Psi dx$$

   Using this what is $\frac{d\langle\hat{p}_x\rangle}{dt}$ equivalent to? The Hamiltonian $\hat{H} = \hat{T} + \hat{V}(x)$.