

Problem set 2

$B = 20.56 \text{ cm}^{-1}$ for HF.

kT at 300K = $\sim 200 \text{ cm}^{-1}$

kT at 600K = $\sim 400 \text{ cm}^{-1}$

kT at 50K = $\sim 35 \text{ cm}^{-1}$

Table showing the calculations for populations at 300K

| J | Energy (E_J) (in units of B) | Energy (E_J) (in units of cm^{-1}) | Value of E_J/kT (at 300K) | Value of $e^{-(E_J/kT)}$ | Value of $2J+1$ | Value of $(2J+1) e^{-(E_J/kT)}$ |
|---|-------------------------------------|---|--------------------------------|--------------------------|-----------------|---------------------------------|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 2B | 41.1 | 0.206 | 0.814 | 3 | 2.4 |
| 2 | 6B | 123.4 | 0.617 | 0.540 | 5 | 2.7 |
| 3 | 12B | 246.7 | 1.234 | 0.291 | 7 | 2.0 |
| 4 | 20B | 411.2 | 2.056 | 0.128 | 9 | 1.2 |
| 5 | 30B | 616.8 | 3.084 | 0.046 | 11 | 0.5 |
| 6 | 42B | 863.5 | 4.318 | 0.013 | 13 | 0.2 |
| 7 | 56B | 1151.4 | 5.757 | 0.003 | 15 | 0.05 |
| 8 | 72B | 1480.3 | 7.402 | 0.0006 | 17 | 0.01 |
| 9 | 90B | 1850.4 | 9.252 | 9.6×10^{-5} | 19 | 0.002 |

J_{max} (from formula: $\{[kT/2hcB]^{1/2} - 0.5\}$) at 300 K = 2

Agrees with the table

Table showing the calculations for populations at 600K

| J | Energy (E_J) (in units of B) | Energy (E_J) (in units of cm^{-1}) | Value of E_J/kT (at 600K) | Value of $e^{-(E_J/kT)}$ | Value of $2J+1$ | Value of $(2J+1) e^{-(E_J/kT)}$ |
|---|--|--|-----------------------------------|-----------------------------|--------------------|------------------------------------|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 2B | 41.1 | 0.103 | 0.903 | 3 | 2.7 |
| 2 | 6B | 123.4 | 0.309 | 0.735 | 5 | 3.7 |
| 3 | 12B | 246.7 | 0.617 | 0.540 | 7 | 3.8 |
| 4 | 20B | 411.2 | 1.036 | 0.355 | 9 | 3.2 |
| 5 | 30B | 616.8 | 1.542 | 0.214 | 11 | 2.4 |
| 6 | 42B | 863.5 | 2.159 | 0.115 | 13 | 1.5 |
| 7 | 56B | 1151.4 | 2.879 | 0.056 | 15 | 0.8 |
| 8 | 72B | 1480.3 | 3.701 | 0.025 | 17 | 0.4 |
| 9 | 90B | 1850.4 | 4.626 | 0.010 | 19 | 0.19 |

J_{max} (from formula: $\{[kT/2hcB]^{1/2} - 0.5\}$) at 600 K = 3

Agrees with the table

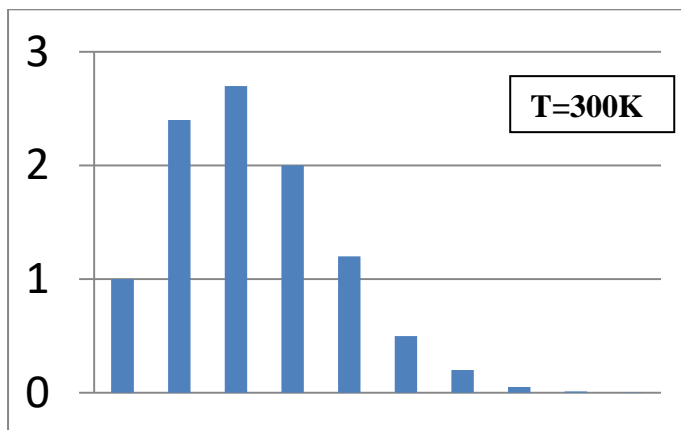
Table showing the calculations for populations at 50K

| J | Energy (E_J) (in units of B) | Energy (E_J) (in units of cm^{-1}) | Value of E_J/kT (at 50K) | Value of $e^{-(E_J/kT)}$ | Value of $2J+1$ | Value of $(2J+1) e^{-(E_J/kT)}$ |
|---|--|--|----------------------------------|-----------------------------|--------------------|------------------------------------|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 2B | 41.1 | 1.175 | 0.309 | 3 | 0.9 |
| 2 | 6B | 123.4 | 3.526 | 0.029 | 5 | 0.1 |
| 3 | 12B | 246.7 | 7.049 | 0.0008 | 7 | 0.006 |
| 4 | 20B | 411.2 | 11.749 | 7.9×10^{-6} | 9 | ~ 0 |
| 5 | 30B | 616.8 | 17.622 | | 11 | |
| 6 | 42B | 863.5 | 24.671 | | 13 | |
| 7 | 56B | 1151.4 | 32.900 | | 15 | |
| 8 | 72B | 1480.3 | | | 17 | |
| 9 | 90B | 1850.4 | | | 19 | |

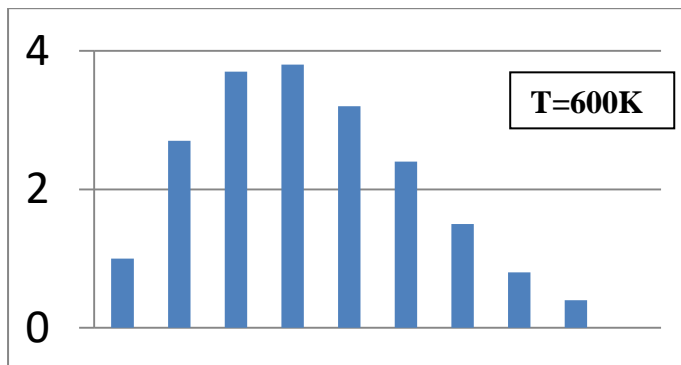
J_{max} (from formula: $\{[kT/2hcB]^{1/2} - 0.5\}$) at 300 K = 1

Agrees with the table

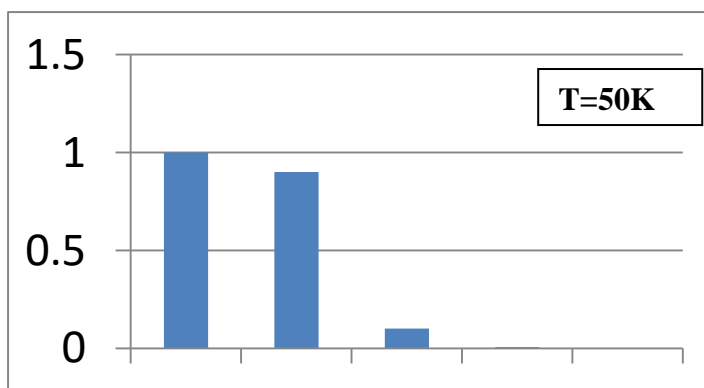
Histogram showing population distribution for various J levels at various temperatures



J **0 1 2 3 4 5 6 7 8**



J **0 1 2 3 4 5 6 7 8**



J **0 1 2 3**

Note how the population shifts to higher J values with increase in temperature

Repeat the exercise for the HI example given in the next problem, to understand the effect of B values on the population.