

Homework-4

①

[X means very important]

1. (i) Show that the curve $\alpha: [0,1] \rightarrow \mathbb{R}^2$,

$\alpha(t) = (t, \sqrt{1-t^2})$ is NOT smooth.

(ii) Do the same for the curve

$\beta: [0,1] \rightarrow \mathbb{R}^2$ given by $\beta(t) = (t, t^{1/3})$.

2. (i) Check that the book's definition of a surface is equivalent to that of ours.

(ii) Any open subset of a surface is a surface. In particular, any open subset of the xy -plane is a surface.

3. If $I, J \subseteq \mathbb{R}$ are intervals and $\varphi: I \rightarrow J$ is a diffeomorphism then $\varphi'(t) \neq 0 \forall t \in I$.

4.* Consider the following curves.

(i) check if they are regular. (ii) Find their unit tangent vector and speed at any time t .

(a) $\alpha: (0, \infty) \rightarrow \mathbb{R}^2$ $\alpha(t) = (t \cos t, t \sin t)$

(b) $\alpha: \mathbb{R} \rightarrow \mathbb{R}^2$ $\alpha(t) = (t - \sin t, 1 - \cos t)$

(c) $\alpha: \mathbb{R} \rightarrow \mathbb{R}^2$ $\alpha(t) = (e^{kt} \cos t, e^{kt} \sin t)$

(d) $\alpha: \mathbb{R} \rightarrow \mathbb{R}^3$ $\alpha(t) = (a \cos t, a \sin t, bt)$
 $a > 0, b > 0$

$$(e) \alpha: (0, \infty) \rightarrow \mathbb{R}^3 \quad \alpha(t) = (t^2, t^2+1, t^2+2) \quad (2)$$

Note: a, b, c are constants,

Example: (d) $\alpha'(t) = (-a \sin t, a \cos t, b)$

$$\Rightarrow \text{Speed} = \|\alpha'(t)\| = \sqrt{a^2 + b^2} \neq 0$$

Hence α is regular also.

Then unit tangent vector at time $t =$

$$\frac{1}{\|\alpha'(t)\|} \cdot \alpha'(t) = \frac{1}{\sqrt{a^2 + b^2}} (-a \sin t, a \cos t, b).$$

* In 4(c), 4(d), 4(e) find arc length parametrizations for base points $t=0$, $t=0$ and $t=1$ respectively.

5. Suppose $\alpha: I \rightarrow \mathbb{R}^3$ is a curve with $\alpha''(t) = 0$. Show that $\alpha(I)$ is contained in a straight line.