Philosophy of Science: Deductive Logic

Reference: Symbolic Logic (I. M. Copi)

1 Validity or Invalidity of an Argument

To determine the validity or invalidity of an argument form, we must examine all possible substitution instances of it to see if any of them have true premisses and false conclusions. This can be verified by making a truth table. However this may turn out to be a very messy process.

1.1 Proving Invalidity

We have established the following way to check invalidity of an argument: Write the truth table of the statements involved in the argument. If it contains at least one row in which truth values are assigned to statements in such a way that the premisses are made true and the conclusion false, the the argument is invalid. Note that you do not need to write the entire truth table for this purpose. As long as you can find one substitution instance that leads to true premisses and false conclusion, you are done with proving the invalidity of the argument. For example, for the argument:

$$\begin{aligned} &p \implies q \\ &r \implies q \\ &\therefore \quad p \implies r \end{aligned}$$

it is enough to see the following:

p	q	r	$p \implies q$	$r \implies q$	$p \implies r$
T	T	F	T	T	F

1.2 Formal Proof of Validity

Example 1.1. *Suppose A, B, C, D, E are statements and consider the following argument:*

$$\begin{aligned} &A \vee (B \implies C) \\ &\sim C \implies (D \implies E) \\ &A \implies C \\ &\sim C \\ &\therefore \quad B \implies E \end{aligned}$$

To establish the validity of this argument, one would require a truth table with 32 rows.

Instead, we will come up with some rules of deduction/inference using which we can establish the validity of an argument. A formal and more concise way of writing out this proof of validity is to list the premisses and the statements deduced from them in one column, which 'justifications" for the latter written besides them.

A **formal proof of validity** *for a given argument is defined to be sequence of statements, each of which is either a premiss of that argument or follows from preceding statements by an elementary valid argument, and such that the last statement in the sequence is a conclusion of. the argument whose validity is being proved.*

1.3 Rules of Inference

We first present a list of nine elementary valid arguments.

1. Modus Ponens (M.P.)

$$\begin{aligned} &p \implies q \\ &p \\ &\therefore\ q \end{aligned}$$

2. Modus Tollens (M.T.)

$$\begin{aligned} &p \implies q \\ &\sim q \\ &\therefore\ \sim p \end{aligned}$$

3. Hypothetical Syllogism (H.T.)

$$\begin{aligned} p &\implies q \\ q &\implies r \\ \therefore\ p &\implies r \end{aligned}$$

4. Disjunctive Syllogism (D.S.)

$$\begin{aligned} &p \vee q \\ &\sim p \\ &\therefore\ q \end{aligned}$$

5. Constructive Dilemma (C.D.)

$$(p \implies q) \cdot (r \implies s)$$
$$p \vee r$$
$$\therefore \quad q \vee s$$

6. Destructive Dilemma (D.D.)

$$(p \implies q) \cdot (r \implies s)$$
$$\sim q \vee \sim s$$
$$\therefore \quad \sim p \vee \sim r$$

7. Simplification (Simp.)

$$p \cdot q$$
$$\therefore \ p$$

8. Conjunction (Conj.)

$$p$$
$$q$$
$$\therefore \ p \cdot q$$

9. Addition (Add.)

$$p$$
$$\therefore \ p \vee q$$

Example 1.2. *We now look at the example 1.1 again and present a formal proof of validity using some of the elementary valid arguments listed above.*

1. $A \vee (B \implies C)$
2. $\sim C \implies (D \implies E)$
3. $A \implies C$
4. $\sim C$

5. $\sim A$ $\quad (Modus\ Tollen(M.T.)\ 3, 4)$
6. $B \implies D$ $\quad (Disjunctive\ Syllogism(D.S.)\ 1, 5)$
7. $D \implies E$ $\quad (Modus\ Ponens(M.P.)\ 2, 4)$
8. $B \implies E$ $\quad (HypotheticalSyllogism(H.S.)\ 6, 7)$

where, by Modus Tollen (M. T.) 3, 4, we mean that using statements (3), (4) and the rule "Modus Tollens", statement (5) follows.

1.4 The Rule of Replacement

In addition to the nine rules of inference, we have the following logical equivalences that come handy in proving validity of arguments:

1. De Morgan's Theorem (De M)

$$\sim(p \cdot q) \equiv (\sim p \vee \sim q)$$
$$\sim(p \vee q) \equiv (\sim p \cdot \sim q)$$

2. Commutation (Com.)

$$(p \vee q) \equiv (q \vee p)$$
$$(p \cdot q) \equiv (q \cdot p)$$

3. Association (Assoc)

$$(p \vee (q \vee r)) \equiv ((p \vee q) \vee r)$$
$$(p \cdot (q \cdot r)) \equiv ((p \cdot q) \cdot r)$$

4. Distribution (Dist.)

$$(p \cdot (q \vee r)) \equiv ((p \cdot q) \vee (p \cdot r))$$
$$(p \vee (q \cdot r)) \equiv ((p \vee q) \cdot (p \vee r))$$

5. Double Negation (D.N.)

$$p \equiv \sim\sim p$$

6. Transposition (Trans.)

$$(p \implies q) \equiv (\sim q \implies \sim p)$$

7. Material Implication (Impl.)

$$(p \implies q) \equiv (\sim p \vee q)$$

8. Material Equivalence (Equiv.)

$$(p \equiv q) \equiv [(p \implies q) \cdot (q \implies p)]$$
$$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$$

9. Exportation (Exp.)

$$[(p \cdot q) \implies r] \equiv [p \implies (q \implies r)]$$

10. Tautology (Taut.)

$$p \equiv p \vee p$$
$$p \equiv p \cdot p$$

Exercise 1.3. *State the rule of inference/replacement by which its conclusion follows from the premiss in the arguments given below:*

(i)

$$(\sim A \implies B) \cdot (C \vee \sim D)$$
$$\therefore (\sim A \implies \sim B) \cdot (\sim D \vee C)$$

(ii)

$$(\sim E \vee F) \cdot (G \vee \sim H)$$
$$\therefore (E \implies F) \cdot (G \vee \sim H)$$

(iii)

$$M \implies \sim (N \vee \sim O)$$
$$\therefore M \implies (\sim N \cdot \sim\sim O)$$