MTH202: Assignment 9

March 10, 2019

1. Suppose the lifetime of a component in a machine is distributed as $Exp(1)$. Then,

   - Find the lifetime L which is typical component is 60 percent certain to exceed.
   - If 5 components are sold, find the probability that at least 1 of them will have a lifetime less than L.

2. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f_X(x) = Cx$$

   where, C is a constant.

   (a) Find C.

   (b) $P(X > 5), P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.

3. The lifetime (in days) of a machine is given by a random variable with density function:

$$f(x) = \begin{cases} 0 \text{ for } x \leq 100 \\ \frac{100}{x^2} \text{ for } x > 100 \end{cases}$$

   There are 5 such machines in a factory. What is the probability that exactly 2 machines will breakdown within the first 150 days?

4. Consider a random variable $X \sim Unif([0, 10])$. Compute the following:

   (a) $P(X < 3)$.

   (b) $P(X > 3)$.

   (c) $P(3 < X < 8)$.

   (d) $E[4X^2 - 2X]$.

   (e) $E[e^X]$.

5. Let $\phi(z)$ denote $P(Z \leq z)$ for a standard normal random variable Z.

   - Let $X \sim \mathcal{N}(2, 4)$ and $Y = 3 - 2X$. Find $P(X > 1), P(2 < Y < 1), P(X > 2|Y < 1)$ in terms of ϕ.

- Let $X \sim \mathcal{N}(0, 2)$. Compute $E[|X|]$.

6. Let $X \sim Exp(\lambda)$. What is the distribution function of X?

7. Let $U \sim Unif([0, 1])$ and $X = -\ln(1 - U)$. Show that $X \sim Exp(1)$.

8. Suppose the number of customers arriving at a store obeys a Poisson distribution with an average of λ customers per unit time. That is, if Y is the number of customers arriving in an interval of length t, then $Y \sim Poi(\lambda t)$. Suppose that the store opens at time $t = 0$. Let X be the arrival time of the first customer. Show that $X \sim Exp(\lambda)$.

9. Test scores on OWLs at Hogwarts are normally distributed with mean 250 and variance 900. Only the top 5 percent students will qualify to become an Auror. What is the minimum score Harry must get in order to qualify?

10. Suppose the number of miles a car can run before the battery wears out is exponential distributed with mean $10, 00$ kms. The owner of the car wants to take 500 kms trip. What is the probability that she will be able to complete the trip without having to replace the battery?