Philosophy of Science: Argument Forms and Truth Tables

Reference: Symbolic Logic (I. M. Copi)

1 Truth and Validity

As mentioned in the previous class, **It is important to understand that the validity or invalidity of an argument has nothing to do with the truth or falsehood of the propositions or statements.**

Logic concerns itself with the validity or invalidity of the arguments, whereas in science we are often concerned with the truthfulness or the falsehood of statements. However, to determine the truth value of statements occurring as inferences of certain premisses, we must. employ arguments that are valid. Recall that **to establish the truth of the conclusion, the argument must be valid and all its premisses must be true.**

In the last lecture we defined three types of statements - conjunction, disjunction and negation and denoted them using symbols $\cdot, \vee, \sim$. Recall that we said that there are two types of disjunctions: inclusive and exclusive. We noted that *at least one disjunct is true* was correct for both inclusive and exclusive 'or'. We introduced the symbol $\vee$ to capture this common partial meaning of 'or' (which is the whole meaning of 'or' when it is inclusive). Finally, we observed that arguments like Disjunctive Syllogism are valid for either interpretation of 'or' thus replacing 'or' by the symbol $\vee$ preserves the validity of such arguments.

2 Conditional Statements

Statements involving if-then are called conditional statements. An example of a conditional statement is "If the train is late then we shall miss our connection". The component between 'if' and 'then' is called *antecedent*. The component that follows 'then' is called *consequent*. A conditional statement does not say anything about the truth-value of the antecedent or the consequent. It asserts that if the antecedent is true then the consequent is also true. In other words, the antecedent *implies* the consequent. We use the symbol $\implies$ to write that a statement p implies statement q ($p \implies q$). Here p is the antecedent and q is the consequent.

Consider the following statement - "If this piece of gold is placed in this solution, then this piece of gold will dissolve". When is this statement false?

The statement is false when the piece of gold is placed in the solution and it doesn't dissolve. Thus, it is false when the antecedent is true but the consequent is false. In other words the statement of the form *if p, then q* is false in case the conjunction $p \cdot (\sim q)$ is true. Similarly, the statement of the form *if p, then q* is true in case the conjunction $\sim (p \cdot (\sim q))$ is true. Thus, we have the following truth table:

p	q	$\sim q$	$p \cdot \sim q$	$\sim (p \cdot \sim q)$	$p \implies q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

Remark 1. *The following are equivalent:*

- $p \implies q$
- *If p then q (or q if p)*
- *p implies q*
- *p only if q*
- *p is a sufficient condition that/for q*
- *q is a necessary condition that/for p*
- *p entails that q*

Example 2.1 (Denying the Antecedent)**.** *Consider the statement "If United States is a peace-loving nation, then I am crow". It is clear that the implication here is neither logical (e.g, in "If all humans and mammals and I am a human, then I am a mammal"), nor definitional (e.g, in "If the figure is a square, then it has four sides"), nor casual/empirical (e.g, in "If I put gold in aqua regia, then it will dissolve"). There is no "real connection" between the antecedent and the consequent. This sort of statement is usually a humorous way of denying the truth of the antecedent, for it typically has a ridiculously false statement as its consequent.*

Note that example above illustrates various "types " of conditional statements. However, because of the similar reason (as in the case of 'or') of a 'common partial meaning' in all of them, we use the symbol $\implies$ for all kinds of implications or conditional statements.

Exercise 2.2. *If X, Y are false statements, is $X \implies (X \implies Y)$ true?*

3 Argument Forms

Let us go back to examples 1.3 and 1.4 from last lecture.

Example 3.1.
If I am a cricket player, then I am famous.
I am not a cricket player.
Therefore, I am not famous.

We said that this argument was invalid by looking at an argument of the same form given by:

Example 3.2.
If Einstein is a cricket player, then Einstein is famous.
Einstein is not a cricket player.
Therefore, Einstein is not famous.

Clearly, the argument is invalid because the premisses are true but the conclusion is not.

Any argument is proved to be invalid if another argument of exactly same form can be constructed with true premisses and false conclusion. This means that the validity or invalidity of an argument is purely *formal* **characteristic of the argument. Thus, two arguments having the same form are either both valid or invalid.**

Therefore, it is convenient to use symbols to represent 'argument forms'. Consider example 2.1 from last lecture:

Example 3.3 (Disjunctive Syllogism)**.**
The United States will become more responsible or there will be a third world war.
The United States will not become more responsible.
Therefore, there will be a third world war.

This can be represent symbolically as:

$$\begin{array}{c} p \vee q \\ \sim \; p \\ \therefore \; q \end{array}$$

In example 3.3 above, $p =$ 'The United States will become more responsible' and $q =$ 'There will be a third world war'.

To determine validity or invalidity of an argument form, we must examine all possible substitutions (of statements) to see if any of them has true premisses and false conclusions. In fact, in determining the validity or invalidity of an argument form, we are only concerned with the truth-values of the statements involved. A convenient way to do this is to construct the truth table. let's write down the truth table for the **Disjunctive Syllogism**.

This represents all the possibilities of substitution instances for statements p and q. The third and fourth columns constitute the premisses of the argument and column two constitutes the conclusion. To determine the validity of this (Disjunctive Syllogism) argument, we need to examine the truth table to see it doesn't have true premisses and false conclusion. The only time when both the premisses are true is the third column and the corresponding conclusion is also true. Thus, we have verified the validity of *Disjunctive Syllogism.*

p	q	$p \vee q$	$\sim p$	$(p \vee q) \cdot (\sim p)$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Exercise 3.4 (Fallacy of Affirming the Consequent)**.** *Show that the following argument is invalid.*

$$
\begin{aligned}
& p \implies q \\
& q \\
& \therefore \quad p
\end{aligned}
$$

4 Statement Forms

We define a statement form to be any sequence of symbols containing statement variables, such that when statements are substituted for statement variables - with the same statement being substituted for every occurrence of the same. statement variable throughout - the result is a statement.

Given a statement form, any statement obtained by substituting statements for statement variables is said to have that form and is called the substitution instance of it.

- **Tautology.** Consider the statement: "Fermat had a proof for the 'Fermat's last theorem' or he didn't". This is a statement of the form $p \vee \sim p$. While the truth-value of the statement p is not known (and is a matter of investigation and/or history of mathematics), the statement $p \vee \sim p$ is always true. This is immediate form the truth table:

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

  Such a statement is called a *tautology.* In other words, a statement form that has only true substitution instances is said to be *tautologous*, or a *tautology.*

- **Contradiction.** Similarly consider the statement: "Fermat had a proof for the 'Fermat's last theorem' and he didn't". This is a statement of the form $p \cdot \sim p$. Again, while the truth-value of the statement p is not known, the statement $p \cdot \sim p$ is always false. The corresponding truth table is given by:

  Such a statement is called a *contradiction.* A statement form that has only false substitution instances is said to be *contradictory*, or a *contradiction.*

p	$\sim p$	$p \cdot \sim p$
T	F	F
F	T	F

- **Contingency.** Statements and statement forms that are neither tautologous or contradictory are said to be *contingent* or *contingencies*. For example, $p, \sim p, p \vee q$ etc. are contingent statement forms.

Two statements are said to be materially equivalent when they have the same truth-value. We symbolise the statement that they are materially equivalent by inserting '$\equiv$' between them. It is defined by the following truth table. To say that two statements are materially equivalent

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

is same as saying that the two statement imply each other. That is, $p \equiv q$ is same as saying $(p \implies q) \cdot (q \implies p)$. Therefore the symbol '$\equiv$' may be read as 'if and only if'. It is also known as a biconditional.

Two statements are said to be **logically equivalent** when the biconditional that expresses their material equivalence is a tautology. Note that if columns in the truth table of statements A and B match, then the statements are logically equivalent (check that this follows from the definition of '$\equiv$' and tautology).

Exercise 4.1. *Show that the statements $\sim (p \cdot q)$ and $\sim p \vee \sim q$ are logically equivalent.*

Hint: You want to write the following truth table and check that last column contains only 'T's.

p	q	$\sim p$	$\sim q$	$p \cdot q$	$\sim (p \cdot q)$	$(\sim p) \vee (\sim q)$	$\sim (p \cdot q) \equiv (\sim p) \vee (\sim q)$