Homework - 6

1) If the curvature of a smooth & regular curve in the xy-plane is constant then show that ~~either~~ it is contained in a straight line or in a circle.

2) Find the unit speed curve $\alpha : \mathbb{R} \to \mathbb{R}^2$ with signed curvature ~~K~~ $x_\beta(t) = \cos t, \forall t \in \mathbb{R}$, where $\alpha(0) = (0,0)$, $\alpha'(0) = (1,0)$.

3) Suppose $A \in M(3, \mathbb{R})$. Show that the following are equivalent:

    (i) $AA^t = I$ ($A^t = $ transpose of A.)

    (ii) $Au \cdot Aw = u \cdot w \quad \forall u, w \in \mathbb{R}^3$

    (iii) $Au \cdot Au = u \cdot u \quad \forall u \in \mathbb{R}^3$

4) (i) Suppose $AA^t = I$, $\det A = 1$. Show that-

     $Au \times Aw = A(u \times w)$.

    (ii) Use this to complete the proofs of the fundamental theorems for plane curves and space curves.

5) Compute $\vec{T}, \vec{n}, \vec{b}, x, \tau$ for the following curves at the given points:

    (i) $\alpha(t) = (t, t^2, t^3)$ at $t = 0$.

    (ii) $\alpha(t) = (t\cos t, t\sin t, t)$ at $t = 0$.

6. Decide if the following curves are planar

    (i) $\alpha(t) = (t, t^2, 1 + t + t^2)$

    (ii) $\alpha(t) = (t, e^t, e^{2t})$