MTH202: Assignment 5

February 11, 2019

1. A couple has two children, the older of which is a boy. What is the probability that they have two boys?

2. Consider the following games and answer the questions.

   (a) Toss a fair coin, repeatedly and independently each time. If it shows HEADS you win a point otherwise you lose a point. What is your expected score after n tosses?

   (b) Toss a fair coin, repeatedly and independently each time. If it shows HEADS you win one point otherwise you win zero points. How many times should you toss the coin to make your expected score larger than 10?

   (c) Toss a coin, repeatedly and independently each time. Play the game described in (a). If after n tosses your expected score is $n/2$, what is the probability of getting HEADS with this coin?

   (d) Toss a coin, repeatedly and independently each time. Play the game described in (a). Is it possible to have an expected score of $2n$ after n tosses?

3. M, H, A denote the event that a student passes in Mathematics, History and Assamese respectively. Represent the following events in terms of M, H, A:

   - Student passes in at least one subject.
   - Student passes in at least two subjects.
   - Student passes in exactly two subjects.
   - Student passes in all subjects.

4. Compute Expectation and Variance of a random variable with following probability mass functions:

   (a) Uniform on $\{1, 2, \ldots, n\}$.

   (b) Poisson with parameter λ.

   (c) Binomial with parameters n and p.

(d) Geometric with parameter q.

(e) $P(X = r) = r/10$ for X taking values in $\{1, 2, 3, 4\}$.

(f) $P(X = r) = P(X = -r) = r/6$ for X taking values in $\{-2, -1, 1, 2\}$.

5. Consider a species with population X_n at time n. At discrete times $t = 1, 2, \ldots$, each individual in the population produces 0 or 2 offsprings (independently of others in the population) with probability p and q respectively and dies immediately. Suppose we start with one individual at time $t = 0$. Consider the event A_n that denotes that the population becomes extinct at time n. Show that this forms an increasing sequence of events.

6. In a supermarket there are N counters. Each counter is open with probability p (independent of the others). Let X denote the number of open counters. What is the probability mass function of X?

7. Suppose $X \sim Poi(1)$ denote the number of earthquakes in a region in one year. What is the probability that there was at least two earthquakes this year?

8. You have a set of 6 keys, and they all look identical. One of them opens a lock. You are going to try them one by one in the order they are arranged on a key-ring. If all orders are equally likely, what is the expected number of keys you will end up trying, until one finally opens the door?

9. An assembly line produces 1 defective product out of every 500 produced. What is the (approximate) probability that there are no defective products in a batch of 80?

10. Consider a set of labelled vertices $V = \{1, 2, \ldots, n\}$. A graph G is defined as a pair $G = (V, E)$, where V denotes the set of vertices and $E \subset V \times V$, denotes the set of undirected edges (i.e. $(i, j) = (j, i)$).

    - How many undirected graphs with m edges are possible on these set of vertices?
    - Suppose that an edge between any two vertices is present with probability p, independently of all the other edges. What is the distribution of $|E|$, where $|E|$ denotes the size of set E, i.e. the number edges in the graph.