

Vectors

Maths Workshop

Problems

1. Find a unit vector in the direction of $3\vec{i} + 12\vec{j} + 4\vec{k}$
2. Find the angle between the vectors $3\vec{i} + 4\vec{j}$ and $4\vec{i} + 4\vec{j} + 2\vec{k}$
3. Let $\vec{A} = 3\vec{i} + 2\vec{j}$ and $\vec{B} = 4\vec{i} - \vec{j}$. What is the projection of \vec{A} along \vec{B} ?
4. Given that $|\vec{A}| = 3$; $|\vec{B}| = 8$; $|\vec{A} \times \vec{B}| = 12$; what is the angle between the two vectors?
5. Let $\vec{A} = 3\vec{i} + 4\vec{j} - 4\vec{k}$. Find a vector in the x-y plane that is perpendicular to \vec{A}
6. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then show that $\vec{a} \perp \vec{b}$
7.
 - (i) Prove that
$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$
 - (ii) Using the previous result prove that $[\vec{a} \ \vec{b} \ \vec{c}]$ is cyclic, i.e $[\vec{a} \ \vec{b} \ \vec{c}] = [\vec{b} \ \vec{c} \ \vec{a}] = [\vec{c} \ \vec{a} \ \vec{b}]$. i.e., $[\vec{a} \ \vec{b} \ \vec{c}] = [\vec{b} \ \vec{c} \ \vec{a}] = [\vec{c} \ \vec{a} \ \vec{b}]$
 - (iii) Find $(2\vec{i} + 3\vec{j}) \cdot [(2\vec{i} + 3\vec{j}) \times (4\vec{j} + 7\vec{k})]$
8.
 - (i) Prove $[\vec{a} + \vec{b} \ \vec{b} + \vec{c} \ \vec{c} + \vec{a}] = 2[\vec{a} \ \vec{b} \ \vec{c}]$
 - (ii) $[\vec{a} \times \vec{b} \ \vec{b} \times \vec{c} \ \vec{c} \times \vec{a}] = [\vec{a} \ \vec{b} \ \vec{c}]^2$

Some Important formulae

- (i) $(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
- (ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
- (iii) $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$
- (iv) $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{a} \cdot \vec{d})(\vec{b} \cdot \vec{c})$
- (v) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a} \ \vec{b} \ \vec{d}]\vec{c} - [\vec{a} \ \vec{b} \ \vec{c}]\vec{d} = [\vec{a} \ \vec{c} \ \vec{d}]\vec{b} - [\vec{b} \ \vec{c} \ \vec{d}]\vec{a}$