

# NFA

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## Formal Defn of an NFA

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times (\Sigma \cup \{\epsilon\}) \rightarrow \mathcal{P}(Q)$$

An NFA  $N$  accepts a string  $s = w_1w_2w_3\dots w_n$  iff  $\exists$  a sequence of  $r_0r_1\dots r_n \in Q$  such that  $r_n \in F$  and  $r_{i+1} \in \delta(r_i, w_i)$

## Equivalence of NFA and DFA

Theorem:  $L$  is regular iff it can be recognized by an NFA.

Proof:

If  $L$  is recognisable by a NFA (let)  $N = (Q, \Sigma, \delta, q_0, F)$

Consider the DFA,  $D = (Q', \Sigma, \delta', q_0', F')$  where -

- $Q' = \mathcal{P}(Q)$
- $\delta' : Q' \rightarrow Q'$  and
  - $\delta'(A, c) \rightarrow \bigcup_{a \in A} E(\delta(a, c))$ , where
    - \*  $a \in A$
    - \*  $E(A) =$  the set of states connected to some  $q \in A$  by  $\epsilon$ . Trivially, some  $q$ s map to themselves via a  $\epsilon$ .
- $q_0' = E(q_0)$
- $F' = \{A' \in Q' \mid A' \cap F \neq \emptyset\}$

If  $S$  is accepted by  $N$  iff  $\exists r_0\dots r_n \in Q$   $r_0=q_0$  and  $r_{i+1} \in \delta(r_i, w_i)$

$\implies r_{i+1} \in R_i = E(\delta(r_i, w_i))$  and  $r_n \in R_n \in F$ .