# PHY102 : Assignment 9

1. (Purcell 7.21) A solenoid of radius $a_1$ and length $b_1$ is located inside a longer solenoid of radius $a_2$ and length $b_2$. The total number of turns is $N_1$ on the inner coil, $N_2$ on the outer. Work out a formula for the mutual inductance $M$. (See figure in Purcell).

2. Compute the self inductance of a solenoid with $N_1$ turns, length $l$ and crosssectional area $A$, with a current $I_1$ flowing through each turn. Now, an insulated coil of $N_2$ turns is wrapped around it. Calculate the mutual inductance $M$, assuming that all the flux from the solenoid passes through the outer coil. Relate the mutual inductance $M$ to the self inductances $L_1$ and $L_2$ of the solenoid and the coil.

3. (Purcell 9.5) Here is a particular electromagnetic field in free space :

$$E_x = 0 \quad E_y = E_0 \sin(kx + \omega t) \quad E_z = 0$$
$$B_x = 0 \quad B_y = 0 \quad B_z = -\frac{E_0}{c} \sin(kx + \omega t)$$

   Show that this field can satisfy Maxwell's equations if $\omega$ and $k$ are related in a certain way.

4. (Purcell 9.7) Show that the electromagnetic field described by

$$\begin{aligned} \mathbf{E} &= E_0 \hat{\mathbf{z}} \cos kx \cos ky \cos \omega t \\ \mathbf{B} &= B_0 (\hat{\mathbf{x}} \cos kx \sin ky - \hat{\mathbf{y}} \sin kx \cos ky) \sin \omega t \end{aligned}$$

   will satisfy Maxwell's equations in vacuum if $E_0 = \sqrt{2} c B_0$ and $\omega = \sqrt{2} ck$.

5. (Purcell section 9.7 and problem 9.12) Show that $\mathbf{E} \cdot \mathbf{B}$ and $E^2 - c^2 B^2$ are invariant under Lorentz field transformations. A plane wave has $\mathbf{B}$ perpendicular to $\mathbf{E}$. What can you conclude ?

6. (Purcell 8.1) How large an inductance, in henrys, should be connected in series with a 120 volts, 60 watt light bulb if it is to operate normally when the combination is connected across a 240 volt, 60 Hz line ?

7. (Purcell 8.4) Consider the resistance $R'$ connected in parallel, rather than in series, with the $LC$ combination. Work out the equation, which applies to this circuit. Find also the conditions on the solution analogous to those that hold in the series $RLC$ circuit. If a series $RLC$ and a parallel $R'LC$ circuit have the same $L, C$ and $Q$, how must $R'$ be related to $R$ ?