

# MTH202: Assignment 6

February 16, 2019

1. A man is known to speak truth 2 out of 3 times. He throws a die and reports that number obtained is a four. Find the probability that the number obtained is actually a four.
2. Find the expectation and variance for the number of heads obtained in 3 coin tosses (You may assume that the coin is fair).
3. Let  $X$  and  $Y$  be distinct real-valued random variables.
  - (a) Show that: for all real numbers  $a$  and  $b$  check that  $a^2E[X^2] + 2abE[XY] + b^2E[Y^2]$  is a non-negative number.
  - (b) What is  $E[X - E[X]]$ ?
  - (c) Show that  $E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$ .
4. Consider a species with population  $X_n$  at time  $n$ . At discrete times  $t = 1, 2, \dots$ , each individual in the population produces  $\{0, 1, 2, 3\}$  offsprings (independently of others in the population) with probabilities  $\{p_0, p_1, p_2, p_3\}$ . Then,
  - (a) Fix an individual. What is the expected number of offsprings produced by the individual at any given time  $t$ ?
  - (b) Suppose you know that the number of individuals at time  $t = 2$  is 5, what is the expected number of offsprings produced at time  $t = 3$ ?
5. Consider an experiment that involves counting the number of  $\alpha$  particles released in one second interval by 1 gram of radioactive substance. We know that, on average, 3.2 such  $\alpha$  particles are released. What is the (approximate) probability that no more than  $2\alpha$  particles will appear? (Hint: Assume that 1 gram of material has  $n$  atoms, where  $n$  is large)
6. Suppose that the number of typographical errors on a single page of your Probability book has a Poisson distribution with parameter  $\lambda = 0.5$ . What is the probability that there is at least one error on the page you are reading? What is the expected number of errors on the page?
7. Consider the set  $S = \{a, a + d, \dots, a + kd\}$  for some fixed  $a, k, d \in \mathbb{N}$ . Let  $X$  be a random variable such that  $X \sim \text{Uniform}(S)$ . Then,

- (a) What is  $P(X = a + rd)$  for  $r \in \{0, 1, \dots, k\}$ ?
- (b) Show that  $E[X] = \frac{\min(X) + \max(X)}{2}$ . Does this hold for any finite set  $S \subset \mathbb{N}$ ?

where  $\min(X)$  and  $\max(X)$  denote the minimum and maximum values attained by  $X$ .

8. Roll a die twice. Let  $X$  be the number of times 4 comes up. Compute  $E[X]$ .
9. Consider  $n$  random variables  $Y_i \sim \text{Geom}(p_i)$  for  $1 \leq i \leq n$ . Compute  $E[Z]$ , where  $Z = \sum_{i=1}^n Y_i$ .
10. Consider a fair die. Answer the following questions:
  - (a) What is the probability of getting the outcome 1 after exactly 10 rolls?
  - (b) What is the probability of getting the outcome 1 after at least 10 rolls?
  - (c) Find the expected value of the times one must throw the die until outcome 1 occurs?
  - (d) Find the expected value of the times one must throw the die until outcome 1 occurs 4 times?
11. A club contains 50 members; 20 are men and 30 are women. A committee of 10 members is chosen at random. Compute the probability mass function of the number of women on the committee. What is the expected number of women on the committee?