

Sets

Dhruva Sambrani

20 January, 2020

Sets

Set is a well defined collection of objects. Well defined here means that an element of the Domain x should be unambiguously belong to or not belong to set S .

$x \in A \equiv$ "x belongs to set A"

$x \notin A \equiv$ "x does not belong to set A"

Properties

- All elements unique.
- Unordered.

Types

Sets need not be of a particular "type" - $\{\{1,2,3\},6,7\}$ May also be infinite. $\{1,2,3,4,\dots\}$

Null/Empty set = $\{\} = \emptyset$

Operations on sets

- Subset: $A \subseteq B$. "A is a subset of B." $x \in A \Rightarrow x \in B$.
- Equality: $A=B$ iff $A \subseteq B \wedge B \subseteq A$. "A is equal to B." $x \in A \Rightarrow x \in B \wedge x \notin A \Rightarrow x \notin B$
- Proper Subset: $A \subset B$ $x \in A \Rightarrow x \in B \wedge A \neq B$.
- Complement: $U \setminus A$ OR A^c . Exactly all x 's in U NOT in A .
- Union: $A \cup B$. "A union B". Exactly all elements in $A \vee B$.
 - If $\{A_\alpha\}$ is a collection of sets indexed by I , then $\bigcup A_\alpha =$ set of x st $x \in A_\alpha$ for some $\alpha \in I$.
- Intersection: $A \cap B$ "A intersection B". Contains exactly all elements in A AND B .
 - $x \in \bigcap A_\alpha$ iff $x \in A_\alpha \forall \alpha \in I$
- Cartesian Product: $A \times B$ "Cartesian Product of A and B". Its the collection of all 2 element sequences (a,b) st $a \in A, b \in B$
 - $A_1 \times A_2 \times \dots \times A_n$ is the collection of all n element sequences (a_i) st $a_i \in A_i$
- Relation operator: aRb "a is related to b". A relation is a subset of $A \times B$.
- Function: $f: A \rightarrow B$. It is a relation such that

1. $\forall a \in A, \exists b \text{ st } aRb.$
2. $aRb \text{ and } aRc \implies b=c.$
 - Equality of functions - $f=g \implies f \subseteq g \wedge g \subseteq f.$

Languages

Σ is a finite set called an “alphabet”.

The finite sequence is called a string.

A language over Σ is a set of strings.