Homework - 5

1) Solve the exercises mentioned in class.

2) For a regular, smooth plane curve α show that

    (i) $\dfrac{d\vec{n_s}}{ds} = -\kappa_s \vec{T}$

3) (ii) Show that κ_s is a smooth function of s.

3) Suppose $\alpha : [a,b] \rightarrow \mathbb{R}^2$ is a smooth regular curve and $\beta : [a,b] \rightarrow \mathbb{R}^2$ be defined by $\beta(t) = \alpha(a+b-t)$. (i) Show that β is regular.

(ii) Suppose their signed curvature functions are κ_1 and κ_2 respectively. Show that

$$\kappa_2(t) = -\kappa_1(a+b-t)$$

Interpret this geometrically.

4) Given an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $a > b > 0$ find a parametrization. Using this find the points where the curvature is maximum and minimum respectively.

5) Draw the following curves and find curvature wherever defined.
    (i) $p = 2 + \cos\theta$ (ii) $p = 1 + 2\cos\theta$,   (iii) $p = \sin 2\theta$

6) Find curvature of the following curves at any point.
    (i) $y = e^x$ (ii) $y = \frac{1}{x}$, $x > 0$ (iii) $y = \log x$.