1. Consider two systems with fundamental relation $S = (NVU)^{1/3}$, where constant of proportionality is taken to be unity, for convenience.Consider N_1 moles and volume V_1 of one system, while N_2 moles and volume V_2 of the second system. These are kept constant. The total energy $U_0 = U_1 + U_2$, is given and is also fixed. Imagine a process in which energy is exchanged between the two systems via a diathermal wall.
   a) Extremize the total entropy and find the equilibrium values of U_1 and U_2.
   b) Verify that the extremum of entropy corresponds to its maximum value.
   c) Show that maximum value of entropy is given by,
   $$S_0 = (N_1 V_1)^{1/3} \left[x^{1/3} + x^{-2/3}\right] \left[\frac{U_0}{(1+x)}\right]^{1/3},$$
   where $x = (N_1 V_1 / N_2 V_2)^{1/2}$.
   d) Find the temperature of each system and of the total system.

2. Now consider the equivalent problem in terms of minimization of energy, given that $U = S^3/NV$ (by inverting the above fundamental relation). Suppose the total entropy is fixed at S_0, the value found above. Taking the values N_1, V_1, N_2, V_2 to be the same as above,
   a) Extremize total energy with respect to variable S_1.
   b) Verify that the extremum corresponds to the minimum of total energy.
   c) Show that the equilibrium value of total energy is U0, as given in the previous problem (hint: use the expression for S_0 found above.)

3. Find the three equations of state for a system described by the funda- mental relation: $U = kS^3/NV$, where k is a positive constant. Verify that the equations of state are homogeneous zero-order (i.e. T, p, μ are intensive). Find μ as a function of T, V, N.

4. A particular system obeys: $u = Av^{-2} \exp(s/R)$, where A is a positive constant and R is the gas constant. N moles of this system at temperature T_0 and pressure P_0, is expanded isentropically, until the pressure is halved. What is the final temperature?

5. Two identical copper blocks in thermal contact with each other (but isolated from their surroundings) are at the same temperature T_f . Show that if they spontaneously separate into hot and cold blocks, this will lead to a reduction in the total entropy and hence will violate the second law.