Tutorial # 4

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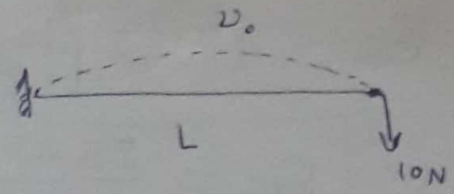


$\mu = \frac{0.01\ Kg}{2.5\ m}$; $T = 10N$

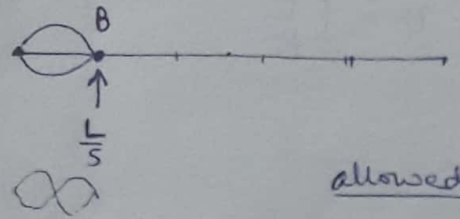
(a)

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10 \times 2.5}{0.01}} = \sqrt{2500}\ m/s$

freq. $\boxed{\nu_0 = \frac{v}{\lambda}}$; for fundamental mode $\frac{\lambda}{2} = L$

$\nu_0 = \frac{v}{2L} = \frac{\sqrt{2500}}{2 \times 2.5} = \sqrt{\frac{2500}{25}} = 10\ Hz$

(b)



; Node at B ($\frac{L}{5}^{th}$ point)

allowed modes: $n \frac{\lambda}{2} = L$

$\boxed{\lambda_n = \frac{2L}{n}}$

$\Rightarrow$ Nearest node $\left(\frac{\lambda_n}{2}\right) = \frac{L}{n}$

$\Rightarrow$ surviving λ_n; $n = 5, 10, 15, \ldots \infty$

$\nu_n = 50, 100, 150, \ldots\ Hz$

Q-2 μ, T

At $t=0$ $y(x,0) = 2 \sin\left(\frac{2\pi x}{L}\right) + 3 \sin\left(\frac{\pi x}{L}\right)$

(a) At $t=0$ all energy is potential (deformation of string)

Potential energy density $\frac{dU}{dx} \simeq \frac{1}{2} T \left(\frac{\partial y}{\partial x}\right)^2$

- Total energy stored in (energy conservation)

$$\boxed{E_{tot} = U(t=0) = \frac{1}{2} T \int_0^L \left(\frac{\partial y}{\partial x}\right)^2 dx}$$

$$\left(\frac{\partial y}{\partial x}\right)^2 = \left[\frac{4\pi}{L} \cos\left(\frac{2\pi x}{L}\right) + \frac{3\pi}{L} \cos\left(\frac{\pi x}{L}\right)\right]^2$$

$\Rightarrow$ Cross terms in $\left(\frac{\partial y}{\partial x}\right)^2$ will NOT contribute to integral $\int_0^L$

PTO

$$E_{tot} = \frac{1}{2} T \frac{\pi^2}{L^2}\left[16\int_0^L \cos^2\left(\frac{2\pi x}{L}\right)dx + 9\int_0^L \cos^2\left(\frac{\pi x}{L}\right)dx\right]$$

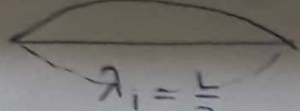
$$\boxed{E_{tot} = \frac{25\pi^2 T}{4L}}$$

(b) Superposition of two - modes : two - standing waves

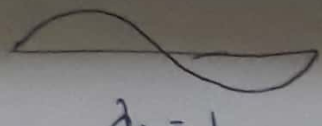
$$y(x,t) = 3\sin\left(\frac{\pi x}{L}\right)\cos\omega_1 t + 2\sin\left(\frac{2\pi x}{L}\right)\cos(\omega_2 t)$$

$$\cancel{\omega = kc \Rightarrow \omega = kc}$$

$$\nu = \frac{v}{\lambda} \Rightarrow \omega = \frac{2\pi}{\lambda} v \quad ; \quad v = \sqrt{\frac{T}{\mu}}$$



$\lambda_1 = \frac{L}{2}$



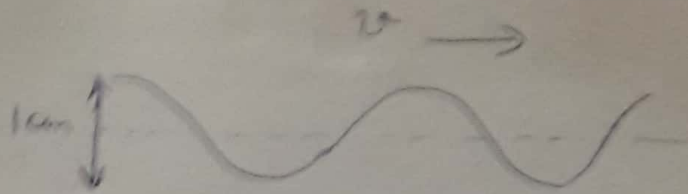
$\lambda_2 = L$

$$\omega_1 = \frac{2\pi v}{\lambda_1} \quad \& \quad \omega_2 = \frac{2\pi v}{\lambda_2}$$

Two standing waves:  & 

$\mu = 0.1\ g/cm = 0.01\ kg/m$

$T = 400\ N$



wave speed $v = \sqrt{\frac{T}{\mu}}$

Amp $A = 0.01\ m$

$\nu = 100\ Hz$

Time average energy flux $\langle P(t) \rangle = 2\pi^2 A^2 \mu \nu^2 v\ J/s$

$= \frac{1}{2}(\mu \omega^2 A^2)\, v$

$\langle P(t) \rangle = 39.48 W$