Symbolic Logic: Introduction

Reference: Symbolic Logic (I. M. Copi)

1 Introduction

This is an introduction to symbolic logic. Our aim is to go over the rules of deductive logic and thus understand the nature of arguments. We do this by learning about methods and principles that help us distinguish between correct/good/sound arguments from incorrect ones. In other words, we want to determine whether an inference made from given premisses follows from it or not. That is, if we assume the premisses to hold, does the inference or the conclusion hold?

To begin we establish some terms and notation. Propositions or Statements can be either true or false. Given a set of Premisses, corresponding to every possible inference is an argument. We are mainly concerned with this "argument".

Example 1.1.
All humans are mortals.
Socrates is human.
Therefore, Socrates is mortal.

Each line represents a proposition or a statement. The same statement that is a premiss in one argument (say the one above) can be a conclusion in some other argument (like the one below).

All animals are mortals.
All humans are animals.
Therefore, all humans are mortals.

Remark 1. *Note that deductive and inductive arguments are different. In all arguments, premisses provide some grounds for the truth of their conclusions, but a deductive argument involves the claim that its premises provide absolutely conclusive grounds. That is, a deductive argument is valid when its premises and conclusions are so related that it is absolutely impossible for the premisses to be true unless the conclusion is also true.*

It is important to understand that the validity or invalidity of an argument has nothing to do with the truth or falsehood of the propositions or statements. This is explained through some examples below.

Example 1.2.
All fish are mammals.
All mammals can fly.
Therefore, all fish can fly.

This is a valid argument but none of the statements/propositions are factually correct or true. Similarly the premisses and conclusions might be true but the argument could be invalid. For example:

Example 1.3.
If I am a cricket player, then I am famous.
I am not a cricket player.
Therefore, I am not famous.

This is not a valid argument. Let's replace "I" by "Einstein" and see what happens.

Example 1.4.
If Einstein is a cricket player, then Einstein is famous.
Einstein is not a cricket player.
Therefore, Einstein is not famous.

Clearly, the argument is invalid because the premisses are true but the conclusion is not.

To establish the truth of the conclusion, the argument must be valid and all its premisses must be true. As mentioned above, we aim to study validity and invalidity of (deductive) arguments.

2 Compound Statements

All statements can be divided into two categories: simple and compound. A simple statement doesn't contain any other statement as its component whereas a compound statement contains another statement (simple or compound) as its component. For example: "We must practise social distancing or COVID-19 will spread quickly" is a compound statement having two simple statements as its components.
As mentioned before, every statement is either true or false, so we can speak of the truth value of a statement, where truth value of a true statement is true and that of a false statement is false. Can we determine the truth value of a compound statement from the truth values of its simple component statements? For example, "Roses are red a violets are blue" is true if both the components are true, and it is false if both the component statements are false. However, in the truth value of statement "Trump believes that Mars in inhabited" is not determined by the truth value of the component "Mars is inhabited".

2.1 Compound Statements and Truth Tables

We denote the statements or propositions by small letters p, q, s etc. Truth values true and false are represented by letters T and F. We discuss two types of compound statements and their truth tables.

- A compound statement obtained by inserting "and" (or "but") between two statements is called a conjunction.

For example, “Roses are red a violets are blue” is a conjunction. However, “and” is also used in statements like “Donald Trump and Narendra Modi are best friends.”, which is not in fact a compound statement but a simple statement asserting a relationship.

A conjunction is represented by a dot. If p and q are two simple statements then $p \cdot q$ denotes their conjunction. Note that:

In case p is true and q is true, $p \cdot q$ is true.

In case p is true and q is false, $p \cdot q$ is false.

In case p is false and q is true, $p \cdot q$ is false.

In case p is false and q is false, $p \cdot q$ is false.

This can be represented as a **truth table**.

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

- A compound statement obtained by inserting a “or” between two statements is called a disjunction. The word “unless” can also be used in expressing the disjunction of two statements.There are two types of “or” you encounter:

  (i) *weak* (also know as *inclusive*) or

  A disjunction that uses *inclusive or* asserts that at least one disjunct is true. For example, ‘Premiums will be waived in the event of sickness or unemployment’. This means that the premiums will waived for people who are sick or umemployed or both.

  (ii) *strong* (also know as *exclusive*) or

  A disjunction that uses *exclusive or* asserts that at least one disjunct is true **but not both are true**. For example, ‘In Rupees 10, you may have coffee, or you may have tea’. The meaning here is that if you pay Rs. 10, you may have either coffee ofr tea but not both.

It is usually difficult to decide which sense of ‘or ” is intended in a disjunction. However, as we shall see, a typical argument that has disjunction as a premiss remains valid on either interpretation of ‘or’. Therefore, irrespective of the type, we represent ‘or’ by $\vee$. To see that *a typical argument that has disjunction as a premiss remains valid on either interpretation of ‘or’*, consider the following example.

Example 2.1.

The United States will become more responsible or there will be a third world war.
The United States will not become more responsible.
Therefore, there will be a third world war.

This is an example of an argument form known as "Disjunctive Syllogism". We will discuss this in more detail later. If p, q are two statements, the truth-value of the disjunction $p \vee q$ can be read from the following **truth table**.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- Negation of a statement p is denoted as $\sim p$. The truth table is given by: Negation of a

p	$\sim p$
T	F
F	T

disjunction is often expressed by using "neither-nor". So, the negation of *'Either Spain or Italy will win'* is negated by the statement *'Neither Spain nor Italy will win'*.

We use parentheses or brackets to resolve any ambiguity arising because of writing compound statements like $p \cdot q \vee r$ or $\sim p \vee q$ (there is an ambiguity because this can either be $(\sim p) \vee r$ or $\sim (p \vee r)$.

Exercise 2.2. *Let A and B denote the statements 'Anil will be elected' and 'Mukesh will be elected' respectively. Write the following statements in the form of symbols A, B, their negations and/or conjunction:*

(i) Anil and Mukesh will not both be elected.

(ii) Anil and Mukesh will both not be elected.

Exercise 2.3. *For statements p, q, let $A = \sim (p \cdot q)$ and $B = (\sim p) \cdot (\sim q)$. Verify that A and B are not same by writing a truth table.*

Hint: You want to write the following truth table and check that last two columns do not match.

p	q	$\sim p$	$\sim q$	$\sim (p \cdot q)$	$(\sim p) \cdot (\sim q)$