MTH 101 - Symmetry
Assignment 2

1. If $A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 1 & 1 \\ 1 & -3 & a \end{pmatrix}$. Find all the solutions of AX=0 by row-reducing A.

2. **Definition** : *Two matrices A, B are said to be row equivalent if B can be obtained from A by a finite sequence of elementary row operations or equivalently, every row vector of one matrix is a linear combination of the row vectors of the other matrix.*
   Prove that the following two matrices are not row-equivalent:
$$\begin{pmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}$$

3. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be 2×2 matrices with real entries. Suppose that A is row-reduced and $a+b+c+d = 0$. Prove that there are exactly three such matrices.

4. Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.

5. Describe explicitly all 3×3 and 2×3 row-reduced echelon matrices.

6. Show that the elementary matrices are invertible and explicitly write down the inverses of the elementary matrices corresponding to the operations $E^{\mathbf{r}}_{c\mathbf{r}}, E^{\mathbf{r}}_{\mathbf{s}}$ and $E^{\mathbf{k}}_{\mathbf{k}+c\mathbf{r}}$.(refer to class notes or assignment 1 for the definition of the elementary row operations.)

7. Let P be the right inverse of a matrix A and Q be the left inverse of the matrix A. Then prove that $P = Q$.

8. If A and B are invertible matrices with inverses A^{-1} and B^{-1} respectively, show that their product AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.

9. Show that if $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2×2 matrix with real entries such that $ad - bc \neq 0$, then A can be written as product of 4 elementary matrices.

10. Let R be an $n \times n$ row-reduced matrix in which the i^{th} row vector is a zero vector. Then prove that R is not invertible.

11. Let C be a row-reduced echelon matrix in which the first pivot element is in coloumn j for $j > 1$. Prove that C cannot be invertible.

∗ If A and B are row equivalent $m \times n$ matrices, prove that $AX = 0$ and $BX = 0$ have exactly the same solution.

∗ Every $m \times n$ matrix is row-equivalent to a row reduced echelon matrix.

∗ **Definition** : *A system of linear equations $AX = b$ is said to be homogeneous if $b = 0$.*
  Prove that if two homogeneous systems of linear equations in two unknowns have the same solutions, then they are equivalent.