MTH202: Assignment 2

January 11, 2019

Axioms of Probability:

- $P(\Omega) = 1$
- $0 \leq P(E) \leq 1$ for every $E \subset \Omega$.
- $E_1, E_2, \ldots \subset \Omega$ be such that $E_i \cap E_j = \phi$ for $i \neq j$, then $P(\cup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$.

1. Consider the following experiments and define explicitly the sample space and the size/cardinality of the sample space.
   - Rolling two dice.
   - Measuring the heights of all trees in a given region.
   - Tossing a coin infinitely many times.
   - Tossing three coins.
2. Consider the experiment of rolling two dice. Write explicitly the following events as subsets of the sample space.
   - $E_1 = \{$All outcomes such that the first roll is odd$\}$
   - $E_2 = \{$All outcomes such that the first roll is even$\}$
   - $E_3 = \{$All outcomes such that the first roll is a prime$\}$
   - $E_4 = \{$All outcomes such that the first roll is an odd prime$\}$
3. Show that $P(E^c) = 1 - P(E)$.
4. Show that if $E_1, E_2, \ldots, E_k$ be k events such that $E_i \cap E_j = \phi$ for all $i, j \in \{1, 2, \ldots, k\}, i \neq j$. Show that $P(\cup_{i=1}^{k} E_i) = \sum_{i=1}^{k} P(E_i)$.
5. Consider a sample space Ω and events $A, B, C \subset \Omega$. Show that:

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

6. Use induction to prove that for events $E_1, E_2, \ldots, E_k$,

$$P(\cup_{i=1}^k E_i) \leq \sum_{i=1}^k P(E_i)$$

7. Let A, B be events such that if $A \subset B$, then show that

$$P(B \setminus A) = P(B) - P(A)$$

8. Let A, B be events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{2}{3}$. Show that $\frac{1}{6} \leq P(A \cap B) \leq \frac{1}{2}$.

9. For $A, B \subset \Omega$, define $A \Delta B = (A \setminus B) \cup (B \setminus A)$. Show that $A \Delta B$ is also an event.

10. For events A, B, prove that:

$$|P(A) - P(B)| \leq P(A \Delta B)$$