

IISER Mohali
MTH102: Analysis in One Variable
Homework Sheet 10
To be discussed during tutorial on April 08, 2016

- Please solve all the problems.
- **Tutorial problems** will be discussed during tutorials.
- If time permits, tutors may also discuss **extra problems** during tutorials.

Tutorial Problems:

- (1) Let $f : (-1, 1) \rightarrow \mathbb{R}$ be given by $f(x) = \frac{1}{1-x}$. Determine the Taylor series expansion of f about 0.
- (2) Let $f : [0, a] \rightarrow \mathbb{R}$ be given by $f(x) = x^3$. Calculate the upper and the lower Riemann integrals of f . Is f Riemann integrable on the interval $[0, a]$?
- (3) Consider the function $f : [0, a] \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Calculate the upper and the lower Riemann integrals of f .
 - (b) Is f Riemann integrable on the interval $[0, a]$?
- (4) Let $f, g : [a, b] \rightarrow \mathbb{R}$ be two functions such that f is Riemann integrable on $[a, b]$ and $f(x) = g(x)$ for all but finitely many $x \in [a, b]$. Show that g is also Riemann integrable and $\int_a^b f(x)dx = \int_a^b g(x)dx$.
Hint: Use induction on the number of points where the two functions differ.
- (5) Give an example of a function $f : [0, 1] \rightarrow \mathbb{R}$ which is not Riemann integrable, but for which the function $|f|$ is Riemann integrable.
Hint: Start with a non-Riemann integrable function you know.

Extra Problems:

- (1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function given by $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$. Determine the Taylor series expansion of f about 0.
- (2) Consider the function $f : [0, a] \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} \frac{x}{5} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Calculate the upper and the lower Riemann integrals of f .
 - (b) Is f integrable on the interval $[0, a]$?