

Homework. 6

- 1) If the curvature of a smooth & regular curve in the xy -plane is constant then show that ~~either~~ it is contained in a straight line or in a circle.
- 2) Find the ^{unit speed} curve $\alpha: \mathbb{R} \rightarrow \mathbb{R}^2$ with signed curvature ~~κ_s~~ $\kappa_s(t) = \cos t$, $\forall t \in \mathbb{R}$, where $\alpha(0) = (0,0)$, $\alpha'(0) = (1,0)$.
- 3) Suppose $A \in M(3, \mathbb{R})$. Show that the following are equivalent:
- (i) $AA^t = I$ ($A^t =$ transpose of A .)
 - (ii) $Au \cdot Aw = u \cdot w \quad \forall u, w \in \mathbb{R}^3$
 - (iii) $Au \cdot Au = u \cdot u \quad \forall u \in \mathbb{R}^3$
- 4) (i) Suppose $AA^t = I$, $\det A = 1$. Show that-
 $Au \times Aw = A(u \times w)$.
- (ii) Use this to complete the proofs of the fundamental theorems for plane curves and space curves.
- 5) Compute $\vec{T}, \vec{n}, \vec{b}, \kappa, \tau$ for the following curves at the given points:
- (i) $\alpha(t) = (t, t^2, t^3)$ at $t = 0$.
 - (ii) $\alpha(t) = (t \cos t, t \sin t, t)$ at $t = 0$.
6. Decide if the following curves are planar
- (i) $\alpha(t) = (t, t^2, 1+t+t^2)$
 - (ii) $\alpha(t) = (t, e^t, e^{2t})$