Solutions to be given in class by tutors

1. Given that,

$$x \equiv \hat{x} \text{ and } p_x \equiv -i\hbar \frac{\partial}{\partial x}$$

Set up quantum mechanical operators for the given dynamical variables:

(a) The Hamiltonian $H = T + V(x)$, where T is the kinetic energy of a particle of mass m and $V(x)$ is the potential energy, in which the particle executes one dimensional motion.

(b) The angular momentum $\vec{L} = \vec{r} \times \vec{p} = L_x \bar{e}_x + L_y \bar{e}_y + L_z \bar{e}_z$. Explicitly write out the operators corresponding to L_x, L_y and L_z in cartesian coordinates.

(c) The magnitude of total angular momentum, $\left|\vec{L}\right|$.

(d) Dipole moment of a charged particle, $\vec{\mu} = e\,|q|\,\vec{r}$, where e is the

(e) $\vec{r} \cdot \vec{p}$. This will be useful to get virial relations, later.

(f) $(xp_y)^2$.

(g) The energy operator in both its forms.

(h) The kinetic energy operator for a particle moving in a 2D cartesian space and 3D cartesian space.

(i) The time evolution operator.

2. Verify the following commutator rules, for the the operators, $\hat{A}$, $\hat{B}$ and $\hat{C}$:

(a) $\left[\hat{A}, \hat{B}\right] + \left[\hat{B}, \hat{A}\right] = 0$

(b) $\left[\hat{A}, \hat{A}\right] = 0$

(c) $\left[\hat{A}, \hat{B} + \hat{C}\right] = \left[\hat{A}, \hat{B}\right] + \left[\hat{A}, \hat{C}\right]$

(d) $\left[\hat{A}\hat{B}, \hat{C}\right] = \hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}$

(e) $\left[\hat{A}, \hat{B}\hat{C}\right] = \left[\hat{A}, \hat{B}\right]\hat{C} + \hat{B}\left[\hat{A}, \hat{C}\right]$

3. What will the sum of commutators $\left[\hat{A}\left[\hat{B}, \hat{C}\right]\right] + \left[\hat{B}\left[\hat{C}, \hat{A}\right]\right] + \left[\hat{C}\left[\hat{A}, \hat{B}\right]\right]$ be equal to?

4. Evaluate the following commutators:

(a) $\left[\hat{L}_x, \hat{T}\right]$

(b) $\left[\hat{x}, \hat{V}(x)\right]$

(c) $\left[\hat{x}, \hat{H}\right]$

(d) $\left[\hat{x}\hat{y}\hat{z}, \hat{p}_x^2\right]$

5. A translation operator $\hat{T}_h$ is defined by $\hat{T}_h f(x) = f(x+h)$. Is $\hat{T}_h$ a linear operator? Evaluate $\left(\hat{T}_1^2 - 3\hat{T}_1 + 2\right)$ acting on x^2. Plot the untransformed function and the function after the operation is done. What does the same operator do with a Gaussian function e^{-x^2}?

6. The function of an operator $\hat{A}$ is defined as:

$$e^{\hat{A}} = \hat{1} + \frac{\hat{A}}{1!} + \frac{\hat{A}^2}{2!} + \frac{\hat{A}^3}{3!} \dots$$

A similar definition is given for $e^{\hat{B}}$. Will $e^{\hat{A}} e^{\hat{B}} = e^{\hat{A}+\hat{B}}$? If not under what conditions will this be true?