

# More on Languages

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## Languages Continued

$\Sigma^*$  = set of all strings over the alphabet  $\Sigma$

A language  $L$  is a subset of  $\Sigma^*$

$\epsilon$  is an empty string.

If  $\omega \in \Sigma^*$ ,  $|\omega|$  = length of the string.

## Examples of languages

1.  $\emptyset$
2.  $\Sigma^*$  > English is a language where the strings are all words in a dictionary and punctuation marks and the collection is grammatically correct > > C++ any string of characters the compiler accepts
3.  $\Sigma = \{a,b\}$ ,  $L = \{x \in \Sigma^* \mid x \text{ begins with } a \text{ \& ends with } b\}$
4.  $\Sigma = \{0,1,\dots,9\}$ ,  $L = \{x \mid x \text{ is the decimal representation of a prime}\}$

## Equality of Languages

Languages are equal if the set of strings are equal.

## Encoding (informal)

A bijective mapping from one set to another. - Pictures are colours encoded as numbers - Similarly movies

## Ordering of a language

$\Sigma$  is finite, hence can be ordered.

$\Sigma^*$  can then be ordered by length first and then the lexicographic order on  $\Sigma$

## Existence of a string in a Language

Graph of a function

$$G(f) = \{x, y \mid y=f(x)\}$$

A string exists in the language if it exists in  $G(f)$  where  $f$  is the defining function of the Language.

There are uncountable languages but a countable set of strings. So we cannot hope to describe all the languages using strings.

## Deterministic Finite State Automaton

Solve the existence of string problem - Read the string one char at a time - Remember something, and react to the next char accordingly. - Do this again and again

Consider -

$$\Sigma = \{0,1\}$$

$$L = \{x \mid x \text{ consists of an even number of 1s}\}$$

Given a string, what do you do?

Go char by char, switching between an “even state” and an “odd state”

DEFN: Deterministic Finite State Automaton is a 5 tuple  $(Q, \Sigma, \delta, q_0, F)$  where

- $Q$  = A finite set called the set of states
- $\Sigma$  = Alphabet
- $\delta$  = is a function  $Q \times \Sigma \rightarrow Q$
- $q_0 \in Q$  and is called the initial state
- $F \subseteq Q$  and is the set of accepted states.