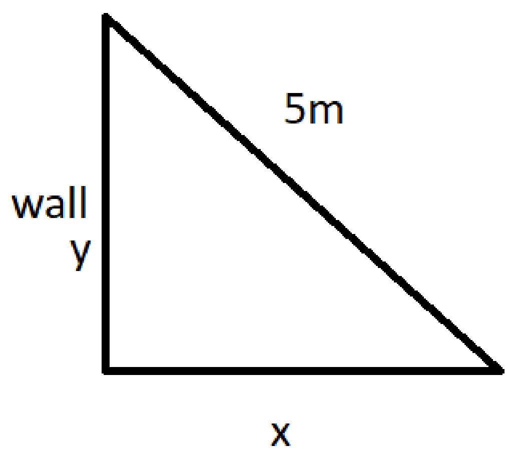
1

Let the side of the square be represented by x.

$$\begin{aligned}
\frac{dx}{dt} &= 4cm/min \\
\text{Area(A)} &= x^2 \\
&= \frac{d(x^2)}{dx} \times \frac{dx}{dt} \\
&= 2x\frac{dx}{dt} \\
&= 64cm^2/min \\
\text{Perimeter(P)} &= 4x \\
\frac{dP}{dt} &= 4\frac{dx}{dt} \\
&= 16cm/min
\end{aligned}$$

2



$$\begin{aligned}
\frac{dx}{dt} &= 2m/s \\
x^2 + y^2 &= 25 \\
\text{Differentiate wrt t} & \\
2x\frac{dx}{dt} + 2y\frac{dy}{dt} &= 0 \text{ \{note that we have used the chain rule here\}} \\
\frac{dy}{dt} &= -\frac{x}{y}\frac{dx}{dt} \\
\text{When x=4, y=3} & \\
\frac{dy}{dt} &= -\frac{8}{3}m/s
\end{aligned}$$

3

$$\begin{aligned}
\frac{dy}{dx} &= 2x \\
dy &= 2xdx \\
\text{Approximating dx by } \Delta x, & \\
\Delta y &= 2x\Delta x
\end{aligned}$$

4

$$\begin{aligned}
\frac{dy}{dx} &= 4x^3 \\
dy &= 4x^3dx \\
\text{Approximating dx by } \Delta x, & \\
\Delta y &= 4x^3\Delta x
\end{aligned}$$

Now, you can put the value of Δx to find the value of Δy and then use the value of Δy to find the new value of y.

5

$$\begin{aligned}
\frac{dT}{dl} &= \frac{\pi}{\sqrt{lg}} \\
&= \frac{T}{2l} \text{ \{A little bit of rearranging\}} \\
\text{Approximating dT by } \Delta T, & \\
\Delta T &= \frac{T\Delta l}{2l}
\end{aligned}$$

6

$$\begin{aligned}
f(x) &= 2x^3 - 24x + 107 \\
f'(x) &= 6x^2 - 24
\end{aligned}$$

To find the critical points, put f'(x)=0

$$\begin{aligned}
\implies x &= \pm 2 \\
f''(x) &= 12x \\
f''(2) &= 24 > 0 \\
f''(-2) &= -24 < 0
\end{aligned}$$

$\therefore$ f has a local maxima at -2 and a local minima at 2

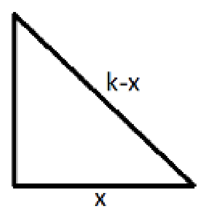
7

Let the numbers be x and y and let S denote the sum of the squares.

$$\begin{aligned}
x+y &= 14 \\
\implies y &= 14-x \\
S &= x^2+y^2 \\
&= x^2+(14-x)^2 \\
&= 2x^2-28x+196 \\
\frac{dS}{dx} &= 0 \\
\implies 4x-28 &= 0 \\
\implies x &= 7 \\
\frac{d^2S}{dx^2} &= 4>0
\end{aligned}$$

$\therefore$ S is minimum when x=y=7.

8



Let us denote the area by A.

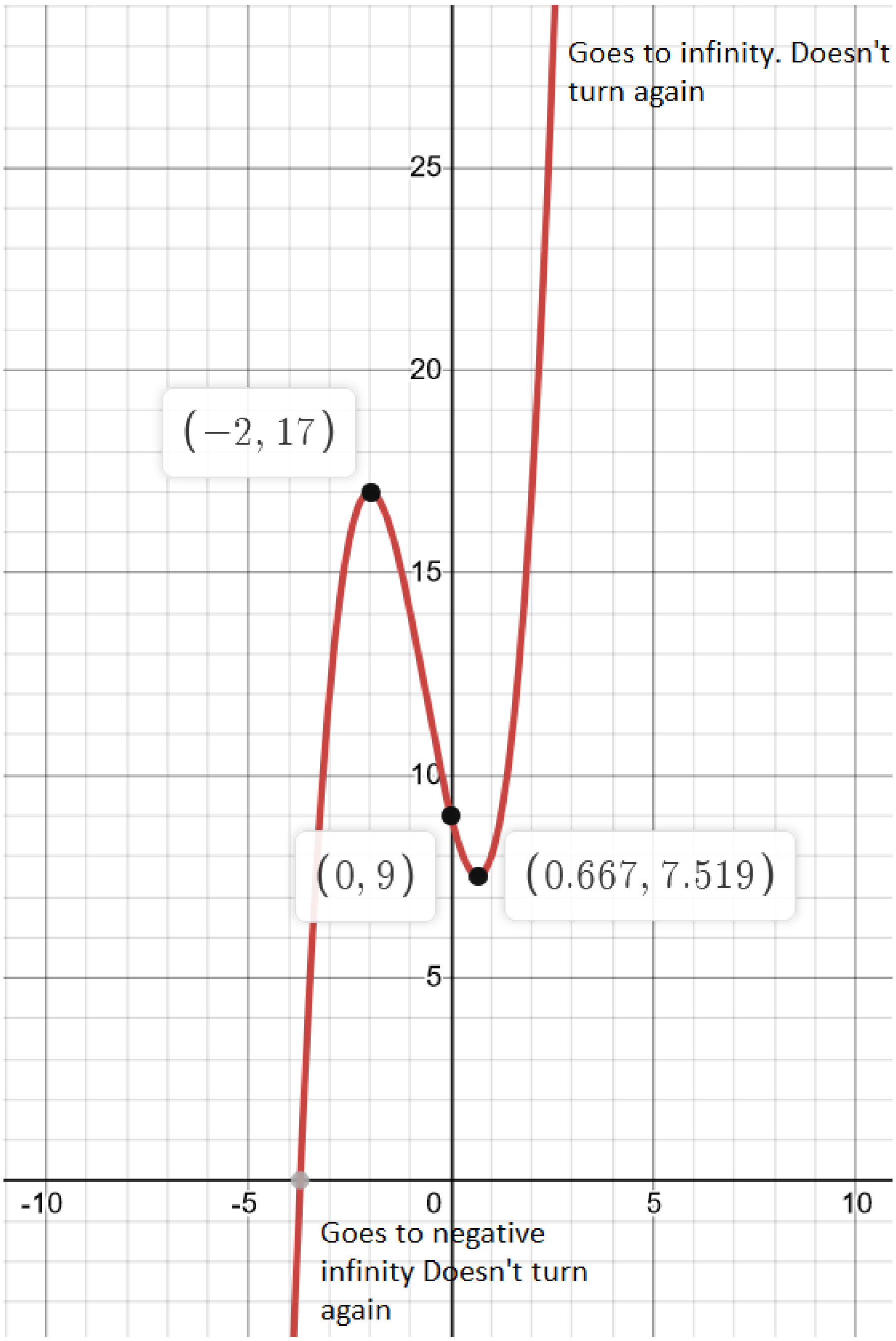
$$\begin{aligned}
\text{Base of the triangle} &= x \\
\text{Height of the triangle} &= \sqrt{(k-x)^2-x^2} \qquad \{\text{By Pythagoras theorem}\} \\
A &= \frac{1}{2}\times \text{base}\times\text{height} \\
A^2 &= \frac{1}{4}\times x^2\times\{(k-x)^2-x^2\} \\
A^2 &= \frac{k^2x^2-2kx^3}{4} \\
\text{Differentiate wrt x} & \\
2A\frac{dA}{dx} &= \frac{2k^2x-6kx^2}{4} \qquad \text{(-18)} \\
\frac{dA}{dx} &= \frac{2k^2x-6kx^2}{4A} \\
\frac{dA}{dx} &= 0 \\
\implies \frac{k^2x-3kx^2}{4A} &= 0 \\
\implies x &= \frac{k}{3} \\
\text{Differentiate eq 18 wrt x again} & \\
2\left(\frac{dA}{dx}\right)^2+2A\frac{d^2A}{dx^2} &= \frac{2k^2-12kx}{4}
\end{aligned}$$

When x=$\frac{k}{3}$, $\frac{dA}{dx}=0$; because we just proved that x=$\frac{k}{3}$ is a critical point. So, we put $\frac{dA}{dx}=0$ and $x=\frac{k}{3}$ and

$$\frac{d^2A}{dx^2} = -\frac{k^2}{4A} < 0$$

$\therefore$ A is maximum when $x=\frac{k}{3}$.

9



x=-2 and x=$\frac{2}{3}$ are critical points.