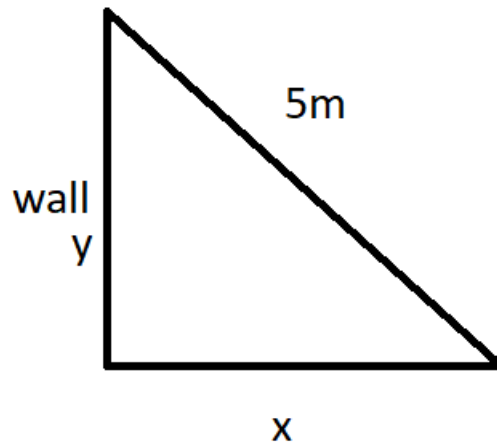


1

Let the side of the square be represented by x.

$$\begin{aligned}
 \frac{dx}{dt} &= 4\text{cm}/\text{min} \\
 \text{Area(A)} &= x^2 \\
 &= \frac{d(x^2)}{dx} \times \frac{dx}{dt} \\
 &= 2x \frac{dx}{dt} \\
 &= 64\text{cm}^2/\text{min} \\
 \text{Perimeter(P)} &= 4x \\
 \frac{dP}{dt} &= 4 \frac{dx}{dt} \\
 &= 16\text{cm}/\text{min}
 \end{aligned}$$

2



$$\begin{aligned}
 \frac{dx}{dt} &= 2\text{m}/\text{s} \\
 x^2 + y^2 &= 25 \\
 \text{Differentiate wrt } t & \\
 2x \frac{dx}{dt} + 2y \frac{dy}{dt} &= 0 \text{ \{note that we have used the chain rule here\}} \\
 \frac{dy}{dt} &= -\frac{x}{y} \frac{dx}{dt} \\
 \text{When } x=4, y=3 & \\
 \frac{dy}{dt} &= -\frac{8}{3}\text{m}/\text{s}
 \end{aligned}$$

3

$$\begin{aligned}\frac{dy}{dx} &= 2x \\ dy &= 2x dx\end{aligned}$$

Approximating dx by Δx ,

$$\Delta y = 2x \Delta x$$

4

$$\begin{aligned}\frac{dy}{dx} &= 4x^3 \\ dy &= 4x^3 dx\end{aligned}$$

Approximating dx by Δx ,

$$\Delta y = 4x^3 \Delta x$$

Now, you can put the value of Δx to find the value of Δy and then use the value of Δy to find the new value of y .

5

$$\begin{aligned}\frac{dT}{dl} &= \frac{\pi}{\sqrt{lg}} \\ &= \frac{T}{2l} \text{ \{A little bit of rearranging\}}\end{aligned}$$

Approximating dT by ΔT ,

$$\Delta T = \frac{T \Delta l}{2l}$$

6

$$\begin{aligned}f(x) &= 2x^3 - 24x + 107 \\ f'(x) &= 6x^2 - 24\end{aligned}$$

To find the critical points, put $f'(x)=0$

$$\begin{aligned}\Rightarrow x &= \pm 2 \\ f''(x) &= 12x \\ f''(2) &= 24 > 0 \\ f''(-2) &= -24 < 0\end{aligned}$$

$\therefore f$ has a local maxima at -2 and a local minima at 2

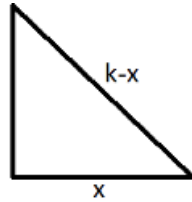
7

Let the numbers be x and y and let S denote the sum of the squares.

$$\begin{aligned}
 x + y &= 14 \\
 \Rightarrow y &= 14 - x \\
 S &= x^2 + y^2 \\
 &= x^2 + (14 - x)^2 \\
 &= 2x^2 - 28x + 196 \\
 \frac{dS}{dx} &= 0 \\
 \Rightarrow 4x - 28 &= 0 \\
 \Rightarrow x &= 7 \\
 \frac{d^2S}{dx^2} &= 4 > 0
 \end{aligned}$$

\therefore S is minimum when x=y=7.

8



Let us denote the area by A.

$$\begin{aligned}
 \text{Base of the triangle} &= x \\
 \text{Height of the triangle} &= \sqrt{(k-x)^2 - x^2} \quad \{\text{By Pythagoras theorem}\} \\
 A &= \frac{1}{2} \times \text{base} \times \text{height} \\
 A^2 &= \frac{1}{4} \times x^2 \times \{(k-x)^2 - x^2\} \\
 A^2 &= \frac{k^2x^2 - 2kx^3}{4} \\
 \text{Differentiate wrt x} \\
 2A \frac{dA}{dx} &= \frac{2k^2x - 6kx^2}{4} \tag{-18} \\
 \frac{dA}{dx} &= \frac{2k^2x - 6kx^2}{4A} \\
 \frac{dA}{dx} &= 0 \\
 \Rightarrow \frac{k^2x - 3kx^2}{4A} &= 0 \\
 \Rightarrow x &= \frac{k}{3}
 \end{aligned}$$

Differentiate eq 18 wrt x again

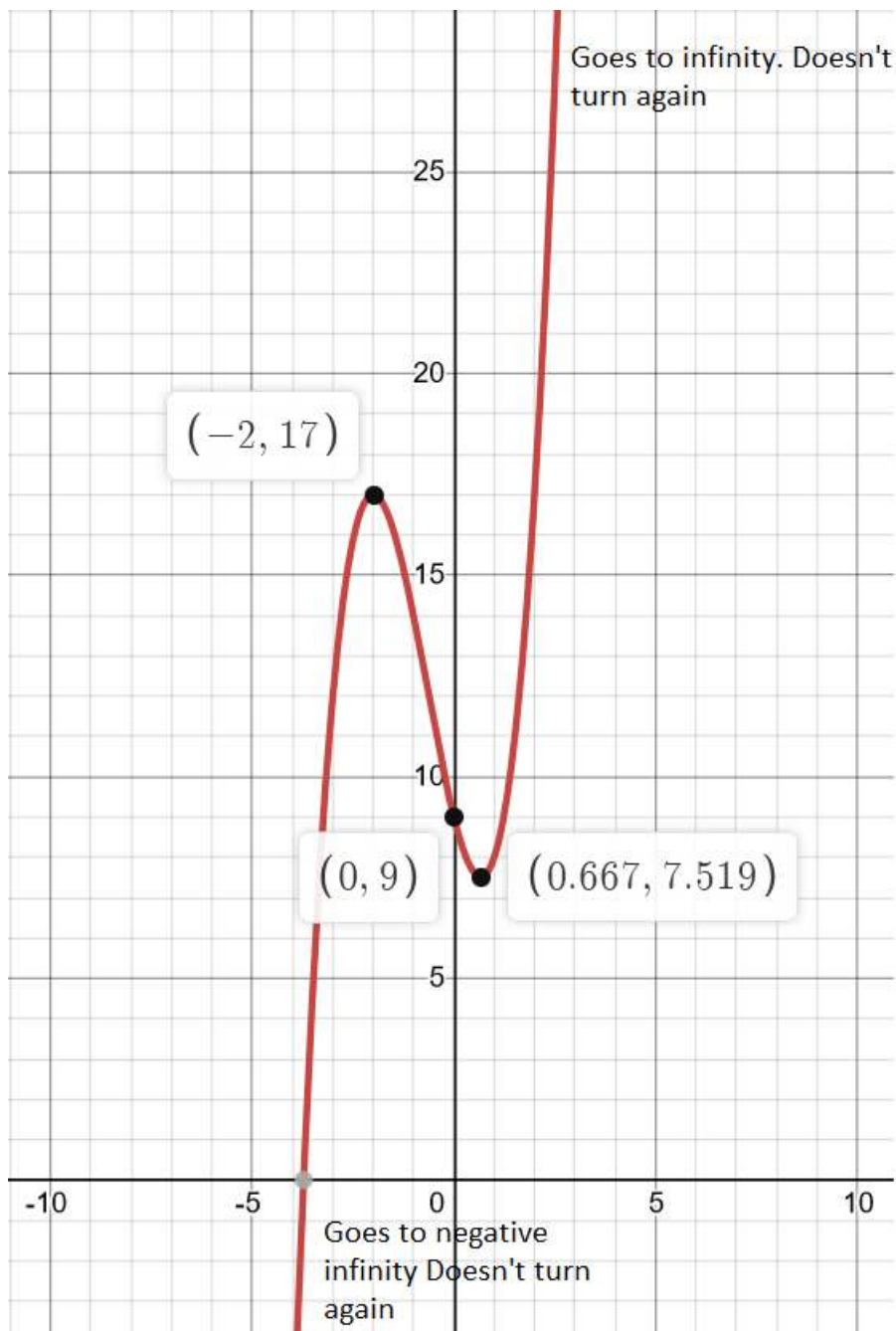
$$2 \left(\frac{dA}{dx} \right)^2 + 2A \frac{d^2A}{dx^2} = \frac{2k^2 - 12kx}{4}$$

When $x = \frac{k}{3}$, $\frac{dA}{dx} = 0$; because we just proved that $x = \frac{k}{3}$ is a critical point. So, we put $\frac{dA}{dx} = 0$ and $x = \frac{k}{3}$ and

$$\frac{d^2 A}{dx^2} = -\frac{k^2}{4A} < 0$$

$\therefore A$ is maximum when $x = \frac{k}{3}$.

9



$x = -2$ and $x = \frac{2}{3}$ are critical points.