

MTH202: Assignment 4, Additional Exercises

January 30, 2019

1. A player has a fair coin, and a fair 4-sided dice and a fair 6-sided dice (all are independent). The player flips the coin, if it shows head, then the 4-sided dice is rolled and its value noted; else if the coin shows tail then the 6-sided dice is rolled and its value noted. Suppose X denote the result obtained. Is X a random variable? What is the p.m.f of X ?
2. A standard (6-sided) fair die is rolled two times (each roll being independent). Let X_i denote the value obtained on the i -th roll. Is this a random variable? What about $S = X_1 - X_2$, the difference of the values rolled? Determine the p.m.f of S .
3. In an experiment, we can observe whether the solution is acidic (event A) and whether the solution is coloured (event B). Assume that $P(B) > 0$. We now carry out the experiment repeatedly (each trial being independent) until B is observed. What is the probability that A is observed at the same time as B ?
4. Suppose k pipes of length 1 meter are attached in a series. Probability that i -th pipe is blocked is p_i (independent of the others). What is the probability that water will flow from one end to another?
5. A system consists of n machines in parallel (so it works as long as one of the machines is working). k -th machine works with probability p_k , independently of the other machines. What is the probability that system functions?
6. Let X be a random variable taking values in $\mathbb{N} \cup \{0\}$ such that $P(X = i) = e^{-s} \frac{s^i}{i!}$ for a fixed $s > 0$. Does this define a p.m.f? Explain why.