MTH202: Assignment 1

January 8, 2019

- **General Principle of Counting:** If r experiments that are to be performed are such that the first one many result in any n_1 possible outcomes; and if for each of these n_1 possible outcomes, there are n_2 possible outcomes of the second experiments; and if, for each of the possible outcomes of the first two experiments there are n_3 possible outcomes of the third experiment and so on $\ldots$, then there is a total of $n_1 \cdot n_2 \ldots n_{r-1} \cdot n_r$ possible outcomes of the r experiments.
- **Permutations:** There are $n!$ permutations or arrangements of n different objects.
- **Combinations:** Number of different ways to select k objects from a total of n objects is $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.
- **Number of integer valued solutions of the equation** $x_1 + \ldots x_r = n$

  Number of positive solutions $= \binom{n-1}{r-1}$.
  Number of non-negative solutions $= \binom{n+r-1}{r-1}$.

1. A license plate consists of English alphabets and any number from the set $\{0, 1, \ldots, 9\}$.
   - How many different 7-place license plates are possible if first 3 places are occupied by letters and final 4 by numbers.
   - How many different license plates of the form defined above are possible if repetition among letters and numbers were prohibited.
2. How many different letter arrangements can be formed from the letters $PEPPER$?
3. In how many ways can 100 identical books be distributed into 10 different bags so that no bag is empty?

4. What is the number of subsets of an n-element set?
5. In how many ways can you order an n-element set?
6. What is the number of all k-element subsets of an n-element set?
7. What is the number of all one-to-one mappings from an n-element set to an m-element set?
8. Compute the number of ways in which k balls can be placed in to n boxes if:
   - Both balls and boxes are distinct.
   - All balls are identical and boxes are distinct.

   Assume that order in which the balls are placed into the boxes.
9. Answer the previous question with additional condition that each box can only hold at most 1 ball.
10. Using a counting argument, show that $2^n = \sum\limits_{k=0}^{n} \binom{n}{k}$.